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Adaptive Receding Horizon Control of a Distributed Collector Solar Field

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Abstract— **This paper presents an adaptive receding horizon control algorithm for a distributed collector solar field which explicitly explores its distributed parameter character. The plant considered is a distributed collector solar field, being described by a nonlinear hyperbolic partial differential equation (PDE) which models the temperature dynamics. A lumped parameter model is obtained by applying Orthogonal Collocation. This model is then used as a basis for controller design. Stability is ensured for the lumped parameter model by resorting to Control Lyapunov function methods. Simulation results using a detailed physically based simulator of the solar field are provided.**

I. INTRODUCTION

Distributed collector solar fields are the subject of a rich bibliography, which investigates many theoretical and practical aspects concerning their dynamics and control. See [4], [9] for a detailed review and references covering a wide variety of control design methods. The interest for such research topic stems from the economic relevance of renewal energies to modern society as well as from pure scientific interests. Indeed, distributed collector solar fields are representative of an important class of distributed parameter plants in which transport phenomena of mass and energy with no diffusion occurs in one dimension. Therefore, studying this problem sheds light on the control of a class of plants with great technological interest. Furthermore, the availability of such a field for open experiments at Plataforma Solar de Almeria (PSA), located in the south of Spain, stimulated the studies performed by several research groups.

Fig. 1. A view of one collector loop with the pipe in the focus of the concentrating mirrors.

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The ACUREX field of PSA, considered here as a case study, is made of curved mirrors which concentrate direct incident sun light in a pipe located at their focus. Inside this pipe flows an oil able to store thermal energy Fig. 1 shows a view of one collector loop with the pipe in the focus of the concentrating mirrors. In broad terms, the control objective consists in manipulating the oil flow such that the temperature of the oil at the outlet of the pipe tracks a prescribed reference value.

This paper presents a still unexplored approach to the control of distributed collector solar fields. Starting from a partial differential equation (PDE) which represents the dominant plant dynamics, a lumped parameter nonlinear (bi-linear) model is obtained by applying Orthogonal Collocation [11], [8]. This lumped model is then used as a basis to design an adaptive controller based on the use of Control Lyapunov Functions [10], [7], as in [1].

Predictive control based on the receding horizon method is a powerful idea, which has been used with success for nonlinear plants [2]. Nevertheless, as discussed in [6], the literature on adaptive receding horizon control for nonlinear systems is scarce, with [1] providing a significant example.

As for MPC (Model Predictive Control) applications to systems described by hyperbolic PDEs, [14] describes a method in which the distributed model is converted into ODEs using PDE characteristic curves. The resulting ODEs are linearized for prediction. The method is applied to a tubular reactor with no diffusion in which two series reactions occur, the manipulated variable being the fluid velocity. According to the authors, one of the advantages is the reduction of the computational load, but this advantage is lost when the model includes more than one variable.

The paper [12] applies MPC with active constraints to a parabolic PDE model of autoclave composite processing. The model approximation is performed implicitly by the optimization routine of the controller, the manipulated variable being a boundary condition.

The paper [13] applies DMC (Dynamic Matrix Control) to a linearized version of a tubular reactor in which a toluen hydro-dealkylation process takes process. The distributed model is reduced to a lumped parameter model using a projection method (Galerkin's method) in which the basis functions are obtained by eigenfunctions, singular value decomposition and the Karhunen-Loève expansion. In this case, the models are of parabolic type with constant coefficients (opposite to the situation considered here in which the coefficient of the first order space derivative is the manipulated variable).

None of the above papers considers neither parameter adaptation nor state estimation. Instead, paper [15] describes the adaptive control of a tubular bio-reactor of hyperbolic type in which the distributed model is reduced to a system of ODEs by orthogonal collocation (as in here) the control being based in output feedback linearization assuming complete access to the state of the lumped model. In [5], a strategy combining approximated feedback linearization and MPC for the control of the ACUREX field is described, but stability is not ensured.

The contribution of the current paper is twofold: First, it provides an application case study for Receding Horizon Adaptive Control. Second, by using Orthogonal Collocation [11], [8], [15] it applies the methods of [7], [1] to the class of distributed parameters described by hyperbolic PDEs.

