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Robust Design of Networked Control Systems with Randomly Varying Delays and Packet Losses

Babak Tavassoli
Control and Intelligent Processing Center of Excellence
Department of Electrical and Computer Engineering
University of Tehran
P.O. BOX 14395-515, Tehran, Iran
tavassoli@ece.ut.ac.ir

Parviz Jabehdar Maralani
Control and Intelligent Processing Center of Excellence
Department of Electrical and Computer Engineering
University of Tehran
P.O. BOX 14395-515, Tehran, Iran
pjabedar@ut.ac.ir

Abstract— **A relatively general framework for designing robust controllers over communication networks is presented. It has the capability of handling randomly time-varying packet transmission delays and probability of packet losses. The presented method can also handle multiple channels of communication simultaneously. Results are based on continuous time methods that use Lyapunov-Krasovskii functionals. It is shown via an example that how state feedback controllers can be designed based on LMIs. Fixed structure output feedback controllers have been designed based on BMIs.**

I. Introduction

Designing control systems for operation over communication networks or networked control systems (NCSs) has been an interesting and very active field of research during the last decade. The subject has been evolved both in its forms of problem statements and also in problem modeling and solution methods. This evolution was highly influenced by the advances in networking technologies together with the new applications of real-time control systems over networks.

Currently NCS research is divided to several fields based on (communication) network types and applications. Some networks have been designed for control applications. For example Foundation™ Fieldbus, PROFIBUS and ControlNet are based on token passing network technologies and use scheduled communication, on the other hand CAN and Industrial Ethernet use CSMA or carrier sensing techniques which result in randomness in communication characteristics [1]. While analysis and design of control systems over scheduled communication networks usually can be performed using available discrete time control theory, the randomness in quantities like transmission delay or presence of data packet losses in some network types are very difficult problems to be dealt with. A solution is application of switched (hybrid) systems theory for modeling loss and varying delays [2,3]. Marcovian jump systems theory provides an alternative solution also [4]. These techniques require to assume that the behavior of NCS is described by a finite set of state space models (each describing a delay value or the case of data loss) where the computational complexity of controller design will be proportional to the number of elements in this set. This is indeed a shortcoming of the mentioned works. In addition to high computational load related to number of models, only a finite set of delay values can be treated. Therefore, these are only approximate methods also.

In this paper based on the recent results of applying Lyapunov-Krasovskii functionals to the problem of controlling time-delay systems [5,6], we will introduce a simple new LMI / BMI based (linear matrix inequality / bilinear matrix inequality) robust control design method where characteristics of delay parameter or loss does not affect the problem complexity. Another interesting property of this work is allowing multiple channels of communication with independent delay parameters to be analyzed simultaneously. This property helps much in the analysis and design of distributed control systems over several different networks or communication channels.

Another advantage of the proposed method is its ability to handle out of order arrival of data packets carrying sampled signal values. For example, a packet containing a signal's value at sampling k arrives after the packet containing the value at sampling k+n. Many other previous methods cannot handle this case because handling delay values larger than sampling period is generally a difficult problem [7].

The organization of the paper is as follows, in section II the problem will be formulated and some basic results will be obtained. In section III a Lyapunov-Krasovskii functional will be introduced for the analysis of the system and the main results will be presented. In section IV some examples will be studied. Conclusions will be made at the end.

II. Problem Statement and Preliminaries

A. Problem Statement

The system under consideration is a linear model with q+1 input and q+1 output channels as below:

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} B_i u_i(t) + Bw(t) \quad \text{(1-a)}$$

$$y_j(t) = C_j x(t) \quad (j = 1,...,q) \quad \text{(1-b)}$$

$$z(t) = C\,x(t) \quad \text{(1-c)}$$

Where $x \in R^n$ is the state vector, $u_i \in R^{m_i}$ is the ith input channel vector, $y_i \in R^{r_i}$ is the ith measurement output channel vector, $w \in R^{m_d}$ is an input disturbance, , $z \in R^{r_d}$ is a controlled output and $A_0 \in R^{n\times n}$, $B_i \in R^{n\times m_i}$, $B \in R^{n\times m_d}$, $C_j \in R^{r_j\times n}$, $C \in R^{r_d\times n}$ are constant matrices.

