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Semi-blind receiver for the fiber-wireless uplink channel

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***Abstract*— Nonlinear distorsion of the radio-over-fiber (ROF) link and multipath dispersion of the wireless link are the major factors limiting the performance of a fiber based wireless system. In order to recover the transmitted signal from the received signal, we propose a semi-blind approach based on an input precoding inducing redundancy. The approach suggested is deterministic and makes it possible to jointly estimate the channel, represented by means of a Wiener model, and the input sequence. The channel is estimated by using a pilot sequence then the symbols are estimated by solving a triangular system of polynomial equations.**

I. INTRODUCTION

The continuing increase in customer demand for broadband applications with mobile cellular and personal communications induces a big challenge for signal processing techniques applied to telecommunications. Optical fiber based wireless access is a key technology because of its potential to increase system capacity and to cover special areas such as tunnels, supermarkets, airports, etc. Such a technology is especially useful for indoor applications with micro and pico cellular architectures [1]. This technology combines two media: radio and optical. Typically, the optical part is used to interconnect a central radio processing facility with a remote radio antenna, the latter providing coverage to wireless broadband users. When the fiber is short and the radio frequency is only a few GHz, effects of fiber dispersion are negligible [2]. However, nonlinear distorsion of the electrical to optical conversion process becomes the major limitation. This distorsion can be modeled as a static nonlinearity.

Typically, the fiber-wireless access is constituted by two parts: the wireless channel and the nonlinear link. The wireless channel is modeled as a tapped delay line filter. The fiber-wireless uplink channel, i.e. from the portable unit to a base station, is then modeled by a linear filter (the wireless channel) followed by a static nonlinearity (the nonlinear link), i.e. a Wiener model.

Wiener model identification was mainly carried out for Gaussian inputs [3]. However, as a Wiener model can be viewed as a particular case of Volterra model with separable kernels, the identification algorithms dedicated to Volterra systems can also be used. In [2] correlation properties of pseudo-noise sequence are used to estimate the fiber wireless uplink channel. Equalization of such a channel is considered, in a supervised mode, by the same authors in [4].

This paper considers a semi-blind approach for identifying Single input Single output (SISO) fiber-wireless uplink channels under the assumption that the input belongs to a finite symbol set. A deterministic method is proposed for both channel estimation and symbol detection using a precoding.

The organization of the paper is as follows: In the next section, the precoding scheme is presented. In section 3 the channel estimation method is explained whereas the symbols detection procedure is described in section 4. The proposed methods are then illustrated by means of simulation results in section 5 before concluding the paper in section 6.

II. CHANNEL MODELING AND PRECODING SCHEME

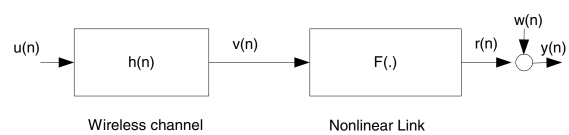

Fig. 1. The Fiber-Wireless Uplink

A representation of the fiber-wireless uplink is shown in figure 1. The wireless channel is modeled as a tapped delay line filter. The nonlinear link function $F(.)$ models the complete optical link from the laser diode to the optical receiver. Assuming that $F(.)$ is continuous within the given dynamic range then from the Weierstrass theorem, $F(.)$ can be approximated to an arbitrary degree of accuracy by a polynomial of degree P such as:

$$r(n) = \sum_{p=1}^{P} c_p v^p(n)$$

The model depicted by figure 1 is a Wiener model that corresponds to a Volterra model with separable and symmetric kernels given by:

$$h_p(i_1, \cdots, i_p) = c_p \prod_{j=1}^{p} h(i_j) \quad (1)$$

The corresponding input/output relation can then be written as:

$$y(n) = \sum_{p=1}^{P} \sum_{i_1=0}^{M} \cdots \sum_{i_p=0}^{M} h_p(i_1, \cdots, i_p) \prod_{j=1}^{p} u(n-i_j) + w(n) \quad (2)$$

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where u and y are respectively the transmitted and the noisy received signals, h_p is the p-th order Volterra kernel and w is the additive noise.

