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Extended Multiple Model Adaptive Estimation for the Detection of Sensor and Actuator Faults

D. Rupp, G. Ducard, E. Shafai, and H.P. Geering, *Fellow, IEEE*

A combination of the multiple model adaptive estimation method (MMAE) with an extended Kalman filter (EKF) is considered. The ability of the MMAE method to detect faults based on a predefined hypothesis and the parameter-estimating ability of an EKF results in a very efficient fault detection approach. This so-called extended multiple model adaptive estimation method (EMMAE) has been investigated on a nonlinear model of an aircraft. The results show that it is a very capable method to detect faults of various types.

I. Introduction

The multiple model adaptive estimation (MMAE) method [1] is based on a bank of parallel Kalman filters (KF), each tuned to describe a particular fault status of the system. The output of each KF is then weighted by its corresponding probability based on the measurement history.

The MMAE method is a good choice for the detection of actuator as well as sensor faults, as long as the expected faults can be hypothesized by a reasonable number of Kalman filters. However, the number of addressable faults is rather restricted, and this method reaches its limits as soon as the actual occurring fault does not closely match the predefined fault hypothesis. This may occur when an actuator is stuck at an unknown position (here called "lock-in-place" fault) that affects the dynamics of the system. We know from KF theory that we have to consider all systematic errors, however, since lock-in-place faults cannot be predicted, they may have detrimental effects on the filter performance. Due to the biased residual, the KF provides a wrong estimation of the state variables, which causes severe problems with the probability calculation. Therefore, neither the fault detection nor the fault isolation works properly.

In order to make MMAE applicable for lock-in-place faults and even for the more general class of varying faults, the MMAE algorithm is combined with extended Kalman filters (EKF), which are able to estimate some (unknown) fault parameters. The resulting method is called in this context "extended multiple model adaptive estimation" (EMMAE).

This paper describes in detail the EMMAE method and shows results of simulation with a nonlinear aircraft model.

II. Actuator/Sensor Fault Detection System

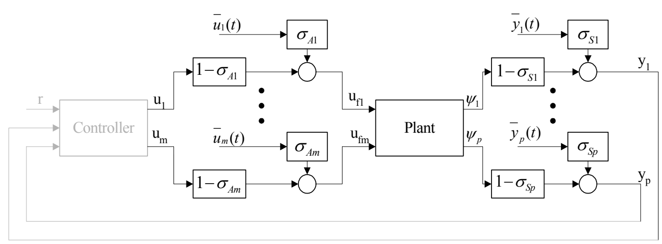

Fig. 1. Control system with actuator and sensor faults.

Consider the control system with actuator and sensor faults shown in Fig. 1.

A. Actuator Faults

As shown in [2] the actuator faults are represented by

$$u_f(t) = u(t) + \sigma_A(\bar{u}(t) - u(t)) \qquad (1)$$

where $u(t)$ is the desired plant input,

$$\bar{u}(t) = \left[\bar{u}_1(t), \bar{u}_2(t), ..., \bar{u}_m(t)\right]^T \qquad (2)$$

is the vector of the (unknown) inputs in case of actuator faults, and

$$\sigma_A = diag\{\sigma_{A1}, \sigma_{A2}, ..., \sigma_{Am}\} \qquad (3)$$

with

$$\sigma_{Aj} = \begin{cases} 1 & \text{if the j}^{\text{th}} \text{ actuator fails} \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Manuscript received March 7, 2005. D. Rupp is with the Measurement and Control Laboratory, Swiss Federal Institute of Technology, Zurich, 8092 Switzerland (corresponding author to provide phone: +41 44 632 24 53; fax: +41 44 632 11 39; e-mail: rupp@imrt.mavt.ethz.ch).
G. Ducard is with the Measurement and Control Laboratory, Swiss Federal Institute of Technology, Zurich, 8092 Switzerland (e-mail: ducard@imrt.mavt.ethz.ch).
E. Shafai is with the Measurement and Control Laboratory, Swiss Federal Institute of Technology, Zurich, 8092 Switzerland (e-mail: shafai@imrt.mavt.ethz.ch).
H. P. Geering is with the Measurement and Control Laboratory, Swiss Federal Institute of Technology, Zurich, 8092 Switzerland (e-mail: geering@imrt.mavt.ethz.ch).

