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Robust Nonlinear Receding-Horizon Control of Induction Motors

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Abstract— **A nonlinear robust receding-horizon control is designed and applied to fifth-order model of induction motor in cascade structure. The control uses only measurement of the rotor speed and stator currents. The rotor flux is estimated by Kalman filter. The controller is based on a finite horizon continuous time minimization of the predicted tracking errors and no online optimization is needed. An integral action is incorporated in external loop to increase the robustness with respect to unknown time-varying load torque. The proposed nonlinear controller permits to achieve asymptotic speed and flux tracking in presence of the unknown load torque and resistances variations. In addition, it assures asymptotic decoupling of the speed and flux subsystems. The controller is applied, via simulation, to a benchmark example.**

I. Introduction

INDUCTION motors, due to their size, low cost and high reliability are the most widespread systems in industrial application, but they represent a highly coupled and nonlinear multivariable system. In recent years, to increase performance of classical control, e.g. field oriented torque control [6], many control strategies have been proposed to achieve better dynamic performance and induction motors have been gradually replacing the DC motors. Among these control strategies, typical approaches include input-output linearization [1]. Recently, indirect field oriented control (IFOC) was proposed in [3] guarantees global exponential speed-flux tracking but under the condition of constant value of load torque and rotor resistance. Note that load torque estimator was utilized in this control scheme. While in [5], authors have proposed a speed/torque and flux tracking adaptive controller without measurements of the rotor fluxes while adapting to the rotor resistance and the unknown load torque. However, to ensure the convergence, the persistent condition should be satisfied [5,9]. It is stated in [10], that the persistent excitation condition is not satisfied when the electric torque is absent due to lack of currents. In this paper, the main advantage of the proposed control scheme is that we do not require exciting signals since we do not need to estimate neither resistances (stator and rotor) value nor load torque. Moreover, asymptotic speed/flux tracking is achieved in presence of these uncertainties.

This work examines the nonlinear receding-horizon control approach with integral action based on a finite horizon dynamic minimization of tracking errors, to achieve torque and rotor flux amplitude tracking objectives. An extension to speed control is realized with a cascaded structure. This is a slightly modified version of the Ping's method [4]. Note that the proposed approach in [4] cannot be applied to induction motor since the derivative of the control signal will appear in the cost function. The advantages of the proposed control law include good tracking performance and good robustness property with respect to both load torque and resistances variations. Moreover, the flux weakening operation has no effect on the speed behavior.

The paper is organized as follows. After the mathematical model of the induction motor developed in section II, a brief overview of the optimal nonlinear receding-horizon control theory is presented in section III. In section IV, we extend the previous scheme to speed control by a cascaded nonlinear control structure. Significant simulation results are given in section V for the nominal and mismatched model of the induction motor with bonded time-varying load disturbance.

II. Mathematical Model

Assuming linear magnetic circuits, the dynamics of induction motor are given by the well-known fifth-order model, see for instance [6] for its derivation and modeling assumptions:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}\,\mathbf{u} \tag{1}$$

With: $\mathbf{x} = \left[i_{s\alpha}\ i_{s\beta}\ \varphi_{r\alpha}\ \varphi_{r\beta}\ \Omega \right]^{\mathrm{T}}$; $\mathbf{u} = \left[u_{s\alpha}\ \ u_{s\beta} \right]^{\mathrm{T}}$

Where: $i_{s\alpha}$, $i_{s\beta}$: stator currents,

$\phi_{r\alpha}$, $\phi_{r\beta}$: rotor fluxes,

Ω : speed,

$u_{s\alpha}$, $u_{s\beta}$: stator voltages.

Vector function $\mathbf{f}(\mathbf{x})$ and constant matrix $\mathbf{g}$ are defined as follows:

$$\mathbf{g} = \left[g_1\ \ g_2 \right] = \begin{bmatrix} \dfrac{1}{\sigma L_s} & 0 & 0\ 0\ 0 \\ 0 & \dfrac{1}{\sigma L_s} & 0\ 0\ 0 \end{bmatrix}^{\mathrm{T}}$$

