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A New Adaptive Fuzzy Iterative Learning Control for Nonlinear Systems with Repeatable Control Tasks

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Abstract— **For the control of uncertain repeatable nonlinear systems with varying initial resetting state errors and non-repeatable input disturbance, a new adaptive fuzzy iterative learning controller is proposed in this paper. The main structure of this learning controller is constructed by a fuzzy learning component and a robust learning component. A new adaptive algorithm combining time-domain and iteration-domain adaptation is derived to search for suitable values of control parameters and then guarantee the closed-loop stability and error convergence. This adaptive algorithm is designed without using projection or deadzone mechanism. With a suitable choice of the weighting gains, the memory size for the storage of parameter profiles can be greatly reduced. It is shown that all the adjustable parameters as well as internal signals remain bounded for all iterations. Moreover, the norm of tracking state error vector will asymptotically converge to a tunable residual set as iteration number tends to infinity.**

I. INTRODUCTION

For a repeated tracking control or a periodic disturbance rejection problem, iterative learning control (ILC) is one of the most effective control strategies. A typical iterative learning controller updates the control input by a learning mechanism using the information of error and input in the previous trial. It has been widely studied in theories [1]-[4] and applied in many practical applications [5]-[7]. Recently, an interesting development in the field of ILC introduces the adaptive control concept into the design of iterative learning controller. This type of learning algorithm, which is often called adaptive iterative learning control or simply adaptive ILC, tunes control parameters instead of the control input itself between successive iterations. Substantial efforts in the area of adaptive ILC have been reported in [8]-[14] for broader applications to uncertain robot manupulators [8], [9], [10], non-Lipschitz nonlinear systems [11], [12], [13], or high relative degree nonlinear systems [14].

But the design and analysis of the above adaptive ILC systems deeply depend on the fact that the unknown parameters are linearly parameterized with known nonlinear functions. If the system nonlinearity can not be linearly parameterized, fuzzy system or neural network based controller has become an important technique for traditional adaptive nonlinear control [15]-[18]. Recently, this concept has been applied to the iterative learning control problem for dealing with the repeated tracking control of unknown nonlinear dynamic systems [19]-[21]. In [19], an adaptive nonlinear compensation ILC by using fuzzy approximation technique was proposed. The initial resetting state errors are assumed to be exactly zero and Lipschitz condition is still required for the system nonlinearities. Similar to [19], two fuzzy systems are designed to approximate the plant nonlinearities in [20]. However, the nonzero initial resetting issue is solved and some strict requirements on the nonlinearities are relaxed. In [21], a direct adaptive learning scheme based on an output recurrent fuzzy neural network is presented. The network is applied to compensate for the certainty equivalent control so that only one network is required. But the control structure is complicated and the size of memory for the storage of parameter profiles is large.

In this paper, a new adaptive fuzzy iterative learning controller for a class of nonlinear systems is proposed. This controller uses only one fuzzy system and possesses simpler structure, especially compared with the related works [19]-[21]. The main structure of this controller is designed by a fuzzy learning component and a robust learning component. For the fuzzy learning component, a fuzzy system is used to compensate for the plant nonlinearity. For the robust learning component, a sliding-mode like strategy is applied to overcome the nonlinear state dependent input gain, non-repeatable input disturbance and fuzzy approximation error. In addition to the new control structure, a new parameter adaptation algorithm combining time-domain and iteration-domain adaptation is derived to update the controller parameters. The boundedness of control parameters and control input can be guaranteed for all the time interval during each iteration without deadzone or projection mechanism as those in [8], [9], [12], [14], [19], [20], [21]. Another interesting feature for our parameter adaptation algorithm is the possible reduction of memory storage size. It is well known that the whole control parameter profiles in the previous iteration must be stored for an adaptive ILC design. This requires large system memory when the number of control parameters is large. If the size of memory is concerned in a real implementation, it is easy to achieve this requirement by setting some of the weighting gains to be zero in our adaptation algorithm such that only time-domain adaptation is executed. We show that the norm of tracking error vector will asymptotically converge to a tunable residual set as iteration goes to infinity even there exist initial state errors. Furthermore, the input disturbance can be totally rejected even it is not periodic or not repeatable.

II. DESIGN APPROACH OF THE ADAPTIVE FUZZY ILC

In this section, we consider a nonlinear system represented as follows :

$$\dot{x}_1^k(t) = x_2^k(t)$$

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$$\begin{aligned}\dot{x}_2^k(t) &= x_3^k(t)\\ &\vdots\\ \dot{x}_n^k(t) &= -f(X^k(t))+b(X^k(t))u^k(t)+d^k(t) \quad (1)\end{aligned}$$

where $X^k(t)=[x_1^k(t),\cdots,x_n^k(t)]^\top\in\mathcal{R}^{n\times 1}$ is the state vector of the system, $u^k(t)$ is the control input, $f(X^k(t))$ and $b(X^k(t))$ are unknown real continuous nonlinear functions of state, and $d^k(t)$ is the input disturbance. Here, k denotes the index of iteration and $t\in[0,T]$. Suppose the given nonlinear system can perform a control process repeatedly over a finite time interval $[0,T]$. The control objective is to force the state vector $X^k(t)=[x_1^k(t),x_2^k(t),\cdots,x_n^k(t)]^\top=[x_1^k(t),\dot{x}_1^k(t),\cdots,x_1^{(n-1),k}(t)]^\top$ to follow some specified bounded vector $X_d^k(t)=[x_d^k(t),\dot{x}_d^k(t),\cdots,x_d^{(n-1),k}(t)]^\top$ for all $t\in[0,T]$ as close as possible when the numbers of learning in iteration domain are large enough. Note that the desired trajectory $X_d^k(t)$ can be varying with respect to different trials similar to the condition in [13]. In order to achieve the control objective, it is required that there exists a positive unknown constant b_L such that $0<b_L\le b(X^k(t))$ for all $X^k(t)$ in a certain controllable compact set $\mathcal{A}_c\in\mathcal{R}^n$. Furthermore, we assume that the non-repeatable input disturbance is bounded for all iterations, i.e., $|d^k(t)|\le d^*$ for some unknown positive constant d^*. In the following, we use four design steps to illustrate the idea of the proposed adaptive fuzzy ILC scheme.

