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H_2 Optimal State–Feedback Control for Systems with Finite Jumps Corrupted by White Noise

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Abstract— **The state–feedback H_2 optimization problem for a class of systems with finite jumps subjected to additive and to multiplicative white noise is considered. It is proved that the optimal solution can be expressed in terms of the stabilizing solution of an appropriate system of coupled Riccati–type equations. An iterative convergent algorithm to compute this stabilizing solution is also presented. The theoretical developments are illustrated by two numerical examples.**

I. INTRODUCTION

The H_2 optimization problem received a major attention in the control literature of the last four decades. This interest is determined by practical reasons. The H_2 control theory is strongly related both to the linear quadratic gaussian problem and to the covariance matrix for the output of a stable system determined by a white noise excitation. The design objective is to minimize this gain using a stabilizing feedback controller. A wide variety of results are actually available in the H_2 optimization problem for deterministic systems (see *e.g.* [6], [12] and their references) and for stochastic systems (*e.g.* [4], [5]). Specific H_2 type control problems for systems with finite jumps have been also studied (*e.g.* [11], [13]). This class of systems is of practical importance since they are used in modelling sampled–data systems resulting when digital controllers are used to control continuous–time plants ([15]). In [7], a linear quadratic problem for sampled–data systems subjected to multiplicative white noise is solved. The multiplicative noise perturbations are frequently considered in the literature to model stochastic uncertainty (see *e.g.* [9], [16]).

The aim of the present paper is to determine the optimal solution of the H_2 state–feedback problem for a class of linear systems with jumps, corrupted by additive and multiplicative white noise. The paper is organized as follows: in Section 2 some definitions and preliminary results are presented and the H_2 optimal state–feedback control problem is formulated. Section 3 is devoted to the computation of the H_2 performance index corresponding to an exponentially stable in mean square system with jumps corrupted by additive and multiplicative white noise. This index is expressed in terms of the bounded solution to a Lyapunov–type system with jumps. The main result of the paper is proved in Section 4 where the optimal solution of the H_2 state–feedback problem is derived. It is shown that this solution is a state–feedback gain which is also optimal in the class of all higher order stabilizing controllers. This gain depends on the stabilizing solution of a specific Riccati–type system with jumps. An iterative convergent procedure to solve such systems together with two illustrative numerical examples are presented in Section 5. The paper ends with a concluding section in which some final remarks and comments are presented.

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II. THE H_2 STATE–FEEDBACK OPTIMIZATION PROBLEM

Consider the following system with finite jumps subjected to additive and multiplicative white noise:

$$\begin{aligned} dx(t) &= A_0x(t)\,dt + A_1x(t)\,dw(t) + Gdv(t)\,, \quad t \neq ih \\ x\left(ih^+\right) &= A_dx(ih) + B_du(i) + G_dv_d(i)\,, \quad i = 0,1,... \\ z(t) &= Cx(t) \\ z_d(i) &= C_dx(ih) + D_du(i) \\ y(ih) &= x(ih) \end{aligned} \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $h > 0$ is the sampling period, $u \in \mathbf{R}^m$ denotes the discrete–time control, $z \in \mathbf{R}^p$ and $z_d \in \mathbf{R}^{p_d}$ are the continuous–time and the discrete–time controlled outputs, respectively. It is assumed that the measured output coincides with the state vector at instants ih, $i = 0,1,...$. One assumes that $w(t) \in \mathbf{R}$, $t \geq 0$ and $v(t) \in \mathbf{R}^r, t \geq 0$ are such that the pair $(v(t), w(t))$ is an $r+1$–dimensional standard Wiener process and $v_d(i) \in \mathbf{R}^{r_d}$, $i = 0,1,...$ is a sequence of independent random vectors on a probability space $\{\Omega, \mathcal{F}, \mathcal{P}\}$. It is also assumed that $v(t), w(t), t \geq 0$ and $v_d(i), i = 0,1,...$ are independent stochastic processes with zero mean and unitary second moments. Notice that the structure of system (1) with the first continuous–time equation not depending on the control variable u corresponds to the structure of a sampled–data system with finite jumps (see *e.g.* [10]). Denote by $\mathcal{F}_t$ the σ–algebra generated by the random variables $v(s), w(s)$ and $v_d(i)$, $0 \leq s \leq t$, $0 \leq ih \leq t$. The class of admissible controls consists of the set of the random vectors $u(i)$ with the following properties:

a) $\sum_{i=0}^{\infty} E\left[|u(i)|^2\right] < \infty$, where $E[\cdot]$ denotes the expectation of the random variable $[\cdot]$ and $|q|^2 := q^Tq$;

b) $u(i)$ is $\mathcal{F}_{ih}$–measurable for $i = 0,1,...$;

c) $\lim_{t\to\infty} E\left[|x_u(t)|^2\right] = 0$, where $x_u(t)$ denotes the solution of (1).