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$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} -\gamma\, i_{s\alpha} + \dfrac{k}{T_r}\varphi_{r\alpha} + p\,\Omega\, k\,\varphi_{r\beta} \\ -\gamma\, i_{s\beta} + \dfrac{k}{T_r}\varphi_{r\beta} - p\,\Omega\, k\,\varphi_{r\alpha} \\ \dfrac{L_m}{T_r} i_{s\alpha} - \dfrac{1}{T_r}\varphi_{r\alpha} - p\,\Omega\,\varphi_{r\beta} \\ \dfrac{L_m}{T_r} i_{s\beta} - \dfrac{1}{T_r}\varphi_{r\beta} + p\,\Omega\,\varphi_{r\alpha} \\ p\dfrac{L_m}{JT_r}\left(\varphi_{r\alpha}\, i_{s\beta} - \varphi_{r\beta}\, i_{s\alpha}\right) - \dfrac{\left(T_L + f\,\Omega\right)}{J} \end{bmatrix}$$

All required parameters above have the following meanings:

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}\ ;\ k = \frac{L_m}{\sigma L_s L_r}\ ;\ \gamma = \frac{1}{\sigma L_s}\left(R_s + R_r\frac{L_m^2}{L_r^2}\right)$$

Where: L_s, L_r are stator and rotor inductances, L_m is the mutual inductance, R_s, R_r are stator and rotor resistances, $T_r = L_r/R_r$ is the rotor time constant, p is the pole pair number, J is the inertia of the machine, f is the friction coefficient, T_L is the load torque considered as an unknown disturbance.

Considering the torque and squared rotor flux modulus as outputs of the A.C. drive, the following equations can be derived, with y_1 as the torque and y_2 as the rotor flux norm:

$$\begin{cases} y_1 = h_1(\mathbf{x}) = p\dfrac{L_m}{L_r}\left(\varphi_{r\alpha} i_{s\beta} - \varphi_{r\beta} i_{s\alpha}\right) \\ y_2 = h_2(\mathbf{x}) = \varphi_{r\alpha}^2 + \varphi_{r\beta}^2 = \varphi_r{}^2 \end{cases} \tag{2}$$

III. NON LINEAR RECEDING-HORIZON LAW

In the receding-horizon control strategy, the following control problem is solved at each $t > 0$ and $\mathbf{x}(t)$:

$$\begin{aligned} &\underset{u(t)}{Min}\, J\big(\mathbf{x}(t), t, \mathbf{u}(t)\big) = \frac{1}{2}\int_t^{t+T} L(\tau)\, d\tau \\ &= \underset{u(t)}{Min}\frac{1}{2}\int_t^{t+T}\left[\mathbf{x}(\tau)^T\,\mathbf{Q}\,\mathbf{x}(\tau) + \mathbf{u}(\tau)^T\,\mathbf{R}\ \ \mathbf{u}(\tau)\right] d\tau \end{aligned} \tag{3}$$

subject to the equation (1) and $x(t+T) = 0$ for some T>0, where **Q** is positive definite matrix and **R** positive semi-definite matrix. To solve a nonlinear dynamic optimization problem with equality constraints is highly computationally intensive, and in many cases it is impossible to be performed within a reasonable time limit, especially for systems with extremely fast dynamics like induction motor. Furthermore, the global optimization solution cannot be guaranteed in each optimization procedure since, in general, it is a non-convex, constrained nonlinear optimization problem.

To avoid the computational burden, we shall approximate the above receding- horizon control problem by Simpson's rule:

$$\begin{aligned} J &= \frac{1}{2}\int_t^{t+T} L(\tau)\, d\tau = \frac{T}{6}\left[L(t) + 4L\left(t + \frac{T}{2}\right) + L(t+T)\right] \\ &= \frac{h}{3}\left[L(t) + 4L(t+h) + L(t+2h)\right] \end{aligned}$$

with $T = 2\,h$ is the prediction horizon.

In order to find the current control that improves tracking error along a fixed interval, the output tracking error $\mathbf{e}(\tau)$ is used instead of the state vector $\mathbf{x}(\tau)$ in the above receding control problem. The above performance index can be written as:

$$\begin{aligned} J = \frac{h}{3}\Big[&\mathbf{e}^{\mathrm{T}}(t)\,\mathbf{Q}\,\mathbf{e}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\,\mathbf{u}(t) \\ &+ 4\mathbf{e}^{\mathrm{T}}(t+h)\mathbf{Q}\,\mathbf{e}(t+h) + 4\mathbf{u}^{\mathrm{T}}(t+h)\mathbf{R}\,\mathbf{u}(t+h) \\ &+ \mathbf{e}^{\mathrm{T}}(t+2h)\,\mathbf{Q}\,\mathbf{e}(t+2h) + \mathbf{u}^{\mathrm{T}}(t+2h)\,\mathbf{R}\,\mathbf{u}(t+2h)\Big] \end{aligned} \tag{4}$$

Where $L(\tau) = \mathbf{e}^{\mathrm{T}}(\tau)\,\mathbf{Q}\,\mathbf{e}(\tau) + \mathbf{u}^{\mathrm{T}}(\tau)\,\mathbf{R}\,\mathbf{u}(\tau)$.