We consider a block transmission system where each $L\times 1$ block $U(n)$ is constituted by a pilot symbol $s(n)$ known by the receiver followed by $(L-1)$ information-bearing symbols $u(nL-i)$, $i=1,\cdots,L-1$ unknown from the receiver:

$$U(n)=\begin{pmatrix} s(n) & u(nL-1) & \cdots & u(nL-L+1)\end{pmatrix}^T \quad (3)$$

Each block $U(n)$ is precoded by a $(K\times L)$ precoding matrix $\mathbf{C}$. Selecting $K>L$ introduces redundancy, which will turn out to be beneficial for identification purpose. The $(K\times 1)$ precoded block is given by:

$$X(n)=\mathbf{C}U(n)=(x(nK)\ x(nK-1)\cdots x(nK-K+1))^T$$

It is obvious that increasing the redundancy ratio defined as $(K-L)/K$, or equivalently decreasing the ratio L/K, decreases the information rate. The precoding matrix used in this paper is that proposed by Choi [5] which ensures a minimal ratio of redundancy:

$$\mathbf{C}=\begin{pmatrix} C_M & 0_{(2M+1)\times(L-1)} \\ 0_{(L-1)\times 1} & \mathbf{I}_{(L-1)\times(L-1)}\end{pmatrix} \quad (4)$$

where C_M is the $(2M+1)\times 1$ vector given by:

$$C_M=\Big(\underbrace{0\cdots0}_{M}1\underbrace{0\cdots0}_{M}\Big)^T \quad (5)$$

The dimension of the data blocks before and after precoding are linked by $K=L+2M$. Note that the matrix $\mathbf{C}$ consists of only ones and zeros. No matrix multiplication is needed to produce the precoded data block, only $2M$ zero-inserting operations are necessary:

$$X(n)=\Big(\underbrace{0\cdots0}_{M}s(n)\underbrace{0\cdots0}_{M}u(nL-1)\cdots u(nL-L+1)\Big)^T \quad (6)$$

Note that the components $x(nK-i)$, $i=0,1,\cdots,K-1$ of $X(n)$ are given by:

$$x(nK-i)=\begin{cases} s(n) & \text{if} \quad i=M \\ u(nL-i+2M) & \text{if} \quad i=2M+1,\cdots,K-1 \\ 0 & \text{otherwise}\end{cases} \quad (7)$$

The insertion of a null guard time interval is not new in digital communications. DVB (Digital Video Broadcasting) systems, for example, send a null guard time interval at the beginning of each frame[6].

Let us consider the $(K\times 1)$ block of data $Y(n)$, received after input precoding, such as $Y(n)=(y(nK)\ y(nK-1)\ \cdots\ y(nK-K+1))^T$, with

$$y(nK-i) = \sum_{p=1}^{P}\sum_{i_1=0}^{M}\cdots\sum_{i_p=0}^{M}h_p(i_1,\cdots,i_p)\prod_{j=1}^{p}x(nK-i-i_j) +w(nK-i) \quad (8)$$

From (7), $x(nK-i_j)$ is different from zero if and only if $i+i_j=M$ or $i+i_j\geq 2M+1$, $j=1,\cdots,P$. Consequently, by taking into account the causality and the finite memory of the kernels, relation (8) can be decomposed into two parts:

- For $i=0,\cdots,M$

$$y(nK-i) = \sum_{p=1}^{P}h_p(M-i,\cdots,M-i)x^p(nK-M) +w(nK-i) \quad (9)$$

- For $i=M+1,\cdots K-1$

$$y(nK-i) = \sum_{p=1}^{P}\sum_{i_1=i}^{M+i}\cdots\sum_{i_p=i}^{M+i}h_p(i_1-i,\cdots,i_p-i) \times\prod_{j=1}^{p}x(nK-i_j)+w(nK-i) \quad (10)$$

As $x(nK-i_j)=0$ for $i_j=i,i+1,\cdots,2M$, $i=M+1,\cdots,K-1$, equation (10) is rewritten as

$$y(nK-i) = \sum_{p=1}^{P}\sum_{i_1=2M+1}^{M+i}\cdots\sum_{i_p=2M+1}^{M+i}h_p(i_1-i,\cdots,i_p-i) \times\prod_{j=1}^{p}x(nK-i_j)+w(nK-i) \quad (11)$$

