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Communication Design for Coordinated Control with a Non-Standard Information Structure

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Abstract— **Feasible suboptimal communication designs for prediction are studied with a non-classical information structure in the coordinated vehicle formation control context. Recasting the prediction design and bandwidth assignment problems as a single Linear Matrix Inequality (LMI) problem is the main consideration of this paper. The non-classical information structure arises because of the limited ability to get information from neighbouring vehicles. A simple example is given and several interesting design issues are then discussed.**

I. INTRODUCTION

Coordinated control of vehicle formation has become a significant topic in control. Typically the size of the entire system prohibits a global solution because the collection of the global state information and the computation of a global control law are overly demanding. In recent years, many improvements in this field have been made. Especially Model Predictive Control (MPC) was successful in dealing with coordinated control problems because it is one of the few control design methods which preserves standard design variables and yet handles constraints. The representative examples of MPC in coordinated vehicle control are [9] and [2]. However little attention has been paid to the following question: "How does each vehicle obtain the other's state information?". Since each vehicle is running in a wireless communication environment, the channel capacity is limited. The most challanging part is that each mobile agent is uncertain about others' control inputs. Therefore the coordinated multi-vehicle formation control problem has a non-classical information architecture. The purpose of this paper is to suggest a technique to construct an estimate of the other mobile agents' states and control inputs and to develop an estimate quality measure or covariance so that the expected control performance can be determined.

We consider a communication system design paradigm for coordinated control with a non-standard information structure. Coordinated control involves the simultaneous control of a fleet of dynamical systems. (We shall refer to them as *vehicles* here.) Because of communication limitations and distributed sensing, a global control approach is not viable. Such a global solution would rely on a supervisor aggregating all measurements and computing control inputs to each of the individual subsystems — the communications bottleneck comes from needing to transfer all measurements at full accuracy to the supervisor and then to disseminate all control values to the subsystems, again at full accuracy. Here we shall study an approach in which the limited availability of communications bandwidth is incorporated into the design. Thus, a global supervisor is not involved and the individual subsystems receive their own measurements at full accuracy and measurements from other subsystems at a lesser accuracy (if at all) via communication through a bit-rate limited channel. Thus the information structure is non-standard.

Witsenhausen, in his landmark paper [8], studied a very simple decentralized control problem with incomplete information passed between the two separate control inputs. He showed that the optimal control was not linear and was very difficult to compute. Significant studies are still reporting attempts to calculate the optimal control [4]. Accordingly, our construction will seek not to find the optimal control, but rather to determine a feasible suboptimal bandwidth assignment in the limited communications channel. We do this by setting up a state estimation procedure based on observers in which the channel bandwidth assignment is reflected as additive measurement noise. We are able to propose a Linear Matrix Inequality (LMI) formulation [1], [3], [6], which reduces to the Kalman filter in the absence of communication constraints.

Recent studies in decentralized control with communication constraints have tended to focus on the best achievable performance [7] and stabilization [5]. Here we build on a MPC approach [9], [2] in which each vehicle solves a local MPC problem with interactions between vehicles defined via constraints to prevent collisions. Constraints from neighboring vehicles are modified (tightened) to reflect the covariance of that vehicle's state estimate from the current location.

We follow the approach of [10] in which the control task is solved locally via MPC with probabilistic non-collision constraints in which the covariance of the state estimates at each vehicle are restricted based on their proximities in the fleet formation. Thus, we assume that we have a given overbound to be achieved with this covariance and we search for feasible suboptimal bit-rate assignments. Details are written in Section 2 followed by a scalar example in Section 3.

In this paper, matrices will be denoted by upper case boldface (e.g., $\mathbf{A}$), vectors will be denoted by lowercaes boldface (e.g., $\mathbf{x}$) and scalars will be denoted by lower case (e.g., y). $\mathbf{A} = blockdiag(\mathbf{A}^i)$ denotes a block diagonal matrix $\mathbf{A}$ with $\mathbf{A}^i$s $i = 1, \ldots, n$.

