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LPV Control for μ-split braking assistance of a road vehicle

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Abstract— **In this paper, a system is developed to control the yaw moment and the lateral deviation in the case of braking with different adhesion conditions on the left and right sides. A driving assistance is obtained by using the front and rear steering angles. The control law is designed using an extension of the H_∞ loop-shaping method to the case of linear parameter-varying (LPV) plants: it allows to obtain a controller whose dynamics depend on the longitudinal speed. The results are evaluated by implementing this controller on a simulation software developed by RENAULT.**

I. INTRODUCTION

During the last years the safety systems have taken more and more place in the vehicles, the most famous of them being the ABS/ASR and the ESP systems.

In this paper, a system is proposed for the control of the yaw moment and the lateral deviation in the case of braking with strong different road adhesion conditions on the left and right sides: this situation is known as the μ-split braking problem. It has been widely studied by the car manufacturers to increase the security of the vehicles. The challenge is to keep the yaw rate as small as possible for the driving pleasure while limiting the lateral deviation for obvious safety reasons.

Different ideas have been proposed: in [1], when the wheels on low adhesion begin to lock, the system keeps their braking pressure and passes the same pressure on the wheels of the other side. This method is also used in [2] but using limited slip differential, and the authors foresee to tune the control when the system is used with other controls systems. Another idea is developed in [3]: when the wheels on one side start to lock, typically on low adhesion, the braking pressures of the wheels on the other side are progressively decreased; with this solution, the stopping distance is less reduced. A similar idea is also used in [4] where the control tries to keep the difference between the pressures under a given threshold. In [5] when a wheel is locking, the pressure increases with a slow rate compared to the case where not all the wheels are locked. In [6] the control is done by associating the braking and the steering: the authors use the differences between the ABS activation signal on each wheel together with a map to generate the correction actions on the rear wheels.

Every time the yaw rate is reduced using the braking as control input, the stopping distance is increased. Therefore, it has been decided in this paper to use only the steering. Furthermore, since the maximal braking force is necessary to minimize the stopping distance, an independent ABS system on each wheel is also used.

The vehicle dynamics are well described by a linear time-invariant (LTI) model for any fixed value of the longitudinal speed, however they vary significantly with the speed, which in addition is measured. Therefore a speed-depending controller can be obtained using the control theory of linear parameter-varying (LPV) plants. To the authors' knowledge, the problem has still not be considered from this perspective. This approach is often used for aerospace plants [7],[8],[9] but it has less been used for car control (see e.g. [10] where a lateral driving assistance system is developed).

The paper is organized as follows. Section II contains a brief summary of the H_∞ loop-shaping design for linear time-invariant (LTI) plants and its extension to the linear parameter-varying (LPV) case. A model of the vehicle in the μ-split context is developed in section III. The design method is applied in Section IV where in addition the results are evaluated by implementing the controller on a simulation software developed by RENAULT SAS. Some conclusions and directions for future works are finally given in the last part.

II. THEORETICAL BACKGROUND

A. H_∞ loop-shaping control for LTI plants

The H_∞ loop-shaping design was introduced by McFarlane and Glover in [11]. This method is interesting for two main raisons: firstly the parameters are chosen according to the classical rules of Automatic control; secondly the method is known to provide interesting robustness properties.

The nominal model $G(s)$ is first shaped with pre- and post-compensators $W_1(s)$ and $W_2(s)$. Both filters are chosen to have an adequate gain and phase behavior of the open-loop plant. The nominal model is then replaced by the new model $G_a(s) = W_2(s)G(s)W_1(s)$. From this augmented model $G_a(s)$, the following H_∞ problem is solved: for γ as small as possible, find a stabilizing controller $K(s)$ such as:

$$\left\| \begin{pmatrix} K(s) \\ I \end{pmatrix} \left(I + G_a(s)K(s)\right)^{-1} \begin{pmatrix} G_a(s) & I \end{pmatrix} \right\|_\infty < \gamma \quad (1)$$

This problem corresponds to the minimization of the H_∞ norm of the transfer function from $(w_1\ w_2)^T$ to $(e_1\ e_2)^T$ in Fig. 1. The final controller is then $K_f(s) = W_1(s)K(s)W_2(s)$.

