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Numerical sensitivity of the backstepping adaptive tuning functions control design

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Abstract— **The backstepping-based adaptive tuning functions design is a control scheme for uncertain systems that ensures reasonably good stability and performance properties of the closed loop. The complexity of the controller makes inevitable the use of digital computers to perform the calculation of the control signal. This paper addresses the issue of the numerical sensitivity of the adaptive tuning functions via computer algebra systems. It is shown that while the increase of the design parameters may be desirable to achieve a good transient performance, it harms the control signal as this increase introduces large high-frequency components due to the numerical errors. Our results suggest that it is necessary a certain compromise between the choice of the design parameters and the numeric precision of the tools involved in the control design.**

I. INTRODUCTION

The last few years witnessed an increasing interest in the backstepping based designs for control of uncertain systems, especially their adaptive version [12]. Unlike most certainty-equivalence designs [10], the adaptive tuning functions offers the designer explicit bounds on the transient behavior of the closed loop [12, Chapter 10]. Since the publication of adaptive tuning functions design applied to linear systems [11], research in this field has focused mainly on robustness issues with respect to unmodelled dynamics and/or external disturbances [7], [8], [9], [17], [18], [27], [23], [24]. The digital implementation of the continuous backstepping adaptive counterpart has been considered in [20]. A modified tuning functions scheme that borrows elements from the certainty-equivalence controllers have been proposed in [28]. The parameter variation has been treated in [4] and schemes that do not assume the knowledge of the high-frequency gain were proposed in [15], [30]. A multivariable version of the tuning functions design was proposed in [13], [25], [2].

With respect to numerically reliable algorithms for solving control problems, we cannot forget that there is a continuing and growing need in the systems and control community for good algorithms and robust numerical software for increasingly challenging applications [22]. This way, in order to contribute to the accurate and efficient numerical solution of problems in control systems analysis and design we must focus on algorithm development, sensitivity and accuracy issues, large-scale computations and high-performance software [6], [3].

In our case, the complexity of the resulting nonlinear controller makes inevitable the use of a computer to deliver the control signal. Due to the representation of real numbers on a finite number of digits, numerical errors propagate during the process of computing the control law. A crucial question that arises naturally is: how sensitive is the control signal to the numerical errors that are generated by the computational process?

This question was considered for the first time in [19] by means of numerical simulations. The sensitivity of the control law with respect to the numerical errors and the influence of the design parameters are analyzed in this paper with the help of symbolic calculus software. The general case is very complex and for the sake of simplicity, we will consider a third order linear system with unknown parameters in closed loop with the standard adaptive tuning functions design [12, Chapter 10]. However, the methodology used for this third order linear system can be applied to any linear system and the results can also be paralleled. With reasonable design parameters, it has been observed that the control signal is corrupted by large high-frequency components.

The paper is organized as follows: Section II presents the plant model and the backstepping tuning functions controller design for this plant, including the differential equations that must be solved simultaneously. The sensitivity of the control law with respect to all its variables is analyzed in Section III with the help of a symbolic calculus software. Finally, the paper will be ended by some conclusions in Section IV.

II. PLANT AND CONTROLLER DESIGN

Consider here a specific system as a prototype. For this system we summarize the control law to be implemented and analyzed.

A. Plant

We consider the linear single-input single-output system

$$y(s) = \frac{b_0}{s(s^2 + a_2 s + a_1)} u(s), \tag{1}$$

where the coefficients $a_1 = 5$, $a_2 = 1$, $b_0 = 1$ are unknown.

The control objective is to ensure the output $y(t)$ to asymptotically track a given smooth reference signal

$$y_r(t) = \frac{1}{p_1(s)p_2(s)} r(t),$$

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where $p_i(s) = \frac{s^2}{\omega_i^2} + \frac{2\xi_i s}{\omega_i} + 1,\ \xi_i = 0.7,\ \omega_i = 4.9\ (i = 1, 2)$ and $r(t)$ is the unit step.