Thus, the problem consists in elaborating a control law $\mathbf{u}(\mathbf{x},t)$ that improves tracking accuracy along the interval $[t; t+T]$, such that $\mathbf{y}(t+T)$ tracks $\mathbf{y}_{ref}(t+T)$. Note that the desired output trajectory is specified by a smooth function $\mathbf{y}_{ref}(t+T)$ for $t \in [t_0; t_f]$. That is, the tracking error is defined by:

$$\mathbf{e}(t+T) = \mathbf{y}(t+T) - \mathbf{y}_{ref}(t+T)$$

A simple and effective way of predicting the influence of $\mathbf{u}(t)$ on $\mathbf{y}(t+T)$ is to expand it into a r_i^{th} order Taylor series expansion, in such a way to obtain, for each component of the vectors:

$$\begin{aligned} y_i(t+T) &= h_i(t) + TL_f h_i + \frac{T^2}{2!}L_f^2 h_i + \\ &+ \ldots\ldots + \frac{T^{r_i}}{r_i!}L_f^{r_i} h_i + \frac{T^{r_i}}{r_i!}L_g L_f{}^{r_i} h_i\, u \end{aligned} \qquad \text{for } i = 1,.,m \tag{5}$$

Where $L_f^k\ h_i$ denote the k^{th} order Lie derivative of h_i with respect to $\mathbf{f}(\mathbf{x})$. r_i is the relative degree of the output y_i, defined to be the nonnegative integer j such that the j^{th} derivative of y_i along the trajectory of equation (1) explicitly depends on $\mathbf{u}(t)$ for the first time.

The expansion of the motor outputs $\mathbf{y}(t+T)$ in a r^{th} (with $r_1 = 1$ and $r_2 = 2$) order Taylor series in compact form is:

$$\mathbf{y}(t+T) = \mathbf{y}(t) + \mathbf{V}_{\mathbf{y}}(\mathbf{x}, T) + \mathbf{\Lambda}(T)\,\mathbf{W}(\mathbf{x})\,\mathbf{u}(t) \tag{6}$$

Where:

$$\mathbf{y}(t) = \begin{vmatrix} y_1 \\ y_2 \end{vmatrix};\quad \mathbf{W}(\mathbf{x}) = \begin{vmatrix} L_{g_1} h_1 & L_{g_2} h_1 \\ L_{g_1} L_f h_2 & L_{g_2} L_f h_2 \end{vmatrix}$$

$$\mathbf{\Lambda}(T) = \begin{vmatrix} T & 0 \\ 0 & \dfrac{T^2}{2} \end{vmatrix};\ \mathbf{V}_{\mathbf{y}}(\mathbf{x}, T) = \begin{vmatrix} TL_f h_1 \\ TL_f h_2 + \dfrac{T^2}{2}L_f{}^2 h_2 \end{vmatrix}.$$

Similarly, $\mathbf{y}_{ref}(t+T)$ may be expanded in a same r^{th} order Taylor series:

$$\mathbf{y}_{ref}(t+T) = \mathbf{y}_{ref}(t) + \mathbf{d}(t,T) \qquad (7)$$

where: $\mathbf{y}_{ref}(t) = \begin{vmatrix} y_{ref_1} \\ y_{ref_2} \end{vmatrix}$, $\mathbf{d}(t,T) = \begin{vmatrix} T\,\dot{y}_{ref_1} \\ T\,\dot{y}_{ref_2} + \dfrac{T^2}{2}\ddot{y}_{ref_2} \end{vmatrix}$

The tracking error at the next instant $(t+T)$ is then predicted as function of the input $\mathbf{u}(t)$ by:

$$\begin{aligned} \mathbf{e}(t+T) &= \mathbf{y}(t+T) - \mathbf{y}_{ref}(t+T) = \\ &= \mathbf{e}(t) + \mathbf{V}_y(\mathbf{x},T) - \mathbf{d}(t,T) + \mathbf{\Lambda}(T)\mathbf{W}(\mathbf{x})\,\mathbf{u}(t) \end{aligned} \qquad (8)$$

