Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Pose Regulation of Robot Manipulators with Dynamic Friction Compensation

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Abstract— **The regulation of end–effector pose of manipulators is addressed in this paper. We present a solution to the operational space pose regulation considering dynamic friction at the robot joints. It is assumed the exact knowledge of the dynamic and kinematic parameters of the robot, including those of the friction. The controller proposed for the pose regulation problem is based on the resolved acceleration control scheme. Experimental results on a two degrees–of–freedom direct–drive arm support the accuracy of the proposed method.**

Index Terms— **Robot pose regulation, Friction observer, Resolved acceleration control, Stability.**

I. INTRODUCTION

Friction is a phenomenon that deteriorates the good motion performance of servomechanisms, such as robots and machine tools. Model–oriented friction compensation techniques are based on the knowledge of suitable friction models that predict the real friction and command an opposed control action to compensate it [1]. Dynamic friction models, such as the Dahl model [2] and LuGre model [3], are able to predict important phenomena that appear at low velocities such as presliding displacement [3]. Thus, using such a friction models, a lot of works concerning friction compensation have been reported in the literature.

On the other hand, pose regulation of robot manipulators has been widely studied. This subject arises from the idea of specifying the motion in the operational space instead of the joint space. The control of manipulators in the operational space considers the robot pose, i.e. the position and orientation of the end–effector, as the output. Roughly speaking, the robot pose regulation problem consists of the robot system reaching, in an asymptotic way, a desired constant pose.

The resolved acceleration controller [4] is an algorithm to solve the pose control of manipulators. This is based on the so–called inverse dynamics technique [5], [6]. Another approach to robot pose regulation without solving the inverse kinematics is by transpose Jacobian–based controllers [7], [8]. Thus, a control problem that requires further consideration is the robot pose regulation without solving the inverse kinematics and using compensation of friction at the robot joints.

The aim of this paper is to present a study on pose regulation of mechanical arms using compensation of friction. Using the resolved acceleration control scheme, and assuming Dahl friction model, we discuss the use of an observer for friction compensation. In the proposed design is demonstrated the asymptotic convergence to zero of the pose error, and exponential convergence of the friction observer.

The paper is organized as follows. Section 2 concerns the robot dynamics and the problem formulation. The results of position regulation and friction compensation are presented in Section 3. Section 4 discusses the use of static friction modeling for compensation. The experimental results are shown in Section 5. Finally, in Section 6 some concluding remarks are drawn.

Throughout this paper the following notation will be adopted. $\lambda_m\{A(\boldsymbol{x})\}$ and $\lambda_M\{A(\boldsymbol{x})\}$ denote the minimum and maximum eigenvalues of a symmetric positive definite matrix $A(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$ for all $\boldsymbol{x} \in \mathbb{R}^n$, respectively. $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ stands for the norm of vector $\boldsymbol{x} \in \mathbb{R}^n$. $\|B(\boldsymbol{x})\| = \sqrt{\lambda_M\{B(\boldsymbol{x})^T B(\boldsymbol{x})\}}$ stands for the induced norm of a matrix $B(\boldsymbol{x}) \in \mathbb{R}^{n\times n}$ for all $\boldsymbol{x} \in \mathbb{R}^n$.

II. POSE REGULATION

The dynamics in joint space of a serial–chain n-link robot manipulator considering the presence of friction at the joints can be written as

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{f}(\boldsymbol{z},\dot{\boldsymbol{q}}) = \boldsymbol{\tau} \qquad (1)$$

where $\boldsymbol{q}$ is the $n \times 1$ vector of joint displacements, $\dot{\boldsymbol{q}}$ is the $n\times 1$ vector of joint velocities, $\boldsymbol{\tau}$ is the $n\times 1$ vector of applied torque inputs, $M(\boldsymbol{q})$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ is the $n \times 1$ vector of centripetal and Coriolis torques, $\boldsymbol{g}(\boldsymbol{q})$ is the $n \times 1$ vector of gravitational torques, and $\boldsymbol{f}(\boldsymbol{z},\dot{\boldsymbol{q}})$ is the $n \times 1$ vector of torques due to the friction which depends of the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^n$ and an unmeasurable internal state $\boldsymbol{z} \in \mathbb{R}^n$ which will be defined below.