II. PROBLEM FORMULATION

A. MODELING

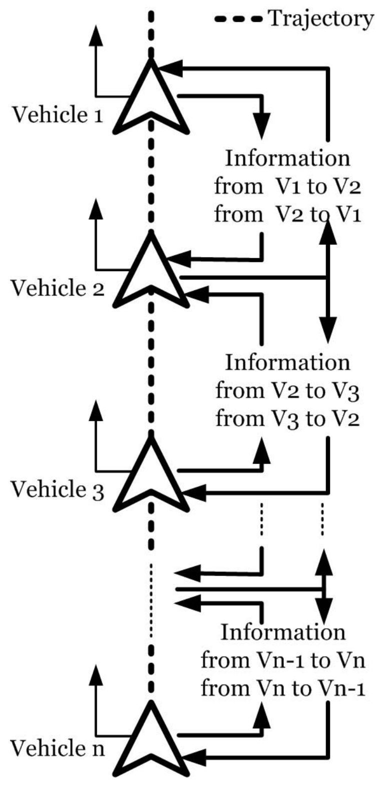

Fig. 1. n vehicle cooperation task

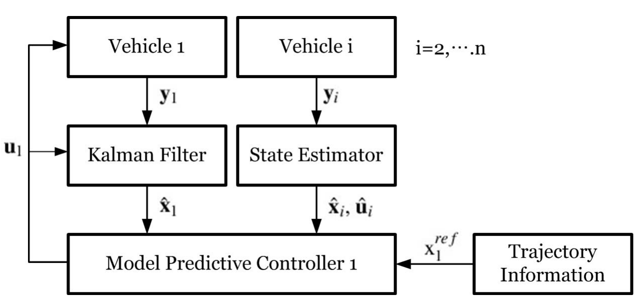

Fig. 2. Control block diagram at Vehicle 1

Consider n autonomous vehicles on the known trajectory with noisy communication as shown in Figure 1. The task is that vehicles track a known trajectory and they try to minimize the distance to the vehicle in front while obeying a no-collision constraint. They need to estimate the states of other vehicles in order to enforce the no-collision control strategy. For ease of presentation, consider a single vehicle, numbered 1, amongst a fleet of vehicles, numbered 2 through n, with states and inputs at time k denoted $\mathbf{x}_k^i$ and $\mathbf{u}_k^i$, $i = 1, \ldots, n$. Information about the states $\mathbf{x}_k^i$ and $\mathbf{u}_k^i$ for $i \neq 1$ are communicated to vehicle 1 via a bit-rate limited channel. Thus a total of N bits per sample time are available for the transfer of measurements of all the $\mathbf{x}_k^i$ and $\mathbf{u}_k^i$ to Vehicle 1. The control and estimation diagram is shown in Figure 2. We assume that information transmission delay is negligibly small between time steps. The other vehicles in the cooperation task will perform in the same way as vehicle 1 does. We assume that one-step-ahead MPC is used and precise dynamical models are available for all the vehicles in the fleet.

$$\mathbf{x}_{k+1}^i = \mathbf{F}^i \mathbf{x}_k^i \quad + \mathbf{G}^i \begin{bmatrix} \mathbf{u}_k^i \\ \mathbf{w}_k^i \end{bmatrix}, \tag{1}$$

where $\mathbf{G}^i = \begin{bmatrix} \mathbf{G}_\mathbf{u}^i & \mathbf{G}_\mathbf{w}^i \end{bmatrix}$.

Note that multi-step-ahead MPC cases can be considered by defining a new state vector with states $\mathbf{x}_k^i$ and input sequences $\mathbf{u}_{k\ldots l}^i$ under appropriate assumptions on inputs in the dynamical model. At each time instant, Vehicle 1 receives a message packet limited to N bits and conveying information to it about the state estimate and control input of each of the other vehicles 2 through n. Using this information, Vehicle 1 computes its own control value (using MPC in our case), which subsequently will be communicated to the other vehicles before being applied. Because both the contol value and the state of Vehicle i are not precisely known at Vehicle 1 and we do not attempt to reconstruct the control input from the state estimate we model the control signal $\mathbf{u}_k^i$ at Vehicle 1 as a white noise with known coavriance $\mathbf{Q}_{\mathbf{u},k}^i$. This attempts to capture the uncertain outcome of an MPC calculation. We model the disturbance to the vehicle $\mathbf{w}_k^i$ as a white noise with normal distribution

$$\begin{bmatrix} \mathbf{u}_k^i \\ \mathbf{w}_k^i \end{bmatrix} \sim N\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{Q}_{\mathbf{u}k}^i & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{\mathbf{w}k}^i \end{bmatrix} \right).$$

The limited message bit-rate and the corresponding assignment to communicated information are captured by the measurement noise covariance model. The measurement equation for $\mathbf{x}_k^i$ and $\mathbf{u}_k^i$ at Vehicle 1 is

$$\mathbf{y}_k^i = \mathbf{H}^{iT} \mathbf{x}_k^i + \mathbf{C}^i \begin{bmatrix} \mathbf{u}_k^i \\ \mathbf{w}_k^i \end{bmatrix} + \mathbf{v}_k^i, \tag{2}$$

where

$$\mathbf{H}^{iT} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}, \ \mathbf{C}^i = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}, \ \mathbf{v}_k^i = \begin{bmatrix} \mathbf{v}_{\mathbf{x},k}^i \\ \mathbf{v}_{\mathbf{u},k}^i \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{R}_k^i\right). \tag{3}$$

We assume that $\mathbf{v}_k^i$ is a white noise. We note, from the structure of (1) and (2), that treating the control input as white noise leads to correlated measerement and process noises in this formulation.

