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MPC-Based Load Shedding for Voltage Stability Enhancement

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Abstract— **Impending voltage collapse can often be avoided by appropriate control of loads. However the traditional form of load control (shedding) is unpopular due to the resulting consumer disruption. Advances in communications and computer systems allow more selective load control though. Individual loads that are sacrificeable in the short-term can be switched with minimal consumer disruption. The paper considers the use of such non-disruptive load control for improving voltage stability. A control strategy that is based on model predictive control (MPC) is proposed. MPC utilizes an internal model to predict system dynamic behaviour over a finite horizon. Control decisions are based on optimizing that predicted response. MPC is a discrete-time form of control, so inaccuracies in predicted behaviour are corrected at the next control interval. A standard 10 bus voltage collapse example is used to illustrate this control strategy.**

I. Introduction

Load control provides an effective means of alleviating voltage collapse. For example the cascading failure of the North American power system in August 2003 could have been avoided by tripping a relatively small amount of load in the Cleveland area [1]. The most effective load shedding strategies are not always so obvious though. Low voltages often provide a good indication of locations where load shedding would assist in relieving system stress [2]. However counter-examples are easy to generate. The simple system of Fig. 1 provides an illustration.

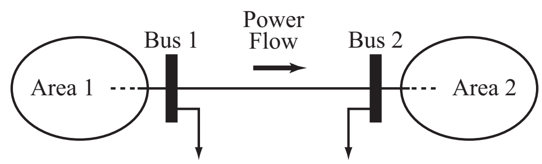

Fig. 1. Illustration of inappropriate undervoltage load shedding.

Consider the situation where the power being exported from Area 1 to Area 2 overloads the corridor between buses 1 and 2. (This may be a consequence of unexpected line tripping between these buses.) As a result of the overload, lines forming the corridor will demand high levels of reactive power, causing voltages at both end buses to fall. Undervoltage load shedding at bus 1, without a matching reduction in Area 1 generation, would likely lead to an increase in power flow over the troublesome corridor. This would exacerbate the situation. Undervoltage load shedding at bus 2 would probably achieve its desired goal. Clearly situations arise where a coordinated approach to load shedding is required. A range of such load shedding schemes have been proposed and/or implemented, see for example [3], [4], [5]. This paper presents preliminary work in the development of an approach based on model predictive control (MPC). It shares some similarities with the MPC-based emergency control scheme of [6].

The traditional form of load control (shedding) is quite disruptive to consumers, and often avoided because of the discontent created. However if it were possible to switch small pieces of load, so that interruptions were effectively unnoticed by consumers, then load control would be more palatable. Recent advances in communications and computer systems facilitate such non-disruptive load control.

The paper is organized as follows. Sections II and III provide overviews of non-disruptive load control and model predictive control respectively. Section IV develops a linear time-varying discrete-time model of power system dynamics, and Section V discusses MPC implementation issues. An example is explored in Section VI. Conclusions are presented in Section VII.

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II. Non-Disruptive Load Control

Many consumer installations consist of loads that are at least partially controllable [7], [8]. Commercial loads typically involve a high proportion of air conditioning and lighting. The thermal time-constant of many commercial buildings is usually quite long. Therefore air conditioning in large multi-storied buildings can be shed with no appreciable short-term effects on building climate. Similarly, a short-term reduction in lighting load is often possible without compromising the building environment. Partial load control within industrial and residential installations is also possible. In the residential case for example, one circuit within a home could be designated for interruptible supply, with a corresponding lower energy charge. That circuit could be used for lower priority loads such as dryers and/or freezers. A similar concept applies for industrial consumers. In the latter case though, it may also be possible to use backup generation to displace grid supply.

The distributed nature of non-disruptive load control implies a need for a hierarchical control structure, as suggested in Fig. 2. A lower (substation) level controller is required

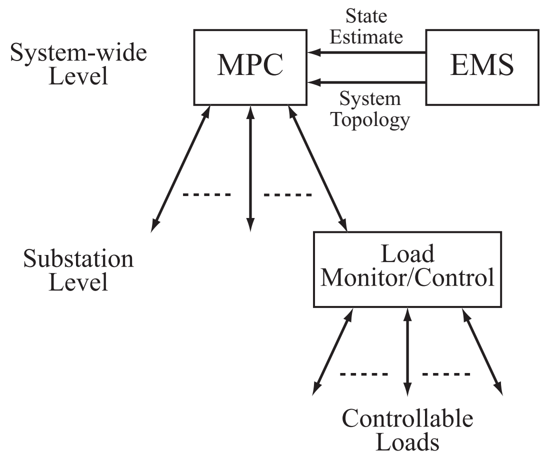

Fig. 2. Hierarchical load control structure.