B. Design procedure

We use the standard tuning functions design of [12, chapter 10], which is schematically outlined in the following control algorithm.

Error variables:

$$z_1 = y - y_r \tag{2}$$
$$z_2 = \lambda_2 - \hat{\varrho}\dot{y}_r - \alpha_1 \tag{3}$$
$$z_3 = \lambda_3 - \hat{\varrho}\ddot{y}_r - \alpha_2 \tag{4}$$

Stabilizing functions:

$$\alpha_1 = \hat{\varrho}\bar{\alpha}_1 \tag{5}$$
$$\bar{\alpha}_1 = -(c_1 + d_1)z_1 - \xi_2 - \bar{\omega}^{\mathrm{T}}\hat{\theta} \tag{6}$$
$$\alpha_2 = -\hat{b}_0 z_1 - \left[c_2 + d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right] z_2 + \beta_2 + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 \tag{7}$$
$$\alpha_3 = -z_2 - \left[c_3 + d_3\left(\frac{\partial\alpha_2}{\partial y}\right)^2\right] z_3 + \beta_3 + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\tau_3 - \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\frac{\partial\alpha_2}{\partial y}z_2\omega \tag{8}$$
$$\beta_2 = \frac{\partial\alpha_1}{\partial y}(\xi_2 + \omega^{\mathrm{T}}\hat{\theta}) + \frac{\partial\alpha_1}{\partial\eta}(A_0\eta + e_3 y) + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + k_2\lambda_1 + \frac{\partial\alpha_1}{\partial\lambda_1}(-k_1\lambda_1 + \lambda_2) + \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}}\right)\dot{\hat{\varrho}} \tag{9}$$
$$\beta_3 = \frac{\partial\alpha_2}{\partial y}(\xi_2 + \omega^{\mathrm{T}}\hat{\theta}) + \frac{\partial\alpha_2}{\partial\eta}(A_0\eta + e_3 y) + \frac{\partial\alpha_2}{\partial y_r}\dot{y}_r + \frac{\partial\alpha_2}{\partial\dot{y}_r}\ddot{y}_r + k_3\lambda_1 + \frac{\partial\alpha_2}{\partial\lambda_1}(-k_1\lambda_1 + \lambda_2) + \frac{\partial\alpha_2}{\partial\lambda_2}(-k_2\lambda_1 + \lambda_3) + \left(\ddot{y}_r + \frac{\partial\alpha_2}{\partial\hat{\varrho}}\right)\dot{\hat{\varrho}} \tag{10}$$

Tuning functions:

$$\tau_1 = (\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)e_1)z_1 \tag{11}$$
$$\tau_2 = \tau_1 - \frac{\partial\alpha_1}{\partial y}\omega z_2 \tag{12}$$
$$\tau_3 = \tau_2 - \frac{\partial\alpha_2}{\partial y}\omega z_3 \tag{13}$$

Parameter update laws:

$$\dot{\hat{\theta}} = \Gamma\tau_3 \tag{14}$$
$$\dot{\hat{\varrho}} = -\gamma_1 \mathrm{sgn}(b_0)(\dot{y}_r + \bar{\alpha}_1)z_1 \tag{15}$$

K-filters:

$$\dot{\eta} = A_0\eta + e_3 y \tag{16}$$
$$\dot{\lambda} = A_0\lambda + e_3 u \tag{17}$$

Adaptive control law:

$$u = \alpha_3 + \hat{\varrho}y_r^{(3)} \tag{18}$$

We present in Figure 1 simulations results of the closed-loop system (1) for two different values of the design parameters. The propagated errors are not significant for $c_i = d_i = 0.5,\ i = 1, 2, 3$ neither in the tracking error $z_1 = y - y_r$ or in the control law u. On the contrary, the control signal is clearly corrupted for $c_i = d_i = 2,\ i = 1, 2, 3$. We will give, in the following lines, a more detailed insight into these numerical issues.