One of the simplest dynamic models —inspired on a bristle deflection interpretation— is the Dahl's model [2] which, including viscous friction, can be described by

$$\dot{\boldsymbol{z}} = -\Psi(\dot{\boldsymbol{q}})\boldsymbol{z} + \dot{\boldsymbol{q}}, \qquad (2)$$

$$\boldsymbol{f}(\boldsymbol{z},\dot{\boldsymbol{q}}) = \Sigma_0\boldsymbol{z} + F_v\dot{\boldsymbol{q}}, \qquad (3)$$

where $F_v = \mathrm{diag}\{f_{v_1},\ldots,f_{v_n}\}$ is a diagonal positive definite matrix of viscous friction coefficient of each joint, $\Sigma_0 = \mathrm{diag}\{\sigma_{0_1},\ldots,\sigma_{0_n}\}$ is a diagonal positive definite matrix of "stiffness" parameter of each joint, and

$$\Psi(\dot{\boldsymbol{q}}) = \mathrm{diag}\left\{\frac{\sigma_{0_1}}{f_{C_1}}|\dot{q}_1|,\ldots,\frac{\sigma_{0_n}}{f_{C_n}}|\dot{q}_n|\right\}$$

is a diagonal positive semidefinite matrix where f_{C_i} denotes the Coulomb parameter for each joint $i = 1,\ldots,n$. It can

Work partially supported by CONACyT, Mexico.

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be shown that this friction model satisfies the requirements stated in [9] to get a passive operator from velocity $\dot{\boldsymbol{q}}$ to friction force $\boldsymbol{f}$.

The manipulator output considered in this paper is the pose (position and orientation) of the end–effector frame with respect to the robot base frame. In this paper we consider non–redundant manipulators.

The pose of the end–effector is characterized by a vector $\boldsymbol{y} \in \mathbb{R}^n$. Both position and orientation of the end–effector are function of the joint displacements, that is to say

$$\boldsymbol{y}(\boldsymbol{q}) = \boldsymbol{h}(\boldsymbol{q}), \tag{4}$$

where $\boldsymbol{h}(\boldsymbol{q}) : \mathbb{R}^n \to \mathbb{R}^n$ denotes the direct kinematics.

Given a desired output pose $\boldsymbol{y}_d$, the regulation aim is to ensure

$$\lim_{t\to\infty} \tilde{\boldsymbol{y}}(\boldsymbol{q}(t)) = \boldsymbol{0} \tag{5}$$

where

$$\tilde{\boldsymbol{y}}(\boldsymbol{q}(t)) = \boldsymbol{y}_d - \boldsymbol{y}(\boldsymbol{q}(t)) = \boldsymbol{y}_d - \boldsymbol{h}(\boldsymbol{q}(t))$$

denotes the pose error.

The time derivative of the direct kinematic model (4) yields the following differential kinematic model

$$\dot{\boldsymbol{y}} = \frac{d}{dt}\boldsymbol{h}(\boldsymbol{q}) = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{q}}\dot{\boldsymbol{q}} = J(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{6}$$

where $J(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the so–called analytical Jacobian matrix [6]. Using the differential kinematics (6) we have that the time derivative of the operational space error $\tilde{\boldsymbol{y}}$ is given by

$$\dot{\tilde{\boldsymbol{y}}} = -J(\boldsymbol{q})\dot{\boldsymbol{q}}.$$

We assume that the Jacobian $J(\boldsymbol{q})$ is full–rank and bounded for all $\boldsymbol{q} \in \mathbb{R}^n$.

III. RESOLVED ACCELERATION CONTROL PLUS FRICTION OBSERVER

Exponential friction observers for mechanical systems have been proposed in [10], for the case of tracking control of one degree–of–freedom second order systems, and in [12] for the case of joint velocity control of rigid robots. In this Section we extend the previous work proposing an exponential friction observer for the case of operational space regulation of mechanical arms.

The controller proposed is based on the resolved–acceleration controller studied in [4], but adding a term for friction compensation. The proposed control law for is written as

$$\begin{aligned}\boldsymbol{\tau} &= M(\boldsymbol{q})J(\boldsymbol{q})^{-1}\left[K_p\tilde{\boldsymbol{y}} - K_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right] \\ &\quad +C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} + \Sigma_0\hat{\boldsymbol{z}}, \end{aligned} \tag{7}$$

where K_p and K_v denotes $n \times n$ symmetric positive definite matrices, and $\dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \frac{d}{dt}J(\boldsymbol{q})$.