By using the predicted tracking error equation (8), at the time $T = h$ and $T = 2h$, the performance index (4) can be written in the conventional quadratic form:

$$\overline{J} = \frac{3}{2h} J = \frac{1}{2}\mathbf{U}^T \mathbf{P}(\mathbf{x},h)\mathbf{U} + \mathbf{U}^T \mathbf{G}(\mathbf{x}) + m(\mathbf{e},\mathbf{Q},h)$$

Where: $\mathbf{U}(t) = \left|\mathbf{u}(t)\ \ \mathbf{u}(t+h)\ \ \mathbf{u}(t+2h)\right|^{\mathrm{T}}$;

$\mathbf{P}(\mathbf{x},h) = diag\left(\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\,\mathbf{K}(\mathbf{x},h)\,\mathbf{W}(\mathbf{x}), 4\mathbf{R}, \mathbf{R}\right)$

$\mathbf{G}^{\mathrm{T}}(\mathbf{x}) = \left|\mathbf{W}^{\mathrm{T}}(\mathbf{x})\left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e} + \mathbf{Z}(\mathbf{x},h)\right)\ 0\ \ 0\right|$;

$\mathbf{\Gamma}(h) = 4\,\mathbf{\Lambda}(h) + \mathbf{\Lambda}(2h)$;

$\mathbf{K}(\mathbf{Q},h) = 4\mathbf{\Lambda}(h)\,\mathbf{Q}\,\mathbf{\Lambda}(h) + \mathbf{\Lambda}(2h)\,\mathbf{Q}\,\mathbf{\Lambda}(2h)$;

$$\begin{aligned} \mathbf{Z}(\mathbf{x},h) = {} & 4\mathbf{\Lambda}(h)\,\mathbf{Q}\left(\mathbf{V}_y(\mathbf{x},h) - \mathbf{d}(t,h)\right) + \\ & \mathbf{\Lambda}(2h)\,\mathbf{Q}\left(\mathbf{V}_y(\mathbf{x},2h) - \mathbf{d}(t,2h)\right) \end{aligned} ;$$

$m(\mathbf{e},\mathbf{Q},h)$ are terms that are independent of $\mathbf{U}(t)$.

The minimization of $\overline{J}$ with respect to $\mathbf{U}(t)$, by setting $\partial\overline{J}/\partial\mathbf{U} = 0$, yields to the optimal control :

$$\mathbf{U}(t) = -\mathbf{P}^{-1}(\mathbf{x},h)\,\mathbf{G}(\mathbf{x},h) \qquad (9)$$

The applied control signal to nonlinear system at time t is given by:

$$\begin{aligned} \mathbf{u}(t) = {} & -\left[\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\,\mathbf{K}(\mathbf{x},h)\,\mathbf{W}(\mathbf{x})\right]^{-1}\mathbf{W}^{\mathrm{T}}(\mathbf{x}) \times \\ & \left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e} + \mathbf{Z}(\mathbf{x},h)\right) \end{aligned} \qquad (10)$$

We notice that the previous output-tracking control law only affects the torque (y_1) and the rotor flux (y_2). In the induction machine, the aim is to control speed and flux, thus an extension to speed control is achieved, in the next section, looking at a cascaded nonlinear receding-horizon control structure.

IV. CASCADED STRUCTURE OF THE NONLINEAR RHC

Cascaded control [2] is typically prescribed for linear systems involving time-scale separation assumption. That is, the inner loop is designed to have a faster dynamic than the outer loop. In this paper, the nonlinear continuous receding-horizon control scheme is extended to control speed by using the cascaded structure (Figure 1). The mechanical equation of the motor is given by:

$$\dot{\Omega}(t) = \frac{1}{J} y_1(t) - \frac{f}{J}\Omega(t) - \frac{1}{J} T_L \qquad (11)$$

The equation (11) allows controlling the speed by acting on the torque y_1. Thus, the initial system can be decomposed into two sub-systems in a cascaded form (Figure 1). The inner loop incorporates torque-flux model and the external loop is the velocity transfer function deduced from the mechanical equation given above.

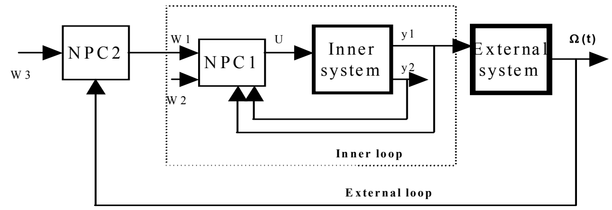

Fig.1. Cascaded control configuration.