The paper is organized as follows: After stating the paper's contributions and relating them to existing literature in this introduction, a reduced complexity solar field model is used in section II, together with Orthogonal Collocation, to obtain a bilinear state space finite dimensional model. The Receding Horizon control law, incorporating a condition which ensures closed loop stability, is obtained in section III and the adaptation law is obtained in section IV. Section V presents simulation results obtained by combining the adaptive receding horizon controller deduced with a detailed physical model. Finally, section VI draws conclusions.

II. A REDUCED COMPLEXITY SOLAR FIELD MODEL

In this section a distributed parameter model of the solar field is presented and reduced to a lumped parameter model by othogonal collocation.

A. Distributed parameter plant model.

In simple terms, the ACUREX distributed collector solar field consists of a pipe located at the focus of parabolical concentrating mirrors. Inside the pipe flows an oil which is to be heated. The manipulated variable is the oil speed (proportional to oil flow) and the aim consists in regulating the outlet oil temperature. The main disturbances are the solar radiation intensity and the inlet oil temperature. See [9] for further details on the plant and the associated control problem.

For the purposes of this paper, the field may be modelled by the following simple hyperbolic PDE:

$$\frac{\partial T(z,t)}{\partial t} + \frac{u(t)}{L}\frac{\partial T(z,t)}{\partial z} = \alpha \; R(t) \qquad (1)$$

Here, *T(z,t)* is the oil temperature at normalized position z measured along the field and at time t, *u* is the oil velocity and *R* is the intensity of solar radiation, assumed to depend only on time t. The parameter L is the pipe length. The actual space coordinate (measured in $[meters]$) is given by zL. The parameter α is assumed to be unknown and it will be called "efficiency"since it is related to mirror efficiency, although it also depends on other factors, such as oil conductivity which is a nonlinear function of temperature. The domains of the variables z and *t*, representing respectively space and time, are:

$$z \in [0,\ 1] \qquad t \in [0,\ +\infty[$$

where L is the length of the pipe. Actually, a detailed model of the field is more complicated [4] and such a type of models has been actually used in this work for the purpose of simulation. The point to be made is that eq. (1) provides a description of the dominant nonlinear dynamics of the plant. As shown in [3], [5], this description together with the adaptation of the parameter α is suitable for controller design.

B. Orthogonal Collocation.

Eq. (1) provides a distributed parameter model of the field and has therefore an infinite dimensional state. According to the approach followed here, before proceeding to controller design, it will be approximated by a lumped parameter model by using the Orthogonal Collocation Method (OCM) [15], [11], [8].

In order to apply the OCM to approximate (1) by a set of ordinary differential equations, it is assumed that the temperature along the pipe $T(z,t)$ is represented by the weighted sum

$$T(z,t) = \sum_{i=0}^{N+1} \varphi_i(z)\, T_i(t) \qquad (2)$$

where the functions $\varphi_i(z)$ are Lagrange interpolation polynomials, orthogonal at the so called interior collocation points z_i for *i*=1,..., *N* and at the boundary collocation points z_0 and z_{N+1}. The polynomials $\varphi_i(z)$ verify thus at the collocation points

$$\varphi_i(z_j) = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \qquad (3)$$

Inserting (2) into eq.(1) results in the ordinary differential equation verified by the time weights $T_i(t)$:

$$\sum_{i=0}^{N+1} \varphi_i(z)\, \frac{dT_i(t)}{dt} = -\frac{u}{L}\sum_{i=0}^{N+1} \frac{d\varphi_i(z)}{dz}\, T_i(t) + \alpha\; R(t) \quad (4)$$

Compute now (4) at each of the collocation points $z = z_j$. Since (3) holds, and individuating the term $i = 0$ corresponding to the boundary conditions, it follows that

$$\frac{dT_j(t)}{dt} = -\frac{u}{L}\left[\sum_{i=1}^{N+1} \frac{d\varphi_i(z_j)}{dz} T_i(t) + \frac{d\varphi_0(z_j)}{dz} T_0(t)\right] + \alpha R(t) \qquad (5)$$

By making $j = 1, \ldots, N+1$, *i. e.* by considering all the collocation points apart from the first, the PDE (1) is therefore approximated by *n*=*N*+1 ordinary differential equations (ODE). The state of this nonlinear ODE system is given by the temperatures at the collocation points.