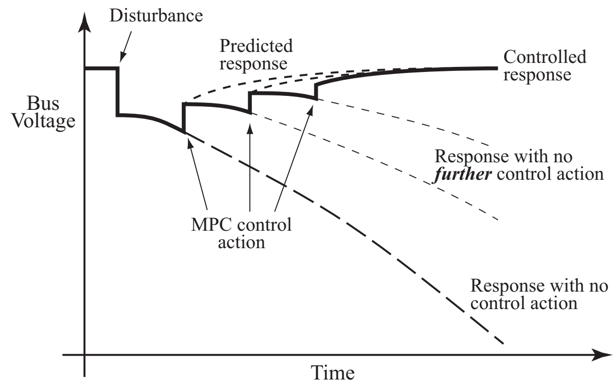

Fig. 3. MPC response.

to coordinate the many small controllable loads. In standby mode, this controller would continually poll loads to track availability of controllable load. Availability information would be passed to the higher level. When load control was required, the higher level controller would specify the desired load change. The substation-level controller would implement that load change by signalling the individual loads. The anticipated and actual load responses may differ. This information would again be coordinated at the lower level and passed to the higher level in preparation for further control action.

The higher-level controller collects system-wide information on controllable-load availability, and formulates feasible responses to disturbances. As suggested earlier, often a simple undervoltage load shedding strategy is inadequate for arresting voltage instability. The higher-level controller must determine the appropriate amount of load to regulate at each location, for arbitrary disturbance scenarios, and in the presence of system and load (actuator) uncertainty. Also, the amount of load disrupted should be minimized. Model predictive control provides an appropriate control strategy for meeting those goals.

III. Model Predictive Control

Model predictive control (MPC) is a discrete-time form of control, with commands issued at periodic intervals [9], [10]. Fig. 3 provides an illustration of the MPC process. Each control decision is obtained by first estimating the system state. This provides the initial condition for prediction (simulation) of subsequent dynamic behaviour. The prediction stage is traditionally formulated as an open-loop optimal control problem over a finite horizon. This results in the corresponding open-loop control sequence. MPC applies the initial control value from that sequence. The process is repeated at the next MPC interval, with the state estimator providing a new initial condition for a new prediction (optimal control) problem.

The optimization problem underlying MPC involves open-loop prediction of system behaviour. Actual behaviour invariably deviates from that predicted response though. However feedback is effectively achieved through the correction applied when the next MPC control signal is issued. This is illustrated in Fig. 3.

Power system dynamic behaviour often involves interactions between continuous dynamics and discrete events, particularly during voltage collapse when many discrete devices switch. Formulation of optimal control problems for such hybrid behaviour is fraught with technical difficulties. However it is shown in Section IV that this problem may be approximated, through the use of trajectory sensitivities, as a linear (time-varying) discrete-time optimal control problem.

IV. Linear Discrete-Time System Model

A. Flows and trajectory sensitivities

An mentioned earlier, power system large disturbance response typically exhibits interactions between continuous dynamics and discrete events. Numerous models for such hybrid systems have been proposed [11], [12]. It is common for the continuous dynamics to be modelled using a differential-algebraic (DAE) representation. Discrete events are incorporated via impulsive mappings and switching within the DAE model.

Independent of the exact form of the underlying model, hybrid system dynamic behaviour can be described by the *flow*

$$x(t) = \phi(x_0, u_0, t) \tag{1}$$

together with the algebraic constraints

$$g(x(t), y(t), u_0) = 0 \tag{2}$$

where x and y are the dynamic and algebraic states respectively of the DAE model, x_0 is the initial value of x, so that $x_0 = x(0) = \phi(x_0, u_0, 0)$, and u_0 describes (constant) parameters. Examples of x states include generator fluxes, y states include bus voltages, and parameters include load magnitudes. It will be shown later that control is realized through piecewise variation of u_0.