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B. Sensor Faults

The sensor faults are represented by

$$y(t) = \psi(t) + \sigma_S(\bar{y}(t) - \psi(t)) \quad (5)$$

where $\psi(t)$ is the actual plant output,

$$\bar{y}(t) = \left[\bar{y}_1(t), \bar{y}_2(t), ..., \bar{y}_p(t)\right]^T \quad (6)$$

is the vector of the (unknown) outputs in case of sensor faults, and

$$\sigma_S = diag\left\{\sigma_{S1}, \sigma_{S2}, ..., \sigma_{Sp}\right\} \quad (7)$$

with

$$\sigma_{Sj} = \begin{cases} 1 & \text{if the jth sensor fails} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

C. Structure of the EMMAE Method

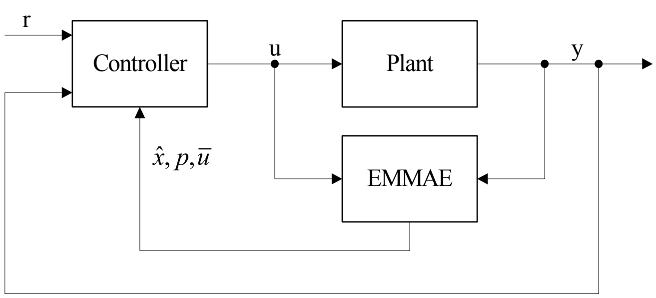

Fig. 2: Scheme of a control system with EMMAE fault detection.

Fig. 2 shows a control system with EMMAE fault detection. The EMMAE block provides the controller with a probability weighted estimation of the state vector $\hat{x}$, the probabilities p of the predefined fault scenarios and an estimation of the corresponding fault parameters ($\bar{u}$ in case of actuator faults and $\bar{y}$ in case of sensor faults).

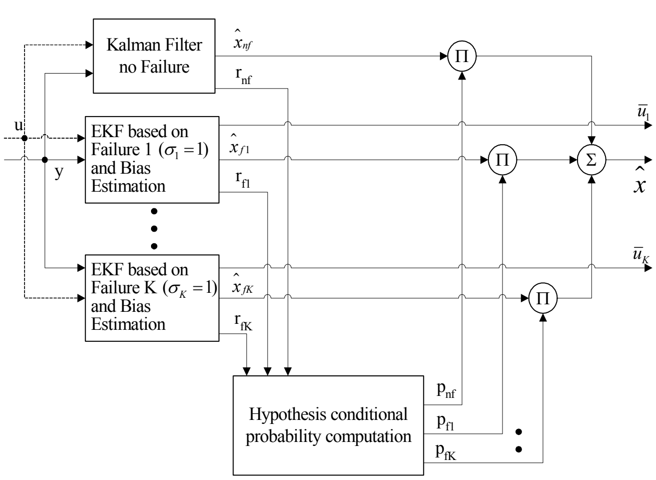

Fig. 3. Scheme of the EMMAE method. EKF are designed for the estimation of state variables and bias estimation.

Fig. 3 shows a block diagram of the EMMAE method. The bank of K EKF blocks is used to estimate the unknown actuator position $\bar{u}_j$ or the unknown sensor bias $\bar{y}_j$ (assuming that j is not known a priori). With the introduction of the EKF blocks, the EMMAE method can be used for all actuator and sensor faults which can be described by the combination of a hard fault (i.e. actuator j cannot be controlled anymore: $\sigma_{Aj} = 1$ or sensor j fails: $\sigma_{Sj} = 1$) as well as a (slowly) varying parameter (i.e. uncontrolled movement of actuator j: $\bar{u}_j(t)$ or meaningless output of sensor j: $\bar{y}_j(t)$).

D. Hypothesis Testing

Given a dynamic system with K possible patterns of faults which may occur during operation, let θ denote the vector of the (uncertain) parameters characterizing the fault status of sensors and actuators of the system. Assume that all K possible conditions can be described by discrete values θ_k of θ (for k = 1, 2, ..., K). Assume further that a bank of K separate (extended) Kalman filters is given (see Fig. 3), each filter representing a hypothesis for a possible fault pattern of the system. We define the conditional probability $p_k(t_i)$ that θ assumes the value θ_k conditioned on the fact that the observed measurement history $Y(t_i)$ assumes the values of Y_i at time t_i [3].