In matrix form, this lumped parameter model is written

$$\dot{x} = -\frac{u}{L}\;(Ax + Bx_0) + C\;\alpha\; R(t) \qquad (6)$$

where

$$x = \begin{bmatrix} T_1 & T_2 & \cdots & T_{N+1} \end{bmatrix}^T \qquad (7)$$

with $T_i(t) \equiv T(z_i, t)$, the matrices A, B and C are given by:

$$A = \begin{bmatrix} \varphi_1'(z_1) & \varphi_2'(z_1) & \cdots & \varphi_{N+1}'(z_1) \\ \varphi_1'(z_2) & \varphi_2'(z_2) & \cdots & \varphi_{N+1}'(z_2) \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_1'(z_{N+1}) & \varphi_2'(z_{N+1}) & \cdots & \varphi_{N+1}'(z_{N+1}) \end{bmatrix} \quad (8)$$

$$B = \begin{bmatrix} \varphi_0'(z_1) \\ \varphi_0'(z_2) \\ \vdots \\ \varphi_0'(z_{N+1}) \end{bmatrix} \qquad C = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad (9)$$

where

$$\varphi_j'(z_i) \equiv \frac{d\varphi_j(z)}{dz}\,|_{z=z_i} \quad (10)$$

and $T_0(t)$ is the boundary condition.

III. The receding horizon controller.

The receding horizon controller (RHC) is now designed as in [7] such as to regulate the state around an equilibrium point, together with a condition which ensures stability in closed loop. For that sake, and motivated by the approximate feedback linearizing control law of [3], define the control law given by

$$u^* = \frac{\alpha\, R^*\, L}{r^* - x_0} \quad (11)$$

where r^* is the set-point of the outlet oil temperature x_n^*. Let x^* be the corresponding equilibrium state and consider the dynamics of the error $e = x - x^*$. This is obtained by subtracting equation (6) and

$$\dot{x}^* = -\frac{u^*}{L}\ (Ax^* + Bx_0) + C\ \alpha\ R^* \quad (12)$$

The error dynamics is given by

$$\dot{e} = \frac{-A}{L}\ u^*\ e + \frac{-A\ e - A\ x^* - B\ x_0}{L}\ \tilde{u} + C\ \alpha\ \tilde{R} \quad (13)$$

where $\tilde{u} = u - u^*$ and $\tilde{R} = R - R^*$. As shown in the Appendix, for $\tilde{u} = 0$ and $\tilde{R} = 0$ the error dynamics is stable whenever the matrix $\bar{A} = -\frac{A}{L}$ is stable. It is not easy to prove a general result concerning the stability of matrix $\overline{A}$ when generated by the OCM and hence, its eigenvalues must be checked for each application.

Define then the RHC for the error dynamics by

$$\min_u J = \int_t^{t+T} e^T(\tau)Pe(\tau) + \gamma\ \tilde{u}^2(\tau)d\tau \quad (14)$$

subject to

$$\dot{e} = \frac{-A}{L}\ u^*\ e + \frac{-A\ e - A\ x^* - B\ x_0}{L}\ \tilde{u} \quad (15)$$

$$V_0(t+T) \geq V_{rhc}(t+T) \quad (16)$$

in which r^* is given by (11),

$$V_0(T) = e_0^T(T)Pe_0(T) \quad (17)$$

where $e_0 = x - x^*$ and x is obtained with $u = u^* + \tilde{u}$ and

$$V_{rhc}(T) = e^T(T)Pe(T) \quad (18)$$

where P is an arbitrary symmetric positive definite matrix.

The constraint (16) is equivalent to impose to the RHC that, at each iteration, the norm of the error at the end of the optimal sequence is bounded by the same norm resulting from the error when $u = u^*$. The existence of a control law, defined for $u = u^*$, which stabilizes the closed loop, allows to interpret V_0 as a Control Lyapunov Function [10] and is a sufficient condition to ensure Global Asymptotic Stability of the loop closed by the RHC [7].

IV. Adaptation law.

In this section a state estimator and an adaptation law which are asymptotically stable when inserted in the closed loop are obtained.

