Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Cooperative Control of Distributed Agents with Nonlinear Dynamics and Delayed Information Exchange: a Stabilizing Receding-Horizon Approach

Elisa Franco, Thomas Parisini, and Marios M. Polycarpou

Abstract— **This paper addresses the problem of cooperative control of a team of distributed agents with nonlinear discrete-time dynamics. Each agent is assumed to evolve in discrete-time based on locally computed control laws and by exchanging delayed state information with a subset of neighboring cooperating agents. The cooperative control problem is formulated in a receding–horizon (RH) framework, where the control laws depend on the local state variables (feedback action) and on delayed information gathered from cooperating neighboring agents (feedforward action). A rigorous stability analysis is carried out exploiting the stabilizing properties of the RH local control laws on one hand and input–to–state stability (ISS) arguments on the other hand. In particular, it is shown that, under suitable assumptions, each controlled agent is ISS under the action of the local control law. The stability of the team of agents is then proved by utilizing small-gain theorem results.**

I. Introduction

The design and analysis of decentralized control systems have been under research investigation for more than thirty years. Many problems falling into this category have been addressed with various mathematical tools, while new related issues continuously arise due to current trends such as the increasing size and complexity of control systems, the availability of spatially distributed sensors and actuators, and the need to come up with more autonomous systems.

When dealing with large scale systems, a key objective is to guarantee closed-loop stability, reducing the computational load stemming from a centralized approach. Starting with the notion of "fixed modes" introduced in the seventies for linear large scale systems [1], other investigations focused on the structure and size of interconnections [2]. Following the reasoning of this latter aspect, adaptive control [3], and more recently model predictive control [4] approaches have been proposed. Specific emphasis on the structural properties of decentralized controlled large-scale systems is given in the research work of D'Andrea and co-workers (see, for instance, [5]), finding applications in fields as flight formation and distributed sensors. Studies on topology independent control have also been recently reported [6].

Another related research direction in decentralized control considers the problem of controlling a team of dynamically decoupled cooperating systems. For instance, there have been some important theoretical results on the stability of swarms [7], but a considerable number of publications in this area focus on specific issues related to Uninhabited Autonomous/Air Vehicles (UAVs) applications. One of the possible approaches is the selection of a suitable cost function to be optimized in a model-predictive control fashion. Such cost can take into account several issues, such as collision avoidance and formation constraints, and may reward the tracking of a certain path. In [8], [9] and [10], the authors consider a two-degrees of freedom team of UAVs assigned to visit a certain number of points. The team of UAVs is controlled in a centralized receding–horizon (RH) framework; by exploiting global potential functions, certain stationarity properties of the generated trajectories are proved in the case of two agents searching for multiple targets. A RH control scheme has also been proposed in [11], [12], where a centralized problem is decomposed to allow local computations and feasibility issues are thoroughly examined; stability is obtained in [11] exploiting a hierarchical decomposition of the team in suitable subgraphs with assigned priorities. Coordination of a large group of cooperating nonlinear vehicles is considered in [13] and related works, where a centralized RH problem is decomposed and solved locally.

To help study the properties of cooperative systems, an ISS analysis has recently been proposed by several authors. In [14], [15] the concept of Leader to Formation Stability is developed. Issues arising in the study of non-holonomic vehicles using ISS are discussed in [16]. ISS tools have been succesfully applied to the case of networked systems with serial communication, where Nesic and Teel propose a new modeling and analysis framework [17], [18].

In this paper we consider a cooperative control problem for a team of distributed agents with nonlinear discrete-time dynamics. The problem formulation is based on a completely decentralized RH control algorithm, analyzed using an ISS approach; we generalize the approach presented in [19] to the nonlinear framework. Each agent evolves in discrete-time by means of locally computed control laws, exchanging delayed state information with a subset of neighboring cooperating agents. The cooperative control problem is first formulated in a RH framework, where the control laws depend on the local state variables (feedback action) and on delayed information gathered from neighboring agents (feedforward action). A rigorous stability analysis is carried out, exploiting the stabilizing properties of the RH local control laws on one hand, and ISS arguments on the other hand. In particular, it is shown that, under suitable assumptions, each locally con-

Corresponding author: Thomas Parisini. This work has been partially supported by the Italian Ministry for University and Research and the AFRL/VA and AFOSR Collaborative Center of Control Science (Grant F33615-01-2-3154).

E. Franco and T. Parisini are with the Dept. of Electrical, Electronic and Computer Engineering, DEEI-University of Trieste, Italy. ({efranco,parisini}@units.it)

M. M. Polycarpou is with the Dept. ECE, University of Cyprus, Nicosia 1678, Cyprus and the Dept. ECECS, University of Cincinnati, Cincinnati OH 45221-0030, USA. (mpolycar@ucy.ac.cy)

trolled agent is ISS under the action of the RH control law. Asymptotic stability of the team of agents is then proved by small–gain theorem reasonings: considering the information flow among the agents as a set of interconnections whose size is weighted through the control action computation, our result confirms that a suitable "interconnection" boundedness is necessary to guarantee stability. The proposed control scheme has a very general form; further investigation would be needed to explicitly consider formation issues typical of any UAV control problem. However, some preliminary simulation results regarding UAV team behavior will illustrate the concepts developed in this paper.

The paper is organized as follows. Section II formulates the multi-agent cooperative control problem in a discrete-time RH framework. The general stability proofs are presented and discussed in Section III and some simulation results are presented in Section IV.