$$p_k(t_i) = \frac{f_{y(t_i)|\theta,Y(t_{i-1})}(y_i|\theta_k, Y_{i-1})p_k(t_{i-1})}{\sum_{j=1}^{K} f_{y(t_i)|\theta,Y(t_{i-1})}(y_i|\theta_j, Y_{i-1})p_j(t_{i-1})} \quad (9)$$

Equation (9) is a recursive representation of Bayes' rule and thus can be used in an "online" algorithm. The conditional density of the measurement y_i at the time t_i, $f_{y(t_i)|\theta,Y(t_{i-1})}(y_i|\theta_k, Y_{i-1})$, can be computed with the information provided by its corresponding Kalman filter

$$f_{y(t_i)|\theta,Y(t_{i-1})}\left(y_i|\theta_k, Y_{i-1}\right) = \frac{1}{(2\pi)^{m/2}\det\left(Q_k(t_i)\right)^{1/2}} \times \exp\left(-\frac{1}{2}r_k^T(t_i)Q_k^{-1}(t_i)r_k(t_i)\right) \quad (10)$$

where m is the dimension of the measurement vector, $r_k(t_i) = y_k(t) - H_k(\theta)\hat{x}_{fk}(t|t-1)$ is the residual with H_k being the measurement matrix, and $Q_k(t_i)$ the residual covariance matrix of the k^th filter at the time t_i.

III. Fault Parameter Estimation

A. Derivation of the System Matrices for the EKF

Consider the nonlinear discrete-time system:

$$\begin{aligned} x(t+1) &= f(x(t), u(t), \theta(t)) + G_v(\theta)v(t) \\ x(0) &= \xi \\ y(t) &= h(x(t), u(t), \theta(t)) + r(t) \end{aligned} \quad (11)$$

where $v(t)$ and $r(t)$ are the system and measurement noises, with the covariance matrices

$$
\begin{aligned}
&E\left\{[v(t)][v(\tau)]^T\right\} = R_v(\theta)\delta_{t\tau} \\
&\quad \text{with } R_v(t,\theta) = R_v^T(\theta) \ge 0 \quad \text{for t}, \tau \ge 0 \\
&E\left\{[r(t)][r(\tau)]^T\right\} = R_r(\theta)\delta_{t\tau} \\
&\quad \text{with } R_r(t,\theta) = R_r^T(\theta) > 0 \quad \text{for t}, \tau \ge 0 \\
&E\left\{[v(t)][r(\tau)]^T\right\} = R_{vr}(\theta)\delta_{t\tau}
\end{aligned} \tag{12}
$$

In order to use a Kalman filter to predict the state variables of the system and also to estimate its parameters, we augment the state vector by the unknown parameters to be estimated [4]:

$$
z = \begin{bmatrix} x \\ \theta \end{bmatrix} \tag{13}
$$

The augmented state vector leads to the following nonlinear state space equations

$$
\begin{aligned}
z(t+1) &= f_z(z(t),u(t)) + \bar{G}_v(t)v(t) \\
y(t) &= h(z(t),u(t)) + r(t)
\end{aligned} \tag{14}
$$

with

$$
f_z(z(t),u(t)) = \begin{bmatrix} f(z(t),u(t)) \\ \theta(t) \end{bmatrix} \tag{15}
$$

and

$$
\bar{G}_v(t) = \begin{bmatrix} G_v \\ 0 \end{bmatrix} \tag{16}
$$

which implies that the system noise is not acting on the parameters.

In order to apply the Kalman filter equations to the nonlinear system (14) to (16), the system has to be linearized continuously around its current working point. Since the current working point is not known for the actual time t_i, we use the prediction that is based on the data of the preceding time step t_{i-1}. Hence the linear system matrices at time t are

$$
\begin{aligned}
F_z(t) &= \frac{\partial}{\partial z} f_z(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} \\
&= \begin{bmatrix} F(\hat{\theta}(t),\hat{x}(t),u(t)) & M(\hat{\theta}(t),\hat{x}(t),u(t)) \\ 0 & I \end{bmatrix}
\end{aligned} \tag{17}
$$

where

$$
F(\hat{\theta}(t),\hat{x}(t),u(t)) = \frac{\partial}{\partial x}\left[f(x(t),u(t),\theta(t))\right]\Big|_{z(t)=\hat{z}(t|t-1)} \tag{18}
$$

and

$$
M(\hat{\theta}(t),\hat{x}(t),u(t)) = \frac{\partial}{\partial \theta}\left[f(x(t),u(t),\theta(t))\right]\Big|_{z(t)=\hat{z}(t|t-1)} \tag{19}
$$