Design Step 1 : Design issue for initial state errors.

Let the state errors $e_1^k(t),\cdots,e_n^k(t)$ be defined as $e_1^k(t)=x_1^k(t)-x_d^k(t), e_2^k(t)=\dot{x}_1^k(t)-\dot{x}_d^k(t),\ \cdots, e_n^k(t)=x_1^{(n-1),k}(t)-x_d^{(n-1),k}(t)$. The initial state errors at each iteration are not necessarily zero, small and fixed, but only assumed to be bounded. As states are measurable, the initial state errors are available for controller design. At first, we define an error function as follows :

$$s^k(t)=c_1e_1^k(t)+c_2e_2^k(t)+\cdots+c_{n-1}e_{n-1}^k(t)+e_n^k(t) \quad (2)$$

where $c_1,\cdots,c_{n-1}$ are the coefficients of a Hurwitz polynomial $\Delta(D)=D^{n-1}+c_{n-1}D^{n-2}+\cdots+c_1$. It is clear that if the learning controller can drive $s^k(t)$ to zero for all $t\in[0,T]$, then the state tracking errors will also asymptotically converge to zero for all $t\in[0,T]$. However, $s^k(0)\neq 0$ due to the possible initial state errors. In order to overcome the uncertainty from the bounded initial state errors, we apply the same technique in our previous work [20], i.e., let ε^k be the available constant satisfying $|s^k(0)|=|c_1e_1^k(0)+c_2e_2^k(0)+\cdots+e_n^k(0)|\equiv\varepsilon^k$, and introduce the auxiliary error function $s_\phi^k(t)$ as

$$\begin{aligned}s_\phi^k(t) &= s^k(t)-\phi^k(t)\mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right) \quad (3)\\ \phi^k(t) &= \varepsilon^k e^{-\eta t},\ \eta>0.\end{aligned}$$

Here **sat** is the saturation function defined as

$$\mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right)=\begin{cases}1 & \text{if } s^k(t)>\phi^k(t)\\ \frac{s^k(t)}{\phi^k(t)} & \text{if } |s^k(t)|\le\phi^k(t)\\ -1 & \text{if } s^k(t)<-\phi^k(t)\end{cases}$$

and $\phi^k(t)$ is a time-varying boundary layer with varying initial value. Note that $\phi^k(t)$ is designed to decrease along time axis with the initial condition chosen as $\phi^k(0)=\varepsilon^k$ for kth iteration and $0<\varepsilon^k e^{-\eta T}\le\phi^k(t)\le\varepsilon^k,\forall t\in[0,T]$. Moreover, $s_\phi^k(t)$ will satisfy $s_\phi^k(0)=0$ and $s_\phi^k(t)\mathbf{sat}(\frac{s^k(t)}{\phi^k(t)})=|s_\phi^k(t)|$ [20]. Now if we let η be suitably large such that $\phi^k(t)$ can be as small as possible for all time interval $[0,T]$, then the control objective will be almost achieved if $\lim_{k\to\infty}s_\phi^k(t)=0,\forall t\in[0,T]$ since $\lim_{k\to\infty}|s^k(t)|\le\phi^\infty(t)$.

Design Step 2. Definition of the control structure.

Using the error function (2) and auxiliary error function (3), we now propose the adaptive fuzzy iterative learning controller as follows :

$$u^k(t)=U_C^k(t)+U_{FL}^k(t)+U_{RL}^k(t) \quad (4)$$

Here $U_C^k(t)$, defined as the feedback component of $u^k(t)$, is expressed as

$$U_C^k(t)=-\eta s^k(t)-\sum_{i=1}^{n-1}c_ie_{i+1}^k(t)+x_d^{(n),k}(t) \quad (5)$$

with the positive constant η the same as that in (3). This component is a typical design for compensation of some known functions. Although the detailed formulations of $U_{FL}^k(t)$ and $U_{RL}^k(t)$ will be specified in next step, we can see the motivation of design concept if we differentiate $(s_\phi^k(t))^2$ along system trajectory (1) with respective to time t as follows :

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left(s_\phi^k(t)\right)^2 =&\ s_\phi^k(t)\Big(\dot{s}^k(t)-\mathbf{sgn}\left(s_\phi^k(t)\right)\dot{\phi}^k(t)\Big)\\ =&\ s_\phi^k(t)\Big\{-\eta s^k(t)-\mathbf{sgn}\left(s_\phi^k(t)\right)\dot{\phi}^k(t)\\ &+U_{FL}^k(t)-f(X^k(t))+U_{RL}^k(t)\\ &+\big(b(X^k(t))-1\big)u^k(t)+d^k(t)\Big\} \quad (6)\end{aligned}$$

Let $U_a^k(t)=U_C^k(t)+U_{FL}^k(t)$, i.e., $u^k(t)=U_a^k(t)+U_{RL}^k(t)$, then (6) can be rewritten as

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left(s_\phi^k(t)\right)^2 =&\ s_\phi^k(t)\Big\{-\eta s^k(t)-\mathbf{sgn}\left(s_\phi^k(t)\right)\dot{\phi}^k(t)\\ &+U_{FL}^k(t)-f(X^k(t))+b(X^k(t))U_{RL}^k(t)\\ &+\big(b(X^k(t))-1\big)U_a^k(t)+d^k(t)\Big\} \quad (7)\end{aligned}$$