II. Problem formulation

We consider a distributed dynamic system made of a set of M agents $\mathcal{A} \triangleq \{\mathcal{A}^i : i = 1, ..., M\}$, each described by the nonlinear time-invariant state equation:

$$x^i_{t+1} = f^i(x^i_t, u^i_t), \qquad t = 0, 1, 2, \dots \tag{1}$$

where $x^i_t \in \mathbb{R}^{n^i}$ and $u^i_t \in \mathbb{R}^{m^i}$ are the state and control vector at time t, for each $i = 1, ..., M$. We assume that all the M agents are synchronized and dynamically decoupled. The coupling arises due to the fact that they operate in the same environment and to the "cooperative" objective imposed on each agent by a cost function defined later on. To achieve some degree of cooperation, each agent $\mathcal{A}^i$ exchanges an information vector I_i with a given set of neighboring agents $\mathcal{G}^i \triangleq \{\mathcal{A}^j : j \in G^i\}$, where G^i denotes the set of indexes identifying the agents belonging to the set $\mathcal{G}^i$. Specifically, at each generic time–instant t, agent $\mathcal{A}^i$, $i = 1, ..., M$, receives from each cooperating neighbor $\mathcal{A}^j \in \mathcal{G}^i$ the value of its local state vector with a delay of Δ_{ji} time steps, that is, agent $\mathcal{A}^i$ receives the vector $x^j_{t-\Delta_{ji}}$ from agent $\mathcal{A}^j \in \mathcal{G}^i$. We group all inputs to agent $\mathcal{A}^i$ into a vector $\bar{v}^i_t$, at each time–instant t, defined as $\bar{v}^i_t \triangleq \mathrm{col}\,[\delta^i(1)\, x^1_{t-\Delta_{i1}}, ..., \delta^i(j)\, x^j_{t-\Delta_{ij}}, \dots, \delta^i(M)\, x^M_{t-\Delta_{iM}}]$, where $\delta^i(j) = 1$ only if $j \in G^i$. The size of vector $\bar{v}^i$ is equal to $n^{tot} = \sum_{i=1}^M n^i$. For each $i = 1, ..., M$ and for a given value of the state vector x^i_t at time–instant t, we now introduce the following finite–horizon (FH) cost function (in general, nonquadratic):

$$J^i_{FH}[x^i_t, \bar{v}^i_t, u^i_{t,t+N^i-1}, N^i, h^i_F(\cdot)] = \sum_{k=t}^{t+N^i-1} \left[h^i(x^i_k, u^i_k) + k^i(x^i_k, \bar{v}^i_t) \right] + h^i_F(x^i_{t+N^i}), \tag{2}$$

where N^i, $i = 1, \dots, M$ are positive integers denoting the lengths of the control horizons. Moreover, for each $i = 1, ..., M$, $h^i_F \in \mathcal{C}^1$ (continuously differentiable) is a suitable terminal cost function, with $h^i_F(0) = 0$. In (2) and in the following, we define $u^i_{t\tau} \triangleq \mathrm{col}\,(u^i_t, \dots, u^i_\tau)$ for both finite and infinite values of τ. At time–instant t, the vector $\bar{v}^i_t$ can be considered as a constant external input in the cost function. Finally, let us assume that $f^i, h^i, k^i \in \mathcal{C}^1$, with $f^i(0,0) = 0$, $h^i(0,0) = 0$, and $k^i(0,0) = 0$.

The local control strategy is based on a RH framework, and is obtained by solving the following problem objective:

Problem *2.1: . At every time instant $t \geq 0$ and for every agent $\mathcal{A}^i$, $i = 1, \dots, M$ described by (1), find the RH optimal control law $u^{i^{RH^\circ}}_t = \gamma^i_{RH^\circ}(x^i_t, \bar{v}^i_t) \in \mathbb{R}^{m^i}$, where $u^{i^{RH^\circ}}_t$ is the first vector of the control sequence $u^{i^{FH^\circ}}_t, \dots, u^{i^{FH^\circ}}_{t+N^i-1}$ (i.e., $u^{i^{RH^\circ}}_t \triangleq u^{i^{FH^\circ}}_t$), that minimizes cost (2) for the state $x^i_t \in \mathbb{R}^{n^i}$ and the cooperation vector $\bar{v}^i_t \in \mathbb{R}^{n^{tot}}$.*

The control objective is twofold, being "local"in the minimization of the partial cost given by the terms $\sum_{k=t}^{t+N^i-1} h^i(x^i_k, u^i_k) + h^i_F(x^i_{t+N^i})$; indeed a "cooperation" objective is the minimization of the remaining terms $\sum_{k=t}^{t+N^i-1} k^i(x^i_k, \bar{v}^i_t)$. Clearly, the dynamic behaviors of the agents are coupled, depending on the specific choice of the partial cost terms h^i, h^i_F and k^i.

III. Stability of the team of cooperating agents

The stability analysis will be carried out in two main steps. In Subsection III-A we shall consider a single agent $\mathcal{A}^i$ analyzing conditions that guarantee asymptotic stability, when a local RH control law is applied without considering the coupling effects. In Subsection III-B, we will prove that each agent is ISS with respect to the input given by the delayed incoming information from its neighbors. Finally, the team will be considered as a single dynamic system resulting from a feedback interconnection of ISS systems.

A. Stability properties of the single agents

Consider a generic agent $\mathcal{A}^i$ whose dynamics are described by (1). We will show that for each $\mathcal{A}^i$, $i = 1, 2, ..., M$, the origin as an equilibrium state of the controlled agent, is globally asymptotically stable (GAS). Moreover, we will also show that each $\mathcal{A}^i$ is ISS with respect to the inputs represented by the information vectors $\bar{v}^i_t$ received from its neighbors at each time–step t. Clearly we are now considering each agent as a "separate" dynamic system in the team: the input vectors $\bar{v}^i_t$ are "external" variables that are assumed not to depend on the behavior of its neighbors (i.e., the coupling between the agents is not taken into account). Let us now introduce some useful notations and assumptions. In general, denote by $\mathcal{Z}$ the class of compact sets, $\mathcal{S} \subset \mathbb{R}^q$, containing the origin as an *internal point*. This means that $\mathcal{S} \in \mathcal{Z} \Leftrightarrow \exists\, \lambda \in \mathbb{R}, \lambda > 0$ such that $N(\lambda) \subset \mathcal{S}$, where $N(\lambda) \triangleq \{x \in \mathbb{R}^q : \|x\| \leq \lambda\}$ and $\|\cdot\|$ is the Euclidean norm. The following assumptions are introduced for each agent $\mathcal{A}^i$, $i = 1, 2, ..., M$:

(i) The linear system $x^i_{t+1} = A^i x^i_t + B^i u^i_t$, obtained via the linearization of system (1) in a neighborhood of the origin, is stabilizable.