As seen in Figure 2, each vehicle has two kinds of state estimator. The Kalman filter is used as a local state estimator as usual. The interesting part is the State Estimator for the other vehicles' states since we do not have exact information of the other vehicles' controls. In the next subsections we will construct a State Estimator under this non-classical information structure and include design bandwidth assignment to achieve the desired estimate and its covariance.

B. PREDICTOR DESIGN

Consider a standard linear state predictor operating at Vehicle 1 to yield a prediction of the state of Vehicle i.

$$\hat{\mathbf{x}}^i_{k+1|k} = \mathbf{F}^i\hat{\mathbf{x}}^i_{k|k-1} + \mathbf{K}^i_p(\mathbf{y}^i_k - \mathbf{H}^{iT}\hat{\mathbf{x}}^i_{k|k-1}), \tag{4}$$

where $\mathbf{K}^i_p = \begin{bmatrix}\mathbf{K}^i_{\mathbf{x},p} & \mathbf{K}^i_{\mathbf{u},p}\end{bmatrix}$.

Define $\tilde{\mathbf{x}}^i_{k+1} = \mathbf{x}^i_{k+1} - \hat{\mathbf{x}}^i_{k+1|k}$. Then the prediction error is governed by

$$\begin{aligned}\tilde{\mathbf{x}}^i_{k+1} =& (\mathbf{F}^i - \mathbf{K}^i_{\mathbf{x},p})\tilde{\mathbf{x}}^i_k + (\mathbf{G}^i_{\mathbf{u}} - \mathbf{K}^i_{\mathbf{u},p})\mathbf{u}^i_k - \mathbf{K}^i_{\mathbf{u},p}\mathbf{v}^i_{\mathbf{u},k} \\ & - \mathbf{K}^i_{\mathbf{x},p}\mathbf{v}^i_{\mathbf{x},k} + \mathbf{G}^i_{\mathbf{w}}\mathbf{w}^i_k.\end{aligned} \tag{5}$$

We note that selecting $\mathbf{K}^i_{\mathbf{u},p} = \mathbf{G}^i_{\mathbf{u}}$ yields an unbiased state estimator (4) at the expense of noise $-\mathbf{K}^i_{\mathbf{u},p}\mathbf{v}^i_{\mathbf{u},k}$. Alternatively, we may maintain $\mathbf{K}^i_{\mathbf{u},p}$ as a design variable and keep the zero-mean white noise assumption on $\mathbf{u}^i_k$ to trade off bias versus variance. Define the direct sum $\tilde{\mathbf{x}}_k = \bigoplus_{i=2}^n(\tilde{\mathbf{x}}^i_k)$, and stack the n equations (5) into a single expression

$$\tilde{\mathbf{x}}_{k+1} = (\mathbf{F} - \mathbf{K}_p\mathbf{H}^T)\tilde{\mathbf{x}}_k + (\mathbf{G} - \mathbf{K}_p\mathbf{C})\begin{bmatrix}\mathbf{u}_k \\ \mathbf{w}_k\end{bmatrix} - \mathbf{K}_p\mathbf{v}_k, \tag{6}$$

where

$\mathbf{F} = blockdiag(\mathbf{F}^i)$, $\mathbf{H}^T = blockdiag(\mathbf{H}^{iT})$,
$\mathbf{G} = blockdiag(\mathbf{G}^i)$, $\mathbf{C} = blockdiag(\mathbf{C}^i)$,
$\mathbf{u}_k = \bigoplus_{i=2}^n(\mathbf{u}^i_k)$, $\mathbf{w}_k = \bigoplus_{i=2}^n(\mathbf{w}^i_k)$,
$\mathbf{v}_k = \bigoplus_{i=2}^n(\mathbf{v}^i_k)$, and $\mathbf{K}_p = blockdiag(\mathbf{K}^i_p)$.

Then, subject to stability of $\mathbf{F} - \mathbf{K}_p\mathbf{H}^T$, the steady state prediction error covariance $\mathbf{P} = Cov(\tilde{\mathbf{x}}_k)$ exists and is given by

$$\begin{aligned}\mathbf{P} =& (\mathbf{F} - \mathbf{K}_p\mathbf{H}^T)\mathbf{P}(\mathbf{F}^T - \mathbf{H}\mathbf{K}_p{}^T) \\ & + (\mathbf{G} - \mathbf{K}_p\mathbf{C})\mathbf{Q}(\mathbf{G}^T - \mathbf{C}^T\mathbf{K}_p{}^T) + \mathbf{K}_p\mathbf{R}\mathbf{K}_p{}^T,\end{aligned} \tag{7}$$

where $\mathbf{Q} = blockdiag(\mathbf{Q}^i)$, and $\mathbf{R} = blockdiag(\mathbf{R}^i)$.