The conditions under which the class of admissible controls is nonempty will be discussed further. By virtue of the standard results from stochastic differential equations ([8], [14]), the system (1) has a unique $\mathcal{F}_t$–adapted solution for any initial condition $x_0 \in \mathbf{R}^n$ and for any admissible control $u(i)$, $i = 0, 1, \ldots$. Moreover, this solution is almost surely left continuous.

Definition 1: a) The stochastic system with jumps (1) is called *exponentially stable in mean square (ESMS)* if for $v(t) = 0, t \geq 0$ and $u(i) = 0, v_d(i) = 0, i = 0, 1, \ldots$ there exist $\alpha > 0$ and $\beta > 0$ such that

$$E\left[|x(t)|^2\right] \leq \beta e^{-\alpha t}|x(0)|$$

for all $x(0) \in \mathbf{R}^n$ and for all $t \geq 0$;

b) The system with jumps and with multiplicative white noise (1) is called *stabilizable* if there exists a state–feedback gain F such that the corresponding closed–loop system

$$\begin{aligned} dx(t) &= A_0 x(t)\,dt + A_1 x(t)\,dw(t), \quad t \neq ih \\ x\left(ih^+\right) &= (A_d + B_d F)\, x(ih) \end{aligned}$$

is ESMS;

c) The system (1) is called *detectable* if there exists a piecewise continuous matrix valued function $H(t)$, $t \geq 0$ and a matrix H_d such that the system:

$$\begin{aligned} dx(t) &= (A_0 + H(t)C)\, x(t)\,dt \\ &\quad + A_1 x(t)\,dw(t), \quad t \neq ih \\ x\left(ih^+\right) &= (A_d + H_d C_d)\, x(ih), \; i = 0, 1, \ldots \end{aligned}$$

is ESMS.

In order to characterize the stability and the stabilizability of a system with jumps corrupted with multiplicative white noise, the following two operators are introduced in [7]:

$$\mathcal{L} : \mathcal{S}_n \longrightarrow \mathcal{S}_n, \; \mathcal{L}_d : \mathcal{S}_n \longrightarrow \mathcal{S}_n$$

defined by

$$\mathcal{L}Y = A_0 Y + Y A_0^T + A_1 Y A_1^T \text{ and } \mathcal{L}_d Y = A_d e^{\mathcal{L}h} Y A_d^T,$$

respectively, where $\mathcal{S}_n$ denotes the space of the $n \times n$ symmetric matrices and $e^{\mathcal{L}t}$ is the exponential of the operator $\mathcal{L}$. The next useful result is proved [7].

Proposition 1: a) The system (1) is ESMS if and only if the eigenvalues of the operator $\mathcal{L}_d$ are located inside the unit disk $|\lambda| < 1$, $\lambda \in \mathbf{C}$;

b) The system (1) is stabilizable if and only if there exist $Y \in \mathcal{S}_n$, $Y > 0$ and $\Gamma \in \mathbf{R}^{m \times n}$ such that:

$$\begin{bmatrix} -Y & A_d e^{\mathcal{L}h} Y + B_d \Gamma \\ \left(A_d e^{\mathcal{L}h} Y + B_d \Gamma\right)^T & -e^{\mathcal{L}h} Y \end{bmatrix} < 0.$$

If the above condition is fulfilled then a stabilizing state–feedback is given by $F = \Gamma\left(e^{\mathcal{L}h} Y\right)^{-1}$.

Consider the following family of discrete–time proper controllers $\mathbf{G}_c$:

$$\begin{aligned} x_c(i+1) &= A_c x_c(i) + B_c u_c(i) \\ y_c(i) &= C_c x_c(i) + D_c u_c(i), \; i = 0, 1, \ldots \end{aligned} \tag{2}$$

where $x_c \in \mathbf{R}^{n_c}$, $u_c \in \mathbf{R}^n$, $y_c \in \mathbf{R}^m$. The controller $\mathbf{G}_c$ is completely defined by the quintuple $(n_c, A_c, B_c, C_c, D_c)$ where the integer $n_c \geq 0$ denotes the controller order. If $n_c = 0$, the controllers coincide with state–feedback gains.