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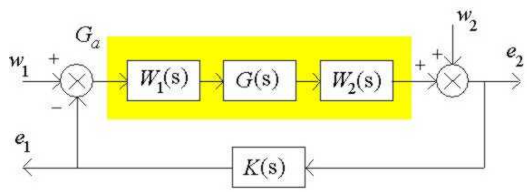

FIG. 1 –. *Loop-shaping H_∞ problem*

B. LPV control

1) Introduction: LPV plants are linear systems whose state-space matrices depend on a vector of varying parameters ρ:

$$\begin{aligned} \dot{x} &= A(\rho)x + B(\rho)u \\ y &= C(\rho)x + D(\rho)u \end{aligned} \tag{2}$$

The results established in [12] are applied by considering plants where:

a. matrices $A(\rho), B(\rho), C(\rho), D(\rho)$ depend affinely on ρ;

b. the time-varying vector ρ varies in a polytope $\mathscr{P}$ of vertices $P_1, \ldots, P_N$:

$$\rho \in \mathscr{C}_0 \{P_1, \ldots, P_N\} \equiv \left\{ \sum_{i=1}^{N} \alpha_i P_i, \alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1 \right\} \tag{3}$$

Let the plant be described by the compact form:

$$S(\rho) = \begin{pmatrix} A(\rho) & B(\rho) \\ C(\rho) & D(\rho) \end{pmatrix}, \qquad \rho \in \mathscr{P} \tag{4}$$

From (a) and (b), $S(\rho)$ belongs to a polytope whose vertices are the images of $P_1, \ldots, P_N$:

$$\{S(\rho), \rho \in \mathscr{P}\} = \mathscr{C}_0 \{S_1, S_2, \ldots, S_N\} \tag{5}$$

$$S_i = \begin{pmatrix} A(P_i) & B(P_i) \\ C(P_i) & D(P_i) \end{pmatrix} \tag{6}$$

2) Control law synthesis: The H_∞ synthesis of section II.A is now extended to LPV plants. The goal is to find a polytopic controller for the polytopic model defined above (figure 2), such that the closed loop plant is stable with $\mathscr{L}_2$ gain less than γ, that is $\|e(t)\|_2 < \gamma.\|w(t)\|_2$ for all trajectory of ρ in $\mathscr{P}$.

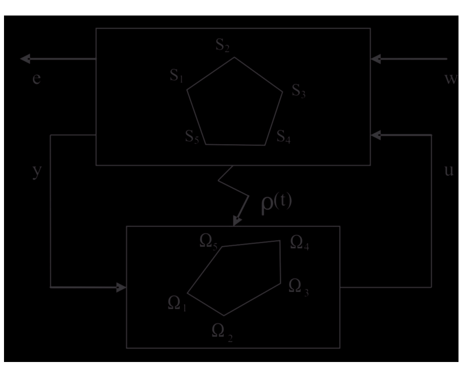

FIG. 2 –. *Closed loop plant*

Let the vertices of the open-loop plant $S(\rho)$ and the controller $K(\rho)$ be described by:

$$S_i = \begin{pmatrix} A_i & B_{1i} & B_2 \\ C_{1i} & D_{11i} & D_{12} \\ C_2 & D_{21} & 0 \end{pmatrix} \tag{7}$$

$$\Omega_i = \begin{pmatrix} A_{K_i} & B_{K_i} \\ C_{K_i} & D_{K_i} \end{pmatrix} \tag{8}$$

Note that matrices B_2, D_{12}, C_2, D_{21} are assumed to be independent of ρ. The vertices of the closed-loop plant are described by:

$$S_{cli} = \begin{pmatrix} A_{cli} & B_{cli} \\ C_{cli} & D_{cli} \end{pmatrix} \tag{9}$$

where

$$A_{cli} = \begin{pmatrix} A_i + B_2 D_{K_i} C_2 & B_2 C_{K_i} \\ B_{K_i} C_2 & A_{K_i} \end{pmatrix} \tag{10}$$

$$B_{cli} = \begin{pmatrix} B_{1i} + B_2 D_{K_i} D_{21} \\ B_{K_i} D_{21} \end{pmatrix} \tag{11}$$

$$C_{cli} = \begin{pmatrix} C_{1i} + D_{12} D_{K_i} C_2 & D_{12} C_{K_i} \end{pmatrix} \tag{12}$$

$$D_{cli} = D_{11_i} + D_{12} D_{K_i} D_{21} \tag{13}$$

A controller exists if and only if a positive definite matrix X exists, which simultaneously satisfies the N inequalities:

$$\Psi_i(X, \Omega_i) = \begin{pmatrix} A_{cli}^T X + X A_{cli} & X B_{cli} & C_{cli}^T \\ B_{cli}^T X & -\gamma I & D_{cli}^T \\ C_{cli} & D_{cli} & -\gamma I \end{pmatrix} < 0 \qquad i = 1, \ldots, N \tag{14}$$