A. State observer

To (6) associate the state estimator with output error re-injection, where $\hat{x}$ is the estimate of x:

$$\dot{\hat{x}} = -\frac{u}{L}\,(A\hat{x} + Bx_0) + C\hat{\alpha}R(t) + K(t)(y - D\hat{x})$$

$$y = Dx = \begin{bmatrix} 0 & 0 & \cdots & 1 \end{bmatrix} x \quad (19)$$

The error dynamics $e_1 := x - \hat{x}$ verifies:

$$\dot{e_1} = -\frac{u}{L}\ A\ e_1 + C\ (\alpha - \hat{\alpha})\ R(t) - K(t)D\ e_1 \quad (20)$$

with $K(t)$ the observer gain, or, else, by

$$\dot{e_1}(t) = A_e e_1 + C\tilde{\alpha}R(t) \quad (21)$$

where

$$A_e := -\frac{u}{L}A - K(t)D \quad (22)$$

B. Lyapunov adaptation law

Consider the candidate Lyapunov function

$$V_1 = e_1^T Q e_1 + \frac{1}{\gamma}\ \tilde{\alpha}^2 \quad (23)$$

where $\gamma > 0$ is a parameter, Q is a positive definite matrix and the parameter estimation error $\tilde{\alpha}$ is defined as $\tilde{\alpha}(t) := \alpha - \hat{\alpha}(t)$ where $\hat{\alpha}$ is the estimate of α. Its derivative is given by:

$$\dot{V}_1 = e_1^T(A_e^T Q + QA_e)e_1 + 2\tilde{\alpha}(CR(t))^T Qe_1 + \frac{2}{\gamma}\tilde{\alpha}\ \dot{\tilde{\alpha}} \quad (24)$$

Stability holds if

$$-M(t) = (A_e^T Q + QA_e) < 0 \ \ and \ \ \dot{\tilde{\alpha}} = -\rho(CR(t))^T Qe_1$$

from which the following adaptation law follows:

$$\dot{\hat{\alpha}} = \rho(CR(t))^T Qe_1 \quad (25)$$

It is remarked that $M(t) > 0$ is ensured by the following choice of the observer gain:

$$K(t) = \frac{u}{L}K_0 \quad (26)$$

Indeed, since the velocity u is bounded above and below by strictly positive values:

$$u_{max} \geq u \geq u_{min} > 0 \tag{27}$$

this choice of $K(t)$ yields

$$\begin{aligned} & A_e^T Q + Q A_e = \\ &= \frac{|u|}{L} \left\{ (-A - K_0 D)^T Q + Q(-A - K_0 D) \right\} = \\ &= -\frac{|u|}{L} M_0 = -M(t) \end{aligned} \tag{28}$$

with the matrix M_0 selected as the positive definite solution of the Lyapunov equation

$$(-A - K_0 D)^T Q + Q(-A - K_0 D) = -M_0 \tag{29}$$

which exists if the pair (A, D) is observable and choosing K_0 such that $-A - K_0 D$ is stable. This ensures stability because, from (28) and the adaptation law (25):

$$\dot{V}_1 = -e_1^T M(t) e_1 = -\frac{|u|}{L} e_1^T M e_1 \leq -\frac{u_{max}}{L}\, e_1^T M e_1 \leq 0 \tag{30}$$

By La Salle's Invariance Principle, it follows that $\lim_{t \to 0} e_1(t) = 0$. The parameter estimation error $\tilde{\alpha}(t)$ will tend to zero if u satisfies a persistency of excitation condition.

C. RHC computational algorithm

A computational efficient version of the the adaptive RHC is thus given by

$$\min_{u_1,\ldots,u_{N_u}} J = \int_t^{t+T} e^T(\tau) P e(\tau) + \gamma \tilde{u}^2(\tau) d\tau \tag{31}$$

subject to

$$\begin{aligned} \dot{x} &= -\frac{u}{L}\ (Ax + Bx_0) + C\ \hat{\alpha}\ R(t) \\ x(t) &= \hat{x}(t) \\ u(\bar{t}) &= seq\{u_1, \ldots, u_{N_u}\} \\ u_{max} &\geq\ u\ \geq\ u_{min} > 0 \\ V_0(x(t+T)) &\geq\ V_{rhc}(xt+T)) \end{aligned}$$

with V_0 computed by (17) with x and α replaced by their estimates.

The estimate of u^* is given by:

$$\hat{u}^* = \frac{\hat{\alpha}\ R(t)\ L}{r(t) - x_0(t)} \tag{32}$$

Here, $\hat{x}$ and $\hat{\alpha}$ are obtained using the state estimator (19), the adaptation law (25), $u(\bar{t})$ is a sequence of step functions with amplitude u_i $(i = 1, \ldots, N_u)$ and duration $\frac{T}{N_u}$ The variable $\bar{t}$ represents time during the minimization horizon $\bar{t} \in [0, T[$.