The lower-right unity matrix of (17) implies that $\theta(t+1) = \theta(t)$. The linearized measurement matrix reads

$$
H_z(t) = \frac{\partial}{\partial z} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = [C_x(t) \quad C_\theta(t)] \tag{20}
$$

where

$$
C_x(t) = \frac{\partial}{\partial x} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} \tag{21}
$$

and

$$
C_\theta(t) = \frac{\partial}{\partial \theta} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} \tag{22}
$$

B. The Plant

We assume that the linearized healthy plant (without faults) can be described by the discrete-time system

$$
\begin{aligned}
x(t+1) &= Fx(t) + G_u u(t) + G_v v(t) \\
x(0) &= \xi \\
y(t) &= Hx(t) + r(t)
\end{aligned} \tag{23}
$$

This plant, together with the fault structure shown in Fig.1, can be described by the "fault" parameters σ_A and $\bar{u}$ from (1) for the actuator faults, and σ_S and $\bar{y}$ from (5) for the sensor faults as follows

$$
\begin{aligned}
x(t+1) &= f(x(t),u(t),\sigma_A(t),\bar{u}(t)) + G_v v(t) \\
y(t) &= h(x(t),\sigma_S(t),\bar{y}(t)) + r(t)
\end{aligned} \tag{24}
$$

C. Actuator Faults

If we only look at actuator faults, we omit the sensor parameters σ_S and $\bar{y}$; hence with the linearized system from (23) the term $f(x(t),u(t),\sigma_A(t),\bar{u}(t))$ from (24) can be written as

$$
\begin{aligned}
f(x(t),u(t),\sigma_A(t),\bar{u}(t)) = \; & Fx(t) + G_u\left[I_m - \sigma_A(t)\right]u(t) \\
& + G_u\sigma_A(t)\bar{u}(t)
\end{aligned} \tag{25}
$$

For the derivation of an EKF for a fault hypothesis of the i[th] actuator, we define $\sigma_{Ai} = 1$ and $\sigma_{Aj} = 0, \bar{u}_j = 0, \; j \ne i$. In order to apply an EKF, the state vector is augmented by the i[th] bias parameter according to Section A.

$$
z = \begin{bmatrix} x \\ \bar{u}_i \end{bmatrix} \tag{26}
$$

The augmented state vector leads to the following state space equations

$$z(t+1) = f_z(z(t),u(t)) + \bar{G}_v(t)v(t)$$
$$y(t) = h(z(t),u(t)) + r(t) \tag{27}$$

with

$$f_z(z(t),u(t)) = \begin{bmatrix} f(z(t),u(t)) \\ \bar{u}_i(t) \end{bmatrix} \tag{28}$$

and

$$\bar{G}_v(t) = \begin{bmatrix} G_v \\ 0 \end{bmatrix} \tag{29}$$

The linearization of the dynamic matrix yields

$$F_z(t) = \frac{\partial}{\partial z} f_z(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = \begin{bmatrix} F & M(\hat{\bar{u}}_i(t),\hat{x}(t),u(t)) \\ 0 & 1 \end{bmatrix} \tag{30}$$

where

$$M(\hat{\bar{u}}_i(t),\hat{x}(t),u(t)) = \frac{\partial}{\partial \bar{u}_i}\left[f(z(t),u(t)) \right]\Big|_{z(t)=\hat{z}(t|t-1)} = G_u^{(i)} \tag{31}$$

with $G_u^{(i)}$ representing the i[th] column of G_u. The input matrix becomes

$$G_z(t) = \frac{\partial}{\partial u} f_z(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = \begin{bmatrix} {}^{(0,i)}G_u \\ 0 \end{bmatrix} \tag{32}$$

with ${}^{(0,i)}G_u$ representing the matrix G_u with the i[th] column set to zero. The linearization of the measurement matrix is

$$H_z(t) = \frac{\partial}{\partial z} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = [C_x(t) \quad C_\theta(t)] \tag{33}$$

where

$$C_x(t) = \frac{\partial}{\partial x} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = H \tag{34}$$

and

$$C_\theta(t) = \frac{\partial}{\partial \bar{u}_i} h(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = 0 \tag{35}$$