One can note that in the equations (9) and (11), the symbols $x(nK-i_j)$, $i_j\geq K$, $j=1,\cdots,P$, do not belong to $X(n)$ but to previous blocks. So, to avoid block-level inter-symbol interference (ISI) we must take $i_j\leq M+i<K$ that means $i=0,\cdots,K-M-1$. Let us define the following truncated version of $Y(n)$ depending only on $X(n)$ and not on previous blocks:

$$\bar{Y}(n)=(y(nK)\ y(nK-1)\ \cdots\ y(nK-K+M+1))^T$$

with

$$y(nK-i)=\sum_{p=1}^{P}h_p(M-i,\cdots,M-i)s^p(n)+w(nK-i) \quad (12)$$

for $i=0,\cdots,M$ and

$$y(nK-i) = \sum_{p=1}^{P}\sum_{i_1,\cdots,i_p=1}^{i-M}h_p(2M-i+i_1,\cdots,2M-i+i_p) \times\prod_{j=1}^{p}u(nL-i_j)+w(nK-i) \quad (13)$$

for $i=M+1,\cdots,K-M-1$.

As the pilot sequence $s(n)$ is known from the receiver, we can first achieve the channel estimation using (12) then detect the information-bearing symbols using the estimated channel and (13).

III. Channel estimation

The relation (12) can be viewed as the input/ouput equation of a Single Input Mutliple outputs (SIMO) static polynomial channel the parameters of the subchannels of which are given by

$$\Theta_i=(h_1(M-i,\cdots,M-i)\cdots h_P(M-i,\cdots,M-i))^T$$

Thus estimating Θ_i, $i = 0, \cdots, M$, provides estimated values only of diagonal kernels coefficients. So another estimation procedure must be investigated for non diagonal kernels coefficients.

A. Estimation of diagonal kernels coefficients

1) Least squares approach: The estimation problem can be formulated as follows:

$$\hat{\Theta}_i = \arg\min_{\Theta_i} \|Y_i - \mathbf{V}\Theta_i\|^2 \quad (14)$$

where Y_i is a $N \times 1$ vector, N being the number of data blocks:

$$Y_i = \begin{pmatrix} y(-i) & y(K-i) & \cdots & y((N-1)K-i) \end{pmatrix}^T$$

$\mathbf{V}$ is the $N \times P$ Vandermonde matrix defined by:

$$\mathbf{V} = \begin{pmatrix} s(0) & \cdots & s^{P-1}(0) & s^P(0) \\ s(1) & \cdots & s^{P-1}(1) & s^P(1) \\ \vdots & \cdots & \vdots & \vdots \\ s(N-1) & \cdots & s^{P-1}(N-1) & s^P(N-1) \end{pmatrix} \quad (15)$$

This method requires the inferiority of the non-linearity order P to N. This condition is not too restrictive because the degree P, generally, doesn't exceed $P = 5$. If the pilot sequence $s(n)$ takes M_s distinct values, the identifiability condition is $M_s > P + 1$.
The optimal parameters are given by:

$$\hat{\Theta}_i = (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T Y_i$$

By considering the QR decomposition of the Vandermonde matrix $\mathbf{V}$:

$$\mathbf{V} = \mathbf{QR} \quad (16)$$

where $\mathbf{Q}$ is an orthogonal matrix of same size than $\mathbf{V}$ and $\mathbf{R}$ is an upper triangular matrix, we get:

$$\mathbf{R}\hat{\Theta}_i = \mathbf{Q}^{\mathbf{T}} Y_i \quad (17)$$

As $\mathbf{R}$ is a triangular matrix, the vector $\hat{\Theta}_i$ is then obtained by solving a triangular system of equations.
One can note that this system of equations (17) is to solve for $i = 0, \cdots, M$, with the same Vandermonde matrix $\mathbf{V}$ for each sub-channel i. So, the QR factorization (16) is carried out only once.