These inequalities are bilinear in the variables X and Ω_i. Some matrix manipulations yield to the following inequalities:

$$\begin{pmatrix} \mathscr{N}_R & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_i R + R A_i^T & R C_{1i}^T & B_{1i} \\ C_{1i} R & -\gamma I & D_{11i} \\ B_{1i}^T & D_{11i}^T & -\gamma I \end{pmatrix} \begin{pmatrix} \mathscr{N}_R & 0 \\ 0 & I \end{pmatrix} < 0 \qquad i = 1, \ldots, N \tag{15}$$

$$\begin{pmatrix} \mathscr{N}_S & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} A_i^T S + S A_i^T & S B_{1i} & C_{1i}^T \\ B_{1i}^T S & -\gamma I & D_{11i}^T \\ C_{1i} & D_{11i} & -\gamma I \end{pmatrix} \begin{pmatrix} \mathscr{N}_S & 0 \\ 0 & I \end{pmatrix} < 0 \qquad i = 1, \ldots, N \tag{16}$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \geq 0 \tag{17}$$

where $\mathscr{N}_R$ and $\mathscr{N}_S$ denote bases of the null spaces of $\left(B_2^T, D_{12}^T\right)$ and (C_2, D_{21}) respectively. These inequalities are affine in variables R, S and γ so that minimizing γ under these constraints is a convex problem.

3) Controller reconstruction: From matrices R and S, matrix X in (14) can be reconstructed by solving:

$$\begin{pmatrix} I & 0 \\ R & M \end{pmatrix} X = \begin{pmatrix} S & N \\ I & 0 \end{pmatrix} \tag{18}$$

where

$$I - RS = MN^T$$

X being determined, each inequality in (14) is now affine in Ω_i and defines a convex set, so that Ω_i can be easily deduced. The final polytopic controller is then given by:

$$K(\rho) = \sum_{i=1}^{N} \alpha_i \Omega_i \tag{19}$$

where the α_i are computed in real time from the actual value of vector ρ as in (3).

III. PROBLEM DEFINITION AND MODEL OF THE CAR

A. The μ-split problem

In this section the problem of the braking action on an asymmetric road is introduced, which is known as the μ-split braking problem. It has first to be noted that this kind of action is done in a dangerous situation, where the driver gives the maximal force on the braking pedal, for instance 500 N during 0.1 s. The second typical condition of the driving situation is the status of the road: the car wheels are submitted to different friction coefficients, for instance 0.2 (ice road) on the left side and 0.7 (dry road) on the right side (a difference of 0.5 at least is necessary to have a μ-split braking situation).

In this situation the forces on the left side of the car are different of the forces of the right side; so a yaw moment is generated on the car, which pushes the car on the side where the coefficient of friction is higher. A line change will therefore occur, so that the car may go out of the road or have a frontal shock with another vehicle coming from the opposite direction. On the other way it is necessary to keep the braking action as high as possible to avoid any collision with the preceding vehicle. So the control action is done using the steering only, while an ABS system is used to avoid the locking of the wheels and so to have the maximal braking force on the wheels.

The control system has to satisfy the following specifications: in order to keep the same lane during braking, the deviation from the right trajectory must be less than an half way (typically $\pm$ 0.75 m) while the yaw rate must be limited to $\pm$ 6 deg /s.

B. Model of the vehicle

When analyzing the condition of the test, it can be seen that all the performances concern the lateral dynamics, so only these dynamics have to be modelled. The control of the stopping distance and of the sliding being in the field of the longitudinal dynamics, it is supposed that an independent ABS system is available on each wheel, which controls the sliding of the wheel without communication with the others wheels.