Once the minimization result $u(\bar{t})$ is obtained, according to a receding horizon scheme u_1 is applied to the plant at $t + \delta$ and the whole process is restarted, δ being an interval of time which is at least the time needed to compute the solution.

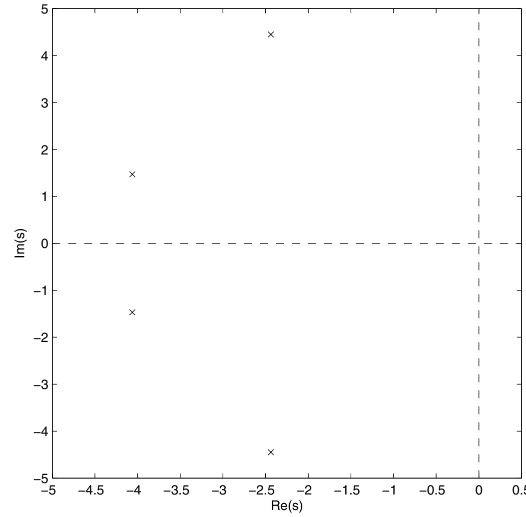

Fig. 2. Eigenvalues of matrix $-A/L$.

If the minimization is not feasible, than one may apply

$$u(t) = u^* \tag{33}$$

which preserves the closed loop stability.

V. Simulation results.

Simulation results of the proposed RHC have been performed in a detailed model of the solar field [4] using experimental sequences for $R(t)$ and $T_0(t)$. The reduced model uses 3 interior collocation points $z = [0.113\ \ 0.500\ \ 0.887]$.

Matrix A is given by

$$A = \begin{bmatrix} 3.87298 & 2.06559 & -1.29099 & 0.67621 \\ -3.22749 & 0.00000 & 3.22749 & -1.50000 \\ 1.29099 & -2.06559 & -3.87298 & 5.32379 \\ -1.87836 & 2.66667 & -14.78831 & 13.00000 \end{bmatrix}$$

and the eigenvalues of $-A/L$ have all negative part (fig 2).

In order to configure the controller. the following parameter choices have been made:

$$\gamma = 1 \times 10^5$$

$$K_0 = [15\ 15\ 15\ 15]^T$$

$$\rho = 1.5 \times 10^{-10}$$

Hereafter, two sets of simulations are reported, with different values of the horizon T and N_u selected accordingly. In the first:

$$T = 60\, s \qquad N_u = 2$$

In the second:

$$T = 180\, s \qquad N_u = 26$$

Figures (3) through (6) show the results for the controller configuration with $T = 60\, s$, while figures (7) through (9) show the results for the configuration with $T = 180\, s$. Apart from the value of N_u (number of "strips"in which the prediction horizon is decomposed for the numerical computation of the control sequence), the major difference between these two controller configurations is the prediction horizon T which is larger in the second case.

As expected, a larger horizon yields smoother input and output signals. Comparing fig. 3 ($T = 60$) with 7 ($T = 180$) it is seen that, by increasing the horizon the output response becomes less oscillatory. This is also true for the manipulated variable (compare fig. 5, $T = 60$, with fig. 8, $T = 180$).

The test planned includes a strong and fast disturbance at time $4.5h$, as seen in the radiation plots of both figs.5 and 8. This disturbance is very well rejected. See fig. 4 for a detail of the response in the case of the horizon $T = 60\,s$.

Comparing the estimates of mirror efficiency α, fig. 6 ($T = 60$) and fig. 9 it is seen that with the larger horizon the convergence is slightly slower, but the reference transitions disturb it less, a consequence of the improvement in control performance.

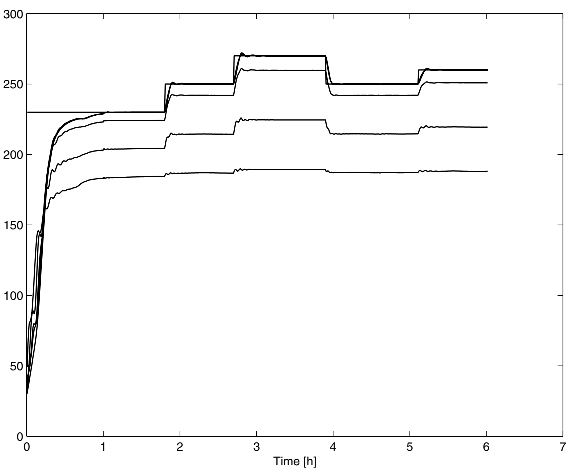

Fig. 3. Closed loop with RHC having $T = 60\,s$. Outlet oil temperature and reference and temperature estimates at the collocation points.