Matrix A_0 can be the result of augmenting several systems. The q input/output pairs are to be connected over q communication networks. The outputs y_i are sampled with different but fixed sampling periods h_i . The sampled value is then transmitted over the network(s), multiplied by constant gain K_i and applied to input u_i so that we can write:

$$u_i(t) = K_i y(t-\tau_i(t)) \quad \Rightarrow \tag{2-a}$$

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t-\tau_i(t)) + Bw(t) \tag{2-b}$$

$$z(t) = C\,x(t)$$

$$A_i = B_i K_i C_i \tag{2-c}$$

The varying parameters $\tau_i(t)$ (i = 1 … q) are results of communication delay and packet losses. If the connections between input/output pairs are established through dynamical systems instead of a constant gains K_i in (2-a), the overall system can still be modeled by equations in the form of (2) provided that system matrices A_0 , B_i , C_i , B , C and K_i are appropriately modified.

It should be noted that some of the calculations in the remainder of paper have been summarized due to limitations in the available space.

B. The Delayed Channel

In this work we are not going to derive a discrete time model for the system and all of our analysis will be performed in continuous time framework. The following lemma establishes a relationship between the probability distribution of quantities $\tau_i(t)$ called *channel delay* and the probability distribution for channel *transmission delay* of data packet communication.

Lemma 2.1: Consider a continuous time signal $y \in R^r$ that is sampled with period h at instants $t_i = kh$ where k is an integer (started from -∞). The sampled values are transmitted over a delayed channel where the last received value is held at the channel output such that at the channel output we will have $y_x(t) = y(t - \tau(t))$. If the probability density function of *transmission delay* variable d is denoted by $p_d(d)$ and delays in consecutive transmissions are assumed to be independent random variables, then we will have the following time-dependent probability density function $p_\tau(\tau,\ t)$ for the continuous time *channel delay* variable $\tau(t)$ (or briefly τ) in terms of Dirac function δ:

$$p_\tau(\tau,t) = \sum_{i=-\infty}^{k_u} a_i(t)\delta\big(\tau-(t-ih)\big) \tag{3-a}$$

$k_u = max\{i\}$ such that $t \geq ih$

$$a_i(t) = P_d(t-ih) \prod_{j=i+1}^{k_u}\big(1-P_d(t-jh)\big) \tag{3-b}$$

$$P_d(d) = \int_0^d p_d(s)ds \tag{3-c}$$

We can also write

$$P_\tau(\tau,t) = \int_0^\tau p_\tau(s,t)ds = \sum_{i=k_l}^{k_u} a_i(t) \tag{3-d}$$

$k_l = max\{i\}$ such that $t - \tau \leq ih$

proof: the proof is straight forward considering the fact that the probability of latest received value at channel output being the sampled value at instant ih, is equal to a_i in (3-b). □

Remark 2.1: It is noticed that out of order arrival of packets carrying the consecutive sampled values do not affect the result in the above lemma.

Now we are going to obtain an upper bound for the expected value of delay τ after time t defined as below to eliminate its dependence on time.

$$E_\tau(t) = \int_0^\infty s\, p_\tau(s,t+s)ds \tag{4}$$

Changing the integration variable and integrating by parts it can be shown that:

$$E_\tau(t) = \int_t^\infty \big(1-P_\tau(s-t,s)\big)ds \tag{5}$$

$P_\tau(s\text{-}t,s)$ is the probability that at least one of packets sent after time t is being received at time s. It is clear that this value has an increasing behavior with respect to s so it is greater than $P_\tau(jh\text{-}t,jh)$ where $j = min\{k\}$ such that $jh>s$. This fact will be used for discretization of the integral in (5). Using this idea, (3-d), probability normalization property of (3-a) and denoting the largest integer smaller than t/h by k_t we can write:

$$E_\tau(t) = \int_t^\infty \left(1-\sum_{t\leq ih\leq s} a_i(s)\right)ds = \int_t^\infty \sum_{ih<t} a_i(s)\,ds \quad \Rightarrow$$

$$E_\tau(t) \leq h\sum_{m=k_t}^{\infty}\ \sum_{ih<t} a_i(mh) = h\sum_{m=k_t}^{\infty}\ \sum_{i=-\infty}^{k_t} a_i(mh) \quad \Rightarrow$$

$$E_\tau(t) \leq h\sum_{m=k_t}^{\infty}\ \sum_{i=-\infty}^{k_t} P_d(mh-ih) \prod_{j=i+1}^{m}\big(1-P_d(mh-jh)\big)$$