(ii) The transition cost functions h^i and k^i are such that there exists a strictly increasing function $\underline{r}^i \in$

$\mathcal{C}[\mathbb{R}^+,\mathbb{R}^+]$, with $\underline{r}^i(0) = 0$, such that[1], letting $\tilde{h}^i(x^i,u^i) \triangleq h^i(x^i,u^i) + k^i(x^i,0)$, we have $\tilde{h}^i(x^i,u^i) \geq \underline{r}^i(\|(x^i,u^i)\|), \forall x^i \in \mathbb{R}^{n^i}, \forall u^i \in \mathbb{R}^{m^i}$, where $(x^i,u^i) \triangleq \mathrm{col}\,(x^i,u^i)$. Moreover, there exist a strictly increasing function $\bar{r}^i \in \mathcal{C}[\mathbb{R}^+,\mathbb{R}^+]$, with $\bar{r}^i(0) = 0$, such that $\tilde{h}^i(x^i,u^i) \leq \bar{r}^i(\|(x^i,u^i)\|), \forall x^i \in \mathbb{R}^{n^i}, \forall u^i \in \mathbb{R}^{m^i}$.

(iii) $h_F^i(\cdot) \in \mathcal{H}(a^i,P^i)$, where $\mathcal{H}(a^i,P^i) \triangleq \{h_F^i(\cdot):\ h_F^i(x^i) = a^i {x^i}^\top P^i x^i\}$, for some $a \in \mathbb{R}$, $a > 0$, and for some positive-definite symmetric matrix $P^i \in \mathbb{R}^{n^i \times n^i}$.

(iv) For every neighborhood $N^i(\lambda^i) \subset \mathbb{R}^{n^i}$ of the origin of the state space, there exists a control horizon $M^i \geq 1$ such that there exists a sequence of control vectors $\left\{u_k^i \in \mathbb{R}^{m^i},\ k = t,\dots,t+M^i-1\right\}$ that yield a state trajectory $x_k^i \in \mathbb{R}^{n^i},\ k = t+1,\dots,t+M^i$ ending in $N^i(\lambda^i)$ (i.e., $x_{t+M^i} \in N^i(\lambda^i)$) for any initial state $x_t^i \in \mathbb{R}^{n^i}$.

(v) The optimal FH feedback control functions $\gamma_{FH^\circ}^i(x_k^i,\bar{v}_t^i,k),\ k = t,\dots,t+N^i-1$, which minimize cost (2), are continuous functions with respect to $x_k^i, \bar{v}_t^i$, for any $x_k^i \in \mathbb{R}^{n^i}$, $\bar{v}_t^i \in \mathbb{R}^{n^{tot}}$ and for any finite integer $N^i \geq 1$.

Denote by $J_{FH^\circ}^i[x_t^i,\bar{v}_t^i,N^i,h_F^i(\cdot)] \triangleq J_{FH}^i[x_t^i,\bar{v}_t^i,u_{t,t+N^i-1}^{i\,\circ},N^i,h_F^i(\cdot)]$ the cost corresponding to the optimal N^i-stage trajectory starting from x_t^i. The following theorem holds.

Theorem *3.1:* Consider agent $\mathcal{A}^i$, $i: 1 \leq i \leq M$. If assumptions (i) to (v) are verified, there exist a finite control horizon $\tilde{N}^i \geq M^i$, a positive scalar $\tilde{a}^i$ and a positive-definite symmetric matrix $P^i \in \mathbb{R}^{n^i \times n^i}$ such that, for every terminal cost function $h_F^i(\cdot) \in \mathcal{H}(a^i,P^i)$, with $a^i \in \mathbb{R},\ a^i \geq \tilde{a}^i$, the following properties hold:

(a) the origin as an equilibriun point of system (1) under the action of the RH optimal control law $\gamma_{RH^\circ}^i$ is GAS for $\bar{v}^i \equiv 0$;

(b) if we furtherly assume that the function f^i in (1) and the optimal RH control law $\gamma_{RH^\circ}^i$ are globally Lipschitz functions with respect to their arguments, then system (1) under the action of the RH optimal control law $\gamma_{RH^\circ}^i$ is ISS with respect to input $\bar{v}_t^i$. □

Part (a) of Theorem 3.1 is a generalization to the global stability case of the early results published in [20] (see also the related works [21], [22] and the references cited therein) showing that closed–loop stability properties are guaranteed by a suitable choice of the local FH cost. In Part (b) it is shown that, under some further assumptions, each agent shows some ISS property.

Proof. Let us consider a generic agent $\mathcal{A}^i$.

Part (a). The proof that 0 is an equilibrium state of the closed-loop system when the RH regulator is applied and when $\bar{v}_t^i = 0$ is straightforward and it is therefore omitted. Now, we show that the function

$$V^i(x^i) \triangleq J_{FH^\circ}^i[x^i,0,N^i,h_F^i(\cdot)], \quad x^i \in \mathbb{R}^{n^i} \tag{3}$$

is a Lyapunov function in $\mathbb{R}^{n^i}$ for system (1) driven by the RH regulator (for now, N^i and $h_F^i(\cdot)$ are not specified). Assumption (v) and the regularity hypotheses on the dynamic system (1) and on cost (2) ensure that $V^i(\cdot)$ is continuous with respect to all its arguments. Moreover, the control sequence $\{u_k^{i^{FH^\circ}} = 0, k = t,t+1,\dots,t+N^i-1\}$ minimizes cost (2) for $x_t^i = 0, \bar{v}_t^i = 0$, thus yielding $J_{FH^\circ}^i[0,0,N^i,h_F^i(\cdot)] = V^i(0) = 0$. By letting $x_t^{i^{FH^\circ}} = x_t^i, \forall x_t^i \in \mathbb{R}^{n^i} \setminus \{0\}$, we obtain

$$\begin{aligned} V^i(x_t^i) &\geq h^i(x_t^i,u_t^i) + k^i(x_t^i,0) = \\ &= \tilde{h}^i(x_t^i,u_t^i) \geq \underline{r}^i\left(\|(x_t^i,u_t^i)\|\right) \geq \underline{r}^i\left(\|x_t^i\|\right) > 0 \end{aligned} \tag{4}$$