III. SENSITIVITY ANALYSIS

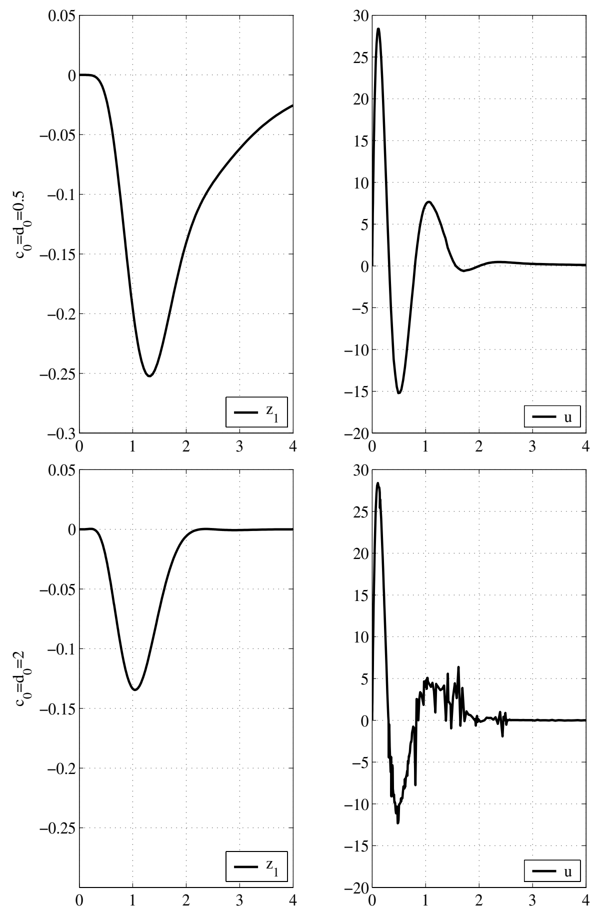

Fig. 1. The propagated errors are not significant for $c_i = d_i = \kappa = 0.5,\ i = 1, 2, 3$, in u (up), but the control signal is clearly corrupted for $c_i = d_i = \kappa = 2,\ i = 1, 2, 3$ (down).

As is well known, a mathematical model comprises independent variables, dependent variables, and relationships (for instance, equations or differential equations) between these quantities.

With respect to the algorithm described in Section II, the numerical methods needed to solve the differential equations of the K-filters (16)-(17) and the parameter update laws (14)-(15) introduce themselves numerical errors. The effects of such errors must be quantified in order to assess the respective model's range of validity.

As considered in [1], the most common procedure for assessing the effects of parameter variations on a model is to vary selected input parameters, rerun the code, and record the corresponding changes in the results, or responses, calculated by the code. The model parameters responsible for the largest relative changes in a response are then considered to be the most important for the respective response.

As can be seen in Section II, for any time t and for fixed design parameters $c_i, d_i > 0,\ i = 1, 2, 3$ and $\gamma > 0$ —these design parameters are chosen by the designer to achieve a good transient performance—, the control signal u can be computed as a function of the reference signal y_r and its derivatives, the K-filters $\eta = (\eta_1, \eta_2, \eta_3)^{\mathrm{T}}$ and $\lambda = (\lambda_1, \lambda_2, \lambda_3)^{\mathrm{T}}$, the estimated parameters $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)^{\mathrm{T}}$ and $\hat{\varrho}$ and the output signal y. We can then write

$$u(t) = f(y_r, \dot{y}_r, \ddot{y}_r, y_r^{(3)}, \eta^{\mathrm{T}}, \lambda^{\mathrm{T}}, \hat{\theta}^{\mathrm{T}}, \hat{\varrho}, y). \qquad (19)$$

The variables $\eta, \lambda, \hat{\theta}, \hat{\varrho}$ and y are explicitly related as can be seen in (14)-(17), that is, these variables are not independent and, for instance, any perturbation ε in the measure of the output y is immediately propagated through the filters and the estimated parameters. However, we are not analyzing the error propagation through the different signals. We are interested in the sensitivity analysis of the control law $u = f$ with respect to small variations in the value of its variables, although they are clearly dependent.