Note that $\mathbf{P}$ is block diagonal, as is $\mathbf{K}_p$, because of the noise structure and corresponding estimate independence (i.e. $E[\tilde{\mathbf{x}}^i_k\tilde{\mathbf{x}}^{jT}_k] = \mathbf{0},\ i \neq j$). If a feasible solution $(\mathbf{P}, \mathbf{K}_p)$ satisfies the following matrix inequality

$$\begin{aligned}-\mathbf{P} +& (\mathbf{F} - \mathbf{K}_p\mathbf{H}^T)\mathbf{P}(\mathbf{F}^T - \mathbf{H}\mathbf{K}^T_p) \\ & + (\mathbf{G} - \mathbf{K}_p\mathbf{C})\mathbf{Q}(\mathbf{G}^T - \mathbf{C}^T\mathbf{K}^T_p) + \mathbf{K}_p\mathbf{R}\mathbf{K}^T_p \leq \mathbf{0}, \\ & \mathbf{P} > \mathbf{0},\end{aligned} \tag{8}$$

then it provides an upper bound on the algebraic solution of (7) and implies that the predictor (4) is stable. Multiplying $\mathbf{P}^{-1}$ both right and left

$$\begin{aligned}-\mathbf{P}^{-1} +& \mathbf{P}^{-1}(\mathbf{F} - \mathbf{K}_p\mathbf{H}^T)\mathbf{P}(\mathbf{F}^T - \mathbf{H}\mathbf{K}^T_p)\mathbf{P}^{-1} \\ & + \mathbf{P}^{-1}(\mathbf{G} - \mathbf{K}_p\mathbf{C})\mathbf{Q}(\mathbf{G}^T - \mathbf{C}^T\mathbf{K}^T_p)\mathbf{P}^{-1} \\ & + \mathbf{P}^{-1}\mathbf{K}_p\mathbf{R}\mathbf{K}^T_p\mathbf{P}^{-1} \leq \mathbf{0}.\end{aligned} \tag{9}$$

Define $\mathbf{Y} = \mathbf{P}^{-1}$ and $\mathbf{L} = \mathbf{Y}\mathbf{K}_p$. The Schur complement of (9) yields the following result, which is a standard construction in prediction derived using LMIs. Although, in this non-standard information structure it includes restrictions on the permissible structure of the solutions. This restriction is easily accommodated in modern software solution packages and is made feasible in this case by the white-noise assumption on the vehicles' control inputs.

Theorem 1: Block diagonal matrices $\mathbf{L}$, $\mathbf{Y}$, and $\mathbf{R}$ satisfying

$$\begin{bmatrix} -\mathbf{Y} & \mathbf{L} & \mathbf{Y}\mathbf{G} - \mathbf{L}\mathbf{C} & \mathbf{Y}\mathbf{F} - \mathbf{L}\mathbf{H}^T \\ \mathbf{L}^T & -\mathbf{R}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}^T\mathbf{Y} - \mathbf{C}^T\mathbf{L}^T & \mathbf{0} & -\mathbf{Q}^{-1} & \mathbf{0} \\ \mathbf{F}^T\mathbf{Y}^T - \mathbf{H}\mathbf{L}^T & \mathbf{0} & \mathbf{0} & -\mathbf{Y} \end{bmatrix} \leq \mathbf{0} \tag{10}$$

exist if and only if block diagonal matrices $\mathbf{P} = \mathbf{Y}^{-1}$, $\mathbf{K}_p = \mathbf{Y}^{-1}\mathbf{L}$, and $\mathbf{R}$ satisfy (8).

C. BANDWIDTH ASSIGNMENT

To explore the bandwidth assignment problem we need to impose a specific structure on $\mathbf{R}^i_k$. We assume that each vehicle knows its own exact control value and applies a Kalman filter own-state observer with precomputable error covariance $\sigma^i_{\mathbf{x}}$. Without loss of generality, from now on, we consider the one-dimensional case. The higher dimensional case can be formulated by adding more elements to $\mathbf{R}^i_k$. Our formulation relies on design using a diagonal $\mathbf{R}^i_k$ matrix. Since the vehicle will transmit its own state estimate, $\hat{\mathbf{x}}^i_k$, and not its true state, to its neighbors, this state estimate error needs to be included into the noise model. Hence $\mathbf{R}^i_k$ has a following structure

$$\mathbf{R}^i_k = Cov(\mathbf{v}^i_k) = \begin{bmatrix}\sigma^i_{\mathbf{x}} + r^i_{\mathbf{x}} & 0 \\ 0 & r^i_{\mathbf{u}}\end{bmatrix}, \tag{11}$$

where $r^i_{\mathbf{x}}$ and $r^i_{\mathbf{u}}$ are sensor noise covariances for measuring state $\mathbf{x}^i_k$ and $\mathbf{u}^i_k$ at vehicle 1 respectively.