Then $V^i(\cdot)$ is positive-definite. Moreover, according to (4) and the properties of function $\underline{r}^i(\cdot)$, it turns out that $V^i(\cdot)$ is radially unbounded, that is $\lim\limits_{\|x^i\|\to\infty} V^i(x^i) = \infty$. We have now to evaluate $\Delta V^i(x_t^i) \triangleq V^i(x_{t+1}^{i^{RH^\circ}}) - V^i(x_t^i)$, for x_t^i and $x_{t+1}^{i^{RH^\circ}}$ belonging to the trajectory generated by the RH regulator and starting from a generic initial state $x_t^i \in \mathbb{R}^{n^i}$. The following identity clearly holds:

$$\begin{aligned} &J_{FH^\circ}^i[x_t^i,0,N^i+1,h_F^i(\cdot)] = \\ &\tilde{h}^i(x_t^i,u_t^{i^{RH^\circ}}) + J_{FH^\circ}^i[x_{t+1}^{i^{RH^\circ}},0,N^i,h_F^i(\cdot)], \end{aligned} \tag{5}$$

$\forall x_t^i \in \mathbb{R}^{n^i}, \forall N^i \geq 1$, where $u_t^{i^{RH^\circ}} = \gamma_{RH^\circ}^i(x_t^i,0) = u_t^{i^{FH^\circ}} = \gamma_{FH^\circ}^i(x_t^i,0)$. We need now the following lemma (the proof is not reported here due to space limitations).

Lemma *3.1:* There exist a positive-definite symmetric matrix $P^i \in \mathbb{R}^{n^i \times n^i}$, a control horizon $\tilde{N}^i \geq M^i$, and a positive scalar $\tilde{a}^i$ such that

$$J_{FH^\circ}^i[x_t^i,0,N^i,h_F^i(\cdot)] \geq J_{FH^\circ}^i[x_t^i,0,N^i+1,h_F^i(\cdot)], \tag{6}$$

$\forall x_t^i \in \mathbb{R}^{n^i}$, $\forall N^i \geq \tilde{N}^i$, $\forall h_F^i(\cdot) \in \mathcal{H}(a^i,P^i)$, with $a^i \in \mathbb{R},\ a^i \geq \tilde{a}^i$. □

It is worth noting that Lemma 3.1 specifies N^i and $h_F^i(\cdot)$ introduced in (3). From (5) and (6), it follows that $J_{FH^\circ}^i[x_t^i,0,N^i,h_F^i(\cdot)] \geq \tilde{h}^i(x_t^i,u_t^{i^{RH^\circ}}) + J_{FH^\circ}^i[x_{t+1}^{i^{RH^\circ}},0,N^i,h_F^i(\cdot)], \quad \forall x_t^i \in \mathbb{R}^{n^i}$, and then

$$\begin{aligned} \Delta V^i(x_t^i) &= J_{FH^\circ}^i[x_{t+1}^{i^{RH^\circ}},0,N^i,h_F^i(\cdot)] - \\ &- J_{FH^\circ}^i[x_t^i,0,N^i,h_F^i(\cdot)] \leq -\tilde{h}^i(x_t^i,u_t^{i^{RH^\circ}}) \leq \\ &\leq -\underline{r}^i\left(\left\|(x_t^i,u_t^{i^{RH^\circ}})\right\|\right) \leq -\underline{r}^i\left(\|x_t^i\|\right), \end{aligned} \tag{7}$$

$\forall x_t^i \in \mathbb{R}^{n^i}, x_t^i \neq 0$, with $\Delta V^i(0) = 0$, thus ending the proof of Part (a).

Part (b). We have to prove that the Lyapunov function $V^i(x^i)$, $i = 1,2,...,M$, is an ISS Lyapunov function, i.e. we have to show that:

($\star$) there exist two functions $\underline{\alpha}^i(\cdot), \bar{\alpha}^i(\cdot)$ of class $\mathcal{K}_\infty$ such that:

$$\underline{\alpha}^i(\|x^i\|) \leq V^i(x^i) \leq \bar{\alpha}^i(\|x^i\|), \quad \forall x^i \in \mathbb{R}^{n^i} \tag{8}$$

[1]When there will be no risk of confusion, notations will be simplified by dropping some subscript and/or superscript from the variables.

($\star\star$) there exist a function $\alpha^i(\cdot)$ of class $\mathcal{K}_\infty$, and a function $\sigma^i(\cdot)$ of class $\mathcal{K}$ such that:

$$\begin{aligned}\Delta V^i_{\bar{v}} &\triangleq V^i(f^i(x^i_t, \gamma^i(x^i_t, \bar{v}^i_t))) - V^i(x^i_t) \\ &= J^i_{FH^\circ}[f^i(x^i_t, \gamma^i(x^i_t, \bar{v}^i_t)), 0, N^i, h^i_F(\cdot)] - \\ &\quad -J^i_{FH^\circ}[x^i_t, 0, N^i, h^i_F(\cdot)] \\ &\leq -\alpha^i(\|x^i_t\|) + \sigma^i(\|\bar{v}^i_t\|)\end{aligned}$$

As to ($\star$), we can set $\underline{\alpha}^i \triangleq \underline{r}^i$ (see (4)) and by letting

$$\bar{\alpha}^i(\|x^i_t\|) \triangleq \sum_{k=t}^{t+N^i-1} \bar{r}^i(\|(x^i_k, u^i_k)\|) + h^i_F(x^i_{t+N^i})$$

we obtain immediately that $V^i(x^i) \leq \bar{\alpha}^i(\|x^i\|), \forall\, x^i \in \mathbb{R}^{n^i}$, thus showing that ($\star$) is satisfied.