The variable $\hat{\boldsymbol{z}} \in \mathbb{R}^n$ involved by (7) is an estimation of the unmeasurable internal friction state $\boldsymbol{z} \in \mathbb{R}^n$. The proposed observer for the unmeasurable internal state $\boldsymbol{z}$ is

$$\begin{aligned}\dot{\boldsymbol{x}} &= K_0J(\boldsymbol{q})^{-1}[K_p\tilde{\boldsymbol{y}} - K_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}] \\ &\quad -\Psi(\dot{\boldsymbol{q}})\hat{\boldsymbol{z}} + \dot{\boldsymbol{q}} \end{aligned} \tag{8}$$

$$\hat{\boldsymbol{z}} = \boldsymbol{x} - K_0\dot{\boldsymbol{q}}, \tag{9}$$

where K_0 is a $n \times n$ diagonal positive definite matrix.

Differentiating (9) with respect to time and substituting (8) in the resulting expression we find that

$$\dot{\hat{\boldsymbol{z}}} = -\Psi(\dot{\boldsymbol{q}})\hat{\boldsymbol{z}} + \dot{\boldsymbol{q}} - K_0M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}}. \tag{10}$$

Using the definition of observation error $\tilde{\boldsymbol{z}} = \hat{\boldsymbol{z}} - \boldsymbol{z}$, and equation (2), we find the observation error dynamics

$$\dot{\tilde{\boldsymbol{z}}} = -\Psi(\dot{\boldsymbol{q}})\tilde{\boldsymbol{z}} - K_0M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}}. \tag{11}$$

The closed–loop system equation is obtained substituting the control action (7) into the robot dynamics (1), and using equation (11):

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\\ \tilde{\boldsymbol{z}}\end{bmatrix} =$$

$$\begin{bmatrix}\dot{\boldsymbol{q}}\\ J(\boldsymbol{q})^{-1}\left[K_p\tilde{\boldsymbol{y}} - K_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right] + M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}}\\ -\Psi(\dot{\boldsymbol{q}})\tilde{\boldsymbol{z}} - K_0M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}}\end{bmatrix}. \tag{12}$$

It can be demonstrated that the equilibrium points of system (12) are given by

$$E = \left\{\begin{bmatrix}\boldsymbol{q}\\ \dot{\boldsymbol{q}}\\ \tilde{\boldsymbol{z}}\end{bmatrix} \in \mathbb{R}^{3n} : \begin{bmatrix}\tilde{\boldsymbol{y}}(\boldsymbol{q})\\ \dot{\boldsymbol{q}}\\ \tilde{\boldsymbol{z}}\end{bmatrix} = \boldsymbol{0}\right\}. \tag{13}$$

A non negative function to demonstrate global convergence of the solutions $[\boldsymbol{q}(t)^T\ \dot{\boldsymbol{q}}(t)^T\ \tilde{\boldsymbol{z}}(t)]^T$ of the closed–loop system to the set E in (13) is given by

$$V(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}}) = \frac{1}{2}\dot{\boldsymbol{q}}^TJ(\boldsymbol{q})^TJ(\boldsymbol{q})\dot{\boldsymbol{q}} + \frac{1}{2}\tilde{\boldsymbol{y}}K_p\tilde{\boldsymbol{y}} + \frac{\alpha}{2}\tilde{\boldsymbol{z}}^T\Sigma_0K_0^{-1}\tilde{\boldsymbol{z}},$$

where α is a strictly positive constant that satisfies

$$\alpha < 2\frac{\sqrt{\lambda_m\{\Sigma_0M(\boldsymbol{q})^{-1}\Sigma_0\}\lambda_m\{J(\boldsymbol{q})^TK_vJ(\boldsymbol{q})\}}}{\|J(\boldsymbol{q})^TJ(\boldsymbol{q})M(\boldsymbol{q})^{-1}\Sigma_0\|} \tag{14}$$