TAB. I –
Names of the variables and the constants

Symbols	Name
y	Lateral deviation
V	Longitudinal speed
ϕ	Roll angle
ψ	Yaw angle
β	Side slip angle
δ_k	Steering angle of wheel k
F_{y_e}	Lateral disturbance force
F_{ϕ_e}	Disturbance force on the roll angle
M_{z_e}	Disturbance moment on the yaw angle
F_{xk}	Longitudinal force on wheel k
x_k	Distance between wheel k and the COG on the x-axis
y_k	Distance between wheel k and the COG on the y-axis.
N_β	Constant depending on the tyre characteristics
$N_{\dot\psi}$	Constant depending on the load and the vehicle parameters
N_{δ_k}	Constant depending on the tyre characteristics
J_z	Inertia of the vehicle on the z axis

By applying the Lagrange method, the following model of the car is obtained [13]:

$$\begin{aligned} m(\dot v - V\dot\psi) &= J_S\ddot\phi + Y_\beta\beta + Y_{\dot\psi}\frac{\dot\psi}{V} + Y_\phi\phi \\ &\quad + \sum_{k=1}^{4}\left(Y_{\delta_k}\delta_k\right) + \sum_{k=1}^{4} F_{xk}\delta_k + F_{y_e} \end{aligned} \tag{20}$$

$$\begin{aligned} J_x\ddot\phi &= (J_{XZS} + m_S ch)\ddot\psi + (\dot v - V\dot\psi)J_S + L_\beta\beta \\ &\quad + L_\phi\phi + L_{\dot\phi}\dot\phi + \sum_{k=1}^{4}\left(L_{\delta_k}\delta_k\right) + L_{\dot\psi}\frac{\dot\psi}{V} + F_{\phi_e} \end{aligned} \tag{21}$$

$$\begin{aligned} J_z\ddot\psi &= N_\beta\beta + N_{\dot\psi}\frac{\dot\psi}{V} + \sum_{k=1}^{4}\left(N_{\delta_k}\delta_k\right) \\ &\quad - \sum_{k=1}^{4}\left(F_{xk}y_k\right) + \sum_{k=1}^{4} x_k F_{xk}\delta_k + M_{z_e} \end{aligned} \tag{22}$$

$$\ddot y = \dot v \tag{23}$$

$$\dot\beta = \frac{\dot v}{V} - \dot\psi \tag{24}$$

where the signification of the time-dependent variables are given on Table I.

The longitudinal speed V is measured and can be considered as a time-varying parameter; this model can therefore be described by a LPV state-space representation, with state vector $X = [\dot y \quad \phi \quad \dot\phi \quad \dot\psi]^T$; the control vector is $u = [\delta_{AV} \quad \delta_{AR}]^T$, where δ_{AV} is the steering angle of the front wheels, and δ_{AR} the steering angle of the rear wheels (i.e. $\delta_1 = \delta_2 = \delta_{AV}$ and $\delta_3 = \delta_4 = \delta_{AR}$). The outputs to be controlled are $\dot\psi$ and y, while only $\dot\psi$ is measured.

IV. PROPOSED SOLUTION

A. Definition of the polytope

The time-varying parameter V affects significantly the dynamics of the plant. A time-invariant controller is therefore not sufficient to obtain satisfying performances in the μ-split context, where the longitudinal speed decreases significantly and quickly. On the other hand, this parameter is measured and is therefore available to enter in a LPV controller.

From equations (20) to (24), it is clear that the state-space matrices of the LPV model depend affinely on V and $1/V$. A time-varying parameter $\rho = (\rho_1,\rho_2)^T = (V,1/V)^T$ is thus considered; the domain of variation of (ρ_1,ρ_2) is therefore the hyperbole $\rho_2 = 1/\rho_1$ (see figure 3), for $\rho_1 \in [V_{min};V_{max}]$, where $[V_{min};V_{max}]$ represents the velocity range.