By coupling the controller (2) to the system with jumps (1) such that $u_c(i) = x(ih)$ and $u(i) = y_c(i)$, $i = 0, 1, \ldots$ one obtains the resulting closed–loop system:

$$\begin{aligned} dx_{cl}(t) &= A_{0,cl} x_{cl}(t)\,dt + A_{1,cl} x_{cl}(t)\,dw(t) \\ &\quad + G_{cl}\,dv(t), \; t \neq ih \\ x_{cl}\left(ih^+\right) &= A_{d,cl} x_{cl}(ih) + G_{d,cl} v_d(i), \; i = 0, 1, \ldots \\ x_{cl}(t) &= C_{cl} x_{cl}(t) \\ z_{d,cl}(ih) &= C_{d,cl} x_{cl}(ih), \end{aligned} \tag{3}$$

where the following notation is used:

$$\begin{aligned} & x_{cl}(t) := \begin{bmatrix} x(t) \\ x_c(i+1) \end{bmatrix}, \; t \in (ih, (i+1)h\,], \\ & A_{0,cl} := \begin{bmatrix} A_0 & 0 \\ 0 & 0 \end{bmatrix}, \; A_{1,cl} := \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}, \; G_{cl} := \begin{bmatrix} G \\ 0 \end{bmatrix}, \\ & A_{d,cl} := \begin{bmatrix} A_d + B_d D_c & B_d C_c \\ B_c & A_c \end{bmatrix}, \; G_{d,cl} := \begin{bmatrix} G_d \\ 0 \end{bmatrix}, \\ & C_{cl} := \begin{bmatrix} C & 0 \end{bmatrix}, \; C_{d,cl} := \begin{bmatrix} C_d + D_d D_c & D_d C_c \end{bmatrix}. \end{aligned} \tag{4}$$

Definition 2: A proper controller of form (2) is called *stabilizing* if the closed–loop system (3) is ESMS.

The H_2 optimal problem considered in this paper consists in determining a stabilizing controller $\mathbf{G}_c$ of form (2), or equivalently a quintuple $(n_c, A_c, B_c, C_c, D_c)$ such that the performance index:

$$J_{\mathbf{G}_c} = \lim_{T \to \infty} \frac{1}{T} E\left[\int_0^T |z(t)|^2\,dt + \sum_{i=0}^{\left[\frac{T}{h}\right]-1} |z_d(i)|^2\right] \tag{5}$$

is minimized, $\left[\frac{T}{h}\right]$ denoting the integer part of $\frac{T}{h}$.

III. PERFORMANCE INDEX COMPUTATION

Consider the stochastic system with jumps obtained by (1) taking $u(i) = 0, i = 0, 1, \ldots$, namely:

$$\begin{aligned} dx(t) &= A_0 x(t)\,dt + A_1 x(t)\,dw(t) \\ &\quad + G\,dv(t), \quad t \neq ih \\ x\left(ih^+\right) &= A_d x(ih) + G_d v_d(i), \quad i = 0, 1, \ldots \\ z(t) &= C x(t) \\ z_d(i) &= C_d x(ih). \end{aligned} \tag{6}$$

Based on the Itô formula (see *e.g.* [14]) one can prove that if (6) is ESMS then the Lyapunov–type system with jumps:

$$\begin{aligned} -\dot{P}(t) &= A_0^T P(t) + P(t) A_0 + A_1^T P(t) A_1 + C^T C, \\ &\qquad t \neq ih \\ P\left(ih^-\right) &= A_d^T P(ih) A_d + C_d^T C_d, \; i = 0, 1, \ldots \end{aligned} \tag{7}$$

has a unique right continuous h-periodic solution $P(t) \geq 0, t \geq 0$. The next result gives the value of the H_2 performance index of form (5) with respect to the solution of the Lyapunov system (7). It represents a generalization of the well–known corresponding results from deterministic and stochastic cases that shows the connection between the H_2 performance and the Gramians.