It is possible to show that the time derivative of $V(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}})$ along of the closed–loop system trajectories (12) is given by

$$\begin{aligned}\dot{V}(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}}) &= -\dot{\boldsymbol{q}}^TJ(\boldsymbol{q})^TK_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} \\ &\quad +\dot{\boldsymbol{q}}^TJ(\boldsymbol{q})^TJ(\boldsymbol{q})M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}} \\ &\quad -\alpha\tilde{\boldsymbol{z}}^T\Sigma_0K_0^{-1}\Psi(\dot{\boldsymbol{q}})\tilde{\boldsymbol{z}} - \alpha\tilde{\boldsymbol{z}}^T\Sigma_0M(\boldsymbol{q})^{-1}\Sigma_0\tilde{\boldsymbol{z}}.\end{aligned}$$

Computing some upper bounds on $\dot{V}(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}})$, it is possible to write

$$\dot{V}(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}}) \le -\begin{bmatrix}\|\dot{\boldsymbol{q}}\|\\ \|\tilde{\boldsymbol{z}}\|\end{bmatrix}^T Q\begin{bmatrix}\|\dot{\boldsymbol{q}}\|\\ \|\tilde{\boldsymbol{z}}\|\end{bmatrix},$$

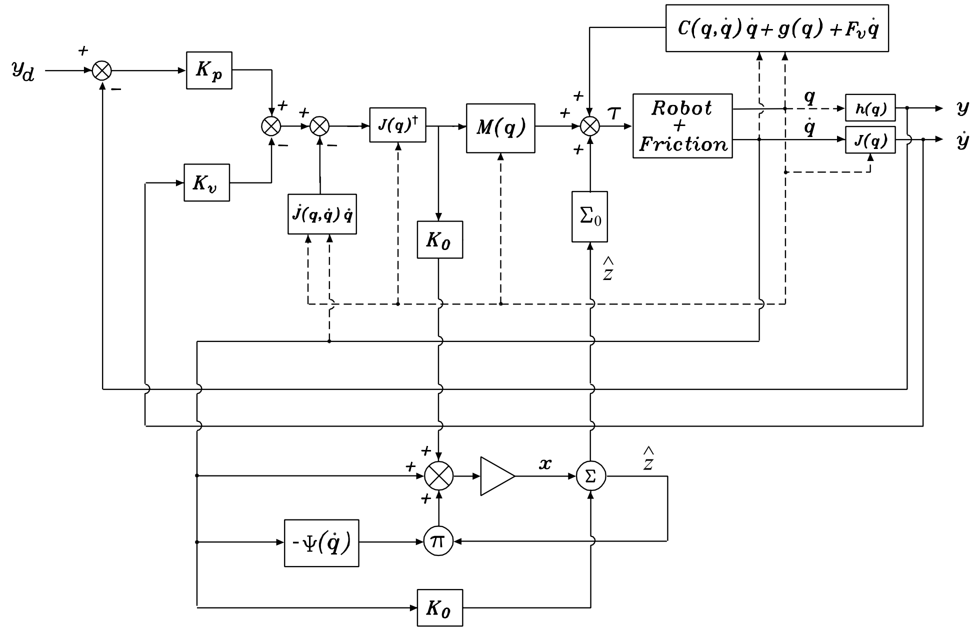

Fig. 1. Block diagram of the proposed controller

where the entries of the matrix Q are given by

$$\begin{aligned} Q_{11} &= \lambda_m\{J(\boldsymbol{q})^T K_v J(\boldsymbol{q})\}, \\ Q_{12} &= -\frac{\alpha}{2}\|J(\boldsymbol{q})^T J(\boldsymbol{q}) M(\boldsymbol{q})^{-1}\Sigma_0\|, \\ Q_{21} &= -\frac{\alpha}{2}\|J(\boldsymbol{q})^T J(\boldsymbol{q}) M(\boldsymbol{q})^{-1}\Sigma_0\|, \\ Q_{22} &= \lambda_m\{\Sigma_0 M(\boldsymbol{q})^{-1}\Sigma_0\}. \end{aligned}$$

It is easy to demonstrate that the symmetric matrix Q is positive definite for any α satisfying (14). This implies that $\dot{V}(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}})$ is a negative semidefinite function. Since the closed–loop system (12) is autonomous, we invoke the LaSalle invariance principle [11] to show that the solutions $[\boldsymbol{q}(t)^T\ \dot{\boldsymbol{q}}(t)^T\ \tilde{\boldsymbol{z}}(t)^T]^T$ converge asymptotically to the set E in (13). To this end, we define the set

$$\Omega = \left\{ \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{z}} \end{bmatrix} \in \mathbb{R}^{3n} : \dot{V}(\boldsymbol{q},\dot{\boldsymbol{q}},\tilde{\boldsymbol{z}}) = 0 \right\}$$

$$= \left\{ \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{z}} \end{bmatrix} \in \mathbb{R}^{3n} : \dot{\boldsymbol{q}} = \boldsymbol{0} = \tilde{\boldsymbol{z}} = \boldsymbol{0},\ \boldsymbol{q} \in \mathbb{R}^n \right\}.$$