The Taylor series expansion of (1) can be expressed as

$$\phi(x_0 + \Delta x_0, u_0 + \Delta u_0, t) = \phi(x_0, u_0, t) + \Phi_x(x_0, u_0, t)\Delta x_0 + \Phi_u(x_0, u_0, t)\Delta u_0 + \text{h.o.t.} \tag{3}$$

where $\Phi_x \triangleq \frac{\partial \phi}{\partial x_0}$ and $\Phi_u \triangleq \frac{\partial \phi}{\partial u_0}$ are trajectory sensitivities. Defining

$$\Delta x(t) = \phi(x_0 + \Delta x_0, u_0 + \Delta u_0, t) - \phi(x_0, u_0, t) \quad (4)$$

and neglecting higher order terms gives rise to

$$\Delta x(t) \approx \Phi_x(x_0, u_0, t)\Delta x_0 + \Phi_u(x_0, u_0, t)\Delta u_0. \quad (5)$$

Differentiating (2) results in

$$g_x \Delta x(t) + g_y \Delta y(t) + g_u \Delta u_0 = 0 \quad (6)$$

where $g_x \triangleq \frac{\partial g}{\partial x}$, $g_y \triangleq \frac{\partial g}{\partial y}$ and $g_u \triangleq \frac{\partial g}{\partial u}$.

It is shown in [13] that the trajectory sensitivities Φ_x, Φ_u are well defined for hybrid systems, provided the underlying flow ϕ is well defined. (This excludes phenomena such as algebraic singularity, sliding modes and Zeno effects.) Furthermore, if simulation utilizes an implicit numerical integration process, then very efficient computation of these sensitivities is possible [14].

B. Model formulation

Based on the flow concept presented in Section IV-A, prediction of behaviour forward from time t_k is possible with knowledge of the state $x_k \triangleq x(t_k)$, and control $u_k \triangleq u(t_k)$. Let T be the period associated with MPC operation, and NT the prediction horizon. Then

$$\begin{aligned} x_{k+N} \triangleq x(t_k + NT) &= \phi(x_{k+N-1}, u_k, T) \\ &= \phi(x_{k+N-2}, u_k, 2T) \\ &\vdots \\ &= \phi(x_k, u_k, NT) \end{aligned}$$

with the corresponding y given by $g(x_{k+i}, y_{k+i}, u_k) = 0$. In other words, the nominal discrete-time trajectory $(x_{k+1}, y_{k+1}), ..., (x_{k+N}, y_{k+N})$ can be obtained by sampling the simulation that begins at the initial value x_k, and that runs for time NT. This trajectory is nominal in the sense that control is held constant at its initial value u_k. The aim of MPC is to determine control adjustments $\Delta u_{k+1}, ..., \Delta u_{k+N-1}$ that achieve desired behaviour in an optimal way.

It follows from (5) that perturbations from the sampled nominal trajectory are given by

$$\begin{aligned} \Delta x_{k+i+1} = \Phi_x(x_{k+i}, u_k, T)\Delta x_{k+i} \\ + \Phi_u(x_{k+i}, u_k, T)\Delta u_{k+i}, \quad 0 \le i \le N-1, \end{aligned} \quad (7)$$

where we have used the fact that $u_k = u_{k+1} = ... = u_{k+N-1}$. Deviations in algebraic states follow from (6),

$$\Delta y_{k+i} = -g_y^{-1}\big(g_x \Delta x_{k+i} + g_u \Delta u_{k+i}\big) \quad (8)$$

where g_x, g_y and g_u are all evaluated at t_{k+i}. (It is assumed that $\Delta u_{k+N} \equiv 0$.)

The linear time-varying discrete-time model therefore becomes

$$\Delta x_{k+i+1} = A_{k+i}\Delta x_{k+i} + B_{k+i}\Delta u_{k+i} \quad (9)$$

$$\Delta y_{k+i} = C_{k+i}\Delta x_{k+i} + D_{k+i}\Delta u_{k+i} \quad (10)$$

where the definitions of A, B, C and D follow directly from (7) and (8). This formulation relates quite closely to [15], though their starting point was a discrete-time model.

As mentioned earlier, implicit numerical integration allows efficient computation of Φ_x and Φ_u. Furthermore, such integration techniques require the formation of g_x, g_y and g_u, and factorization of g_y. Therefore the model (9),(10) can be compute efficiently, even for large-scale systems such as power systems.