It can be easily shown that the last expression is in fact independent of k_t , changing roles of variables as $i \rightarrow i\text{-}k_t$, $j \rightarrow j\text{-}k_t$, $m \rightarrow m\text{-}k_t$ we have:

$$E_\tau(t) \leq h\sum_{m=0}^{\infty}\ \sum_{i=-\infty}^{0} P_d(mh-ih) \prod_{j=i+1}^{m}\big(1-P_d(mh-jh)\big) \tag{6}$$

The above calculations can be summarized as the following lemma:

Lemma 2.2: The time varying expected delay value of the channel delay described in Lemma 2.1 which is defined by (4) or alternatively (5) is bounded by the time invariant expression in (6) which depends only on the probability distribution of channel packet *transmission delay* and sampling period.

Remark 2.2: For modeling a channel with a certain packet loss probability p_{loss} we can consider a transmission delay probability density function as $p_d(d) = f_d(d) + p_{\text{loss}}\delta(d-d_1)$ where d_1 tends toward $+\infty$ and δ is the Dirac function. This kind of functions can be easily handled in the framework of this section because we only need P_d in (3-c) that will have only a step discontinuity at infinity.

Remark 2.3: dependence of upper bound in (6) on sampling period h is a result of discretization of the integral in (5) and the inequality in (6) which gets closer to equality if h becomes smaller.

III. Main Results

Based on the results of [5], the following descriptor form of (2-b) is helpful for constructing Lyapunov-Krasovskii functionals for its study. This form is derived using Newton-Leibniz formula.

$$\begin{aligned}&\dot{x}(t) = y(t)\\&0 = -y(t) + \sum_{i=0}^{q} A_i x(t) - \sum_{i=1}^{q} A_i \int_{t-\tau_i}^{t} y(s)ds + Bw(t)\end{aligned} \quad (7)$$

A. The Lyapunov-Krasovskii functional

The following Lyapunov-Krasovskii functional will be utilized in this paper:

$$\begin{aligned}&V(t) = V_1(t) + V_2(t)\\&V_1(t) = \bar{x}^T(t) E P \bar{x}(t)\\&V_2(t) = \sum_{i=1}^{q} \int_{-\infty}^{t} f_i(t,s) y^T(s) R_i y(s) ds\end{aligned} \quad (8)$$

where the functions $f_i(t,s)$ will be determined later and

$$\bar{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad E = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, P_1 = P_1^T$$

Decreasing behavior of V together with its positive definiteness can guarantee stability of (2-b). It may seem that time dependency of functions $f_i(t,s)$ can disturb the common proof of stability using Lyapunov-Krasovskii functionals if for example, $f_i(t,s)$ decreases with time. However this is not true. Two independent reasons are as the following: First, the V_1 part of V in (8) is still independent of time. Second, the functions $f_i(t,s)$ will have periodic and thus non-vanishing behavior in steady state. A detailed proof is eliminated due to limited space. In the remainder of paper the dependence of variables on time t may be omitted for the sake of brevity.

The derivative of the V can be calculated as below. The calculation for V_1 part is similar to [5,6].

$$\dot{V}_1(t) = \bar{x}^T \Psi_1 \bar{x} - \mu_i - \mu_i^T + \theta \quad (10\text{-a})$$

$$\mu_i = \sum_{i=1}^{q} \int_{t-\tau_i(t)}^{t} y^T(s)ds \begin{bmatrix} A_i^T P_2 & A_i^T P_3 \end{bmatrix} \bar{x} \quad (10\text{-b})$$

$$\theta = \bar{x}^T P^T \begin{bmatrix} 0 \\ B \end{bmatrix} w + w^T \begin{bmatrix} 0 & B^T \end{bmatrix} P \bar{x} \quad (10\text{-c})$$

$$\Psi_1 = \begin{bmatrix} P_2^T \sum_{i=0}^{q} A_i + \sum_{i=0}^{q} A_i^T P_2 & P_1 - P_2^T + \sum_{i=0}^{q} A_i^T P_3 \\ P_1 - P_2 + P_3^T \sum_{i=0}^{q} A_i & -P_3^T - P_3 \end{bmatrix} \quad (10\text{-d})$$

$$\dot{V}_2(t) = \sum_{i=1}^{q} \left(f_i(t,t) y^T(t) R_i y(t) + \nu_i \right) \quad (11\text{-a})$$

$$\nu_i = \int_{-\infty}^{t} \frac{\partial}{\partial t} f_i(t,s) y^T(s) R_i y(s) ds \quad (11\text{-b})$$