Coming to ($\star\star$), f^i and γ^i being globally Lipschitz by assumption, from the previous definition $\Delta V^i_{\bar{v}} = J^i_{FH^\circ}[f^i(x^i_t, \gamma^i(x^i_t, \bar{v}^i_t)), 0, N^i, h^i_F(\cdot)] - J^i_{FH^\circ}[x^i_t, 0, N^i, h^i_F(\cdot)]$, it follows that

$$\begin{aligned}\Delta V^i_{\bar{v}} &\leq \bar{\alpha}^i(\|f^i(x^i_t, \gamma^i(x^i_t, \bar{v}^i_t))\|) - \\ &\quad -\underline{\alpha}^i(\|x^i_t\|) \leq \bar{\alpha}^i(L^i_f\, L^i_\gamma \|\bar{v}^i_t\|) - \underline{\alpha}^i(\|x^i_t\|), \\ &\leq \sigma^i(\|\bar{v}^i_t\|) - \underline{\alpha}^i(\|x^i_t\|)\end{aligned}$$

where L^i_f and L^i_γ denote the Lipschitz constants associated with f^i and γ^i, respectively. Then, also ($\star\star$) is satisfied and therefore the closed–loop system is ISS with respect to the input $\bar{v}^i_t$. ■

B. Stability properties of the team of agents

Let us now consider the agents as a team $\mathcal{A} = \{\mathcal{A}^i, i = 1, ..., M\}$ where each $\mathcal{A}^i$ is controlled by the locally–stabilizing RH control law solving Problem 2.1. Therefore, we can write $x^i_{t+1} = \tilde{f}^i(x^1_t, \bar{v}^i_t) \triangleq f^i(x^i_t, \gamma^i(x^i_t, \bar{v}^i_t)),\ i = 1, ..., M$. Then, let us rewrite the team of dynamical systems as a suitable interconnection of two composite systems. To this end, let $X_t \triangleq \mathrm{col}\,(x^1_t, \cdots, x^M_t)$ and $\bar{\mathcal{V}}_t \triangleq \mathrm{col}\,(\bar{v}^1_t, \cdots, \bar{v}^M_t)$. Hence the following state equation can be written, where $\tilde{F}(X_t, \bar{\mathcal{V}}_t) \triangleq \mathrm{col}\,[\tilde{f}^1(x^1_t, \bar{v}^1_t),\ \tilde{f}^2(x^2_t, \bar{v}^2_t),\ \ldots,\ \tilde{f}^M(x^M_t, \bar{v}^M_t)]$,

$$X_{t+1} = \tilde{F}(X_t, \bar{\mathcal{V}}_t) \tag{9}$$

Vector $\bar{\mathcal{V}}_t$ can be easily characterized as the output of a dynamic system taking into account the delayed state information exchanged between the agents. First, we set $\Delta \triangleq \max\{\Delta^{ij}, i, j = 1, ..., M, i \neq j\}$. Then we let $\mathcal{Z}_t \triangleq \mathrm{col}\,(X_t,\, \rho^1_t, \cdots, \rho^\tau_t, \cdots, \rho^\Delta_t)$, where the variables ρ are introduced to store the delayed states; specifically $\rho^1_{t+1} = X_t$ and $\rho^\tau_{t+1} = \rho^{\tau-1}_t, \tau = 2, \ldots, \Delta$. Hence, it follows that

$$\begin{cases}\mathcal{Z}_{t+1} = A\,\mathcal{Z}_t + B\,X_t, \\ \bar{\mathcal{V}}_t = C\,\mathcal{Z}_t.\end{cases} \tag{10}$$

The definition of the involved matrices is trivial, and thus omitted. Summing up, the state equation describing the dynamics of the team of agents can be written as a feedback interconnection between the dynamic systems (9) and (10). Let us now prove the following lemma.

Lemma *3.2:* Let us suppose that Assumptions in Theorem 3.1 are verified. Then dynamic systems (9) and (10) are provided with suitable ISS Lyapunov functions $V(X_t)$ and $V^D(\mathcal{Z}_t)$, respectively. □

Proof. Consider the Lyapunov function candidate $V(X_t) \triangleq \sum_{i=1}^M V^i(x^i_t)$ for the lumped system (9). From (8), it follows that $\sum_{i=1}^M \underline{\alpha}^i(\|x^i_t\|) \leq V(X_t) \leq \sum_{i=1}^M \bar{\alpha}^i(\|x^i_t\|)$. Clearly $\|x^i_t\| \leq \|X_t\|$, and thus $V(X_t) \leq \sum_{i=1}^M \bar{\alpha}^i(\|x^i_t\|) \leq \sum_{i=1}^M \bar{\alpha}^i(\|X_t\|) \leq \bar{\alpha}(\|X_t\|)$, where we set $\bar{\alpha}(\|X_t\|) \triangleq \sum_{i=1}^M \bar{\alpha}^i(\|X_t\|)$. Moreover $\sum_{i=1}^M \|x^i_t\| \leq \sum_{i=1}^M \|X_t\| = M\|X_t\|$. Then $\|X_t\| \geq \frac{1}{M}\sum_{i=1}^M \|x^i_t\|$ and $\|X_t\| \leq \sum_{i=1}^M \|x^i_t\|$. Hence, it follows immediately that (recall that for any $\mathcal{K}$ function γ it is always true that $\gamma(a+b) \leq \gamma(2a) + \gamma(2b)$ where $a, b > 0$) $\underline{\alpha}^i(\|X_t\|) \leq \underline{\alpha}^i(\sum_{i=1}^M \|x^i_t\|) \leq \sum_{i=1}^M \underline{\alpha}^i(M\|X_t\|)$ and then $\underline{\alpha}^i(\|X_t\|/M) \leq \underline{\alpha}^i(\frac{1}{M}\sum_{i=1}^M \|x^i_t\|) \leq \sum_{i=1}^M \underline{\alpha}^i(\|X_t\|)$ Therefore, letting $\underline{\alpha}(\|X_t\|) \triangleq \underline{\alpha}^i(\|X_t\|/M)$ for an arbitrarily chosen index i, we showed that $\underline{\alpha}(\|X_t\|) \leq V(X_t) \leq \bar{\alpha}(\|X_t\|)$. Let us now write

$$\begin{aligned}\Delta V &\triangleq \sum_{i=1}^M V^i(\tilde{f}^i(x^i_t, \bar{v}^i_t)) - \sum_{i=1}^M V^i(x^i_t) \leq \\ &\leq -\sum_{i=1}^M \underline{\alpha}^i(\|x^i_t\|) + \sum_{i=1}^M \sigma^i(\|\bar{v}^i_t\|)\end{aligned}$$