Now investigate the first and second terms in the right hand side of (7) by using (3) as follows

$$\begin{aligned}&s_\phi^k(t)\Big\{-\eta s^k(t)-\mathbf{sgn}\left(s_\phi^k(t)\right)\dot{\phi}^k(t)\Big\}\\ &= s_\phi^k(t)\Big\{-\eta s_\phi^k(t)-\eta\phi^k(t)\mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right)\\ &\quad -\mathbf{sgn}\left(s_\phi^k(t)\right)\dot{\phi}^k(t)\Big\} = -\eta\left(s_\phi^k(t)\right)^2 \quad (8)\end{aligned}$$

Substituting (8) into (7), it yields

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left(s_\phi^k(t)\right)^2 =&\ -\eta\left(s_\phi^k(t)\right)^2+s_\phi^k(t)\Big\{U_{FL}^k(t)-f(X^k(t))\\ &+b(X^k(t))U_{RL}^k(t)+\big(b(X^k(t))-1\big)U_a^k(t)+d^k(t)\Big\}\end{aligned} \quad (9)$$

We can consider (9) as a system error equation with zero initial condition $s_\phi^k(0)=0$. Since $f(X^k(t))$, $b(X^k(t))$ and $d^k(t)$ are in general unknown, the fuzzy learning component $U_{FL}^k(t)$ and robust learning component $U_{RL}^k(t)$ will be designed to compensate for these unknown nonlinearities and disturbance.

Design Step 3 : Design of $U_{FL}^k(t)$ and $U_{RL}^k(t)$.

In order to overcome the unknown nonlinear function $f(X^k(t))$, we apply the fuzzy approximation technique to design our iterative learning controller. The fuzzy system $\widehat{f}(X^k(t),W^k(t))$ which performs as the approximator of $f(X^k(t))$ is described as follows:

$$\widehat{f}(X^k(t),W^k(t)) = \frac{\sum_{\ell=1}^{m} w_\ell^k(t)\prod_{i=1}^{n}\mu_{f_{i\ell}}(x_i^k(t))}{\sum_{\ell=1}^{m}\prod_{i=1}^{n}\mu_{f_{i\ell}}(x_i^k(t))} = \sum_{\ell=1}^{m} w_\ell^k(t) z_\ell(X^k(t)) \equiv W^k(t)^\top Z(X^k(t)) \quad (10)$$

In the representation of (10), m is the number of fuzzy rules, $w_\ell^k(t)$ is the consequent parameter, and $\mu_{f_{i\ell}}(x_i^k(t))$ is the fuzzy membership function for the fuzzy system $\widehat{f}$. The fuzzy system is expressed as a series of radial basis function expansion with the basis functions as $z_\ell(X^k(t))$. It is well known that the fuzzy system (10) can uniformly approximate real continuous nonlinear function $f(X^k(t))$ on a compact set $\mathcal{A}_c \subset \mathcal{R}^{n\times 1}$. An important aspect of the above approximation property is that there exist optimal weights W^* such that the function approximation error between the optimal fuzzy system $\widehat{f}(X^k(t),W^*)$ and function $f(X^k(t))$ can be bounded by prescribed constant ϵ^* on the compact set $\mathcal{A}_c$. More precisely, if we let $f(X^k(t)) = \widehat{f}(X^k(t),W^*)+\epsilon(X^k(t))$, then the approximation error will satisfy $|\epsilon(X^k(t))|\le\epsilon^*$, $\forall X^k(t)\in\mathcal{A}_c$.

Since there exists a lower bound b_L for the input nonlinear gain $b(X^k(t))$, we apply two different ways to overcome the nonlinearities. In this step, a fuzzy learning control $U_{FL}^k(t)$ based on the fuzzy system and a robust learning control $U_{RL}^k(t)$ based on sliding-mode like strategy will be proposed to compensate for $f(X^k(t))$ and $b(X^k(t))$, respectively. Both components are specified as follows :

$$U_{FL}^k(t) = \widehat{f}(X^k(t),W^k(t)) = W^k(t)^\top Z(X^k(t)) \quad (11)$$

$$U_{RL}^k(t) = -\mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right)\psi^k(t) - \mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right)\left(\theta^k(t)+1\right)|U_a^k(t)| \quad (12)$$

where $W^k(t)\in R^{m\times 1}$ is fuzzy parameter vector of the given fuzzy system and $\psi^k(t)\in R^1$, $\theta^k(t)\in R^1$ are called the bounding control parameters. To explain the key point of the design concept in this step, let us apply $U_{FL}^k(t)$ (11) and $U_{RL}^k(t)$ (12) to equation (9) and show that

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\left(s_\phi^k(t)\right)^2 &= -\eta\left(s_\phi^k(t)\right)^2 + s_\phi^k(t)\Big\{\widehat{f}(X^k(t),W^k(t)) \\
&\quad -f(X^k(t)) - \mathbf{sat}\left(\frac{s^k(t)}{\phi^k(t)}\right)\psi^k(t) + d^k(t)\Big\} \\
&\quad + s_\phi^k(t)\Big\{-\mathbf{sat}\left(\frac{s^k(t)}{\phi(t)}\right)b(X^k(t))(\theta^k(t)+1)|U_a^k(t)| \\
&\quad + \Big(b(X^k(t))-1\Big)U_a^k(t)\Big\} \\
&\le -\eta\left(s_\phi^k(t)\right)^2 + s_\phi^k(t)\big(W^k(t)-W^*\big)^\top Z(X^k(t)) \\
&\quad -|s_\phi^k(t)|\left(\psi^k(t)-\psi^*\right) - \theta^k(t)b(X^k(t)|s_\phi^k(t)||U_a^k(t)| \\
&\quad +|s_\phi^k(t)||U_a^k(t)| \qquad (13)
\end{aligned}$$

where $\psi^* = \epsilon^* + d^*$.