It should be emphasized that the linear model (9),(10) is not a linearization around an equilibrium point, but rather a linearization around a (possibly) large disturbance nonlinear, non-smooth trajectory. During periods of normal power system operation though, when the system is close to equilibrium, the properties of trajectory sensitivities [13] ensure that the model (9),(10) effectively reverts to a time-invariant linearization around the equilibrium point. This model is therefore suited to both small disturbance regulation and large disturbance emergency control.

V. MPC Implementation

The general model of Section IV must be tailored to the specific requirements of non-disruptive load control. Those details are outlined below.

A. Load model

MPC implementation is not limited to any particular load model. It is important though that the effect of load control action is incorporated into the model. For example voltage dependent load could be modelled as

$$P(V, \lambda) = (1 - \lambda)P_0 \left(\frac{V}{V_0}\right)^{\alpha} \quad (11)$$

and similarly for reactive power $Q(V, \lambda)$. No load shedding (full load) corresponds to $\lambda = 0$, while complete load shedding is given by $\lambda = 1$. In fact, if the load were partially served by local distributed generation, it is (theoretically) possible for the bus to become a net exporter of energy, corresponding to $\lambda > 1$. For this load model, the parameter λ is equivalent to the control u in the general model.[1] Emergency control requires periodic adjustment of λ.

The MPC algorithm must take account of the limits on the amount of load that is available for control. Let the maximum amount of load that can be shed at a particular location j be λ^j_{max}. Then at any interval over the MPC prediction horizon,

$$0 \le \lambda^j + \Delta\lambda^j_{k+i} \le \lambda^j_{max} \quad (12)$$

where λ^j is the actual load previously shed, i.e., the load shed at the beginning of the MPC prediction process, and $\Delta\lambda^j_{k+i}$ is a proposed load change at time t_{k+i}. Note that this assumes previously shed load can be restored. If that is not the case, i.e., there is a latching mechanism, then (12) becomes simply $0 \le \Delta\lambda^j_{k+i} \le \lambda^j_{max} - \lambda^j$. It may also be appropriate to limit the load change at any interval, according to $\Delta\lambda_{minchange} \le \Delta\lambda \le \Delta\lambda_{maxchange}$. For example, such

[1] The notational difference is deliberate, in order to differentiate this specific implementation from the more general form.

limits may be necessary to avoid excessive voltage steps. All these limits can be combined together to give

$$\Delta\lambda^j_{min} \leq \Delta\lambda^j_{k+i} \leq \Delta\lambda^j_{max} \tag{13}$$

where $\Delta\lambda^j_{min}$ and $\Delta\lambda^j_{max}$ are the most stringent minimum and maximum limits respectively.

B. Voltage constraints

The aim of the MPC process is to shed just enough load that bus voltages recover to within acceptable voltage bounds $[V_l \;\; V_u]$. Driving voltages to within these bounds terminates the voltage collapse process. This requirement is implemented in the MPC optimization formulation by placing constraints on the voltages at the prediction horizon,

$$V_l \leq V(t_{k+N}) \leq V_u \tag{14}$$

where V is a subset of the algebraic states y.

C. MPC optimization

The MPC optimization process seeks to determine minimal load changes $\Delta\lambda^j$ that ensure voltage constraints (14) are satisfied. This results in a nonlinear, constrained, dynamic embedded optimization problem. An iterative process is required to solve such problems, with each iteration involving simulation over the prediction horizon. However by using the (approximate) model of Section IV, the solution process can be substantially simplified. The voltages at the prediction horizon, required to ensure (14) is satisfied, become simply

$$V(t_{k+N}) \approx V_{pred}(t_{k+N}) + \Delta V_{k+N} \tag{15}$$

where V_{pred} describes the voltages predicted by simulation of the nominal trajectory, and from (10),

$$\Delta V_{k+N} = C_{k+N}\Delta x_{k+N}. \tag{16}$$

The errors in this approximation will result in (slightly) sub-optimal load controls $\Delta\lambda$ being applied by MPC. However the effects of that sub-optimality will only persist until the subsequent MPC cycle. At that time, the whole optimization process is repeated.