The desired reference models, chosen in continuous time, are given by:

- For the torque trajectory (y_1):

$$\frac{y_{ref\,1}(s)}{w_1(s)} = \frac{\omega_0}{s+\omega_0}$$

- For the flux trajectory (y_2):

$$\frac{y_{ref\,2}(s)}{w_2(s)} = \frac{\omega_f^{\,2}}{s^2 + 2\xi_f \omega_f s + \omega_f^{\,2}}$$

- For the velocity trajectory (Ω):

$$\frac{\Omega_{ref}(s)}{w_3(s)} = \frac{\omega_v^{\,2}}{s^2 + 2\xi_v \omega_v s + \omega_v^{\,2}} .$$

Assuming that the torque y_1 tracks the reference signal $\mathbf{y}_{ref1}$, the global prediction model of the external loop is calculated, including the torque closed loop, in the following manner:

$$\Omega(s) = \frac{1}{Js+f} y_1(s) \approx \frac{\omega_0}{(Js+f)(s+\omega_0)} w_1(s)$$

The control objective of the external loop is the tracking of $\Omega(t)$ to a desired reference $\Omega_{\text{ref}}(t)$. The unknown load torque T_L disturbs the mechanical equation (11) and induces a steady state error in the rotor speed response. To eliminate this steady state error an integral action is introduced in the external loop by minimizing the predicted rotor position tracking error, instead of rotor speed, with the external control signal. Then from the above equations, the predicted tracking error of the rotor position can be expressed by:

$$e_\theta(t+T) = e_\theta + T\,e_v(t) + V_\theta(\Omega,T) - d_\theta(t,T) + W_\theta(T) W_1(t) \qquad (12)$$

Where: $e_\theta = \int e_v(\tau)\,d\tau$, $e_v(t) = \Omega(t) - \Omega_{ref}(t)$,

$$V_\theta(\Omega,T) = \frac{T^2}{2}\frac{f}{J}\left(\frac{T\,f}{3\,J} - 1\right)\Omega + \frac{T^2}{2\,J}\left(1 - \frac{T}{3}\left(\frac{f}{J} + w_0\right)\right) y_{ref1}$$

$d_\theta(t,T) = \dfrac{T^2}{2}\dot{\Omega}_{ref} + \dfrac{T^3}{6}\ddot{\Omega}_{ref}$ and $W_\theta(T) = \dfrac{T^3}{6}\dfrac{w_0}{J}$.

The control objective is the tracking of θ to a desired reference θ_{ref} and the tracking of y_1 and y_2 to desired reference signals $\mathbf{y}_{ref1}$ and $\mathbf{y}_{ref2}$. The performance indexes for the nonlinear system are:

- Inner loop:

$$J_1 = \frac{1}{2}\int_t^{t+T} L(\tau)\,d\tau = \frac{1}{2}\int_t^{t+T} \mathbf{e}^T(\tau)\,\mathbf{Q}\,\mathbf{e}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\,\mathbf{u}(\tau)\,d\tau \quad (13)$$

- External loop:

$$J_2 = \frac{1}{2}\int_t^{t+T_v} L_\theta(\tau)\,d\tau = \frac{1}{2}\int_t^{t+T_v} q_\theta\, e_\theta^2(\tau) + r_\theta\, W_1^2(\tau)\,d\tau$$
$$= \frac{h_v}{3}\left|L_\theta(t) + 4\,L_\theta(t+h_v) + L_\theta(t+2h_v)\right| \quad (14)$$

with $T_v = 2\,h_v$.

Also in this case, the cost function (14) can be expressed in quadratic form:

$$J_2 = \frac{h_v}{3}\left| m_\theta(e_\theta, q_\theta, h_v) + \overline{\mathbf{W}}_1^T \mathbf{G}_\theta(\Omega, h_v) + \frac{1}{2}\overline{\mathbf{W}}_1^T \mathbf{P}_\theta(\Omega, h_v)\,\overline{\mathbf{W}}_1 \right|$$

Where: $\overline{\mathbf{W}}_1(t) = \left|W_1(t)\;\; W_1(t+h_v)\;\; W_1(t+2h_v)\right|^{\mathrm{T}}$,

$\mathbf{G}_\theta(\Omega, h_v) = \left|8q_\theta W_\theta(h_v)\left(3e_\theta + 5h_v\, e_v + K_\theta - \overline{d}\right)\;\; 0 \;\;\; 0\right|^{\mathrm{T}}$