2) Estimation method using a complex periodic pilot sequence: Let us assume that $\{s(n)\}$ is a $T \geq P$ periodic pilot sequence given by:

$$s(n) = e^{j2\pi n/T}, \quad n = 0, 1, \cdots, N-1$$

Such a sequence has been proposed for semi-blind identification of linear communication channels [7]. By considering the noiseless case, the relation (12) can be written as follows:

$$y(nK-i) = \sum_{p=1}^{P} h_p(M-i, \cdots, M-i) e^{j\frac{2\pi}{T}pn}$$

Let us define $e_p(n) = e^{j\frac{2\pi}{T}pn}$. By defining the following scalar product [8]:

$$< e_p, e_q >= \frac{1}{T} \sum_{n=0}^{T-1} e_p(n) e_q^*(n)$$

one can conclude that the sequences $e_p(n)$, $p = 1, \cdots, P$, $n = 0, \cdots, T-1$ are orthonormal, i.e.

$$< e_p, e_q >= \begin{cases} 1, & \text{if } p = q \\ 0, & \text{otherwise} \end{cases}$$

Let us consider the LS criterion:

$$J = \sum_{n=0}^{N-1} \left| y(nK-i) - \sum_{p=1}^{P} h_p(M-i, \cdots, M-i)) e_p(n) \right|^2 \quad (18)$$

We assume that $N = N'T$ where N' is an integer. By setting to zero the derivative of J with respect to $h_p(M-i, \cdots, M-i)$, $p = 1, \cdots, P$, we get:

$$\sum_{n=0}^{N'T-1} \sum_{q=1}^{P} h_q(M-i, \cdots, M-i) e_p^*(n) e_q(n) = \sum_{n=0}^{N'T-1} y(nK-i) e_p^*(n)$$

or equivalently

$$\sum_{j=0}^{N'-1} \sum_{t=0}^{T-1} \sum_{q=1}^{P} h_q(M-i, \cdots, M-i) e_p^*(jT+t) e_q(jT+t) = \sum_{n=0}^{N'T-1} y(nK-i) e_p^*(n)$$

Knowing that $e_p(jT+t) = e_p(t)$, we get:

$$\sum_{j=0}^{N'-1} \sum_{t=0}^{T-1} \sum_{q=1}^{P} h_q(M-i, \cdots, M-i) e_p^*(t) e_q(t) = \sum_{n=0}^{N'T-1} y(nK-i) e_p^*(n)$$

Then

$$N'T h_p(M-i, \cdots, M-i) = \sum_{n=0}^{N'T-1} y(nK-i) e_p^*(n)$$

In other words, the diagonal coefficients of the Wiener model are obtained as follows: for $p = 1, \cdots, P$ and $i = 0, \cdots, M$

$$h_p(M-i, \cdots, M-i) = \frac{1}{N} \sum_{n=0}^{N-1} y(nK-i) e^{-j\frac{2\pi}{T}pn} \quad (19)$$

This method is very elegant and simple to implement. Theoretically, the *a priori* knowledge of the order P is not necessary. As the estimation of the coefficients $h_p(M-i, \cdots, M-i)$ is decoupled, if P is the effective non-linearity order, then $h_p(M-i, \cdots, M-i) = 0$, for $p > P$. However, it is necessary to note that this development was carried out in the noiseless case. In presence of noise, the estimation of the Wiener model is biased by the additive noise. This bias is equal to:

$$b_p = \frac{1}{N} \sum_{n=0}^{N-1} w(nK-i) e^{-j\frac{2\pi}{T}pn}$$

B. Estimation of non diagonal kernels coefficients

The knowledge of the diagonal kernels coefficients and the separability of the kernels are now used to develop a procedure for estimating the non diagonal kernels coefficients. The main result is stated by the following theorem:

Theorem: *The non diagonal kernels coefficients of a Wiener model are obtained from the diagonal kernels coefficients $h_p(i,\cdots,i)$,$p=1,\cdots,P$, as follows:*

$$h_p(i_1,\cdots,i_p)=\frac{1}{M+1}\sum_{i=0}^{M}\frac{h_p(i,\cdots,i)}{(h_1(i))^p}\prod_{j=1}^{p}h_1(i_j) \qquad (20)$$