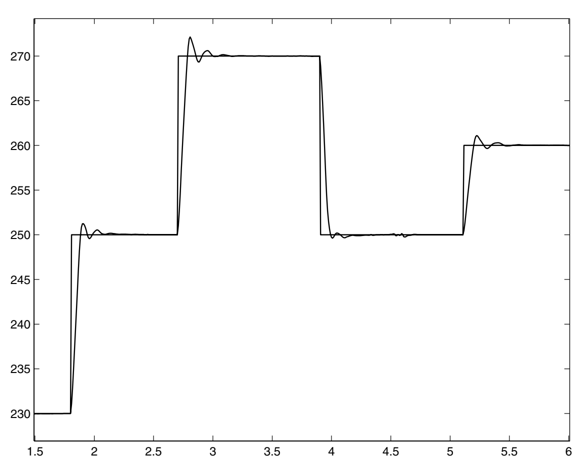

Fig. 4. Closed loop with RHC having $T = 60\,s$. Outlet oil temperature and reference. Detail

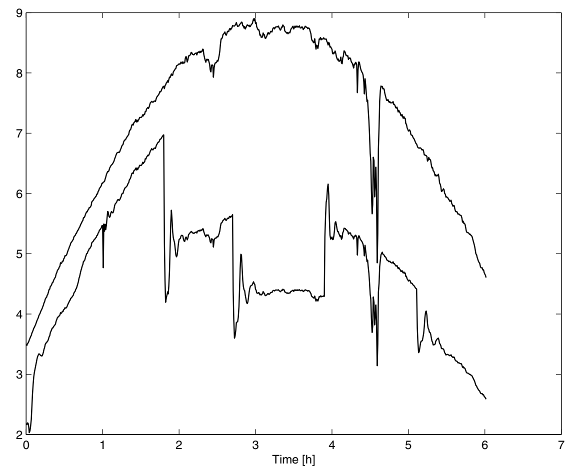

Fig. 5. Closed loop with RHC having $T = 60\,s$. Radiation (disturbance – above) ($\times 1 \times 10^{-2}$) $[W/m^2]$ and oil flow (manipulated variable – below) ($\times 1 \times 10^{4}$) l/s.

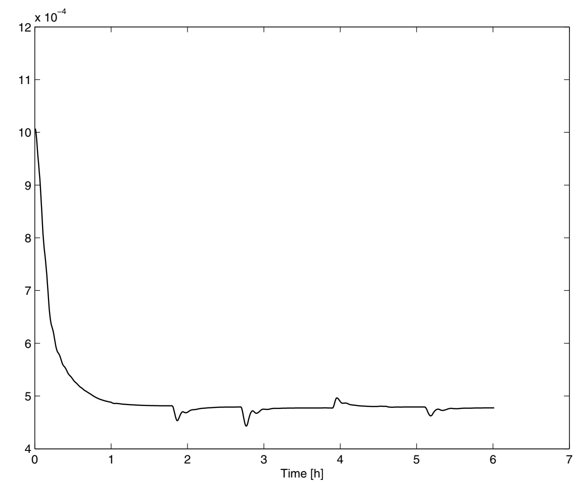

Fig. 6. Closed loop with RHC having $T = 60\,s$. Mirror efficiency estimate, $\hat{\alpha}$.

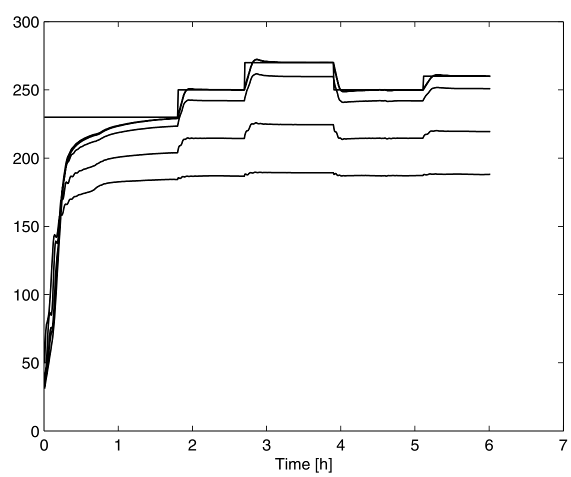

Fig. 7. Closed loop with RHC having $T = 180\,s$. Outlet oil temperature and reference and temperature estimates at the collocation points.