$\mathbf{P}_\theta(\Omega, h_v) = diag\left(r_\theta + 68\; q_\theta\, W_\theta^2(h_v),\; 4r_\theta,\; r_\theta\right);$

$K_\theta(\Omega, h_v) = V_\theta(\Omega, h_v) + 2\,V_\theta(\Omega, 2h_v)$,

$\overline{d}(\Omega, h_v) = d_\theta(t, h_v) + 2\,d_\theta(t, 2\,h_v)$ and $m_\theta(e_\theta, q_\theta, h_v)$ represents terms that are independent of $\overline{W}_1$.

From the minimization of the performance indexes (J_1 and J_2), and by taking only the control signal applied at time t (receding-horizon principle), we obtain:

- For the external loop:

$$W_1(t) = -\frac{4\,q_\theta W_\theta(h_v)\left(3e_\theta + 5\,h_v\, e_v + K_\theta(\Omega, h_v) - \overline{d}(\Omega, h_v)\right)}{r_\theta + 68\,q_\theta\, W_\theta^2(h_v)}$$
$$W_2 = \phi_{rnom} \quad (15)$$

- For the inner loop:

$$\mathbf{u}(t) = -\left[\mathbf{R} + \mathbf{W}^{\mathrm{T}}(\mathbf{x})\,\mathbf{K}(\mathbf{x}, h)\,\mathbf{W}(\mathbf{x})\right]^{-1}\mathbf{W}^{\mathrm{T}}(\mathbf{x}) \times \left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e} + \mathbf{Z}(\mathbf{x}, h)\right) \quad (16)$$

Tracking performance:

For the external loop: the equation (15) with $r_\theta = 0$, gives using the second order derivative of Ω the following position tracking error dynamics:

$$\dddot{e}_\theta + \frac{27}{17h_v}\ddot{e}_\theta + \frac{30}{17h_v^2}\dot{e}_\theta + \frac{18}{17h_v^3}e_\theta = 0 \quad (17)$$

For the internal loop: we assume that $\mathbf{W}(\mathbf{x})$ has a full rank. Let $\mathbf{Q} = q\,\mathbf{I}_2$, $\mathbf{R} = 0$ in the controller (16), we obtain:

$$\mathbf{u}(t) = -\mathbf{W}(\mathbf{x})^{-1}\,\mathbf{K}(\mathbf{Q}, h)^{-1}\left(\mathbf{\Gamma}(h)\,\mathbf{Q}\,\mathbf{e}(t) + \mathbf{Z}(\mathbf{x}, h)\right)$$

Differentiating the output y_1 one time and the output y_2 twice and by using the above control equation, we can show that the tracking errors dynamics are:

- For the torque:

$$\dot{e}_1(t) + \frac{3}{2h}e_1(t) = 0 \quad (18)$$

- For the flux:

$$\ddot{e}_2(t) + \frac{6}{5h}\dot{e}_2(t) + \frac{2}{h^2}e_2(t) = 0 \quad (19)$$

The above dynamics equations are linear and time invariant. Thus, the proposed tracking controller design technique leads to feedback linearization and we can easily verify the asymptotic stability of the tracking errors dynamics of the overall system.

Zero dynamics: The error dynamics (18) and (19) are linear and time invariant. Thus, the relative degrees are respectively 1 and 2. The sum is three, leading to a two order unobservable dynamics. By using the method given in [1] or in [7], we can show that these two zeros dynamics are stable provided the decoupling matrix $\mathbf{W}(\mathbf{x})$ is nonsingular. Notice that the decoupling matrix is only singular at the start up ($\det \mathbf{W}(\mathbf{x}) = -2pkR_r(\varphi_{r\alpha}^2 + \varphi_{r\beta}^2)$), this singularity can be avoided by using a flux observer with initial condition $\hat{\phi}(0) \neq 0$.

Flux observer: Rotor fluxes are difficult to measure and several papers are devoted to this problem. Since Kalman filter is less sensitive to noise and model inaccuracy and has a good behavior in the presence of resistance variations. The recursive form of the Kalman filter, used in this paper, is derived from the electrical equations of the induction motor model (1) [7].