Due to the dependency of μ_i in (10-b) on random variable $\tau_i(t)$, we need to calculate the expected value of $\dot{V}$ or actually μ_i. Integrating by parts we can write:

$$\begin{aligned}\underset{\tau_i}{E}\{\mu_i\} &= \underset{\tau_i}{E}\left\{ \sum_{i=1}^{q} \int_{t-\tau_i(t)}^{t} y^T(s)ds \begin{bmatrix} A_i^T P_2 & A_i^T P_3 \end{bmatrix} \bar{x} \right\}\\ &= \sum_{i=1}^{q} \left(\int_{0}^{\infty} p_{\tau_i}(\tau_i, t) \int_{t-\tau_i}^{t} y^T(s) ds \, d\tau_i \right) \begin{bmatrix} A_i^T P_2 & A_i^T P_3 \end{bmatrix} \bar{x}\\ &= \sum_{i=1}^{q} \left(\int_{-\infty}^{t} (1 - P_{\tau_i}(t-s, t)) y^T(s) ds \right) \begin{bmatrix} A_i^T P_2 & A_i^T P_3 \end{bmatrix} \bar{x}\end{aligned} \quad (12)$$

The following relations are both the definition of functions g_i and determination of functions f_i as free parameters of V_2:

$$g_i(t,s) = -\frac{\partial}{\partial t} f_i(t,s) = 1 - P_{\tau_i}(t-s, t) \quad \Rightarrow \quad (13\text{-a})$$

$$f_i(t,s) = \int_{t}^{\infty} \left(1 - P_{\tau_i}(u-s, u)\right) du \quad (13\text{-b})$$

Using (10) and (11) we can write:

$$E\{\dot{V}(t)\} = \bar{x}^T\Psi_1\bar{x} + \theta + \sum_{i=1}^{q}\left(f_i(t,t)y^T(t)R_i y(t) - \underset{\tau_i}{E}\{\mu_i\} - \underset{\tau_i}{E}\{\mu_i^T\} + \nu_i\right) \quad (14)$$

At this step we are going to replace $-E\{\mu_i\}$-$E\{\mu_i^T\}+\nu_i$ in (14) with an upper bound based on some results in [8] that is

$$-2aNb \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad (15\text{-a})$$

$$\text{where} \quad \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0 \quad (15\text{-b})$$

Based on (11-b), (12) and (13) and by setting $a=y(s)$, $b=\bar{x}(t)$, $N=Y=\begin{bmatrix} A_i^T P_2 & A_i^T P_3 \end{bmatrix}$, $X=R_i$, $Z=Z_i$ and multiplying the above relation by $g_i(t,s)$ and integrating the result over $s\in(-\infty,t]$ we will have the following:

$$-\underset{\tau_i}{E}\{\mu_i\} - \underset{\tau_i}{E}\{\mu_i^T\} + \nu_i \le \int_{-\infty}^{t} g_i(t,s)ds\, \bar{x}^T Z_i \bar{x} = f_i(t,t)\bar{x}^T Z_i \bar{x}$$

Therefore assuming that (15-b) holds we have

$$E\{\dot{V}(t)\} = \bar{x}^T\Psi_1\bar{x} + \sum_{i=1}^{q} f_i(t,t)\left(y^T R_i y + \bar{x}^T Z_i \bar{x}\right) + \theta$$

Comparing (13-b) with (5), $f_i(t,t)$ is exactly the expected value of channel delay which is bounded due to Lemma 2.2 by constant values that we will denote by α_i . The above relation can then be rewritten as:

$$E\{\dot{V}(t)\} \le \bar{x}^T\Psi_1\bar{x} + \sum_{i=1}^{q} \alpha_i\left(y^T R_i y + \bar{x}^T Z_i \bar{x}\right) + \theta \quad (16)$$

Where θ is defined in (10-c).