The study of the bit-rate assignment problem devolves to the consideration of how to determine the specific values for $r^i_{\mathbf{x}}$ and $r^i_{\mathbf{u}}$. The key idea is the following. If we round binary numbers to the Nth binary place, then the magnitude of the rounding error is 2^{-N}. Our technique is to capture the bit-rate limitations by modifying the measurement noise covariance to satisfy

$$\prod_{i=2}^n(r^i_{\mathbf{x}} \times r^i_{\mathbf{u}}) \geq 2^{-N}. \tag{12}$$

This equation describes the limitation of the total bit-rate into Vehicle 1 from communicating Vehicles 2 through n. There are N bits per sample time and these must be assigned to each part of the communications stream. We model this as contributing to the measurement noises of these components, as explained above — smaller bit-rate implying larger measurement noise. Our approach is to construct state predictions, $\hat{\mathbf{x}}^i_{k+1|k}$, for all the vehicles at Vehicle 1 using the above measurement description.

From (10-12) we have the following matrix inequality, whose solution $(\mathbf{Y}, \mathbf{L}, r^i_{\mathbf{x}}, r^i_{\mathbf{u}})$ determines the bandwidth assignment.

$$\prod_{i=2}^{n}(r^i_{\mathbf{x}} \times r^i_{\mathbf{u}}) \geq 2^{-N},$$

$$\begin{bmatrix} -\mathbf{Y} & \mathbf{L} & \mathbf{YG}-\mathbf{LC} & \mathbf{YF}-\mathbf{LH}^T \\ \mathbf{L}^T & -\mathbf{R}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}^T\mathbf{Y}-\mathbf{C}^T\mathbf{L}^T & \mathbf{0} & -\mathbf{Q}^{-1} & \mathbf{0} \\ \mathbf{F}^T\mathbf{Y}^T-\mathbf{HL}^T & \mathbf{0} & \mathbf{0} & -\mathbf{Y} \end{bmatrix} \leq \mathbf{0}, \tag{13}$$

where $\mathbf{R} = blockdiag\left(\begin{bmatrix} \sigma^i_{\mathbf{x}} + r^i_{\mathbf{x}} & 0 \\ 0 & r^i_{\mathbf{u}} \end{bmatrix}\right)$.

Note that (13) represents the general case at Vehicle 1 predicting n vehicles' states and control values. However, since (12) is not convex, this matrix inequality cannot be easily solved. Hence we seek an approximation of the $\mathbf{R}^{-1}$. Now, for simplification of the main idea, we consider for the moment Vehicle 1 estimating a single vehicle i on 1-D real line. Take the logarithm base 2 of (12) and denote $z^i_{\mathbf{x}} = -\log_2(r^i_{\mathbf{x}})$ and $z^i_{\mathbf{u}} = -\log_2(r^i_{\mathbf{u}})$ so that $z^i_{\mathbf{x}}$ and $z^i_{\mathbf{u}}$ are actually the bit rate assignments to each transmission. Then (12) yields

$$z^i_{\mathbf{x}} + z^i_{\mathbf{u}} \leq N. \tag{14}$$

Using the inequality $x \geq 1 + \ln x$ with $x = 2^z$, we have

$$\begin{aligned} \mathbf{R}^{-1} &= \begin{bmatrix} \frac{1}{\sigma^i_{\mathbf{x}}+r^i_{\mathbf{x}}} & 0 \\ 0 & \frac{1}{r^i_{\mathbf{u}}} \end{bmatrix} = \begin{bmatrix} \frac{2^{z^i_{\mathbf{x}}}}{2^{z^i_{\mathbf{x}}}\sigma^i_{\mathbf{x}}+1} & 0 \\ 0 & 2^{z^i_{\mathbf{u}}} \end{bmatrix} \\ &\geq \begin{bmatrix} \frac{1}{\sigma^i_{\mathbf{x}}+1} + z^i_{\mathbf{x}}\left(\frac{\ln 2}{(\sigma^i_{\mathbf{x}}+1)^2}\right) & 0 \\ 0 & 1 + z^i_{\mathbf{u}}(\ln 2) \end{bmatrix}. \end{aligned} \tag{15}$$

We may now return to considering the full complexity estimation problem by defining

$$\mathbf{R}_a^{-1} = blockdiag\left(\begin{bmatrix} \frac{1}{\sigma^i_{\mathbf{x}}+1} + z^i_{\mathbf{x}}\left(\frac{\ln 2}{(\sigma^i_{\mathbf{x}}+1)^2}\right) & 0 \\ 0 & 1 + z^i_{\mathbf{u}}(\ln 2) \end{bmatrix}\right). \tag{16}$$

Note that (16) is now linear in $z^i_{\mathbf{x}}$ and $z^i_{\mathbf{u}}$. From (10), (14), and (15), we may replace (13) with the following inequalities, noting that the new solution set is a subset of the previous one.