Design Step 4 : The parameter adaptation laws.

The adaptive algorithms combining time-domain and iteration-domain adaptation without deadzone or projection mechansim are proposed as follows

$$(1-\gamma_1)\dot{W}^k(t) = -\gamma_1 W^k(t) + \gamma_1 W^{k-1}(t) - \beta_1 s_\phi^k(t) Z(X^k(t)) \quad (14)$$

$$(1-\gamma_2)\dot{\psi}^k(t) = -\gamma_2\psi^k(t) + \gamma_2\psi^{k-1}(t) + \beta_2|s_\phi^k(t)| \quad (15)$$

$$(1-\gamma_3)\dot{\theta}^k(t) = -\gamma_3\theta^k(t) + \gamma_3\theta^{k-1}(t) + \beta_3|s_\phi^k(t)||U_a^k(t)| \quad (16)$$

where $W^k(0)=W^{k-1}(T), \psi^k(0)=\psi^{k-1}(T), \theta^k(0)=\theta^{k-1}(T)$ for $k\ge 1$, and $0<\gamma_1,\gamma_2,\gamma_3<1$, $\beta_1,\beta_2,\beta_3>0$. In this adaptive law, $\gamma_1,\gamma_2,\gamma_3$ and β_1,β_2,β_3 are defined as the weighting gains and learning gains, respectively. The initial fuzzy parameter vector $W^0(t)=W^{ini}$ will be set according to any expert knowledge on the nonlinear function $f(X^k(t))$ or simply to be zero. On the other hand, the initial control parameters $\psi^0(t),\theta^0(t)$ will be chosen as $\psi^0(t)=\psi^{ini}$ and $\theta^0(t)=\theta^{ini}, \forall t\in[0,T]$ where ψ^{ini} and θ^{ini} are some small positive constants. It is noted that under the adaptation laws (15) and (16), it is easily shown that $\psi^k(t)>0$ and $\theta^k(t)>0, \forall t\in[0,T]$ and $\forall k\ge 1$. Since $b(X^k(t))\ge b_L>0$ and $\theta^k(t)>0$ for all $k\ge 1$ and $t\in[0,T]$, this implies the last two terms of (13) can be rewritten as

$$\begin{aligned}
&-\theta^k(t)b(X^k(t))|s_\phi^k(t)||U_a^k(t)| + |s_\phi^k(t)||U_a^k(t)| \\
&\le -b_L\left(\theta^k(t)-\frac{1}{b_L}\right)|s_\phi^k(t)||U_a^k(t)| \qquad (17)
\end{aligned}$$

Define parameter errors as $\widetilde{W}^k(t)=W^k(t)-W^*, \widetilde{\psi}^k(t)=\psi^k(t)-\psi^*$ and $\widetilde{\theta}^k(t)=\theta^k(t)-\theta^*$ with $\theta^*=\frac{1}{b_L}$. Substituting (17) into equation (13), we have

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\left(s_\phi^k(t)\right)^2 &\le -\eta\left(s_\phi^k(t)\right)^2 + s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) \\
&\quad -\widetilde{\psi}^k(t)|s_\phi^k(t)| - b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)| \qquad (18)
\end{aligned}$$

III. Stability and Convergence

In this section, we use the concept of $L_{pe}[0,T]$to denote the set of Lebesgue measurable (or piecewise continuous) real valued (vector) functions with L_{pe} norm as defined in [10]. To begin with, the boundedness of internal signals at first iteration will be first established.

Lemma 1 : Consider the unknown nonlinear uncertain system (1) satisfying the requirements of $b(X^k(t))\ge b_L>0$

and $|d^k(t)| \leq d^*$ and give a desired bounded state vector $X_d^k(t)$. The adaptive fuzzy iterative learning controller (4), (5), (11), (12) and adaptation laws (14), (15), (16) ensure that all the internal signals at first iteration are bounded, i.e., $s_\phi^1(t), s^1(t), e_1^1(t), \cdots, e_n^1(t), W^1(t), \psi^1(t), \theta^1(t), u^1(t), \dot{W}^1(t), \dot{\psi}^1(t), \dot{\theta}^1(t) \in L_{\infty e}[0,T]$.

Proof : Let us choose a Lyapunov function as

$$V_a^k(t) = \frac{1}{2}(s_\phi^k(t))^2 + \frac{(1-\gamma_1)}{2\beta_1}\widetilde{W}^k(t)^\top \widetilde{W}^k(t) + \frac{(1-\gamma_2)}{2\beta_2}(\widetilde{\psi}^k(t))^2 + \frac{(1-\gamma_3)b_L}{2\beta_3}(\widetilde{\theta}^k(t))^2 \quad (19)$$

and compute its derivative with respective to time t along (18) as follows :

$$\begin{aligned}\dot{V}_a^k(t) \;\leq\; & -\eta(s_\phi^k(t))^2 + s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) \\ & -\widetilde{\psi}^k(t)|s_\phi^k(t)| - b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)| \\ & +\frac{1}{\beta_1}\widetilde{W}^k(t)^\top\left[(1-\gamma_1)\dot{\widetilde{W}}^k(t)\right] \\ & +\frac{\widetilde{\psi}^k(t)}{\beta_2}\left[(1-\gamma_2)\dot{\widetilde{\psi}}^k(t)\right] + \frac{b_L\widetilde{\theta}^k(t)}{\beta_3}\left[(1-\gamma_3)\dot{\widetilde{\theta}}^k(t)\right]\end{aligned} \quad (20)$$