The influence of the perturbed variables with respect to the control signal will be different, as we will see in the next subsections.

A. Base of the analysis

It has been shown that the control signal u is a function of 15 variables, see equation (19). However, the reference signal y_r and its derivatives are external signals so that it will be assumed to be known exactly.

Our methodology is to perturb slightly the signals that the control law uses as inputs and observe the behavior of the closed loop. For this purpose, we use computer algebra systems that are packages that facilitate symbolic mathematics. The main advantages of this software are that they use exact arithmetic and do not suffer from loss of precision or significance; these packages also work symbolically and are speedy, efficient and reliable tools for performing long and tedious calculations.

In order to fully understand the sensitivity of the control law u, let us consider a *perturbation function* $\varepsilon : [a, b] \subset \mathbb{R}^+ \to \mathbb{R},\ 0 < a < b$. This function is introduced to simulate the effect of the numerical errors that are generated during the computational process or the measurements of the signals.

On one side, we consider the ideal case when the control problem is solved ideally without numerical errors. We consider then that $u(t)$ is the control law, $\eta(t)$ and $\lambda(t)$ are the K-filters, $\hat{\theta}(t)$ and $\hat{\varrho}(t)$ are the estimated parameters and $y(t)$ is the output signal.

On the other side, consider any of the variables of the function f, for instance y. For any time $t \in [a, b]$ we define the function $\tilde{u}_y$ as

$$\tilde{u}_y(t) = f(y_r(t), \cdots, \hat{\varrho}(t), y(t) + \varepsilon(t)).$$

For any time t, $\tilde{u}_y(t)$ can be considered as the perturbed control generated if *only* the variable $y(t)$ is perturbed by a quantity defined by $\varepsilon(t)$. For any other variable, the functions $\tilde{u}_x(t)$ are defined in a similar way.

Finally, we define the sensitivity function, $D_x(t)$ of u with respect to the variable x and associated to the perturbed function $\varepsilon(t)$ as

$$D_x(t) = \frac{|u(t) - \tilde{u}_x(t)|}{m},$$

where m is the maximum absolute value of the control signal u,

$$m = \max_{t \in [a,b]} |u(t)|.$$

As said before, the function $\varepsilon(t)$ can be considered as a function that, for any $t \in [a, b]$, contains random numerical errors that are generated by the computational process. This way, $D_x(t)$ is a certain measure of the relative error with respect to the control signal.

B. First consideration

In Figure 2 they are depicted the time history representations of the sensitivity functions $D_{\eta_1}, D_{\eta_2}, D_{\eta_3}, D_{\lambda_1}, D_{\lambda_2}, D_{\lambda_3}, D_{\hat{\theta}_1}, D_{\hat{\theta}_2}, D_{\hat{\theta}_3}, D_{\hat{\varrho}}$ and D_y in a logarithmic scale, with the choice of a constant perturbed function $\varepsilon(t) = 10^{-7}$ and design parameters $c_i = d_i = \gamma = 2,\ i = 1, 2, 3$.

Tables I and II contains the maximum values of the sensitivity functions $D_x(t)$, for all the variables, for a particular (constant) choice of the perturbed function and for both stable and unstable systems, respectively. Even for very small values of $\varepsilon(t)$, the sensitivity of the variable η is shown to be considerable, specially η_1, with relative error values near $4 \cdot 10^3\%$.

As a conclusion, although all the variables have a certain influence, the variable of the control law responsible for the largest relative changes is η_1 and it must be then considered to be the most important for the sensitivity analysis.

C. Second consideration

At this point, we have observed that the variable η_1 is the responsible for the largest relative changes.

A question that can arise naturally at this point is: how is the influence of the choice of the design parameters in the control signal u, if we reduce our analysis to the fact that only the perturbations over η_1 are considerable?