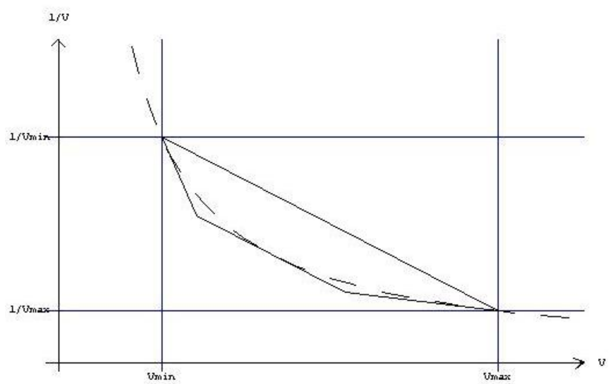

FIG. 3 –. *Structure of the polytope*

This domain has to be included in a polytope which has to be chosen as small as possible: to this end the trapezoidal domain shown on figure 3 is considered. This domain is first limited by the segment $[(V_{min},1/V_{min}),(V_{max},1/V_{max})]$; two other edges are the tangents at points $(V_{min},1/V_{min})$ and $(V_{max},1/V_{max})$; the last edge is the line which is parallel to the first segment and tangent at the hyperbole.

Obviously one cannot choose $V_{min} = 0$ in order to have a finite domain. Furthermore the value of V_{min} has to be chosen high enough to reduce the surface of the polytope. $V_{min} = 30$ km /h is therefore chosen : this means that for all velocities smaller than V_{min}, the controller of the vertex $V_{min},1/V_{min}$ will be applied. Since the dynamics are not significantly varying at low speed, this is not a severe limitation.

B. Design of the control law

The first step is to perform a H_∞ loop-shaping design for each vertex of the polytope defined above, by choosing a pre-compensator $W_1(s)$ and a post-compensator $W_2(s)$. In order to simplify the design, the same filters $W_1(s)$, $W_2(s)$ are chosen for all vertices (in addition, matrices B_2, D_{12}, C_2, D_{21} will be independent of ρ as required).

The choice of these filters is very important. According to [11], the pre-compensator $W_1(s)$ generally includes low-pass filters to reduce the measurement noise and to have a good robustness against neglected dynamics. The goal of this filter is also to avoid having significant action at high frequencies to reduce the actuators' activity. The post-compensator $W_2(s)$ allows to have a high open-loop gain at low frequencies to obtain a small tracking error: a PI filter is therefore associated to a phase lead compensator to increase the stability margins.

The structure of the augmented model $G_a(s)$ is therefore the one given on figure 4, with the following filters: [1].

$$W_1^{AV}(s) = \frac{K_1^{AV}}{\left(1+\tau_1^{AV}s\right)} \qquad W_1^{AR}(s) = \frac{K_1^{AR}}{\left(1+\tau_1^{AR}s\right)} \tag{25}$$

$$W_2^{\dot\psi}(s) = K_2^{\dot\psi}\frac{1+\tau_2^{\dot\psi}s}{s\left(1+\tau_2'^{\dot\psi}s\right)} \text{ with } \tau_2^{\dot\psi} > \tau_2'^{\dot\psi} \tag{26}$$

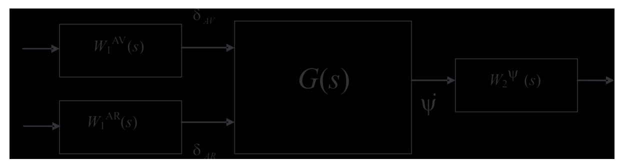

FIG. 4 –. *Structure of the augmented model*

The controller for each vertex is then determined according to the method explained in section II.B. To obtain the LPV controller, one has to compute in real time, from the velocity V, the coefficients α_i which express $(\rho_1,\rho_2) = (V,1/V)$ as a barycenter of the polytope defined in section IV.A. Let $(x_i,y_i), i = 1,...,4$ be the vertices of the polytope. One has then to solve in $\alpha_1,...,\alpha_4$ the linear system:

$$\begin{aligned} 1/V &= \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 + \alpha_4 y_4 \\ V &= \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 \\ 1 &= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \end{aligned} \tag{27}$$

This system having an infinite number of solutions, a particular way has to be defined to obtain a solution which insures a smooth evolution of the α_i. The following algorithm allows to compute the LPV controller for all speed V of the vehicle greater than 30 km /h: [2]

Algorithm

1) given the speed V, solve system (27) using a least square method to minimize the difference between the previous α_i and the actual ones.
2) test if all α_i are positive or zero:
   - if some α_i is negative, add the condition $\alpha_i = 0$ and solve system (27) again with this new condition
   - else go to 3
3) compute the state-space matrices of the controller using (19)
4) go to 1 until $V < 30$ km /h

1. For reason of confidentiality the numerical values of the parameters cannot be given

2. It has been observed that this algorithm always gives a solution with non negative values for α_i, although no guarantee is offered. Another solution should be to minimize the distance between α and a center point, e.g. $[1/4 \quad 1/4 \quad 1/4 \quad 1/4]$.