Since $-\gamma_1 W^k(t) + \gamma_1 W^{k-1}(t) = -\gamma_1\widetilde{W}^k(t) + \gamma_1\widetilde{W}^{k-1}(t)$, $-\gamma_2\psi^k(t) + \gamma_2\epsilon^{k-1}(t) = -\gamma_2\widetilde{\psi}^k(t) + \gamma_2\widetilde{\psi}^{k-1}(t)$ and $-\gamma_3\theta^k(t) + \gamma_3\theta^{k-1}(t) = -\gamma_3\widetilde{\theta}^k(t) + \gamma_3\widetilde{\theta}^{k-1}(t)$, $\dot{V}_a^k(t)$ in (20) can be simplified by using (14), (15) and (16) as

$$\begin{aligned}\dot{V}_a^k(t) \;\leq\; & -\eta(s_\phi^k(t))^2 - \frac{\gamma_1}{\beta_1}\widetilde{W}^k(t)^\top\widetilde{W}^k(t) \\ & + \frac{\gamma_1}{\beta_1}\widetilde{W}^k(t)^\top\widetilde{W}^{k-1}(t) - \frac{\gamma_2}{\beta_2}(\widetilde{\psi}^k(t))^2 \\ & + \frac{\gamma_2}{\beta_2}\widetilde{\psi}^k(t)\widetilde{\psi}^{k-1}(t) - \frac{\gamma_3 b_L}{\beta_3}(\widetilde{\theta}^k(t))^2 \\ & + \frac{\gamma_3 b_L}{\beta_3}\widetilde{\theta}^k(t)\widetilde{\theta}^{k-1}(t)\end{aligned} \quad (21)$$

Note that $\widetilde{W}^0(t) = W^0(t) - W^* = W^{ini} - W^* \equiv \overline{W}^0$, $\widetilde{\psi}^0(t) = \psi^0(t) - \psi^* = \psi^{ini} - \psi^* \equiv \overline{\psi}^0$ and $\widetilde{\theta}^0(t) = \theta^0(t) - \theta^* = \theta^{ini} - \theta^* \equiv \overline{\theta}^0$ are bounded for all $t \in [0,T]$ so that if $k=1$, (21) can be rewritten as

$$\begin{aligned}\dot{V}_a^1(t) \;\leq\; & -\eta(s_\phi^1(t))^2 - \frac{\gamma_1}{2\beta_1}\widetilde{W}^1(t)^\top\widetilde{W}^1(t) \\ & - \frac{\gamma_2}{2\beta_2}(\widetilde{\psi}^1(t))^2 - \frac{\gamma_3 b_L}{2\beta_3}(\widetilde{\theta}^1(t))^2 \\ & - \frac{\gamma_1}{2\beta_1}\big(\widetilde{W}^1(t) - \overline{W}^0\big)^\top\big(\widetilde{W}^1(t) - \overline{W}^0\big) \\ & - \frac{\gamma_2}{2\beta_2}(\widetilde{\psi}^1(t) - \overline{\psi}^0)^2 - \frac{\gamma_3 b_L}{2\beta_3}(\widetilde{\theta}^1(t) - \overline{\theta}^0)^2 \\ & + \frac{\gamma_1}{2\beta_1}\overline{W}^{0\top}\overline{W}^0 + \frac{\gamma_2}{2\beta_2}(\overline{\psi}^0)^2 + \frac{\gamma_3 b_L}{2\beta_3}(\overline{\theta}^0)^2 \\ \leq\; & -\lambda V_a^1(t) + \overline{\lambda}^0\end{aligned} \quad (22)$$

where $\lambda = \min\{2\eta, \frac{\gamma_1}{1-\gamma_1}, \frac{\gamma_2}{1-\gamma_2}, \frac{\gamma_3}{1-\gamma_3}\}$, $\overline{\lambda}^0 = \frac{\gamma_1}{2\beta_1}\overline{W}^{0\top}\overline{W}^0 + \frac{\gamma_2}{2\beta_2}(\overline{\psi}^0)^2 + \frac{\gamma_3 b_L}{2\beta_3}(\overline{\theta}^0)^2$. Note that the initial value $V_a^1(0)$ is bounded since $s_\phi^1(0) = 0$, $\widetilde{W}^1(0) = W^1(0) - W^* = W^0(T) - W^* = \overline{W}^0$, $\widetilde{\psi}^1(0) = \psi^1(0) - \psi^* = \psi^0(T) - \psi^* = \overline{\psi}^0$ and $\widetilde{\theta}^1(0) = \theta^1(0) - \theta^* = \theta^0(T) - \theta^* = \overline{\theta}^0$. Together with the result of (22), it readily implies $V_a^1(t), s_\phi^1(t), \widetilde{W}^1(t), \widetilde{\psi}^1(t), \widetilde{\theta}^1(t) \in L_{\infty e}[0,T]$. Since the basis function vector $Z(X^k(t))$ is bounded for all $k \geq 1$, we conclude that $s^1(t)$ (by (2)), $s_\phi^1(t)$ (by (3)), $u^1(t)$ (by (4), (11), (12)), $\dot{e}^1(t), \cdots, \dot{e}_n(t)$ (by (2)), $\dot{W}^1(t)$ (by (14)), $\dot{\psi}^1(t)$ (by (15)), $\dot{\theta}^1(t)$ (by (16)) $\in L_{\infty e}[0,T]$. This completes the proof. Q.E.D.

Based on the results in lemma 1, the error function $(s_\phi^k(t))^2$ converges in the $L_{1e}[0,T]$ norm sense and $(s_\phi^k(T))^2$ in the $L_{\infty e}[0,T]$ norm sense will be shown in the following lemma 2.