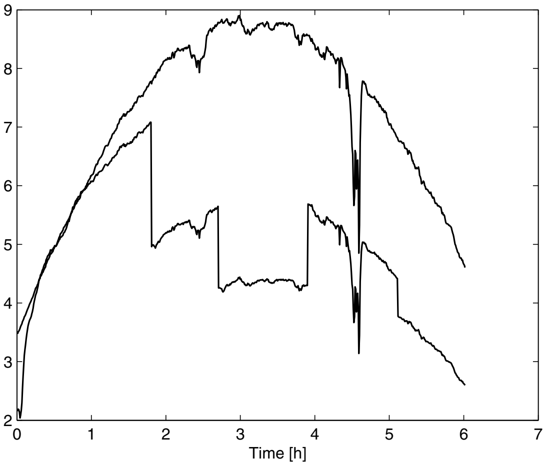

Fig. 8. Closed loop with RHC having $T = 180\,s$. Radiation (disturbance – above) $(\times 1 \times 10^{-2})$ $[W/m^2]$ and oil flow (manipulated variable – below) $(\times 1 \times 10^4)$ l/s.

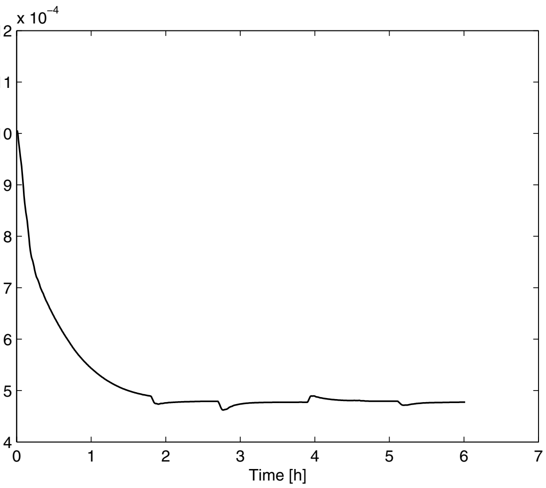

Fig. 9. Closed loop with RHC having $T = 180\,s$. Mirror efficiency estimate, $\hat{\alpha}$.

VI. Discussion and Conclusions.

Adaptive nonlinear receding horizon control of a distributed collector solar field has been address. A distributed parameter model which represents the dominant dynamics of the field is used as a basis for control design. In order to apply receding horizon control, a lumped parameter model is generated by using the Orthogonal Collocation method. This results in a low order finite state bilinear model. For the same level of approximation, finite difference methods would require a much higher order. Receding horizon minimization of a functional with adequate constraints is then performed. It should be mentioned that stability of the closed loop with the RHC is ensured only when the state is available for direct measurement. The approach has been tested in a detailed physical plant model with good results.

The interest of the work reported is twofold: First, it provides an approach which can be used in a class of plants of technological interest. Furthermore, it provides a case study on adaptive nonlinear receding horizon control, extending recent algorithms to distributed parameter plants.

VII. Appendix

Consider the linear time varying equation

$$\dot{e}(t) = \bar{A}u^*(t)e(t) \tag{34}$$

with

$$0 < u_{min} \leq u^*(t) \leq u_{max} \quad \forall\, t \geq 0 \tag{35}$$

and such that $\bar{A} = -A/L$ is a stability matrix with $\bar{\lambda}$ a negative number greater than the largest real part of its eigenvalues. Use the Gronwall-Bellman inequality [16] to conclude that

$$\|e(t)\| \leq \|e(0)\| exp\{\int_0^t \|\bar{A}u^*(\tau)\| d\tau\} \tag{36}$$

Considering (35), this implies

$$\|e(t)\| \leq \|e(0)\| e^{\bar{\lambda} u_{min} t} \tag{37}$$

Since $\bar{\lambda} < 0$, this establishes asymptotic stability of (34).

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