Using (30) to (36) the linearized system can be written as

$$\begin{bmatrix} x(t+1) \\ \bar{u}_i(t+1) \end{bmatrix} = \begin{bmatrix} F & G_u^{(i)} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ \bar{u}_i(t) \end{bmatrix} + \begin{bmatrix} {}^{(0,i)}G_u \\ 0 \end{bmatrix} u(t)$$
$$y(t) = [H \quad 0] \begin{bmatrix} x(t) \\ \bar{u}_i(t) \end{bmatrix} \tag{36}$$

D. Sensor Faults

In case of sensor faults we omit the actuator parameters $\sigma_A(t)$ and $\bar{u}(t)$. The output equation is

$$h(x(t),\sigma_S(t),\bar{y}(t)) = \left[I_p - \sigma_S(t) \right] Hx(t) + \sigma_S(t)\bar{y}(t) \tag{37}$$

and hence the output may be written as

$$y(t) = \left[I_p - \sigma_S(t) \right] Hx(t) + \sigma_S(t)\bar{y}(t) + r(t) \tag{38}$$

For the derivation of an EKF for a fault hypothesis of the i[th] sensor we define $\sigma_{Si} = 1$ and $\sigma_{Sj} = 0, \bar{y}_j = 0, \; j \neq i$. In order to apply an EKF, the state vector is again augmented by the i[th] bias parameter according to Section A.

$$z = \begin{bmatrix} x \\ \bar{y}_i \end{bmatrix} \tag{39}$$

The augmented state vector leads to the following state space equations

$$z(t+1) = f_z(z(t),u(t)) + \bar{G}_v(t)v(t)$$
$$y(t) = h(z(t),u(t)) + r(t) \tag{40}$$

with

$$f_z(z(t),u(t)) = \begin{bmatrix} f(z(t),u(t)) \\ \bar{y}_i(t) \end{bmatrix} \tag{41}$$

and

$$\bar{G}_v(t) = \begin{bmatrix} G_v \\ 0 \end{bmatrix} \tag{42}$$

The linearization of the dynamic matrix thus yields

$$F_z(t) = \frac{\partial}{\partial z} f_z(z(t),u(t))\Big|_{z(t)=\hat{z}(t|t-1)} = \begin{bmatrix} F & M(\hat{\bar{y}}_i(t),\hat{x}(t),u(t)) \\ 0 & I \end{bmatrix} \tag{43}$$

where

$$M(\hat{\bar{y}}_i(t),\hat{x}(t),u(t)) = \frac{\partial}{\partial \bar{y}_i}\left[f(z(t),u(t)) \right]\Big|_{z(t)=\hat{z}(t|t-1)} = 0 \tag{44}$$

The input matrix becomes

$$G_z(t) = \frac{\partial}{\partial u} f_z(z(t),u(t))\bigg|_{z(t)=\hat{z}(t|t-1)} = \begin{bmatrix} G_u \\ 0 \end{bmatrix} \quad (45)$$

The linearization of the measurement matrix is

$$H_z(t) = \frac{\partial}{\partial z} h(z(t),u(t))\bigg|_{z(t)=\hat{z}(t|t-1)} = \begin{bmatrix} C_x(t) & C_\theta(t) \end{bmatrix} \quad (46)$$

where

$$C_x(t) = \frac{\partial}{\partial x} h(z(t),u(t))\bigg|_{z(t)=\hat{z}(t|t-1)} = {}^{(i,0)}H \quad (47)$$

and

$$C_\theta(t) = \frac{\partial}{\partial \bar{y}_i} h(z(t),u(t))\bigg|_{z(t)=\hat{z}(t|t-1)} = [0 \;\; \ldots \;\; 0 \;\; \underset{i^{th} entry}{1} \;\; 0 \;\; \ldots \;\; 0]^T \quad (48)$$

with ${}^{(i,0)}H$ representing the matrix H in which the i[th] row is set to zero. Using (43) to (48) the linearized system can be written as

$$\begin{bmatrix} x(t+1) \\ \bar{y}_i(t+1) \end{bmatrix} = \begin{bmatrix} F & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ \bar{y}_i(t) \end{bmatrix} + \begin{bmatrix} G_u \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [{}^{(i,0)}H \;\vdots\; [0 \;\; \ldots \;\; 0 \;\; \underset{i^{th} entry}{1} \;\; 0 \;\; \ldots \;\; 0]^T] \begin{bmatrix} x(t) \\ \bar{y}_i(t) \end{bmatrix} \quad (49)$$