Lemma 2 : Consider the problem set-up in lemma 1, the proposed adaptive fuzzy iterative learning controller ensures $\widetilde{W}^k(T)$, $\widetilde{\psi}^k(T)$ and $\widetilde{\theta}^k(T)$ are bounded for all $k \geq 1$, and

$$\lim_{k\to\infty}\int_0^T (s_\phi^k(t))^2 dt = 0, \text{ and } \lim_{k\to\infty}(s_\phi^k(T))^2 = 0. \quad (23)$$

Proof : Define the positive function $V^k(T)$ as

$$\begin{aligned}V^k(T) = & \int_0^T\left[\frac{\gamma_1}{2\beta_1}\widetilde{W}^k(t)^\top\widetilde{W}^k(t) + \frac{\gamma_2}{2\beta_2}(\widetilde{\psi}^k(t))^2 + \frac{\gamma_3 b_L}{2\beta_3}(\widetilde{\theta}^k(t))^2\right]dt + \frac{1-\gamma_1}{2\beta_1}\widetilde{W}^k(T)^\top\widetilde{W}^k(T) \\ & + \frac{(1-\gamma_2)}{2\beta_2}(\widetilde{\psi}^k(T))^2 + \frac{(1-\gamma_3)b_L}{2\beta_3}(\widetilde{\theta}^k(T))^2,\end{aligned} \quad (24)$$

the difference between $V^k(T)$ and $V^{k-1}(T)$ can be derived by using integration by parts as follows :

$$\begin{aligned}& V^k(T) - V^{k-1}(T) \\ & = \int_0^T\Big[-\frac{\gamma_1}{2\beta_1}\big(\widetilde{W}^k(t) - \widetilde{W}^{k-1}(t)\big)^\top\big(\widetilde{W}^k(t) - \widetilde{W}^{k-1}(t)\big) \\ & \quad -\frac{\gamma_2}{2\beta_2}(\widetilde{\psi}(t) - \widetilde{\psi}^{k-1}(t))^2 - \frac{\gamma_3 b_L}{2\beta_3}(\widetilde{\theta}(t) - \widetilde{\theta}^{k-1}(t))^2 \\ & \quad - s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) + \widetilde{\psi}^k(t)|s_\phi^k(t)| \\ & \quad + b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)|\Big]dt \\ & \leq \int_0^T\Big[-s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) + \widetilde{\psi}^k(t)|s_\phi^k(t)| \\ & \quad + b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)|\Big]dt\end{aligned} \quad (25)$$

If we define $V_b^k(t) = \frac{1}{2}(s_\phi^k(t))^2$, we can easily derive the following inequality by similar argument in lemma 1 that

$$\begin{aligned}\dot{V}_b^k(t) \;\leq\; & -\eta(s_\phi^k(t))^2 + s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) \\ & -\widetilde{\psi}^k(t)|s_\phi^k(t)| - b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)|\end{aligned} \quad (26)$$

Integrating both side of (26) from 0 to T gives

$$\int_0^T\Big[-s_\phi^k(t)\widetilde{W}^k(t)^\top Z(X^k(t)) + b_L\widetilde{\theta}^k(t)|s_\phi^k(t)||U_a^k(t)| + \widetilde{\psi}^k(t)|s_\phi^k(t)|\Big]dt \leq -\int_0^T \eta(s_\phi^k(t))^2 dt - V_b^k(T) \quad (27)$$

where we use the property of $V_b^k(0) = \frac{1}{2}(s_\phi^k(0))^2 = 0$. Substituting (27) into (25), it yields

$$V^k(T) - V^{k-1}(T) \le -\int_0^T \eta(s_\phi^k(t))^2 dt - \frac{1}{2}(s_\phi^k(T))^2 \quad (28)$$

Since $V^1(T)$ is bounded by lemma 1, and $V^k(T)$ is positive and monotonically decreasing, $V^k(T)$ is bounded for all $k \ge 1$ and will converge as k approaches infinity to some limit value V_T (independent of k). The boundedness of $V^k(T)$ also ensures the boundedness of $\widetilde{W}^k(T), \widetilde{\psi}^k(T)$ and $\widetilde{\theta}^k(T)$ for all $k \ge 1$. On the other hand, (28) implies

$$\int_0^T \eta(s_\phi^k(t))^2 dt \le V^{k-1}(T) - V^k(T) \le V^1(T) \quad (29)$$

$$\frac{1}{2}(s_\phi^k(T))^2 \le V^{k-1}(T) - V^k(T) \le V^1(T) \quad (30)$$

for all $k \ge 1$. The boundedness of $(s_\phi^k(T))^2$ for all iterations is then established from (30). Finally, as $\lim_{k\to\infty} V^{k-1}(T) - V^k(T) = 0$, it concludes (23) of lemma 2. Q.E.D.

The convergence of $s_\phi^k(t), s^k(t), e_1^k(t), \cdots, e_n^k(t)$ and boundedness of all internal signals for all $k \ge 1$ are now established in the following theorem.

Theorem 1 : Consider the system set-up in lemma 1 and define $E^k(t) = [e_1^k(t), e_2^k(t), \cdots e_{n-1}^k(t)]^\top$. The proposed adaptive fuzzy iterative learning controller guarantees the tracking performance and system stability as follows :

(t1) $s_\phi^k(t),\ s^k(t),\ e_1^k(t), \cdots, e_n^k(t),\ W^k(t),\ \psi^k(t),\ \theta^k(t),\ u^k(t),\ \dot{W}^k(t),\ \dot{\psi}^k(t),\ \dot{\theta}^k(t) \in L_{\infty e}[0,T]$, for all $k \ge 1$.

(t2) $\lim_{k\to\infty} (s_\phi^k(t))^2 = (s_\phi^\infty(t))^2 = 0$, for all $t \in [0,T]$.

(t3) $\lim_{k\to\infty} |s^k(t)| = |s^\infty(t)| \le \phi^\infty(t)$, for all $t \in [0,T]$.