For any time instant t, fixed design parameter $c_i = d_i = \gamma = \kappa > 0,\ i = 1, 2, 3$, and if we consider

$$\tilde{u}_{\eta_1}(t) = f(y_r, \dot{y}_r, \ddot{y}_r, y_r^{(3)}, \eta^{\mathrm{T}} + e_1^{\mathrm{T}}\varepsilon, \lambda^{\mathrm{T}}, \hat{\theta}^{\mathrm{T}}, \hat{\varrho}, y),$$

the absolute error of the control design $|u(t) - \tilde{u}_{\eta_1}(t)|$ can be expressed as

$$|u(t) - \tilde{u}_{\eta_1}(t)| = \sum_{i=1}^{9} \varepsilon^i \sum_{j=0}^{12} \nu_{ij}(t)\kappa^j. \qquad (20)$$

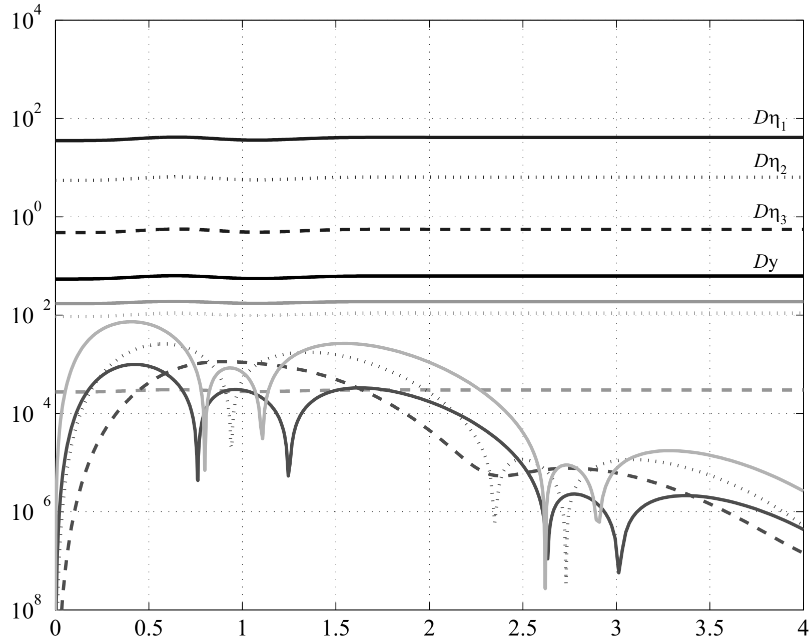

Fig. 2. Time history representation of $D_{\eta_1}, D_{\eta_2}, D_{\eta_3}, D_{\lambda_1}, D_{\lambda_2}, D_{\lambda_3}, D_{\hat{\theta}_1}, D_{\hat{\theta}_2}, D_{\hat{\theta}_3}, D_{\hat{\varrho}}$ and D_y in a logarithmic scale, with $\varepsilon(t) = 10^{-7}$ and $c_i = d_i = \gamma = 2,\ i = 1, 2, 3.$

TABLE I
MAXIMUM VALUES OF THE SENSITIVY FUNCTIONS $D_x(t)$, FOR $\varepsilon(t) = 10^{-7}$ AND $a_1 = 5, a_2 = b_0 = 1$ (STABLE SYSTEM).

variable	$\max_{t\in[0,4]} D_x(t)$	
x	$c_i = d_i = \gamma = 0.5$	$c_i = d_i = \gamma = 2$
η_1	$5.9634 \cdot 10^{-2}$	$4.1743 \cdot 10^{+1}$
η_2	$1.9197 \cdot 10^{-2}$	$6.4881 \cdot 10^{+0}$
η_3	$3.8149 \cdot 10^{-3}$	$5.6228 \cdot 10^{-1}$
λ_1	$4.6353 \cdot 10^{-4}$	$1.8945 \cdot 10^{-2}$
λ_2	$9.8813 \cdot 10^{-6}$	$1.1022 \cdot 10^{-2}$
λ_3	$7.4611 \cdot 10^{-6}$	$3.0353 \cdot 10^{-4}$
$\hat{\theta}_1$	$6.6353 \cdot 10^{-6}$	$9.9091 \cdot 10^{-4}$
$\hat{\theta}_2$	$5.1341 \cdot 10^{-6}$	$2.5940 \cdot 10^{-3}$
$\hat{\theta}_3$	$2.4707 \cdot 10^{-6}$	$1.1271 \cdot 10^{-3}$
$\hat{\varrho}$	$2.8861 \cdot 10^{-5}$	$7.3046 \cdot 10^{-3}$
y	$4.8361 \cdot 10^{-4}$	$6.3120 \cdot 10^{-2}$