IV. Simulation Results

The fault detection with the EMMAE method was tested on a nonlinear aircraft model of tenth order. Simulations were performed in Matlab/Simulink on an open-loop control architecture, where an input sinusoidal signal excited the process. For test purposes two (redundant) ailerons were "equipped" with an EKF. Fig. 4 depicts the first simulation in which two ailerons are excited by a sine signal. After 4.5 s the second aileron gets stuck at an offset position. Fig. 4.b clearly shows that the roll rate is still estimated correctly despite the fault. Fig. 4.c shows the estimation of the aileron bias. Up to the aileron fault the estimation follows the actual aileron movement.

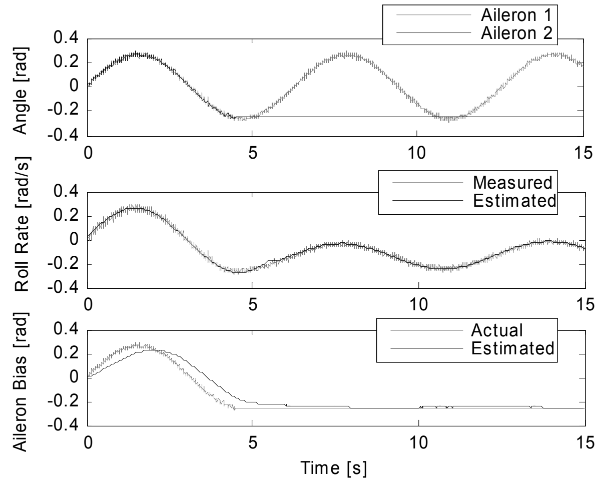

Fig. 4. (a) aileron input signal and roll rate, (b) measured and estimated roll rate, (c) estimated aileron bias. Lock-in-place fault of aileron 2 at 4.5 s.

Fig. 5 shows the probabilities (compare (9)) of the no-fault hypothesis as well as the fault hypothesis of the two ailerons. The fault is detected and isolated after about 1.5 s.

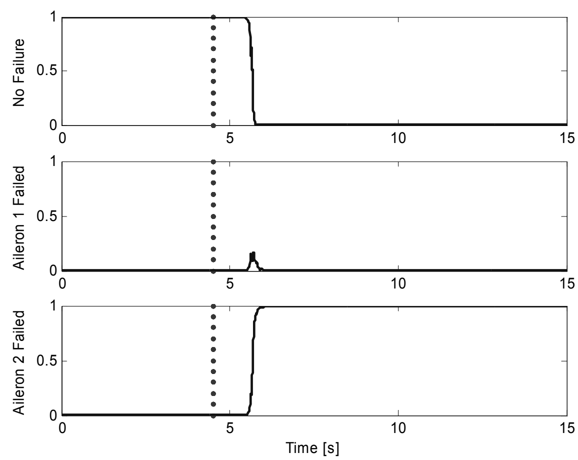

Fig. 5. Probability that either no fault or a fault of ailerons 1 or 2 has occurred.

In the second simulation shown in Fig. 6 the second aileron fails after 5 s, but then still moves in the manner of a square signal. Fig. 6b and Fig. 6c show that the roll rate as well as the fault signal is estimated quite well. Fig. 7 shows that the fault is detected immediately, however, during the first 5 s after the fault there is an ambiguity with the fault hypothesis of the first aileron.

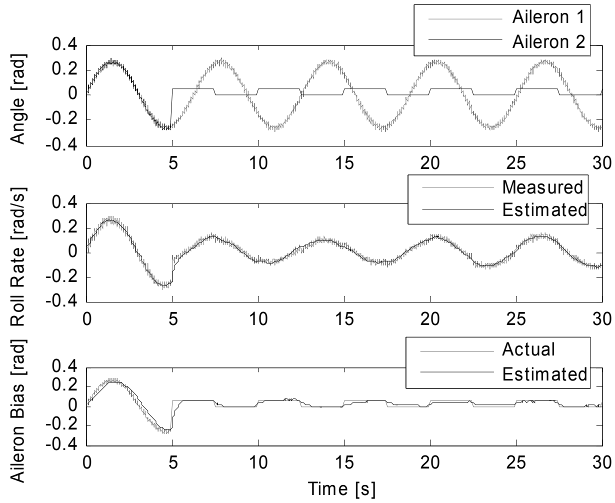

Fig. 6. (a) Aileron input signal and roll rate, (b) measured and estimated roll rate, (c) estimated ailreon bias. Varying lock-in-place fault of aileron 2 starting at 5 s.