C. Results

The complete structure of the closed-loop system is given on figure 5: in a general situation, the driver uses the steering of the front wheels, whereas a feedforward path interprets the driver's action to define the desired yaw rate $\dot{\psi}_d$. The controller doesn't erase the driver's action but adds a supplementary angle on the front wheels and also uses the rear wheels to reject the disturbances. By tuning the controller, it was decided to limit the steering angles of the front steering wheels and that of the rear wheels: the first limitation is of the same magnitude as the maximal angle given by the driver, while the second one is smaller to act mainly by the front wheels; both values avoid saturation of the tyres.

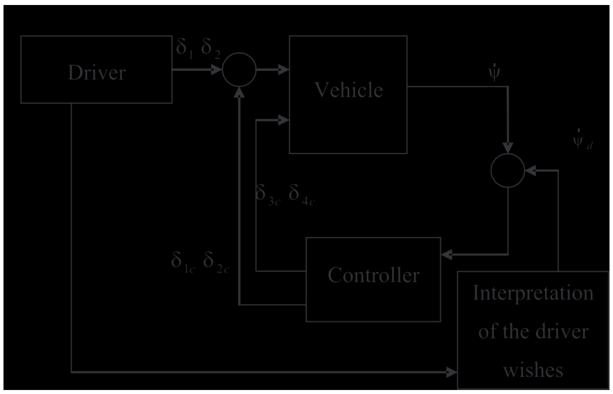

FIG. 5 –. *Structure of the control system*

TAB. II –
Numerical values

Variables and symbols	values	physical units
Weight of the vehicle - m	1700	kg
Footing - l	2,741	m
Front way - t_1	1525	mm
Rear way - t_2	1480	mm
Inertia on the z axis - J_z	3000	kg m^2
Front coefficients of drift - C_1 C_2	1250	N /deg
Rear coefficients of drift - C_3 C_4	1085	N /deg
F_{xk}	300	N

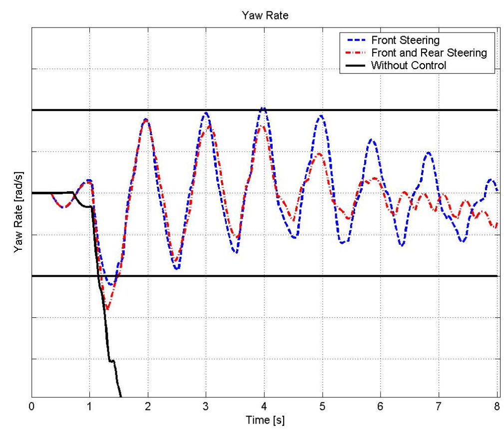

FIG. 6 –. *Yaw rate performance*

The results obtained when applying the LPV control on a μ-split situation are now presented. The initial speed of the vehicle is 100 km /h and it is supposed to have a friction coefficients equal to 0.7 on the right side of the car and 0.2 on the other side. In this condition, the car has a tendency to go on the right side. The car is supposed to move on a straight line, so the driver doesn't use the steering and the desired yaw rate is zero.

The second condition that is used is to have always the same pressure of braking on the rear wheels, which is the common situation.

Two control strategies are compared: in the first one only the front steering wheels are used by the controller, the angle being generated by an electrical motor on the steering column. Minimizing γ under (15)-(17) for V in the set $[30-110]$ km /h gives in this case $\gamma = 1.93$. In the second case the front and the rear steering wheels are used (for generating the rear steering angle, a particular system developed by RENAULT is included): in this case, $\gamma = 2.66$ is obtained. Since values between 2 and 3 are considered as acceptable for LTI plants [11], obtaining such values in a LPV context is very satisfying. They have been obtained using the MATLAB solver [14].

All the results shown below are obtained with a complete non linear simulator of the vehicle dynamics developed by RENAULT. The typical values of the car considered are shown in table II.