Theorem 1: If the stochastic system with jumps (6) is ESMS then:

$$\lim_{T\to\infty}\frac{1}{T}E\left[\int_0^T |z(t)|^2 dt+\sum_{i=0}^{\left[\frac{T}{h}\right]-1}|z_d(i)|^2\right]=\frac{1}{h}\left[Tr\,G_d^T P(0)G_d+Tr\,G^T\left(\int_0^h P(s)\,ds\right)G\right]$$

where $P(t)\geq 0, t\geq 0$ denotes the unique bounded solution of (7) and Tr denotes the trace.

Proof. The proof follows applying the Itô formula ([14]) for the function $V(x(t))=x^T(t)P(t)x(t)$ and integrating its differential between ih and $(i+1)h$.

IV. MAIN RESULT

The key role in solving the H_2 optimal state–feedback problem is played by the following system of coupled Riccati–type equations with finite jumps:

$$\begin{aligned}-\dot{X}(t) &= A_0^T X(t)+X(t)A_0+A_1^T X(t)A_1\\ &\quad +C^T C\\ X(ih^-) &= A_d^T X(ih)A_d-\left[A_d^T X(ih)B_d+C_d^T D_d\right]\\ &\quad\times\left[D_d^T D_d+B_d^T X(ih)B_d\right]^{-1}\\ &\quad\times\left[B_d^T X(ih)A_d+D_d^T C_d\right]+C_d^T C_d\end{aligned}\tag{8}$$

Definition 3: A symmetric solution $X(t), t\geq 0$ of the system (8) is called *stabilizing* if $D_d^T D_d+B_d^T X(ih)B_d>0$ and if the system with finite jumps and with multiplicative white noise:

$$\begin{aligned}dx(t) &= A_0x(t)dt+A_1x(t)dw(t),\quad t\neq ih\\ x(ih^+) &= \mathcal{A}_d x(ih),\ i=0,1,...\end{aligned}$$

with

$$\begin{aligned}\mathcal{A}_d &:= A_d-B_d\left[D_d^T D_d+B_d^T X(ih)B_d\right]^{-1}\\ &\quad\times\left[B_d^T X(ih)A_d+D_d^T C_d\right]\end{aligned}$$

is ESMS.

Remark 1: The stabilizing solution $X(t), t\geq 0$ of the system (8) is h-periodic.

The next result whose proof follows by standard arguments as in the deterministic and in the stochastic frameworks, gives sufficient conditions for the existence of the stabilizing solution of the system (8).

Proposition 2: If the system (1) is stabilizable and detectable then the Riccati–type system with jumps (8) has a stabilizing solution.

The next lemma will be used in the sequel. Its proof follows by direct algebraic computations.

Lemma 1: For any matrix $M\in\mathbf{R}^{m\times n}$, the Riccati–type system (8) can be written in the equivalent form:

$$\begin{aligned}-\dot{X}(t) &= A_0^T X(t)+X(t)A_0+A_1^T X(t)A_1\\ &\quad +C^T C,\ t\neq ih\\ X(ih^-) &= (A_d+B_dM)^T X(ih)(A_d+B_dM)\\ &\quad+(C_d+D_dM)^T(C_d+D_dM)\\ &\quad-(F-M)^T\left[D_d^T D_d+B_d^T X(ih)B_d\right]\\ &\quad\times(F-M)\qquad i=0,1,...\end{aligned}\tag{9}$$

where

$$\begin{aligned}F &:= -\left[D_d^T D_d+B_d^T X(ih)B_d\right]^{-1}\\ &\quad\times\left[B_d^T X(ih)A_d+D_d^T C_d\right].\end{aligned}\tag{10}$$

The solution of the H_2 optimal state–feedback problem is given by the following result:

Theorem 2: Assume that the Riccati–type system (8) has a stabilizing solution $X(t), t\geq 0$. Then the minimal H_2 performance index (5) is:

$$\min_{\mathbf{G}_c\ stabilizing} J_{\mathbf{G}_c} = \frac{1}{h}\left[Tr\,G_d^T X(0)G_d+Tr\,G^T\left(\int_0^h X(s)\,ds\right)G\right]\tag{11}$$

and it is provided by the optimal control $u(i)=Fx(ih), i=0,1,...$ where the gain F is given by (10).