Since any solution evolving in Ω implies that $\ddot{\boldsymbol{q}} = \boldsymbol{0}$, the largest invariant set in Ω is given by

$$S = \left\{ \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{z}} \end{bmatrix} \in \mathbb{R}^{3n} : \begin{bmatrix} \tilde{\boldsymbol{y}}(\boldsymbol{q}) \\ \dot{\boldsymbol{q}} \\ \tilde{\boldsymbol{z}} \end{bmatrix} = \boldsymbol{0} \right\} = E$$

Therefore, according to the LaSalle invariance principle, the solutions of the closed–loop system (12) approaches E as $t \to \infty$ for any initial condition $[\boldsymbol{q}(0)^T\ \dot{\boldsymbol{q}}(0)^T\ \tilde{\boldsymbol{z}}(0)^T]^T \in \mathbb{R}^{3n}$. Thus, the pose regulation control objective is attained since we have demonstrated that

$$\lim_{t\to\infty} \begin{bmatrix} \tilde{\boldsymbol{y}}(t) \\ \dot{\boldsymbol{q}}(t) \\ \tilde{\boldsymbol{z}}(t) \end{bmatrix} = \boldsymbol{0} \qquad (15)$$

is satisfied. This result is valid as long as the Jacobian $J(\boldsymbol{q})$ be nonsingular. Otherwise, the control objective can be achieved in a local sense, i.e. for suitable initial conditions.

Let us notice that the observer (8)–(9) is exponentially convergent. In order to show that, consider the function

$$V_0(\tilde{\boldsymbol{z}}) = \frac{1}{2}\tilde{\boldsymbol{z}}^T \Sigma_0 K_0^{-1} \tilde{\boldsymbol{z}},$$

whose time derivative satisfies

$$\dot{V}_0(\tilde{\boldsymbol{z}}) \le -\lambda_m\{\Sigma_0 M(\boldsymbol{q})^{-1}\Sigma_0\}\|\tilde{\boldsymbol{z}}\|^2.$$

This implies that —see e.g. Vidyasagar [11]— the inequality

$$\|\tilde{\boldsymbol{z}}(t)\| \le c_1 e^{-c_2 t},$$

where c_1 and c_2 are strictly positive constants, is satisfied for all $t \ge 0$. Thus, observer (8)–(9) is exponentially convergent.

In practice, the assumption that $J(\boldsymbol{q})$ is full–rank for all $\boldsymbol{q} \in \mathbb{R}^n$ may not be satisfied. Thus the controller and the desired pose $\boldsymbol{y}_d$ are constrained to a singularity free region of the operational space where the Jacobian $J(\boldsymbol{q})$ is full–rank.

IV. RESOLVED ACCELERATION CONTROL PLUS VISCOUS AND COULOMB FRICTION

For comparison purpose during experimental tests, we resort to the simple static model of friction adopted by the control and robotics communities

$$\boldsymbol{f}(\dot{\boldsymbol{q}}) = F_v\dot{\boldsymbol{q}} + F_C\text{sign}(\dot{\boldsymbol{q}}) \qquad (16)$$

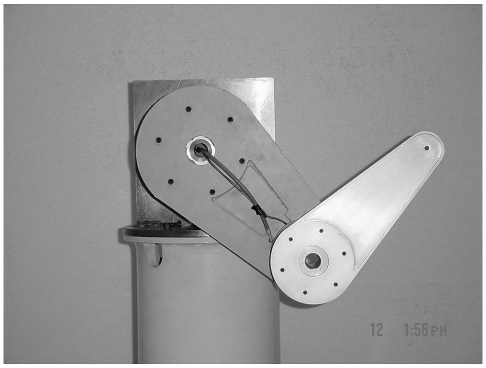

Fig. 2. Experimental arm

where F_v is the diagonal positive definite matrix that contains the viscous friction coefficients for each joint, F_C is the diagonal positive definite matrix that contains the Coulomb friction coefficients for each joint, and

$$\mathrm{sign}(\dot{\boldsymbol{q}}) = \begin{bmatrix} \mathrm{sign}(\dot{q}_1) \\ \vdots \\ \mathrm{sign}(\dot{q}_n) \end{bmatrix},$$

where $\mathrm{sign}(x)$=1 for $x > 0$, and $\mathrm{sign}(x)$= –1 for $x < 0$.