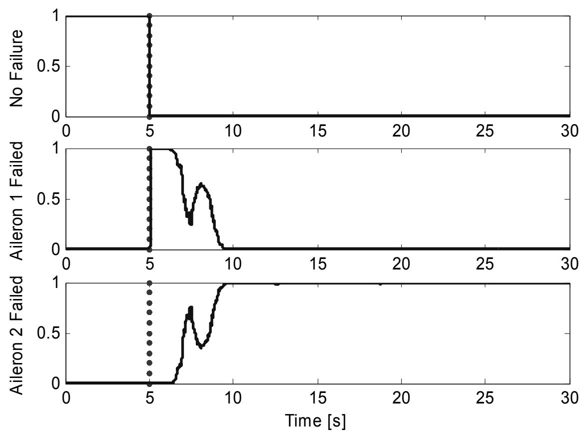

Fig. 7. Probability that either no fault or a fault of ailerons 1 or 2 has occurred.

V. Conclusion

The MMAE method has been combined with a parameter-estimating EKF in order to extend the class of detectable faults. Every fault is described by two parameters, one having binary characteristic, estimated by the MMAE part of the EMMAE algorithm and the other one being a (time-varying) real value estimated by the EKF part of the EMMAE algorithm.

Simulations show the ability of the EMMAE method to detect faults that can be described by these two parameters. The proposed EMMAE method will be further tested in a closed-loop control architecture, and robustness properties of the FDI algorithm will also be investigated.

Appendix

A. Model

For the simulations a six-degree-of-freedom body-motion model was used. In order to compute change of the attitude of the aircraft in time, quaternions have been used. The state space is described by ten state variables:

$$x(t)=\begin{bmatrix} x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\\ x_5(t)\\ x_6(t)\\ x_7(t)\\ x_8(t)\\ x_9(t)\\ x_{10}(t)\end{bmatrix} \triangleq \begin{bmatrix} p(t)\\ q(t)\\ r(t)\\ q_0(t)\\ q_1(t)\\ q_2(t)\\ q_3(t)\\ u(t)\\ v(t)\\ w(t)\end{bmatrix} \begin{matrix} \text{roll rate } [\text{rad/s}]\\ \text{nick rate } [\text{rad/s}]\\ \text{yaw rate } [\text{rad/s}]\\ \text{Euler parameter } [\text{-}]\\ \text{"}\\ \text{"}\\ \text{"}\\ \text{longitudinal velocity } [\text{m/s}]\\ \text{lateral velocity } [\text{m/s}]\\ \text{normal velocity } [\text{m/s}]\end{matrix}$$

The inputs with two redundant ailerons are given as

$$u(t)=\begin{bmatrix} u_1(t)\\ u_2(t)\\ u_3(t)\\ u_4(t)\end{bmatrix} \triangleq \begin{bmatrix} \xi_1(t)\\ \xi_2(t)\\ \eta(t)\\ \zeta(t)\end{bmatrix} \begin{matrix} \text{aileron 1 angle [rad]}\\ \text{aileron 2 angle [rad]}\\ \text{elevator angle [rad]}\\ \text{rudder angle [rad]}\end{matrix}$$

and the outputs

$$y(t)=\begin{bmatrix} y_1(t)\\ y_2(t)\\ y_3(t)\\ y_4(t)\\ y_5(t)\\ y_6(t)\end{bmatrix} \triangleq \begin{bmatrix} p(t)\\ q(t)\\ r(t)\\ u(t)\\ v(t)\\ w(t)\end{bmatrix} \begin{matrix} \text{roll rate } [\text{rad/s}]\\ \text{nick rate } [\text{rad/s}]\\ \text{yaw rate } [\text{rad/s}]\\ \text{longitudinal velocity } [\text{m/s}]\\ \text{lateral velocity } [\text{m/s}]\\ \text{normal velocity } [\text{m/s}]\end{matrix}$$

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