V. Simulation Results

Computer simulations have been performed to check the behaviour of the proposed controller. The plant under control is a 1.1 kW induction machine used in [8] with the following parameters**:**

$R_r = 3.6\ \Omega$,	$R_s = 8\ \Omega$,	$L_r = 0.47$ H,
$L_s = 0.47$ H,	$L_m = 0.452$ H,	$J = 0.015$ kgm^2,
$p = 2$,	$f = 0.005$,	$T_{nom} = 5$N,
$\Omega_{nom} = 73.3$ rad/s		$\phi_{r\alpha\beta} = 1.14$ Wb .

The parameters values of the three reference models are chosen as follows:

$\xi_f = 1$, $\omega_f = 20$ rad/s for the flux trajectory

$\xi_v = 1$, $\omega_v = 10$ rad/s for the speed trajectory

$\omega_0 = 40$ rad/s for the torque trajectory.

To examine the flux and the speed tracking performances, it was considered that speed must reach the value $\Omega = 70$ rad/s in the interval of time 0.02-2 s; and $\Omega = 140$ rad/s in the interval 2-4 s; and $\Omega = 60$ rad/s for $t > 4$. The flux must reach the nominal value $\phi_{nom} = 1.14$ Wb in the interval of time 0-2s. As it is stated in [1] and [5], the flux reference will need to be reduced from the nominal value as the speed reference is increased above the rated speed in order to keep the required field voltages within the limits. Thus, the flux is reduced to 0.5 Wb in the interval 2-4s. To test the disturbance rejection, a 5 Nm unknown load torque is applied between t = 0.8 s and t = 1.4 s, afterwards it is decreased to 2Nm. All initial conditions of the motor are set to zero except for the flux observer: $\phi_r(0) = 0.02$ Wb.

After several trials, the control parameters are chosen as:

$\mathbf{Q} = 10^4\,\mathbf{I}_2$, $\mathbf{R} = 10^{-2}\,\mathbf{I}_2$, $h = 0.002$,

$q_\theta = 1e4$, $h_v = 3h$, $r_\theta = 0.0001$.

Figure 2 shows that the behavior of the actual rotor flux is very close to the flux reference. It also appears that the rotor speed fits to the speed reference trajectory. The applied load torque has no effect on the flux and its effect on the speed is rapidly compensated since it is well estimated (Figure 4). Figure 3 depicts the variations of the admissible stator voltage ($u_{s\alpha}$, $u_{s\beta}$) and the stator current i_s that is also admissible, within the saturations limits [8].

In the mismatched case, Figure 5 shows the resistances variations (R_r and R_s), the induced rotor time constant variation and time-varying load torque. The simulation results on Figure 6 shows that a good tracking performance is achieved and the above results demonstrate that the proposed controller has strong robustness properties in the presence of load disturbance and parameter variations.

Consequently, the use of the proposed feedback nonlinear receding-horizon scheme under cascade structure can solve the control problem of induction machines in the presence of uncertainties in load torque and resistance parameters variations without rotor resistance estimation. We remind that when a decoupling control algorithm is used, the variation in load torque and rotor resistance R_r causes the loss of input-output decoupling property and this can deteriorate the transient response. This calls for adaptive version of the algorithm where the convergence of the estimator is under persistency excitation of the induction machine [5, 9, 10]. Thus, the main advantage of the proposed algorithm is that we do not need to estimate neither load torque nor resistances and good performances are achieved. Moreover, in order to keep the required field voltages within the limits, the flux reference was reduced from its nominal value as the speed reference was increased above the rated speed. Operation in this flux weakening regime will excite the coupling between flux and speed in field oriented control, causing undesirable speed fluctuation and perhaps instability [1]. However, with the proposed simpler control strategy, the flux weakening operation has no effect on the speed behavior.

VI. Conclusion

This paper has presented approximate nonlinear receding-horizon controller for induction motors in cascade structure. It was assumed that only the stator currents and the rotor speed were available for measurements. To increase the robustness of the controller to time-varying load torque an integral action is introduced in external loop. The results for the benchmark were found to be satisfactory. Indeed, the proposed controller allows achieving higher dynamic tracking performance and improved robustness with respect to resistances variations and unknown load torque. In addition, the nonlinear controller ensures asymptotic decoupling of the speed and flux subsystems.. Additional research should be oriented first towards a nonlinear sensorless control scheme to reduce anymore the cost, secondly to discrete time-implementation of the proposed nonlinear receding-horizon controller. Analysis of the influence of sampling rate, truncation errors, measurement noise and saturations are all worthy of further investigation.