Proof: The pth-order Volterra kernel coefficients associated with the Wiener model are given by (1):

$$h_p(i_1,\cdots,i_p)=c_p\prod_{j=1}^{p}h(i_j), \quad p=1,\cdots,P$$

where h is the impulse response of the linear filter and c_p are the polynomial coefficients. The diagonal kernels coefficients are such as:

$$h_p(i,\cdots,i)=c_p(h(i))^p, \quad i=0;\cdots,M$$

We deduce that:

$$c_p=\frac{h_p(i,\cdots,i)}{(h(i))^p}, \quad i=0;\cdots,M$$

By summing this relation for all the possible values of i, we get:

$$c_p=\frac{1}{M+1}\sum_{i=0}^{M}\frac{h_p(i,\cdots,i)}{(h(i))^p}$$

Knowing that $h_1(i)=c_1h(i)$, we deduce the following relation :

$$\frac{c_p}{c_1^p}=\frac{1}{M+1}\sum_{i=0}^{M}\frac{h_p(i,\cdots,i)}{(h_1(i))^p}$$

We have also:

$$\begin{aligned}h_p(i_1,\cdots,i_p) &= c_p\prod_{j=1}^{p}h(i_j)\\ &= c_p\prod_{j=1}^{p}\frac{h_1(i_j)}{c_1}\\ &= \frac{c_p}{c_1^p}\prod_{j=1}^{p}h_1(i_j)\\ &= \frac{1}{M+1}\sum_{i=0}^{M}\frac{h_p(i,\cdots,i)}{(h_1(i))^p}\prod_{j=1}^{p}h_1(i_j)\end{aligned}$$

■

IV. Symbols estimation

Once the channel matrix determined, the information-bearing symbols are to be estimated. A deterministic method based on the solution of a triangular system of polynomial equations and called TPS-root (*Triangular Polynomial System root*) is proposed.

Taking the causality of the kernels h_p into account, it is obvious that (13) has a triangular structure and can be solved by back substitution to estimate the transmitted symbols $u(nL-i)$, $i=1,\cdots,L-1$. In the noiseless case, the corresponding system of equations can be written as:

$$\begin{aligned}P_1\left(u(nL-1)\right) &= 0\\ P_2\left(u(nL-1),u(nL-2)\right) &= 0\\ \vdots &= \vdots\\ P_{L-1}\left(u(nL-1),\cdots,u(nL-L+1)\right) &= 0\end{aligned}$$

where each P_{i-M}, $i=M+1,\cdots,L+M-1$ is a polynomial given by:

$$\begin{aligned}&P_{i-M}(u(nL-1),\cdots,u(nL-i+M))\\ &\qquad=\sum_{p=1}^{P}y_p(nK-i)-y(nK-i)\end{aligned} \qquad (21)$$

with

$$\begin{aligned}y_p(nK-i) &= \sum_{i_1,\cdots,i_p=1}^{i-M}h_p(2M-i+i_1,\cdots,2M-i+i_p)\\ &\quad\times\prod_{j=1}^{p}u(nL-i_j)\end{aligned}$$

Let us assume that the first $(i-M-1)$ equations have been already solved with $\hat{u}(nL-k)$, $k=1,\cdots,i-M-1$, as respective solutions. By substituting these values in the expression of $P_{i-M}(u(nL-1),\cdots,u(nL-i+M))$ we get a polynomial in the single variable $u(nL-i+M)$:

$$\begin{aligned}&\tilde{P}_{i-M}(u(nL-i+M))\\ &\quad=P_{i-M}(\hat{u}(nL-1),\cdots,\hat{u}(nL-i+M+1),u(nL-i+M))\end{aligned}$$