Theorem 2: If feasible solutions $\mathbf{Y}$,$\mathbf{L}$, $z^i_{\mathbf{x}}$, and $z^i_{\mathbf{u}}$ exist for

$$\sum_{i=2}^{n} z^i_{\mathbf{x}} + z^i_{\mathbf{u}} \leq N,$$

$$\begin{bmatrix} -\mathbf{Y} & \mathbf{L} & \mathbf{YG}-\mathbf{LC} & \mathbf{YF}-\mathbf{LH}^T \\ \mathbf{L}^T & -\mathbf{R}_a^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}^T\mathbf{Y}-\mathbf{C}^T\mathbf{L}^T & \mathbf{0} & -\mathbf{Q}^{-1} & \mathbf{0} \\ \mathbf{F}^T\mathbf{Y}^T-\mathbf{HL}^T & \mathbf{0} & \mathbf{0} & -\mathbf{Y} \end{bmatrix} \leq \mathbf{0} \tag{17}$$

where
$\mathbf{R}_a^{-1} = blockdiag\left(\begin{bmatrix} \frac{1}{\sigma^i_{\mathbf{x}}+1} + z^i_{\mathbf{x}}\left(\frac{\ln 2}{(\sigma^i_{\mathbf{x}}+1)^2}\right) & 0 \\ 0 & 1 + z^i_{\mathbf{u}}(\ln 2) \end{bmatrix}\right)$.

then they are feasible solutions of (13).

Corollary 1: The predictor (4)

$$\hat{\mathbf{x}}^i_{k+1|k} = \mathbf{F}^i\hat{\mathbf{x}}^i_{k|k-1} + \mathbf{K}^i_p(\mathbf{y}^i_k - \mathbf{H}^{iT}\hat{\mathbf{x}}^i_{k|k-1})$$

is stable if there exist feasible $\mathbf{Y}$, $\mathbf{L}$, $z^i_{\mathbf{x}}$, and $z^i_{\mathbf{u}}$ for (17) with $\mathbf{Y}$ and $\mathbf{L}$ block diagonal.

One may seek feasible solutions of (17), which minimize $\mathbf{tr}(\mathbf{P}) = \mathbf{tr}(\mathbf{Y}^{-1})$. To do this we introduce a new variable $\mathbf{W}$ such that

$$\mathbf{W} > \mathbf{P}, \tag{18}$$

and then minimize the trace of $\mathbf{W}$. The Schur compliment of (18) is

$$\begin{bmatrix} -\mathbf{W} & \mathbf{I} \\ \mathbf{I} & -\mathbf{Y} \end{bmatrix} < \mathbf{0}. \tag{19}$$

This yields the following convex LMI optimization problem to provide a solution $\mathbf{K}_p$, $\mathbf{P}$, $z^i_{\mathbf{x}}$ and $z^i_{\mathbf{u}}$ for coordinated control with non-standard information structure.

Theorem 3: The block diagonal solution $\mathbf{K}_p = \mathbf{Y}^{-1}\mathbf{L}$ and bit-rates $z^i_{\mathbf{x}}$ and $z^i_{\mathbf{u}}$ solving the convex optimization below, yield a stable state estimator with covariance $\mathbf{P} < \mathbf{W}$ when bit-rates $z^i_{\mathbf{x}}$ and $z^i_{\mathbf{u}}$ are assigned for the communication of $\mathbf{x}^i_k$ and $\mathbf{u}^i_k$ from Vehicle i to Vehicle 1.