B. The Stability and Robustness Results

The main result of the paper can now be stated as the following theorem:

Theorem 3.1: The system in (2-b) rewritten in (17) is asymptotically stable and H_∞ norm from $w(t)$ to $z(t)$ is less than γ if matrices $P_1^T=P_1>0$, P_2, P_3, R_i, Z_{1i}, Z_{2i}, Z_{3i}, (i=1…q) exist such that the set of LMIs in (18) hold. α_i s in (18-a) are calculated from (18-c) where P_{di} is cumulative distribution function of ith channel's packet *transmission delay* defined in (3-c) and h_i is the sampling period at ith channel.

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t-\tau_i(t)) + Bw(t) \quad (17\text{-a})$$

$$A_i = B_i K_i C_i \quad (17\text{-b})$$

$$z(t) = C\,x(t) \quad (17\text{-c})$$

$$\Psi_{21} = \begin{bmatrix} P_2^T \sum_{i=0}^{q} A_i + \sum_{i=0}^{q} A_i^T P_2 & P_1 - P_2^T + \sum_{i=0}^{q} A_i^T P_3 & P_2^T B \\ P_1 - P_2 + P_3^T \sum_{i=0}^{q} A_i & -P_3^T - P_3 & P_3^T B \\ B^T P_2 & B^T P_3 & -\gamma^2 I \end{bmatrix}$$

$$\Psi_{22} = \begin{bmatrix} C^T C + \sum_{i=1}^{q}\alpha_i Z_{1_i} & \sum_{i=1}^{q}\alpha_i Z_{2_i} & 0 \\ \sum_{i=1}^{q}\alpha_i Z_{1_i}^T & \sum_{i=1}^{q}\alpha_i (Z_{3_i} + R_i) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Psi_2 = \Psi_{21} + \Psi_{22} > 0 \quad (18\text{-a})$$

$$\begin{bmatrix} R_i & A_i^T P_2 & A_i^T P_3 \\ P_2^T A_i & Z_{1_i} & Z_{2_i} \\ P_3^T A_i & Z_{2_i}^T & Z_{3_i} \end{bmatrix} > 0 \quad (18\text{-b})$$

$$\alpha_i = h_i \sum_{m=0}^{\infty} \sum_{i=-\infty}^{0} P_{d_i}(mh - ih) \prod_{j=i+1}^{m}\left(1 - P_{d_i}(mh - jh)\right) \quad (18\text{-c})$$

Proof: Assuming that (18) holds, the sub-matrix of Ψ_2 composed of first two rows and first two columns will be also negative definite. Removing positive term C^TC from top left element of this sub-matrix, the resulting matrix indicated by Ψ_{11} will be also negative definite. If w=0 then θ=0 and right-hand side of (16) can be written as $\bar{x}^T\Psi_{11}\bar{x}$ which means that expectation of time derivative of functional V is always negative if (18-b) holds, which is in fact (15-b). On the other hand (18-b) implies that $R_i>0$. From (13-b) we have also $f_i(t,s)>0$. These two inequalities together with $P_1>0$ imply that V in (8) is a positive definite functional which proves the asymptotic stability of (17).

For studying robustness, the performance index (19) will be utilized.

$$J = \int_{0}^{\infty}\left(z^T z - \gamma^2 w^T w\right)dt \quad (19)$$

H_∞ norm from w to z will be less than γ if J is guaranteed to be negative for L_2-norm bounded w signals and zero initial condition. These limitations imply that $V(0)=V(\infty)=0$ based on stability of system. So for negativeness of J we require the following expression to be negative:

$$\int_{0}^{\infty}\left(z^T z - \gamma^2 w^T w\right)dt + E\{V(\infty) - V(0)\} \quad =$$

$$\int_0^\infty \left(z^T z - \gamma^2 w^T w + E\{\dot{V}\}\right) dt \le \int_0^\infty \begin{bmatrix} \bar{x} \\ w \end{bmatrix}^T \Psi_2 \begin{bmatrix} \bar{x} \\ w \end{bmatrix} dt$$

According to last expression above, $\Psi_2<0$ implies $J<0$ and ensures the H_∞ norm bound. Note that (18-c) also follows from (6) as discussed in derivation of (16).