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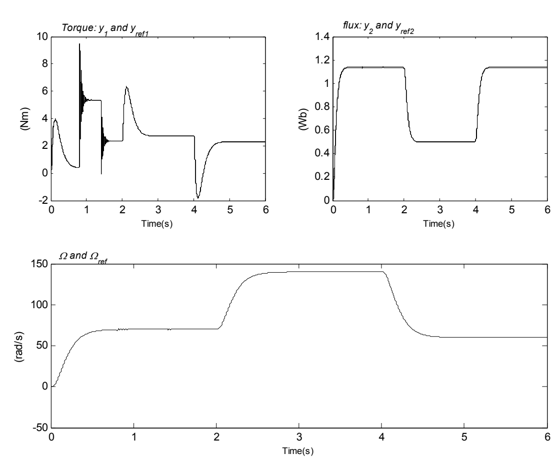

Fig. 2. Rotor torque, rotor flux and speed tracking performance.

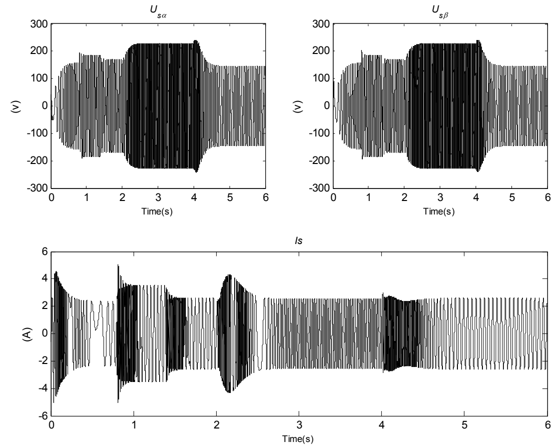

Fig. 3. Stator voltage ($u_{s\alpha}$, $u_{s\beta}$) and stator current i_s.

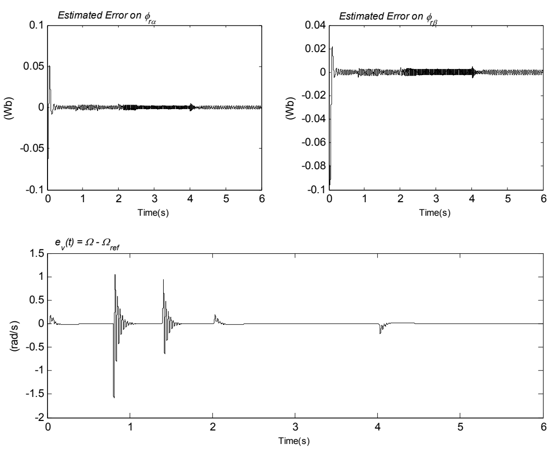

Fig. 4. Flux and rotor speed tracking error.

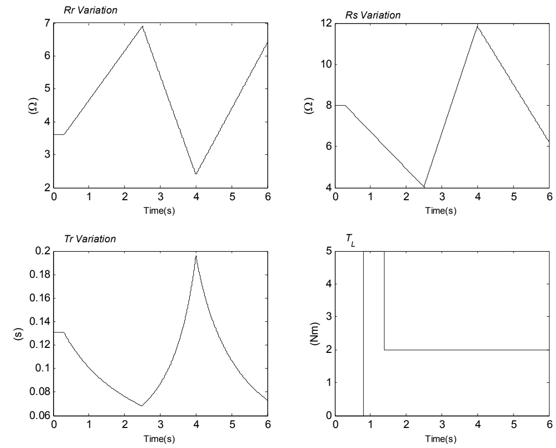

Fig. 5. Electrical parameter variations.

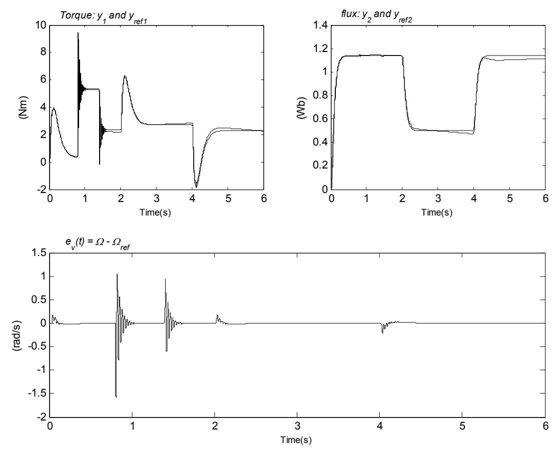

Fig. 6. Rotor torque, rotor flux and speed tracking performance in the mismatched case.