(t4) Let $C^\top = [c_1, \cdots, c_{n-1}]$ and δ be the positive constant such that $\Delta(D-\eta)$ is still a Hurwitz polynomial, then

$$\lim_{k\to\infty} \|E^k(t)\| \le m_1 e^{-\delta t}\|E^\infty(0)\| + m_1\varepsilon^\infty \frac{e^{-\eta t} - e^{-\delta t}}{\delta - \eta} \quad (31)$$

$$\lim_{k\to\infty} |e_n^k(t)| \le |C^\top E^\infty(t)| + e^{-\eta t}\varepsilon^\infty \quad (32)$$

for some positive constant m_1 and $t \in [0,T]$.

Proof :

(t1) Since $s_\phi^1(t), \widetilde{W}^1(t), \widetilde{\psi}^1(t), \widetilde{\theta}^k(t) \in L_{\infty e}[0,T]$ as shown in lemma 1, if we assume $s_\phi^{k-1}(t),\ \widetilde{W}^{k-1}(t),\ \widetilde{\psi}^{k-1}(t),\ \widetilde{\theta}^{k-1}(t) \in L_{\infty e}[0,T]$, then derivative of Lyapunov function $V_a^k(t)$ in (21) can be easily shown to satisfy

$$\dot{V}_a^k(t) \le -\lambda V_a^k(t) + \overline{\lambda}^{k-1} \quad (33)$$

where $\overline{\lambda}^{k-1} = \frac{\gamma_1}{2\beta_1}\overline{W}^{k-1\top}\overline{W}^{k-1} + \frac{\gamma_2}{2\beta_2}(\overline{\psi}^{k-1})^2 + \frac{\gamma_3 b_L}{2\beta_3}(\overline{\theta}^{k-1})^2$ and $\overline{W}^{k-1}$, $\overline{\psi}^{k-1}$, $\overline{\theta}^{k-1}$ are the upper bounds on $|\widetilde{W}^{k-1}(t)|$, $|\widetilde{\psi}^{k-1}(t)|$ and $|\widetilde{\theta}^{k-1}(t)|$, respectively. Since the initial value $V_a^k(0)$ of the Lyapunov function $V_a^k(t)$ is bounded for all $k \ge 1$ due to lemma 2, we conclude from (33) that $s_\phi^k(t),\ \widetilde{W}^k(t),\ \widetilde{\psi}^k(t),\ \widetilde{\theta}^k(t) \in L_{\infty e}[0,T]$. Using the same argument in lemma 1, it is easily shown that $s^k(t), e_1^k(t), \cdots e_n^k(t), u^k(t),\ \dot{W}^k(t), \dot{\psi}^k(t), \dot{\theta}^k(t) \in L_{\infty e}[0,T]$. Hence, (t1) of theorem 1 is achieved by using mathematical induction since $V_a^k(0)$ is bounded for all $k \ge 1$.

(t2) According to (t1) of this theorem, we have $(s_\phi^k(t))^2 \in L_{\infty e}[0,T]$ and $\frac{d}{dt}(s_\phi^k(t))^2 = 2s_\phi^k(t)(\dot{s}^k(t) - \mathbf{sgn}(s_\phi^k(t))\dot{\phi}^k(t)) \in L_{\infty e}[0,T]$ for all $k \ge 1$. These facts imply that $(s_\phi^k(t))^2$ is uniformly continuous over $[0,T]$ for all $k \ge 1$. On the other hand, $(s_\phi^k(t))^2$ satisfies $\lim_{k\to\infty}\int_0^T (s_\phi^k(t))^2 dt = 0$ from (23) of lemma 2. We can now conclude, by using similar argument for Barbalat's lemma (e.g., Lemma 3.2.6 in [22]), that $\lim_{k\to\infty}(s_\phi^k(t))^2 = 0$ for all $t \in [0,T]$.

(t3) Since $\lim_{k\to\infty} s_\phi^k(t) = 0$, we have the bound of $s^\infty(t)$ by equation (3) as $\lim_{k\to\infty}|s^k(t)| = |s^\infty(t)| \le \phi^\infty(t) = e^{-\eta t}\varepsilon^\infty$, $\forall\ t \in [0,T]$. This proves (t3) of theorem 1.

(t4) In order to investigate the state tracking performance in the final iteration when (t1), (t2) and (t3) of the main theorem are achieved, we consider the state space equation

$$\dot{E}^\infty(t) = A_c E^\infty(t) + B_c s^\infty(t) \quad (34)$$

by the definition of error function $s^k(t)$ in (2). Note that the characteristic equation of A_c is equal to $\Delta(D) = 0$. Solution of (34) in time domain is given as

$$E^\infty(t) = \Phi(t)E^\infty(0) + \int_0^t \Phi(t-\tau)B_c s^\infty(\tau)d\tau \quad (35)$$

where the state transition matrix $\Phi(t)$ satisfies $\|\Phi(t)\| \le m_1 e^{-\delta t}$ for some suitable positive constant m_1 and $\|B_c\| = 1$. Taking norms on (35), it concludes (31) of (t4). Finally, tracking performance of $e_n^\infty(t)$ which is shown in (32) can be easily derived by using the definition of (2). Q.E.D.

Remark 1: In this adaptive law, the whole parameter profiles $W^{k-1}(t), \psi^{k-1}(t)$ and $\theta^{k-1}(t)$ in the previous iteration must be stored as most of the adaptive iterative learning algorithms [8]-[14], [19]-[21]. This requires large system memory, especially when compared with the traditional PID-type ILC. If the size of memory is concerned in a real implementation, we can adopt the modification of our adaptation laws by, for example, setting $\gamma_1 = 0$ since the numbers of fuzzy parameters are in general large. In other words, only time-domain adaptation for fuzzy parameter vector $W^k(t)$ so that the profile of previous iteration $W^{k-1}(t)$ is not required. It is easily shown that all the technical results obtained in theorem 1 still remain if $\gamma_1 = 0$.