Proof. Let $\mathbf{G}_c$ be a stabilizing n_c–degree controller of form (2). Denote by:

$$P_{cl}(t)=\begin{bmatrix}P_{11}(t) & P_{12}(t)\\ P_{12}^T(t) & P_{22}(t)\end{bmatrix},\ P_{11}(t)\in\mathbf{R}^{n\times n},\ P_{12}(t)\in\mathbf{R}^{n\times n_c}, P_{22}(t)\in\mathbf{R}^{n_c\times n_c}, t\geq 0\tag{12}$$

the solution of the Lyapunov–type system of form (7) associated to the closed–loop system (3) obtained by coupling the controller $\mathbf{G}_c$ to the system (1). Then $P_{cl}(t)$ satisfies the system:

$$\begin{aligned}-\dot{P}_{cl}(t) &= A_{0,cl}^T P_{cl}(t)+P_{cl}(t)A_{0,cl}+A_{1,cl}^T P_{cl}(t)A_{1,cl}\\ &\quad+C_{cl}^T C_{cl},\ t\neq ih\\ P_{cl}(ih^-) &= A_{d,cl}^T P_{cl}(ih)A_{d,cl}+C_{d,cl}^T C_{d,cl},\ i=0,1,...\end{aligned}\tag{13}$$

where the matrices corresponding to the closed–loop system are defined by (4). Using Theorem 1 it results that the H_2 performance index given by $\mathbf{G}_c$ is:

$$J_{\mathbf{G}_c}=\frac{1}{h}\left[Tr\,G_d^T P_{11}(0)G_d+Tr\,G^T\left(\int_0^h P_{11}(s)\,ds\right)G\right].\tag{14}$$

Denote by:

$$U_{11}(t):=P_{11}(t)-X(t), t\geq 0$$

where $X(t), t\geq 0$ is the stabilizing solution of the system (8) and by:

$$U(t):=\begin{bmatrix}U_{11}(t) & P_{12}(t)\\ P_{12}^T(t) & P_{22}(t)\end{bmatrix}, t\geq 0.\tag{15}$$

Applying Lemma 1 for $M = D_c$, the Riccati–type system (8) can be written as:

$$\begin{aligned} -\dot{X}(t) &= A_0^T X(t) + X(t) A_0 + A_1^T X(t) A_1 \\ &\quad + C^T C,\ t \neq ih \qquad (16) \\ X(ih^-) &= (A_d + B_d D_c)^T X(ih)(A_d + B_d D_c) \\ &\quad + (C_d + D_d D_c)^T (C_d + D_d D_c) \\ &\quad - (F - D_c)^T \left[D_d^T D_d + B_d^T X(ih) B_d\right] \\ &\quad \times (F - D_c) \qquad i = 0, 1, ... \end{aligned}$$

with F given by (10). Using the partition (12) of $P_{cl}(t)$ and the expressions (4) one obtains from (13), six equations with respect to $P_{11}(t), P_{12}(t)$ and $P_{22}(t), t \geq 0$. Then direct computations based on these equations together with (16) show that $U(t), t \geq 0$ verifies the system:

$$\begin{aligned} -\dot{U}(t) &= A_{0,cl}^T U(t) + U(t) A_{0,cl} + A_{1,cl}^T U(t) A_{1,cl}, \\ &\quad t \neq ih \qquad (17) \\ U(ih^-) &= A_{d,cl}^T U(ih) A_{d,cl} \\ &\quad + \Theta^T \left[D_d^T D_d + B_d^T X(ih) B_d\right] \Theta,\ i = 0, 1, ... \end{aligned}$$

where:

$$\Theta := \begin{bmatrix} -F + D_c & C_c \end{bmatrix}$$

with F given by (10). Since the closed–loop system is stable by assumption, it results that $U(t) \geq 0,\ t \geq 0$ and therefore $P_{11}(t) \geq X(t), t \geq 0$. Then from (14) one directly obtains (11) and thus the proof ends.

Remark 2: From Theorem 2 it results that the optimal solution to the H_2 state–feedback problem is a static gain ($n_c = 0$) which is also optimal in the class of all higher order stabilizing controllers of form (2).

V. COMPUTATIONAL ASPECTS

An iterative method to determine the stabilizing solution of the system of coupled Riccati–type equations with jumps (8) is given by the next result.