The first term of friction model (16) represents the viscous friction, while the second one represents the Coulomb friction. Compensation of friction according to this model yields the following controller

$$\begin{aligned} \boldsymbol{\tau} &= M(\boldsymbol{q})J(\boldsymbol{q})^{-1}\left[K_p\tilde{\boldsymbol{y}} - K_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right] \\ &\quad +C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} + F_C\mathrm{sgn}(\dot{\boldsymbol{q}}). \end{aligned} \tag{17}$$

On the other hand, the controller given by

$$\begin{aligned} \boldsymbol{\tau} &= M(\boldsymbol{q})J(\boldsymbol{q})^{-1}\left[K_p\tilde{\boldsymbol{y}} - K_vJ(\boldsymbol{q})\dot{\boldsymbol{q}} - \dot{J}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\right] \\ &\quad +C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) + F_v\dot{\boldsymbol{q}} \end{aligned} \tag{18}$$

results whether the Coulomb friction is neglected.

V. EXPERIMENTAL RESULTS

We carried out experiments of pose regulation using the direct–drive two–link robot shown in Figure 2. The complete description of the robot dynamics can be found in [12]. The pose of the robot system (only Cartesian position is of concern in this setup) is given by the following kinematic model:

$$\boldsymbol{h}(\boldsymbol{q}) = \begin{bmatrix} 0.26\,\sin(q_1) + 0.26\,\sin(q_1+q_2) \\ -0.26\,\cos(q_1) - 0.26\,\cos(q_1+q_2) \end{bmatrix}.$$

The initial configuration of the robot manipulator was given by $\begin{bmatrix} y_1(0) \\ y_2(0) \end{bmatrix} = \begin{bmatrix} 0.345 \\ 0 \end{bmatrix}$ [m].
The desired position was

$$\boldsymbol{y}_d = \begin{bmatrix} 0.1 \\ 0.35 \end{bmatrix} \quad [\mathrm{m}].$$

The gains for controller (7) were set as follows:

$$\begin{aligned} K_p &= diag\{55.0, 55.0\} \quad [1/\mathrm{sec}^2], \\ K_v &= diag\{15.0, 30.0\} \quad [1/\mathrm{sec}], \end{aligned}$$

while for the observer (8)–(9) the selected gains were

$$K_0 = diag\{0.0075, 0.0025\} \quad [\mathrm{sec}\cdot\mathrm{rad}/[\mathrm{kg}\cdot\mathrm{m}^2]].$$

The friction parameters in matrices F_v and F_c were obtained from experimental procedures following guidelines described in [13].

A. Viscous friction compensation

The first experiment was for controller (18), which is based on compensation of only viscous friction. The resulting Cartesian positions are shown in Figure 3, where it is observed that the steady state position of $y_1(t)$ is 0.075 [m], and of $y_2(t)$ is 0.3 [m], respectively. This represents a relative error with respect to the desired position $\boldsymbol{y}_d$ of 25% for y_1, and 14% for y_2.

B. Viscous and Coulomb friction compensation

The second experiment was for controller (17), which uses compensation of both viscous and Coulomb friction. The Cartesian positions are depicted in Figure 4, where oscillations of small amplitude are observed. One reason for this behavior is due to the presence of the discontinuous function $\mathrm{sign}(\boldsymbol{q})$ into the control law (17). The oscillations are presented with bigger amplitude for $y_1(t)$. In steady state, the Cartesian positions are held around 0.08 [m] for y_1, and 0.33 [m] for y_2, a 20% and 5.7%, respectively, of relative error with respect to the desired position $\boldsymbol{y}_d$.

C. Viscous and Dahl friction compensation

Finally, Figure 5 describes the time evolution of the robot Cartesian positions using the Dahl–based controller (7)–(9). In steady state the Cartesian positions are 0.095 [m] for y_1, and 0.347 [m] for y_2. This represents a relative error of 5% and 0.9%, respectively. Thus, the best performance is obtained using Dahl–based friction compensation.