IV. Simulation Example

In this example, we apply the proposed adaptive fuzzy iterative learning controller to an inverted pendulum system [20]. In the following discussions, $x_1^k(t)$ and $x_2^k(t)$ will be used to denote the angular displacement and velocity of the pole, respectively. The plant parameters of the inverted pendulum are chosen as $m_c = 1$ kg, $m = 0.1$ kg, and $\ell = 1$ m. The control objective is to make the state vector $X^k(t) = [x_1^k(t), x_2^k(t)]^\top$ to track the desired trajectory $X_d(t) = [x_d(t), \dot{x}_d(t)]^\top = [\sin(t), \cos(t)]^\top$ for $t \in [0, 10]$. The design parameters are chosen as $c_1 = 5$, $\eta = 5$, $\gamma_1 = \gamma_2 = \gamma_3 = 0.5$ and $\beta_1 = \beta_2 = \beta_3 = 10$, respectively. The fuzzy system in (11) is designed by the following linguistic description : Rule ℓ : IF $x_1^k(t)$ is $\mathcal{A}_{1\ell}$ and $x_2^k(t)$ is $\mathcal{A}_{2\ell}$, Then $f(x^k(t))$ is near $singleton\_w_\ell^k$, where $\mathcal{A}_{1\ell}, \mathcal{A}_{2\ell}$ are the fuzzy sets with membership function $\mu_{\mathcal{A}_{i\ell}}(x_i^k) =$

$exp[-a_{i\ell}(x_i^k(t)-b_{i\ell})^2]$ and $singleton\_w_\ell^k = w_\ell^k(t)$ is a crisp tunable value. Four rules are designed with the following parameter values $[a_{11}, a_{21}, a_{12}, a_{22}, a_{13}, a_{23}, a_{14}, a_{24}] = [1,1,1,1,1,1,1,1]$, $[b_{11}, b_{21}, b_{12}, b_{22}, b_{13}, b_{23}, b_{14}, b_{24}] = [-1,-1,1,1,-1,1,1,-1]$, $W^0(t) = [w_1^0(t), w_2^0(t), w_3^0(t), w_4^0(t)]^\top = [-1,1,-1,1]^\top$, $\forall t \in [0,10]$. The initial values of $\psi^0(t)$ and $\theta^0(t)$ are set to be $\psi^0(t) = 0.1$ and $\theta^0(t) = 0.1$ for all $t \in [0,10]$.

In this simulation, we arbitrarily set the initial position and velocity of the pole for the first ten iterations. The input disturbance is assumed to be the form of $d^k(t) = d_1 \sin w_1 t + d_2 \cos w_2 t$ where amplitude $d_1, d_2 \in (0,1)$ and frequency $w_1, w_2 \in (0,5)$. The supremum value and rms value of $s_\phi^k(t)$ with respective to iteration number k are shown respectively in Figure 1 (a) and (b). It is clearly observed that the asymptotic convergence proved in (t2) of theorem 1 is achieved. Since the supremum and rms values of $s_\phi^{10}(t)$ are small enough, we show the trajectory of $s^{10}(t)$, which satisfies (t3) of theorem 1, in Figure 1 (c). The boundedness of the control force $u^{10}(t)$ is given in Figure 1 (d). Even there exist varying initial state errors and random input disturbances, the nice state tracking performances after ten trials are found in Figure 1 (e) and (f).

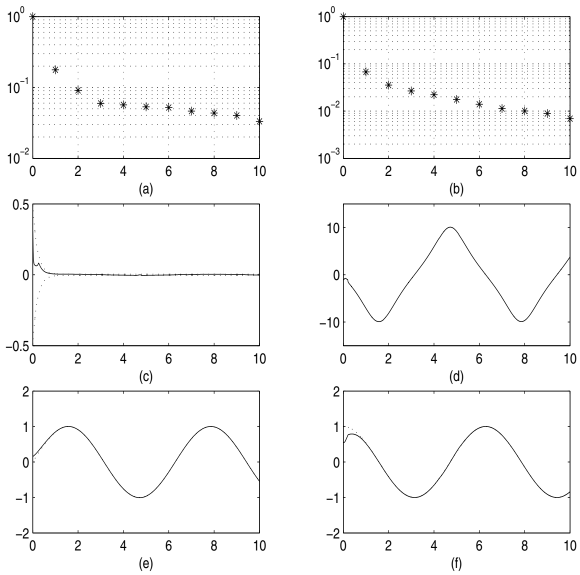

Figure 1 : design parameter $\gamma_i = 0.5$, $\beta_i = 10$,
(a) The supremum value of $s_\phi^k(t)$ versus iteration k;
(b) The rms value of $s_\phi^k(t)$ versus iteration k;
(c) $s^{10}(t)$ (solid) and $\pm\phi^{10}(t)$ (dotted) versus time t;
(d) Control input trajectory $u^{10}(t)$ versus time t;
(e) $x_d(t)$ (dotted) and $x_1^{10}(t)$ (solid) versus time t ;
(f) $\dot{x}_d(t)$ (dotted) and $x_2^{10}(t)$ (solid) versus time t.

V. CONCLUSIONS

To solve the repeated state tracking control problem of nonlinear systems with initial state resetting error and non-repeatable input disturbance, a novel adaptive fuzzy iterative learning control strategy is presented in this paper. Compared with the existing related works, the structure of the proposed learning controller is simpler and the adaptation algorithm can be designed without projection or deadzone mechanism. If the size of memory for storage of parameter profile is concerned in realization, it is easy to achieve this requirement by setting some weighting gains of the adaptation algorithm to be zero. Under this adaptive fuzzy iterative learning controller, we show that the norm of tracking error vector will asymptotically converge to a tunable residual set with all the internal signals and control parameters remaining bounded, even there exist large, varying initial state errors and non-repeatable input disturbance. Finally, an inverted pendulum system is used as a simulation example to demonstrate all the theoretical results derived.

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