First, we have $-\sum_{i=1}^M \underline{\alpha}^i(\|x^i_t\|) \leq -\underline{\alpha}^i(\|X_t\|/M)$ and $\sum_{i=1}^M \sigma^i(\|\bar{v}^i_t\|) \leq \sum_{i=1}^M \sigma^i(\|\bar{\mathcal{V}}_t\|)$. Then, letting $\underline{\alpha}(\|X_t\|) \triangleq \underline{\alpha}^i(\|X_t\|/M)$ and $\sigma(\|\bar{\mathcal{V}}_t\|) \triangleq \sum_{i=1}^M \sigma^i(\|\bar{\mathcal{V}}_t\|)$, it follows that $\Delta V \leq -\underline{\alpha}(\|X_t\|) + \sigma(\|\bar{\mathcal{V}}_t\|)$ thus showing that V is an ISS Lyapunov function for the lumped system (9).

System (10), describing the effects of the time–delays in the information exchange variables, is ISS being an asymptotically stable linear system and a candidate ISS Lyapunov function is $V^D(\mathcal{Z}_t) \triangleq \|\mathcal{Z}_t\|^2$. It is easy to find two positive constants $\underline{a}^D$ and $\bar{a}^D$ such that $\underline{a}^D\|\mathcal{Z}_t\|^2 \leq V^D(\mathcal{Z}_t) \leq \bar{a}^D\|\mathcal{Z}_t\|^2$ and thus the first part of the definition of ISS Lyapunov function holds by defining $\underline{\alpha}^D(\|\mathcal{Z}_t\|) \triangleq \underline{a}^D\|\mathcal{Z}_t\|^2$ and $\bar{\alpha}^D(\|\mathcal{Z}_t\|) \triangleq \bar{a}^D\|\mathcal{Z}_t\|^2$, which are two $\mathcal{K}-$functions. Moreover

$$\begin{aligned}V^D(\mathcal{Z}_{t+1}) - V^D(\mathcal{Z}_t) = \|A\mathcal{Z}_t + BX_t\|^2 - \|\mathcal{Z}_t\|^2 \leq \\ \leq \|\mathcal{Z}_t\|^2_Q + \|X_t\|^2_{B^\top B}\ .\end{aligned}$$

Then $\Delta V^D \leq -\alpha^D(\|\mathcal{Z}_t\|) + \sigma^D(\|X_t\|)$ (the definitions of $\alpha^D(\cdot)$ and $\sigma^D(\cdot)$ are straightforward). ■

Recalling from (10) that $\bar{\mathcal{V}}_t = C\,\mathcal{Z}_t$, from the proof of Lemma 3.2, it follows immediately that the ISS Lyapunov functions $V(X_t)$ and $V^D(\mathcal{Z}_t)$ satisfy

$$V(X_{t+1}) - V(X_t) \leq -\tilde{\alpha}(V(X_t)) + \tilde{\sigma}(V^D(\mathcal{Z}_t)), \tag{11}$$

$$V^D(\mathcal{Z}_{t+1}) - V^D(\mathcal{Z}_t) \leq -\tilde{\alpha}^D(V^D(\mathcal{Z}_t)) + \tilde{\sigma}^D(V(X_t)), \tag{12}$$

where $\tilde{\alpha}(\cdot)$ and $\tilde{\alpha}^D(\cdot)$ are $\mathcal{K}_\infty$ functions, and $\tilde{\sigma}(\cdot)$ and $\tilde{\sigma}^D(\cdot)$ are $\mathcal{K}$ functions, respectively. It is easy to show that $\tilde{\alpha} \triangleq \alpha \circ (\bar{\alpha})^{-1}$, $\tilde{\sigma} \triangleq \sigma \circ (\underline{\alpha}^D)^{-1}$, $\tilde{\alpha}^D \triangleq \alpha^D \circ (\bar{\alpha}^D)^{-1}$, and $\tilde{\sigma}^D \triangleq \sigma^D \circ (\underline{\alpha})^{-1}$. Now, the following result about the stability properties of the team of cooperating agents can be immediately proved.

Theorem *3.2:* Suppose that Assumptions in Theorem 3.1 are verified. Let us also suppose that the following small gain condition holds (Id denotes the identity operator):

$$\tilde{\alpha}^{-1} \circ \tilde{\sigma} \circ \left(\tilde{\alpha}^D\right)^{-1} \circ \tilde{\sigma}^D < \mathrm{Id}\,. \tag{13}$$

Then the team of cooperating agents described by the interconnected dynamic equations (9) and (10) is GAS. □

Proof. The proof is very simple. Owing to the Assumptions made in Theorem 3.1, by Lemma 3.2 it follows that systems (9) and (10) are provided with ISS Lyapunov functions V and V^D satisfying inequalities (11) and (12). Then, Corollary 4.2 in [23] can be directly used showing that, if the small gain condition (13) is verified, then the feedback system resulting from the interconnection between systems (9) and (10) is GAS thus ending the proof. ■

Remark. It is worth noting that the small–gain condition (11) may turn out to be conservative in practice as it is typical of these kind of results. On the other hand, the generality of the problem makes it rather difficult to obtain tighter conditions without making restrictive assumptions on the structure of the agents' dynamics and on the cost function.