□

In the case of a single delay channel, theorem 3.1 can be used for LMI based delayed state feedback design for (17) in a manner similar to that in [6]. In this case we have $q=1$ and $C_1=I$. This result is summarized in the following corollary. The proof is omitted here. It includes changes of variables (e.g. $Q=P^{-1}$), application of Schur complements and a system augmentation. The reader is referred to [6].

Corollary 3.1: Considering the system (20), a stabilizing state feedback over a delayed channel that keeps the H_∞ norm from w to z below γ can be designed if there exit $Q_1^T=Q_1>0$, Q_2, Q_3, Y, R, Z_1, Z_2, Z_3 such that the set of LMIs in (21) hold. α_1 is obtained from (18-c). The feedback gain K_1 is obtained from (22).

$$\dot{x}(t) = A_0 x(t) + B_1 K_1 x(t-\tau_1(t)) + Bw(t) \qquad \text{(20-a)}$$

$$z(t) = C\,x(t) \qquad \text{(20-b)}$$

$$\Psi_3 = \alpha_1 \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{bmatrix} + \begin{bmatrix} Q_2^T + Q_2 & Q_3 - Q_2^T + Q_1 \bar{A}^T + Y^T \bar{B}^T \\ Q_3^T - Q_2 + \bar{A} Q_1 + \bar{B} Y & -Q_3^T - Q_3 + \gamma^{-2} \bar{B}_1 \bar{B}_1^T \end{bmatrix}$$

$$\begin{bmatrix} \Psi_3 & & \alpha_1 Q_2^T & Q_1 \bar{C}^T \\ & & \alpha_1 Q_3^T & 0 \\ \alpha_1 Q_2 & \alpha_1 Q_3 & -\alpha_1 R & 0 \\ \bar{C} Q_1 & 0 & 0 & -I \end{bmatrix} < 0 \qquad \text{(21-a)}$$

$$\begin{bmatrix} R & 0 & R\bar{A}_1^T \\ 0 & Z_1 & Z_2 \\ \bar{A}_1 R & Z_2^T & Z_3 \end{bmatrix} > 0 \qquad \text{(21-b)}$$

$$K_1 = \left(I - Y Q_1^{-1} \begin{bmatrix} 0 \\ I_{m_1} \end{bmatrix} \right)^{-1} Y Q_1^{-1} \begin{bmatrix} I_n \\ 0 \end{bmatrix} \qquad \text{(22)}$$

$$\bar{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & -\rho I \end{bmatrix} \quad \bar{A}_1 = \begin{bmatrix} 0 & B_1 \\ 0 & 0 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad \bar{B}_1 = \begin{bmatrix} 0 \\ \rho I \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} C & 0 \end{bmatrix} \quad \bar{A} = \bar{A}_0 + \bar{A}_1$$

The above matrices are in fact the result of augmenting a fast system to the input of original system that tends toward identity matrix I when ρ is large (hence in design process ρ should be selected to be a large value).

Remark 3.1: It can be easily shown that if a set of solutions of (18-a) and (18-b) exists for one set of $\alpha_i = \bar{\alpha}_i$, then the same solution set satisfies (18-a) and (18-b) for any other set of α_i where $\alpha_i \le \bar{\alpha}_i$. Although there will be some degree of conservativeness, but for designing a delayed state feedback control over multiple delayed channels that carry different parts of state measurement information, one can consider the value $max\{\alpha_i\}$ as α_1 in Corollary 3.1 for calculating a state feedback gain.

Remark 3.2: In calculation of α_i from (18-c), transmission delay distribution function P_d may contain uncertainty. According to remark 3.1, in such a case, the maximum value of α_i should be calculated over the uncertainty space.

IV. Design Example

In this section we consider the following example plant

$$\dot{x} = \begin{bmatrix} -2 & 0.3 & 0 \\ -0.3 & -1 & 0.2 \\ 0 & -0.2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} w$$

$$y_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \qquad y_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x \qquad z = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x$$

Sensor measurements for y_1 and y_2 are available through communication channels No.1 and No.2 respectively. As an example we assume that channel No.1 has a cumulative delay probability distribution function P_d shown in figure (1) and a packet loss probability of 0.2 (Remark 2.2). Note that the value of loss probability can also be deduced from the final value of P_d in this figure. Sampling period for y_2 will be h_2=0.12 sec.