TABLE II
MAXIMUM VALUES OF THE SENSITIVY FUNCTIONS $D_x(t)$, FOR $\varepsilon(t) = 10^{-7}$ AND $a_1 = -1, a_2 = b_0 = 1$ (UNSTABLE).

variable	$\max_{t\in[0,4]} D_x(t)$	
x	$c_i = d_i = \gamma = 0.5$	$c_i = d_i = \gamma = 2$
η_1	$5.8476 \cdot 10^{-2}$	$4.0583 \cdot 10^{+1}$
η_2	$1.8739 \cdot 10^{-2}$	$6.2983 \cdot 10^{+0}$
η_3	$3.7151 \cdot 10^{-3}$	$5.4416 \cdot 10^{-1}$
λ_1	$4.5985 \cdot 10^{-4}$	$1.8632 \cdot 10^{-2}$
λ_2	$1.0446 \cdot 10^{-5}$	$1.0809 \cdot 10^{-2}$
λ_3	$7.4021 \cdot 10^{-6}$	$2.9852 \cdot 10^{-4}$
$\hat{\theta}_1$	$8.0584 \cdot 10^{-6}$	$1.0365 \cdot 10^{-3}$
$\hat{\theta}_2$	$5.6264 \cdot 10^{-6}$	$2.8132 \cdot 10^{-3}$
$\hat{\theta}_3$	$2.9390 \cdot 10^{-6}$	$1.2153 \cdot 10^{-3}$
$\hat{\varrho}$	$3.9099 \cdot 10^{-5}$	$7.9106 \cdot 10^{-3}$
y	$4.7352 \cdot 10^{-4}$	$6.1479 \cdot 10^{-2}$

We assume ε to be small. This way, we are specially interested in the magnitude of both the coefficients $\nu_{1,12}$ of the term $\varepsilon\kappa^{12}$ (when $\kappa > 1$) and $\nu_{1,0}$ of the term $\varepsilon\kappa^0$ (when $\kappa \leq 1$), which are the most important terms responsible of the error propagation. The explicit expressions of these coefficients are:

$$\begin{aligned}\nu_{1,12}(t) =& \hat{\varrho}^9 \big(9.81 \cdot 10^5 - 8.24 \cdot 10^8 (y - y_r)\eta_1 \\ & -8.07 \cdot 10^7 (y - y_r)\eta_2 \\ & -1.28 \cdot 10^5 (y - y_r)\eta_3 \\ & -3.23 \cdot 10^6 y y_r + 2.26 \cdot 10^6 y^2 \\ & +9.70 \cdot 10^5 y_r^2 - 1.62 \cdot 10^5 \hat{\theta}_2 \\ & +1.60 \cdot 10^4 \hat{\theta}_3\big) \end{aligned} \quad (21)$$

$$\begin{aligned}\nu_{1,0}(t) =\ & 6.24 \cdot 10 \left(-\hat{\varrho}\hat{\theta}_3^2 + \hat{\theta}_1\hat{\theta}_3 + \hat{\varrho}\hat{\theta}_2^2\hat{\theta}_3\right) \\ & + 6.31 \cdot 10^2 \left(-\hat{\theta}_1\hat{\theta}_2 - \hat{\varrho}\hat{\theta}_2^3\right) - 4.00 \cdot 10^4 \hat{\varrho}\hat{\theta}_2 \\ & + 3.83 \cdot 10^3 \hat{\theta}_1 + 2.81 \cdot 10^5 \hat{\varrho} + 1.26 \cdot 10^3 \hat{\varrho}\hat{\theta}_2\hat{\theta}_3 \\ & + 3.96 \cdot 10^3 \hat{\varrho}\hat{\theta}_3 \end{aligned} \quad (22)$$