Min Cov:

$$\min_{L,W,Y,z^i_x,z^i_u} \mathbf{tr}(\mathbf{W})$$

subject to:

$$\begin{bmatrix} -\mathbf{W} & \mathbf{I} \\ \mathbf{I} & -\mathbf{Y} \end{bmatrix} < \mathbf{0},$$

$$\sum_{i=2}^{n}(z^i_{\mathbf{x}} + z^i_{\mathbf{u}}) \leq N,$$

$$\begin{bmatrix} -\mathbf{Y} & \mathbf{L} & \mathbf{YG}-\mathbf{LC} & \mathbf{YF}-\mathbf{LH}^T \\ \mathbf{L}^T & -\mathbf{R}_a^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{G}^T\mathbf{Y}-\mathbf{C}^T\mathbf{L}^T & \mathbf{0} & -\mathbf{Q}^{-1} & \mathbf{0} \\ \mathbf{F}^T\mathbf{Y}^T-\mathbf{HL}^T & \mathbf{0} & \mathbf{0} & -\mathbf{Y} \end{bmatrix} \leq \mathbf{0}, \tag{20}$$

where $\mathbf{Y}$ and $\mathbf{W}$ are block diagonal and

$$\mathbf{R}_a^{-1} = blockdiag\left(\begin{bmatrix} \frac{1}{\sigma^i_{\mathbf{x}}+1} + z^i_{\mathbf{x}}\left(\frac{\ln 2}{(\sigma^i_{\mathbf{x}}+1)^2}\right) & 0 \\ 0 & 1 + z^i_{\mathbf{u}}(\ln 2) \end{bmatrix}\right).$$

Note that the new minimizing solution $\mathbf{P}$ will overbound the minimizing solution for $\mathbf{P}$ subject to (13).

III. EXAMPLE

In this section, the result from Section 2 will be demonstrated with a *scalar* example. The notations remain the same as in Section 2 with only the matrix operation eased to the scalar calculation. Consider the following four vehicle cooperating task. Vehicles track a known $1-D$ trajectory

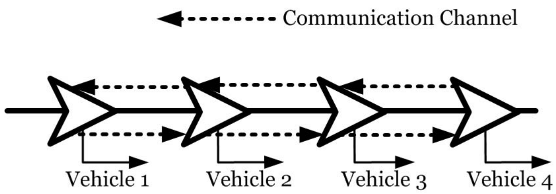

Fig. 3. Four vehicle cooperation task showing data links.

and each vehicle is controled by a local MPC controller to minimize the distance to the neighbouring vehicles while obeying a no-collision constraint. Each vehicle's dynamics are identical and are described by

$$\begin{aligned} x^i_{k+1} &= x^i_k + \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} u^i_k \\ w^i_k \end{bmatrix}, \qquad i = 1,2,3,4, \\ w^i_k &\sim N(0,10). \end{aligned} \tag{21}$$

The model-based observers running in each vehicle will use (21) as a model with the control input replaced by a zero-mean white noise,

$$u^i_k \sim N(0,1).$$

Every vehicle has 64 available bits per time sample for receiving data about the other vehicles' states and inputs. To apply the technique of Section 2 we need to have the local state estimate covariance, σ^i_k, for Vehicle i at Vehicle i, required in (16). This is computed by solving the Kalman filtering Algebraic Riccati Equation accounting for local position errors – here for all vehicles we take this value to be $\sigma^i_{k|k} = 0.03$.

As in Section 2, we model the measurement dynamics as

$$\begin{aligned} y^i_k &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} x^i_k + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u^i_k \\ w^i_k \end{bmatrix} + v^i_k, \\ v^i_k &\sim N\left(0, \begin{bmatrix} r^i_x & 0 \\ 0 & r^i_u \end{bmatrix}\right). \end{aligned} \tag{22}$$

We solve the predictor design problem at each vehicle to predict the others' states and control inputs all in the formulation of a single LMI. Note that the vehicle formation has the relay communication structure illustrated in Figure 3. Vehicle 1 receives all information about Vehicles 2 to 4 by relay from Vehicle 2. Similarly, Vehicle 2 receives information from Vehicle 4 only by relay via Vehicle 3. Hence the measurement noise covariance of Vehicle 3 state and input at Vehicle 1 will be bigger than it is at Vehicle 2. That is, the measurement noise covariance matrix $\mathbf{R}$, associated with measuring Vehicle 3's states and control values at Vehicles 1 and 2, has the following structure. Note that, as shown in Figure 3, the same information transmission process occurs in the opposite direction. We only solve vehicle 1's estimation problem. We can treat that of the others using the same method. The formulation of the measurement noise covariance at vehicle 1 captures the information structure inherent in the relay communication architecture. [Note that this example has all vehicles communicating to all others. If we introduce stable models in place of (21) the same solutions will exist with incomplete connectivity.]

$$\mathbf{R} = blockdiag(\mathbf{R}^i)$$

$$\mathbf{R^3} = \begin{bmatrix} \sigma + r^3_{2,x} & 0 & 0 & 0 \\ 0 & r^3_{2,u} & 0 & 0 \\ 0 & 0 & (\sigma + r^3_{2,x}) + r^3_{1,x} & 0 \\ 0 & 0 & 0 & r^3_{2,u} + r^3_{1,u} \end{bmatrix}$$

where $\sigma = \sigma^2 = \sigma^3$, and r^i_j denotes the measurement noise covariance for vehicle i at vehicle j.