Proposition 3: Assume that the system (1) is stabilizable and detectable. Then the stabilizing solution of the Riccati–type system (8) can be determined by the following iterative procedure:

$$\begin{aligned} X_{k+1}(ih) &= e^{A_0^T h} X_{k+1}(ih^-) e^{A_0 h} \\ &\quad + \int_0^h e^{A_0^T s} C^T C e^{A_0 s} ds \\ &\quad + \int_0^h e^{A_0^T s} A_1^T X_k(s) A_1 e^{A_0 s} ds \\ X_{k+1}(ih^-) &= (A_d + B_d F_k)^T X_{k+1}(ih)(A_d + B_d F_k) \\ &\quad + (C_d + D_d F_k)^T (C_d + D_d F_k) \end{aligned}$$

with

$$\begin{aligned} F_k &= -\left[D_d^T D_d + B_d^T X_k(ih) B_d\right]^{-1} \\ &\quad \times \left[B_d^T X_k(ih^-) A_d + D_d^T C_d\right] \end{aligned}$$

$k = 0, 1, ...$ where $X_0(t) = 0,\ t \in (0, h)$, F_0 is a stabilizing state–feedback chosen such that the system of form (2) is ESMS and $X_k(t),\ t \in (0, h),\ k \geq 1$ is the solution of the differential equation (4) with the final condition $X_k(ih)$ at $t = ih$.

If the system (1) is not ESMS then an initial stabilizing state–feedback F_0 can be found using Proposition 1 b).

In order to illustrate the iterative numerical procedure defined in the above proposition, two numerical examples are presented.

Numerical example 1. Consider the following second order system with jumps corrupted with multiplicative white noise:

$$\begin{aligned} A_0 &= \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}, \\ A_d &= \begin{bmatrix} 0.8 & 0.7 \\ 0.4 & -0.5 \end{bmatrix},\ G = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \\ B_d &= \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ G_d = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 2 \end{bmatrix},\ C_d = \begin{bmatrix} 1 & 3 \end{bmatrix},\ D_d = 0 \end{aligned}$$

and the sampling period $h = 0.3$. Since the above system is ESMS one can choose $F_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}$. Then applying the iterative procedure given in Proposition 3 one obtains after 10 iterations, $F = \begin{bmatrix} -0.2866 & 0.1129 \end{bmatrix}$. This value of the of the optimal state–feedback gain has been computed for the accuracy level $\varepsilon = 10^{-5}$. Using formula (11) it results that the optimal H_2 index performance given by this state–feedback gain is $J = 296.3699$.

Numerical example 2. For this example the following third order system with jumps and state dependent noise is considered:

$$\begin{aligned} &A_0 = \begin{bmatrix} -2 & 1 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}, \\ &A_d = \begin{bmatrix} 2 & 0.3 & 0.4 \\ -0.3 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{bmatrix},\ B_d = \begin{bmatrix} 1 \\ 0.3 \\ 0.2 \end{bmatrix}, \\ &G = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ G_d = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \\ &C = \begin{bmatrix} 1 & 3 & 7 \end{bmatrix},\ C_d = \begin{bmatrix} 1 & 2 & 5 \end{bmatrix},\ D_d = 0. \end{aligned}$$

The sampling period is $h = 0.1$. Based on Proposition 1 one can see that the above system is not ESMS but it is stabilizable. Indeed, $F_0 = \begin{bmatrix} -1.75 & 0 & 0 \end{bmatrix}$ is a stabilizing state–feedback gain. Starting with this initial gain, the iterative procedure provides after 14 iterations the H_2 optimal state-feedback gain $F_0 = \begin{bmatrix} -1.7047 & -0.4029 & -0.5704 \end{bmatrix}$, determined with the accuracy level $\varepsilon = 10^{-5}$.

VI. CONCLUDING REMARKS

An H_2 optimization problem for systems with finite jumps corrupted by white noise is considered. It was shown that the H_2 performance index associated with an ESMS stochastic system with jumps and with multiplicative and additive white noise depends on the bounded solution of a Lyapunov–type system. This performance index is minimized by a state–feedback gain expressed in terms of the stabilizing solution of an appropriate system of coupled Riccati equations with jumps. The proof of this result stated in Theorem 2 reveals that the state–feedback gain is also optimal in the class of all higher–order stabilizing controllers. The numerical iterative algorithm given in Proposition 3 one allows to determine the stabilizing solution to the Riccati–type system with jumps and therefore the H_2 optimal solution.

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