VI. CONCLUSIONS

The end–effector pose regulation for n degrees–of–freedom manipulators has been addressed in this paper. The exact dynamic and kinematic parameters of the robot, including those for friction model, are assumed to be known. Resorting to the resolved acceleration structure a controller/observer algorithm has been proposed for solving the pose regulation problem of robot manipulators. Experimental results on a two degrees–of–freedom direct–drive robot arm showed better results for Dahl friction compensation that for the classical static model of viscous and Coulomb friction. The pose regulation considering friction compensation deserves further study, like to consider transpose Jacobian and energy shaping [14] based controllers, different ways of representing the orientation of the robot end–effector —e.g. the unit quaternion—, among other designs.

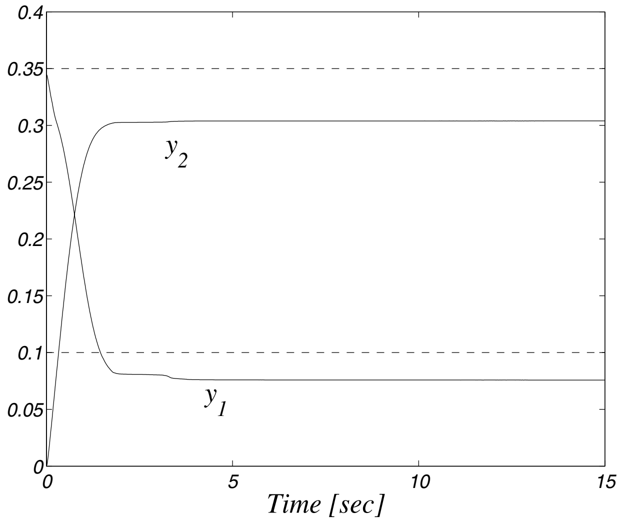

Fig. 3. Cartesian positions using only viscous friction compensation

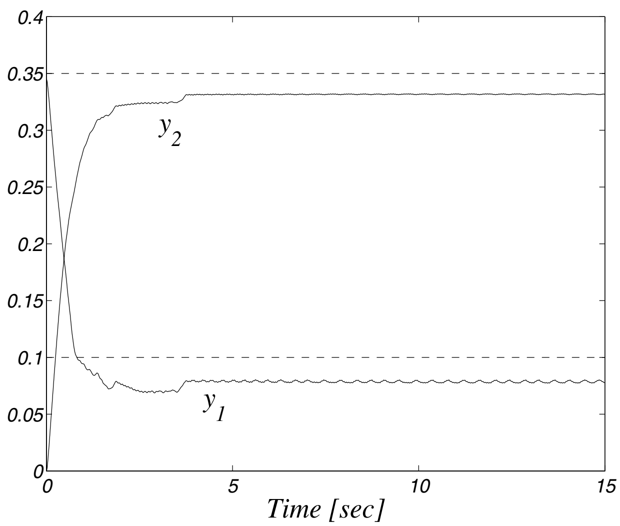

Fig. 4. Cartesian positions using both viscous and Coulomb friction compensation

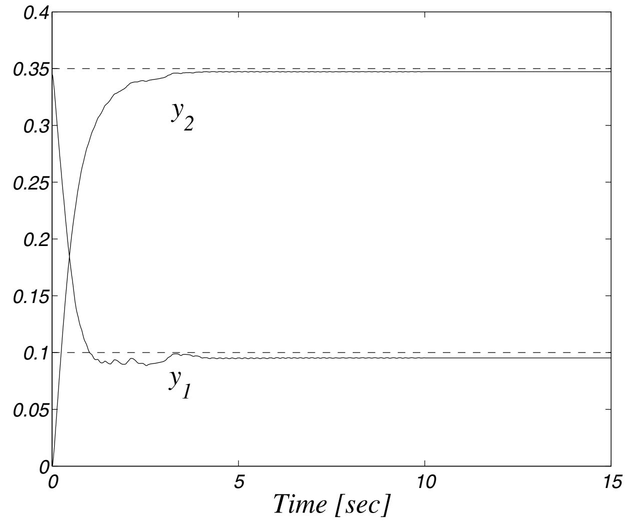

Fig. 5. Cartesian positions using Dahl based friction compensation

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