The time history of the coefficients $\nu_{1,0}$ and $\nu_{1,12}$ are depicted in Figure 3. It can be seen that $\nu_{1,0} \approx 3 \cdot 10^5$ and $\nu_{1,12} \approx 10^6$ and only with a choice of $0 < \kappa < 1$ the expression $|u(t) - \tilde{u}_{\eta_1}(t)|$ will be small. In the same sense, a choice of $\kappa > 1$ leads to a big absolute error of the control signal. This phenomena can be appreciated in Figure 1 for two different values of the design parameters, $\kappa = 0.5 < 1$ and $\kappa = 2 > 1$.

The explicit analytical expressions of $\tilde{u}$ in function of the perturbation function $\varepsilon(t)$ and the design parameters

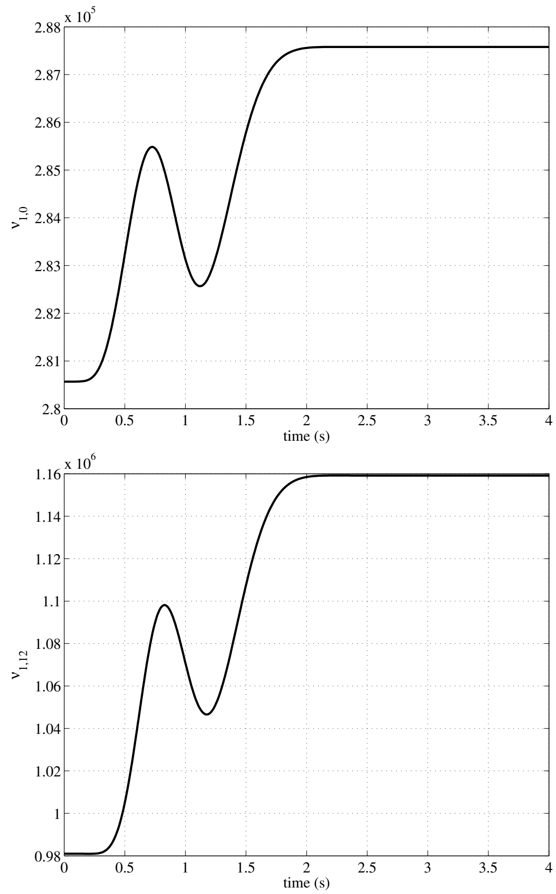

Fig. 3. Time history of the coefficients $\nu_{1,0}$ (up) and $\nu_{1,12}$ (down).

$c_i, d_i,\ i = 1, 2, 3$, and γ, are generated using the symbolic calculus software Maple[1].

IV. CONCLUDING REMARKS

This paper has focused on a analysis via numerical simulations and with the help of symbolic calculus of the sensitivity of the adaptive backstepping tuning functions control design for a relative degree three linear system —in both stable and unstable plants. Improving the transient behavior is done generally by increasing the design gains c_0, d_0 and γ. It has been observed that for $c_0 = d_0 = \gamma < 1$ the effect of small numerical error propagation remains negligible, while for $c_0 = d_0 > 1$ the control signal is corrupted by noise arising from the computational process. The analysis of the phenomena led to the following conclusion: to improve the tracking error performance $y - y_r$, an increase of the gains c_0 and d_0 is desirable. However, this increase the effect of the numerical errors in the inherent computations. The effect of the error propagation is an unnecessary actuator effort and a large control amplitude. The tuning function design has thus shown to be sensitive to the numerical errors that are generated by the computational process.

[1]Maple is a trademark of Waterloo Maple Inc.

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