The optimization objective is to shed the minimal amount of load, $\min_{\Delta\lambda}\sum_j \left|\lambda^j + \Delta\lambda^j\right|$. By observing (12), this can be restated

$$\min_{\Delta\lambda}\sum_j \left(\lambda^j + \Delta\lambda^j\right). \tag{17}$$

For each MPC optimization, the amount of load previously shed at each bus λ^j is a constant. Therefore the objective function can be further simplified. Collecting together the objective and constraints gives the linear programming (LP) problem

$$\min_{\Delta\lambda}\sum_j \Delta\lambda^j \tag{18}$$

subject to the linear model (9),(10) and inequality constraints

$$\Delta\lambda_{min} \leq \Delta\lambda_{k+i} \leq \Delta\lambda_{max}, \quad 0 \leq i < N \tag{19}$$

$$V_l \leq V_{pred}(t_{k+N}) + \Delta V_{k+N} \leq V_u. \tag{20}$$

Such problems can be solved efficiently, even for very large sets of equations.

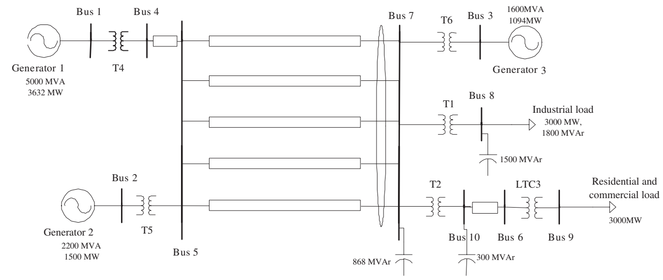

Fig. 4. Voltage collapse test system.

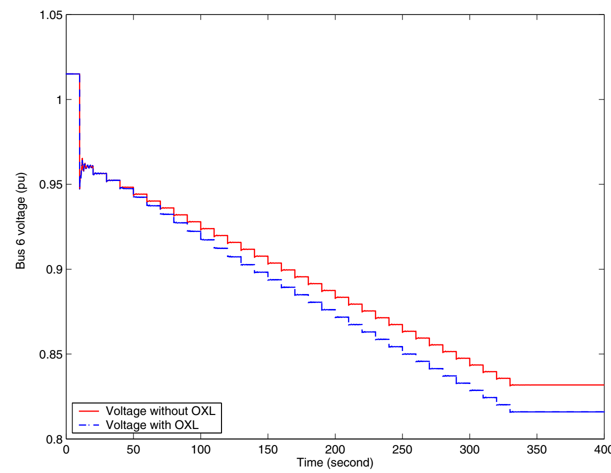

Fig. 5. Voltage behaviour without MPC.

VI. Example

The small system of Fig. 4 is well established as a benchmark for exploring voltage stability issues [2], [16], [17]. An outage of any one of the feeders between buses 5 and 7 results in voltage collapse behaviour. This is illustrated in Fig. 5 for a line outage at 10 seconds. In response to the line trip, voltages across the right-hand network dropped. This caused load tap changers (LTCs) to respond in an attempt to restore load bus voltages. However tap changing actually drove voltages lower, resulting in voltage collapse.

Two situations were considered, 1) no over-excitation limiter (OXL) on generator 3 (solid red curve), and 2) inclusion of an OXL on generator 3 (dashed blue line.) Both exhibit undesirable voltage behaviour, though the OXL clearly induced a more onerous response. The reactive support provided by generator 3, for the two cases, is shown in Fig. 6. The OXL ensures that reactive demand does not rise to a damaging level.

The studies presented subsequently explore the MPC model detail required to achieve adequate control. To enable this comparison, the system was modelled precisely. A sixth order model (two axes, with two windings on each axis) [18] was used for each generator, and IEEE standard models `AC4A` and `PSS1A` for all AVRs and PSSs respectively. The OXL model was taken from [17]. A standard induction motor model [17] was used for the industrial load at bus 8, and a static voltage dependent representation for the bus 9 load.

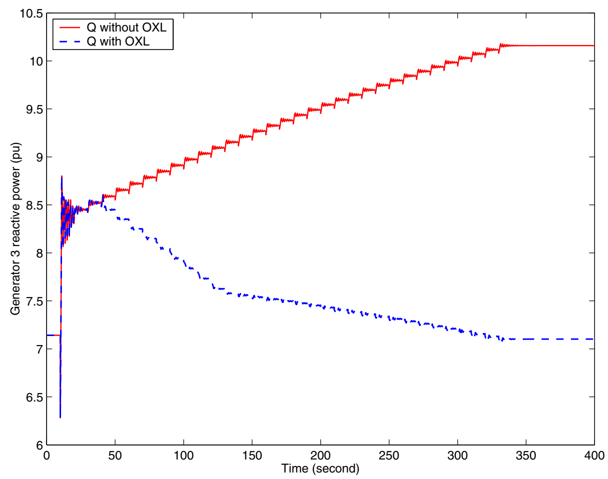

Fig. 6. Reactive support provided by generator 3, without MPC.