Thus the symbol $u(nL-i+M)$ is a root of $\tilde{P}_{i-M}(u(nL-i+M))$. As $\tilde{P}_{i-M}(u(nL-i+M))$ is a Pth-order polynomial we can determine the set of its roots $\mathscr{R}_{i-M,n}=\left\{r_{p,n}\right\}_{p=1}^{P}$. Knowing the alphabet $\left\{\alpha_j\right\}_{j=1}^{Q}$ of the transmitted signal, the estimated symbol $\hat{u}(nL-i+M)$ is then obtained as:

$$\hat{u}(nL-i+M)=\arg\min_{r\in\mathscr{R}_{i-M,n}}\prod_{j=1}^{Q}(r-\alpha_j)(r-\alpha_j)^* \qquad (22)$$

Note that the principle of symbol estimation by solving polynomial equation was first suggested by Redfern and Tong Zhou [9] for Volterra systems. However in their method, the channel coefficients are assumed to be known. In the best of our knowledge the method proposed in this paper is the first one that allows to jointly estimate the channel and the symbols.

The TPS-root method can be applied for systems having a minimal phase first order kernel or not. However a possible drawback is due to its DFE (Decision Feedback Equalizer) nature. Each estimation error is then propagated on the other estimated symbols. This propagation is hopefully limited to the current block of data.

V. Simulation results

The estimation procedure described in this paper was used for jointly estimating the channel and the symbols in a semi-blind way. The simulated channel is constituted by a linear channel followed by a static nonlinearity. The linear filter is defined by $h = \begin{pmatrix} 1 & 0.5 & -0.2 \end{pmatrix}^T$ and the static nonlinearity is represented by the polynomial $P(u) = u + 0.3u^2 + 0.1u^3$. The input signal is a 4-PAM signal. The size of each transmitted block is equal to $L = 6$. The pilot sequences used in this paper are: a periodic complex signal, a 4-level pseudo-random sequence (PRS) and a gaussian sequence. The performances of the proposed estimation procedure are evaluated in terms of the NMSE (Normalized Mean Square Error):

$$NMSE = \frac{\sum_{p=1}^{P}\sum_{i_1=0}^{M}\cdots\sum_{i_p=i_{p-1}}^{M}(h_p(i_1,\cdots,i_p)-\hat{h}_p(i_1,\cdots,i_p))^2}{\sum_{p=1}^{P}\sum_{i_1=0}^{M}\cdots\sum_{i_p=i_{p-1}}^{M}h_p^2(i_1,\cdots,i_p)}$$

where $h_p(i)$ and $\hat{h}_p(i)$ are respectively the actual and the estimated channel coefficients.

The true and the estimated values of the Volterra kernels coefficients are given in tables I, II and III. These results were obtained by averaging 100 independent experiment with a periodic pilot sequence of period $T = 4$, a number of blocks $N = 40$ and a Signal to Noise Ratio SNR=30 dB.

TABLE I

First order kernel estimation

	True	Estimate (mean $\pm$ standard deviation)
$h_1(0)$	1.0	0.999 ± 0.004
$h_1(1)$	0.5	0.500 ± 0.002
$h_1(2)$	-0.2	$-0.200 \pm 6\times10^{-4}$

TABLE II

Second order kernel estimation

	True	Estimate (mean $\pm$ standard deviation)
$h_2(0,0)$	0.30	0.300 ± 0.012
$h_2(0,1)$	0.15	0.150 ± 0.006
$h_2(0,2)$	-0.06	-0.060 ± 0.002
$h_2(1,1)$	0.075	0.075 ± 0.003
$h_2(1,2)$	-0.03	-0.030 ± 0.001
$h_2(2,2)$	0.012	$0.012 \pm 5\times10^{-4}$

From tables I, II and III we can note the accuracy of the proposed estimation method. By considering the transmission of different data blocks number, we compare the use of the complex periodic sequence as pilot sequence with a 4-level PRS and a Gaussian sequence. For different values of SNR, we can note that for $N = 40$, the NMSE obtained with the periodic and the Gaussian pilot sequences are similar whereas the 4-level PRS exhibits lower performances (Table IV). By increasing the data number we get similar performances for periodic and 4-level PRS. That is particularly the case for high SNRs (Table V). However, the estimation method using a periodic pilot sequence is simpler to implement than the QR-based LS algorithm. Note again, that when only $N = T$ data are used, for $T = 4$, the QR-based method failed for both Gaussian and 4-level PRS whereas the pilot sequence allowed to get $NMSE = 2.62\times10^{-1}$ for SNR=20 dB.