In our working example the prediction error covariance variable $\mathbf{P} = \mathbf{Y}^{-1}$ is a 6 by 6 block diagonal matrix with diagonal elements corresponding to the state estimates

$$\hat{x}^4_1, \hat{x}^3_1, \hat{x}^2_1, \hat{x}^4_2, \hat{x}^3_2, \hat{x}^4_3,$$

and $\mathbf{R}$ is a 12 by 12 block diagonal matrix with elements determined by

$$z^4_{1,x}, z^4_{1,u}, z^3_{1,x}, z^3_{1,u}, z^2_{1,x}, z^2_{1,u}, z^4_{2,x}, z^4_{2,u}, z^3_{2,x}, z^3_{2,u}, z^4_{3,x}, z^4_{3,u}$$

We set up this LMI problem as in (20) and solve this using `lmitool` in `matlab` with a block diagonal structure on $\mathbf{W}$, $\mathbf{Y}$ and $\mathbf{L}$ to yield: predictor gains $\mathbf{K}^i_p$, bit-rate assignments $z^i_{j,x}$ and $z^i_{j,u}$, and an upper bound $\mathbf{P} = \mathbf{Y}^{-1}$ on the prediction error covariances.

Some results of this calculation are presented in the following tables.

Bit-rate to Vehicle 1	Assigned bits for states	Assigned bits for control inputs
Vehicle 4	5	4
Vehicle 3	11	9
Vehicle 2	18	17

TABLE I

BANDWIDTH ASSIGNMENT AT VEHICLE 1

	Predictor Gains at Vehicle 1
for Vehicle 4	[0.9868 0.8692]
for Vehicle 3	[0.9852 0.8539]
for Vehicle 2	[0.9922 0.9234]

TABLE II

RESULTING PREDICTOR GAINS $\mathbf{K}^i_p$ AT VEHICLE 1

Remarks:

- The numbers of bits assigned, which are shown at Table 1, were actually decimal real numbers. We rounded them to natural numbers. Although, fractional bit-rates

	Corresponding Prediction Error at Vehicle 1	Actual Prediction Error Covariance
for Vehicle 4	10.2753	10.1257
for Vehicle 3	10.3050	10.0545
for Vehicle 2	10.1587	10.0360

TABLE III

ERROR COVARIANCE BOUND $\mathbf{Y}^{-1}$ AND ACTUAL ERROR COVARIANCE FROM (7) AT VEHICLE 1

Bit-rate for Vehicle 4	Assigned bits for states	Assigned bits for control inputs
at Vehicle 3	34	30
at Vehicle 2	9	7
at Vehicle 1	5	4

TABLE IV

BANDWIDTH ASSIGNMENT FOR VEHICLE 4 FROM VEHICLE 1,2 ,AND 3

are achievable by sharing bits between channels over time.

- Since at Vehicle 1 the quality of information required about Vehicle 4 is very low, Vehicle 1 does not assign many bits for measuring Vehicle 4's states and control values.
- Table 3 shows that the resulting covariances from (20) bound above the results from (7) with $\mathbf{K}_p^i$ at Table 2, $\mathbf{Q}$ at (21) and $\mathbf{R}$ from Table 1.
- Table 4 shows the effect of relay communication structure. The farther is the Vehicle i away from Vehicle 4, the less bit-rate is assigned at Vehicle i for Vehicle 4.

Several design problems naturally arise at this stage. Firstly, for MPC controllers, the no-collision constraint should be considered. Each vehicle needs to avoid the uncertain area of other vehicles' state described by its prediction error covariance. This yields a numerical control state-constraint. Secondly, for estimators, the geometry of the coordinated fleet comes into play through the adjacencies of the different vehicles, which using the methods of [10] can be incorporated via specified limits on the covariances of the variaous state estimates. Thus, if Vehicle 2 is close to Vehicle 1 then its position needs to be known accurately, while if Vehicle 3 is more remote, then its covariance at Vehicle 1 can be permitted to be greater. This can be embodied as a constraint on the final state estimate covariances at Vehicle 1, which is easily incorporated into the LMI formulation.

IV. CONCLUSION AND FUTURE WORK

This paper studies the state predictor formulation in the vehicle coordinated control problem. Under the non-standard information architecture, predictors can be designed using LMIs. Moreover, by approximation techniques, bandwidth assignment to achieve the required prediction performance can be derived. Future work will include the consideration of this paper's results with MPC. One of the challenges when communication is incorporated into MPC is that we may face timing problems since each vehicle needs the others' current states and control input sequences, which might not yet be available to predict their future states and controls.

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