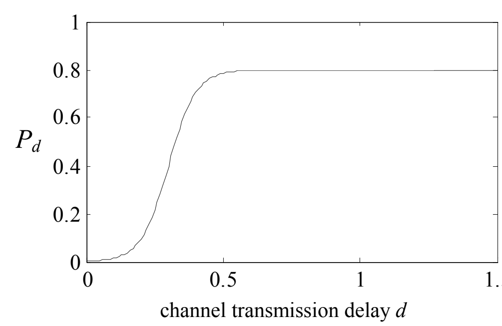

Fig.1 P_d for channel transmission delay of y_1

Applying lemma 2.2 we obtain α_2 = 0.5, we assume that corresponding value for channel No.1 is also α_1 = 0.2 .

The realistic assumption about an NCS is that only partial measurement is available over such a distributed system. The observer based controller design is not very practical in this case both due to its decentralized nature and also its large scale. The static output feedback design is a very general framework that can include many new and traditional types of controllers as its special cases. But this framework usually leads to bilinear matrix inequalities (BMIs) which are difficult to solve compared to LMIs. There are several global and local methods for solving BMIs [9,10]. At this step we will use (18) and (17-b) from theorem 3.1 as a BMI in original variables of the theorem plus K_i to design both proportional controllers and proportional-derivative (PD) controllers for channels No.1 and No.2. The control strategy is shown in figure (2).

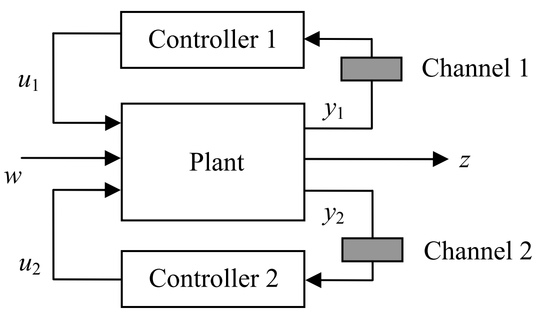

Fig. 2. A control strategy for the example plant

A difficulty in optimization under BMIs is finding a feasible initial condition. Because of stability of the example plant K_i =0 can be initial conditions for optimization in this case.

The results of designing proportional controllers for the above strategy are:

K_1= 0.0445, K_2= -65.2 $\|H_{zw}\|_\infty = 0.722$

Where $\|H_{zw}\|_\infty$ is the *minimized* H_∞ gain from w to z.

Because C_iB_i=0 for both channels a PD controller can be designed for each channel by replacing output matrices C_i with $[C_i^T \ C_i^T A^T]^T$ which can achieve a lower attenuation level. The results are:

K_1 = [102.44 53.28]

K_2 = [115.25 38.77] $\|H_{zw}\|_\infty = 0.51$

Higher order controllers with arbitrary structures can be designed by augmenting integrators to the example plant.

Now assuming that z is also measurable with an expected cannel delay of α_3<0.5, we are going to design a delayed state feedback for the control input u_1 based on LMIs of corollary 3.1 with ρ =1000. Note that we have to replace all α_i values with their maximum 0.5 due to remark 3.1. The results are:

K_1 = [2.8 -17.6 -1.32] $\|H_{zw}\|_\infty = 0.615$

The gain is multiplied by $x=[y_1 \ z \ y_2]^T$.

It worth mentioning that a state feedback design for delay free system has the following results:

K_1 = [90.7 -8267 -59.5] $\|H_{zw}\|_\infty = 0.038$

However, applying theorem 3.1 the stability delay margin for this design is only 0.017 seconds.

V. CONCLUSION

A method is presented for handling distributed linear control systems containing multiple delay channels as a result of system variables being carried through possibly different network connections. The channel characteristics were presented in a very general format so that they can model various network effects like time varying packet transmission delay, packet losses or possibly other problems. The method is based on a special kind of Lyapunov-Krasovskii functionals. Application of other results on these functionals can be also studied in future.

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