The yaw rate of the vehicle during the braking action and its lateral deviation are shown on figure 6 and 7, where the horizontal lines indicate the maximal allowable values: the solid line corresponds to the behavior obtained without control, which is obviously not satisfactory; the dotted line shows the case where both the front and the rear steering are used, while for the dashed line only the front steering is used. The controls inputs are shown on figures 8 and 9 (of course the rear steering input appears in the second case only).

As expected, the vehicle goes first on the right as can be seen on figure 7, but the lateral deviation remains below the specified value. The performances of the yaw rate and of the lateral deviation are conflicting: good performances for the first one induce bad performances on the other one, as can be seen on figures 6 and 7 where a smaller yaw rate amplitude is associated with a bigger lateral deviation. The first variable is a performance of driving pleasure while the second is a performance of security. It can be seen that the maximal value of the yaw rate is somewhat higher than the required value but the overtaking is acceptable in terms of driving pleasure. Finally one can check that the control inputs remains below the chosen specified limitations.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, a possible solution of the μ-split braking problem has been presented using the LPV method. This solution allows to obtain good performances using only the

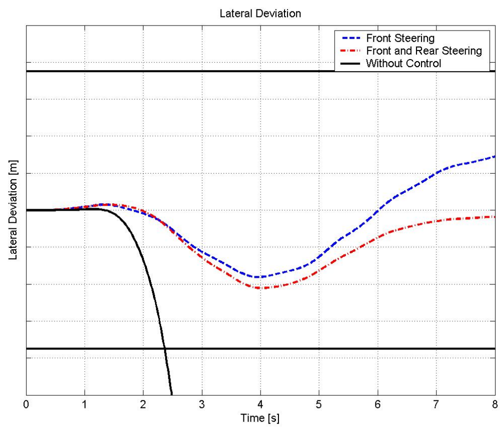

FIG. 7 –. *Lateral movement performance*

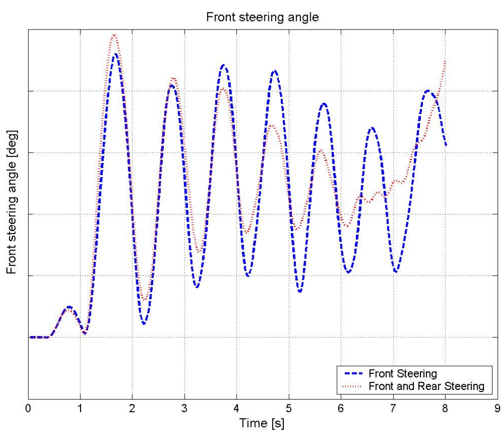

FIG. 8 –. *Front steering activity*

steering angles and thus without using the braking forces. The choice of using front and rear wheels has allowed us to reduce the control action on both actuators, to have a higher yaw torque and also to synthesize the control on a wide set of speed. Furthermore the control system accepts a wider difference of adhesion between both sides compared to the case when both actuators are used separately. Using this solution can increase the performances: the car can turn more quickly with a smaller radius of curve, which is necessary to have faster responses.

In the future, one can think to use a small braking action to erase the overtaking of the yaw rate or actuators on the suspensions. A problem that could be found using the LPV control in this case and in the future updates is the choice of the loop-shaping filters; this problem can be considered using non convex optimization. On the other side, adding new control inputs increases the number of decision variables and can induce numerical difficulties: so if the braking is used, it is better to use only a control on the pressure of the front steering to reduce such problems. In a more general way it is possible, as suggested by some author for the yaw

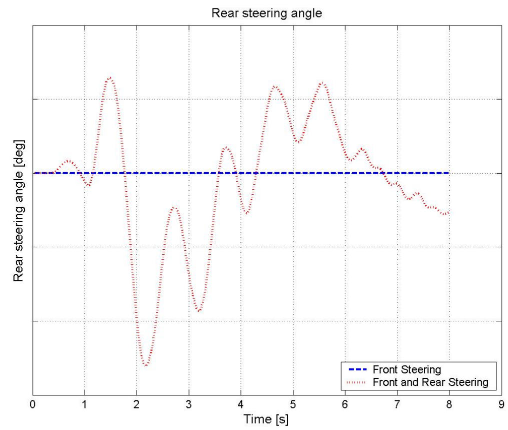

FIG. 9 –. *Rear steering activity*

rate control [15], to use an electrical motor to generate either a motor torque or a braking torque on the wheels to have quickly an higher yaw moment on the vehicle and thus to have a faster control.

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