IV. Simulation results

This section shows some results on the applicability of the proposed methodology to the cooperative control problem of a set of UAVs. In Subsection IV-A an LQ framework is considered (see [19]),where the local control laws can be determined analytically. In Subsection IV-B, the case of UAVs with nonlinear dynamics is taken into account. No conceptual changes have to be considered owing to the generality of the approach; difficulties were though obviously encountered, due to the local minima arising in the on–line minimization of the cost functions.

A. Team of linear agents

In this subsection, a team composed by LTI systems will be considered, given by a set of simplified UAV's moving in $\mathbb{R}^3$. The objective of the distributed cooperative controller is to reach a certain formation around the origin, on the plane $z = 0$, maintaining the formation through the whole trajectory. The discretized state equations for each UAV take on the linear structure $x^i_{t+1} = A^i x^i_t + B^i u^i_t$, obtained from the discretization of linear damped double integrator equations, with sampling time $T = 0.1s$. The physical parameters, identical for all the agents, are the mass $m = 0.75\,Kg$ and the viscosity $\mu = 0.01\,Kg/m\,s$. Recalling that Δ^{ij} is the delay occurring in the information received by agent i from agent j, we set $\Delta^{ij} = 2T$ for all the agents except $\Delta^{21} = 6T$, $\Delta^{32} = \Delta^{54} = 4T$, $\Delta^{24} = 3T$. Agent 1 does not cooperate nor communicate, being the leader of the formation; moreover $\delta^{ii} = 0$. For the local cost function, we set the prediction horizons as $N^i = N = 5$ and equal weighting matrices for all the agents, limiting the cooperation terms. The RH control law can be derived analytically [19]. In Figure 1 the team trajectories are reported: the objective is to attain a formation along a line of $45°$ as followers of the leader (red agent), on the plane $z = 0$. The dashed lines represent the desired trajectories, while the solid lines render the actual behavior of the agents. The colored circles represent the positions of the vehicles taken each second, while the black circles are the desired position at the same time instants.

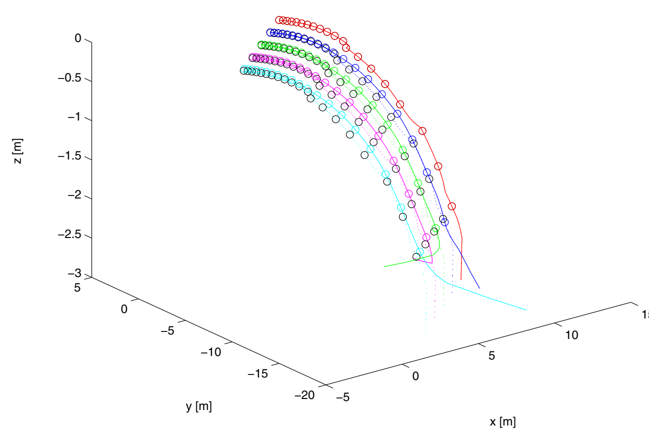

Fig. 1. Linear case: behavior of a team of 5 agents.

B. Team of nonlinear agents

In this section we will show some simulation results concerning a team of UAVs with nonlinear dynamics. As it is evident from the theoretical formulation of our problem, we are not explicitly considering a formation equilibrium point, nor constraints in the MPC algorithm; moreover, the information flow is limited and does not include the predicted trajectory exchange. A team of M vehicles will be considered, whose models and data are taken according to [24]; we will briefly recall the nonlinear equations describing the system:

$$\begin{aligned} m\ddot{x}^i &= -\mu_1 \dot{x}^i + (u^i_R + u^i_L)\cos(\theta^i), \\ m\ddot{y}^i &= -\mu_1 \dot{y}^i + (u^i_R + u^i_L)\sin(\theta^i), \\ J\ddot{\theta}^i &= -\mu_2 \dot{\theta}^i + (u^i_R - u^i_L) r_v. \end{aligned} \tag{14}$$

All the members of the team have the same physical parameters: the mass is $m = 0.75\,Kg$, the inertia is $J = 0.00316\,Kgm^2$, the linear friction coefficient is $\mu_1 = 0.15\,Kg/s$ and the rotational friction coefficient is $\mu_2 = 0.005\,Kgm^2/s$ and finally the radius of the vehicle is $r_v = 8.9cm$. The above equations have been discretized with a sampling time $T = 0.1s$. The state vector (defined in the usual way) of each agent will be from now on denoted as z^i_t. The communication topology is stationary, and the locally minimized cost function is quadratic; we set equal costs for all the agents, $N^i = N = 9$; the delays have been all set to $\Delta^{ij} = \Delta = 2$. The leader moves with a thrust of $0.1N$ on both sides, which is limited to the range $[0, 0.7]$. In Figure 2, the leader (blue agent) tracks a given moving strategy, and the followers are maintaining a triangular formation; the coloured dashed lines denote their desired trajectories; the triangles represent snapshots of the position and orientation of the vehicles at $t = 2s$, $t = 8s$, $t = 18s$, and the black triangles represent the desired configurations at the same time instants. Due to local minima that the non convex local cost function may have, a functional analysis of the costs should be done to guarantee the formation, assuming certain properties of the reference trajectories for each agent [25].

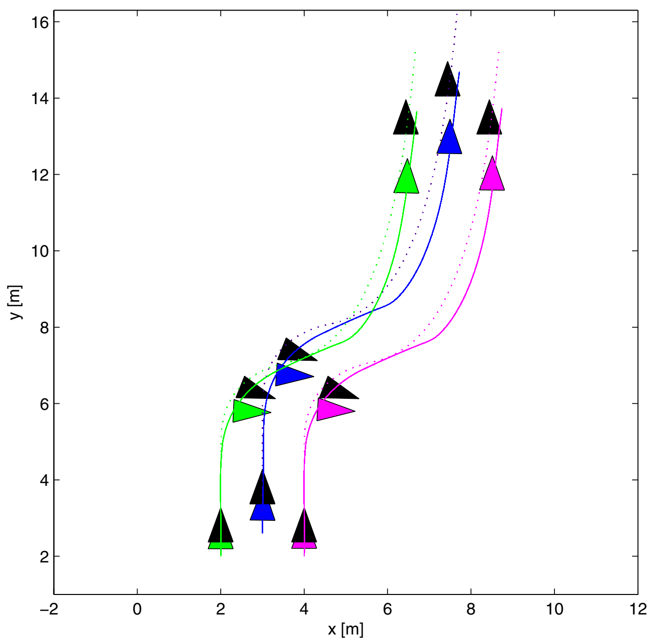

Fig. 2. Nonlinear case: behavior of the team following a leader.