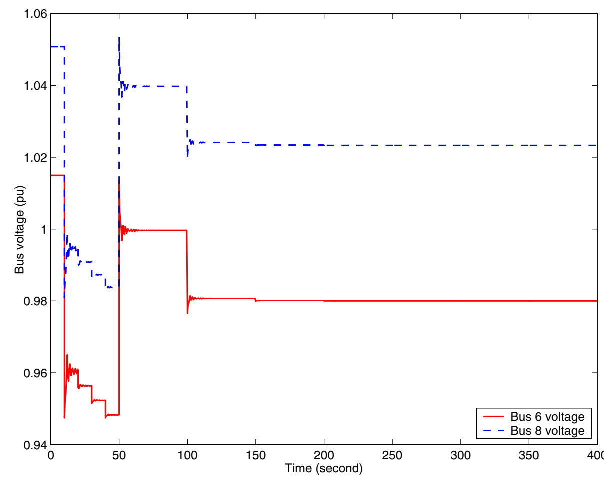

Fig. 7. Voltage behaviour, perfect MPC model.

The AVR of transformer LTC3 was represented by a model that captured switching events associated with deadbands and timers [11].

In all cases MPC was set to run every $T = 50$ seconds, with an horizon time of $2T = 100$ seconds. The control objective was to restore the voltages of buses 6 and 8 above 0.98pu by shedding minimum load at buses 8 and 9. (These two sets of buses were chosen to avoid symmetry between load-shed buses and voltage-regulated buses.) This objective was achieved by solving the LP optimization problem (18)-(20) for the corresponding values of $\Delta\lambda$ at each MPC step.

A. Perfect MPC model

This initial investigation considered the ideal (though unrealistic) situation where the internal MPC model exactly matched the real system. The voltages at the regulated buses are shown in Fig. 7. It is apparent that in response to the initial MPC load control command, both voltages rose above their specified minimum values. The initial MPC command therefore over-compensated for the collapsing voltages by shedding too much load. This was a consequence of approximating perturbed trajectories in (20) using trajectory sensitivities. The voltage overshoot was corrected with the second MPC control command though, with the bus 6 voltage falling to its lower limit of 0.98 pu. At this step all of the bus 9 load was actually restored; see Fig. 8 for the load shedding commands. Note that negligible MPC action is required beyond the second control interval.

B. Imprecise load response

The nature of non-disruptive load control means there will always be some uncertainty in the amount of load that is actually available for control. To investigate this situation, the load control signals generated by MPC were randomly perturbed by up to $\pm 10\%$. Fig. 9 shows that performance was only slightly degraded.

C. Realistic implementation

It is unrealistic to expect that the MPC controller could maintain a complete, accurate system representation. To

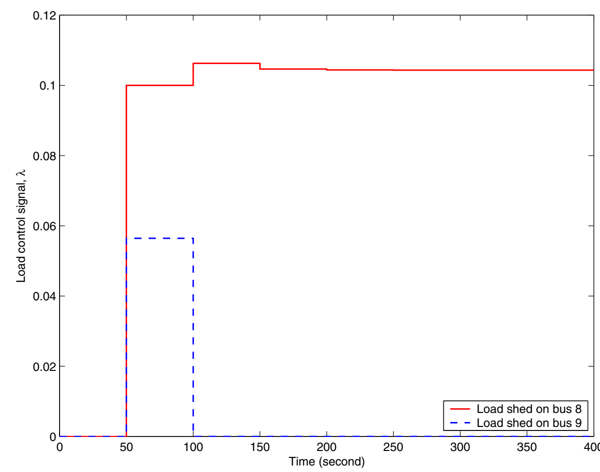

Fig. 8. Load control signals, perfect MPC model.

investigate this case, the MPC internal model was altered to make use of a simplified generator representation. Also the OXL was removed from the MPC model. Furthermore, load uncertainty was incorporated, as in Section VI-B. Voltage response and load control signals are shown in Figs. 10 and 11 respectively. It is apparent that model approximation did not adversely affect the quality of MPC regulation.