V. Conclusions

The problem of cooperative control of a team of distributed agents with nonlinear discrete-time dynamics has been considered. The local control law takes on a feedback–feedforward structure and has been determined in a nonlinear RH framework where the cooperation objective has been embedded in the local cost functions to be minimized locally by each agent belonging to the team. Under some assumptions, the stability of the team of agents under the action of the local RH controllers has been shown using ISS and small–gain theorem arguments. Future research efforts will be devoted towards devising more constructive procedures for the determination of stabilizing RH controllers (see [22] in a standard centralized RH framework) and to consider the case where disturbances and uncertainties affect the communication between the agents of the team.

References

[1] S. H. Wang and E. Davidson, "On the stabilization of decentralized control systems," *IEEE Transactions on Automatic Control*, vol. 18, no. 5, pp. 473–478, 1973.

[2] D. D. Siljak, *Large-Scale Dynamic Systems*. New York: North-Holland, 1978.

[3] P. Ioannou, "Decentralized adaptive control of interconnected systems," *IEEE Trans. on Automatic Control*, vol. 31, no. 4, pp. 291 – 298, 1986.

[4] E.Camponogara, D. Jia, B. H. Krogh, and S. Talukdar, "Distributed model predictive control," *IEEE Control System Magazine*, vol. 22, no. 1, pp. 44–52, February 2002.

[5] R. D'Andrea and G. E. Dullerud, "Distributed control design for spatially interconnected systems," *IEEE Trans. on Automatic Control*, vol. 48, no. 9, pp. 1478–1495, 2003.

[6] R. Cogill and S. Lall, "Control design for topology-independent stability of interconnected systems," in *Proc. of the IEEE American Control Conference*, Boston, 2004, pp. 3717 – 3722.

[7] Y. Liu, K. Passino, and M. Polycarpou, "Stability analysis of M-dimensional asynchronous swarms with fixed communication topology," *IEEE Trans. on Automatic Control*, vol. 48, no. 1, pp. 76–95, 2003.

[8] W. Li and C. G. Cassandras, "A receding horizon approach for solving some cooperative control problems," in *Proc. of the IEEE Conf. on Decision and Control*, Las Vegas, 2002, pp. 3760–3765.

[9] ——, "Stability properties of a cooperative receding horizon controller," in *Proc. of the IEEE Conf. on Decision and Control*, Hawaii, 2003, pp. 492–497.

[10] ——, "Stability properties of a receding horizon controller for cooperating UAV's," in *Proc. of the IEEE Conf. on Decision and Control*, Bahamas, 2004, pp. 2905–2910.

[11] T. Kevicky, F. Borrelli, and G. J. Balas, "Hierarchical design of decentralized receding horizon controllers for decoupled systems," in *Proc. of the IEEE Conf. on Decision and Control*, Bahamas, 2004, pp. 1592–1597.

[12] ——, "A study on decentralized receding horizon control for decoupled systems," in *Proc. of the IEEE American Control Conference*, Boston, 2004, pp. 4921–4926.

[13] W. B. Dunbar and R. M. Murray, "Receding horizon control of multi-vehicle formation: a distributed implementation," in *Proc. of the IEEE Conf. on Decision and Control*, Bahamas, 2004, pp. 1995–2002.

[14] H. G. Tanner, G. J. Pappas, and V. Kumar, "Input-to-state stability on formation graphs," in *Proc. of the IEEE Conf. on Decision and Control*, Las Vegas, 2002, pp. 2439–2444.

[15] ——, "Leader to formation stability," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 3, pp. 2129–2136, 2004.

[16] X. Chen and A. Serrani, "Remarks on ISS and formation control," in *Proc. of the IEEE Conf. on Decision and Control*, Bahamas, 2004, pp. 177–182.

[17] D. Nesic and A. R. Teel, "Input-output stability properties of networked control systems," *IEEE Trans. on Automatic Control*, vol. 49, no. 10, pp. 1650–1667, 2004.

[18] ——, "Input to state stability properties of networked control systems," *Automatica*, vol. 40, no. 12, pp. 2121–2128, 2004.

[19] E. Franco, T. Parisini, and M. Polycarpou, "Cooperative control of discrete–time agents with delayed information exchange: a receding–horizon approach," in *Proc. of the IEEE Conf. on Decision and Control*, Bahamas, 2004, pp. 4274–4279.

[20] T. Parisini and R. Zoppoli, "A receding-horizon regulator for nonlinear systems and a neural approximation," *Automatica*, vol. 31, no. 10, pp. 1443–1451, 1995.

[21] G. De Nicolao, L. Magni, and R. Scattolini, "Stabilizing receding-horizon control of nonlinear time-varying systems," *IEEE Trans. on Automatic Control*, vol. 43, no. 6, pp. 1030–1036, 1998.

[22] T. Parisini, M. Sanguineti, and R. Zoppoli, "Nonlinear stabilization by receding horizon neural regulators," *Int. J. of Control*, vol. 70, no. 3, pp. 341–362, 1998.

[23] Z. P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica*, vol. 37, no. 6, pp. 857–869, 2001.

[24] Z. Jin, S. Waydo, E. B. Wildanger, M. Lammers, H. Scholze, P. Foley, D. Held, and R. M. Murray, "The second generation Caltech multi-vehicle wireless testbed," in *Proc. of the IEEE American Control Conference*, Boston, 2004, pp. 5321 – 5326.

[25] M. Egerstedt and X. Hu, "A study on decentralized receding horizon control for decoupled systems," *IEEE Trans. on Robotics and Automation*, vol. 17, no. 6, pp. 947–951, 2001.