These results are encouraging, though certainly not definitive. The degree to which MPC can tolerate model inaccuracy is core to practical power system implementation. This is the focus of on-going research.

VII. Conclusions

Many consumer installations include components that can be tripped with negligible short-term effects. Consolidation of such load fragments provides a non-disruptive load control capability that can be used to alleviate voltage collapse. The paper explores a hierarchical control structure which consists of a lower level controller (consolidator) that communicates with loads, together with a higher level controller that formulates coordinated responses to threats of voltage instability.

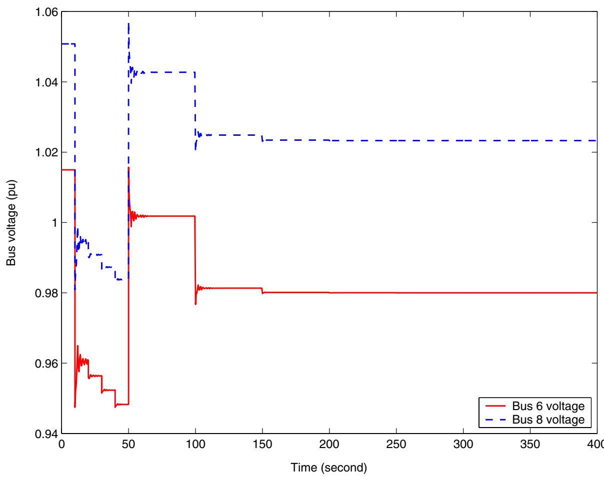

Fig. 9. Voltage behaviour, imprecise load response.

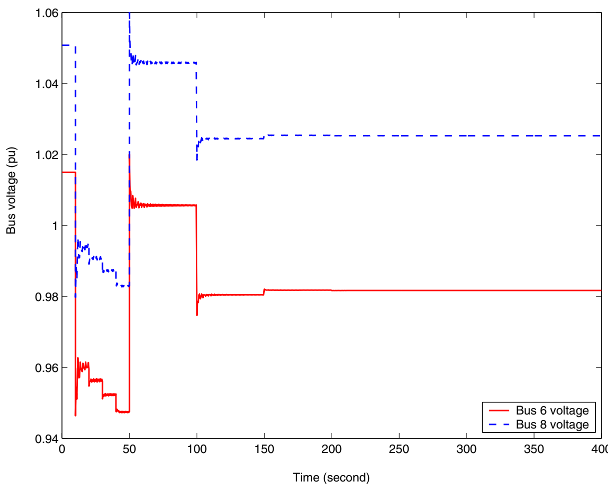

Fig. 10. Voltage behaviour, approximate MPC model.

It has been shown in the paper that model predictive control (MPC) provides a very effective higher-level control strategy.

MPC utilizes an internal model of the system to predict response to a disturbance. A dynamic embedded optimization problem is formulated to determine the minimum load shedding required to restore voltages to acceptable levels. It is shown that the use of trajectory sensitivities allows this optimization to be reduced to a linear programming problem, even though the system exhibits hybrid dynamics. This simplification, together with MPC model approximations, gives rise to discrepancies between predicted and actual system behaviour. Hence the derived controls are slightly sub-optimal. However errors are corrected by subsequent repetition of the MPC prediction/optimization algorithm. The MPC control strategy is therefore practical for large-scale power system applications.

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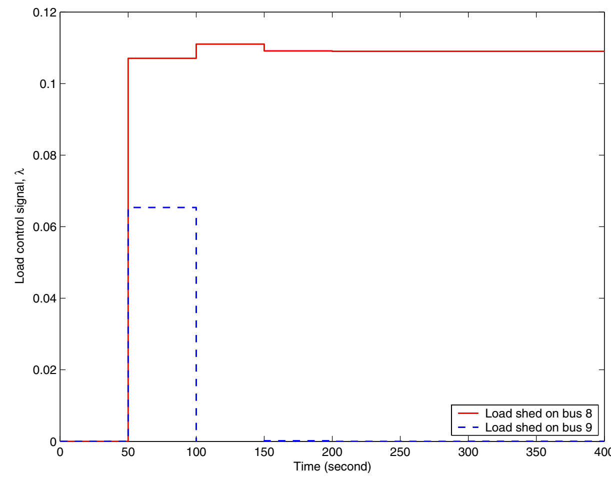

Fig. 11. Load control signals, approximate MPC model.

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