TABLE III

Third order kernel estimation

	True	Estimate (mean $\pm$ standard deviation)
$h_3(0,0,0)$	0.100	0.099 ± 0.025
$h_3(0,0,1)$	0.050	0.049 ± 0.013
$h_3(0,0,2)$	-0.02	-0.020 ± 0.005
$h_3(0,1,1)$	0.025	0.025 ± 0.006
$h_3(0,1,2)$	-0.01	-0.010 ± 0.003
$h_3(0,2,2)$	0.004	0.004 ± 0.001
$h_3(1,1,1)$	0.012	0.012 ± 0.003
$h_3(1,1,2)$	-5×10^{-3}	-0.005 ± 0.001
$h_3(1,2,2)$	0.002	$0.002 \pm 5\times10^{-4}$
$h_3(2,2,2)$	-8×10^{-4}	$-8\times10^{-4} \pm 2\times10^{-4}$

TABLE IV

NMSE evaluation ($N = 40$)

Pilot sequence	SNR		
	10 dB	20 dB	30 dB
4-level PRS	9.498×10^{-1}	2.010×10^{-2}	1.500×10^{-3}
Gaussian	9.200×10^{-2}	7.200×10^{-3}	6.537×10^{-4}
Périodic ($T = 4$)	4.700×10^{-2}	8.700×10^{-3}	8.093×10^{-4}

To evaluate the symbol estimation procedure, we plot the SER (Symbol Error Rate) (Figure 2). We consider the transmission of $N = 100$ blocks of data. The pilot sequence is periodic with period $T = 4$. The proposed method is compared with the p-th order inverse method and the fixed point method respectively suggested in [10] and [11]. By evaluating the SER (Symbol Error Rate), we can conclude that the TPS-root method, suggested in this paper, outperforms the other methods at all SNRs. This result confirms that presented in [9].

VI. Conclusion

In this paper, joint channel and symbols estimation is carried out for Wiener type channels by means of a semi-blind approach. Using a specific input precoding induces a redundancy in the received signal that allows successive channel estimation and symbols detection. The channel estimation is solved in two steps. The diagonal kernels coefficients are

TABLE V

NMSE evaluation (SNR=30dB)

Pilot sequence	N		
	80	120	160
4-level PRS	8.400×10^{-3}	5.100×10^{-3}	3.600×10^{-3}
Gaussian	1.900×10^{-3}	1.700×10^{-3}	8.112×10^{-4}
Périodic ($T = 4$)	4.500×10^{-3}	4.100×10^{-3}	2.300×10^{-3}

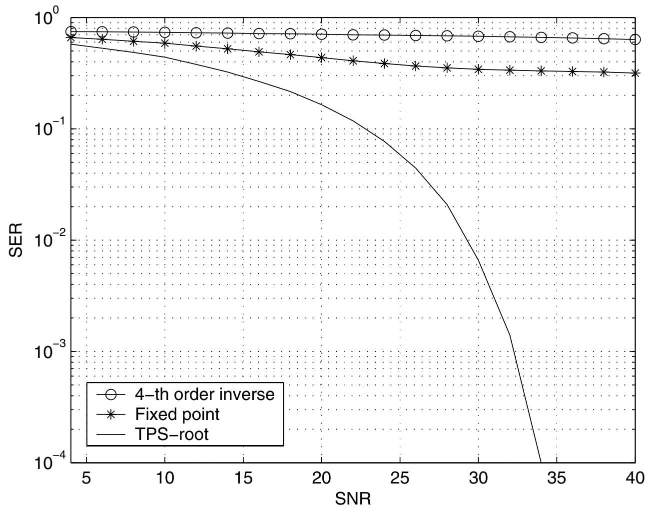

Fig. 2. SER versus SNR

first estimated by means of a Least Squares algorithm, then the estimated non diagonal kernels are deduced from the estimated diagonal ones. The symbols estimation is obtained in solving a triangular system of polynomial equations. The proposed methods are simple to implement and give good results with the drawback to reduce the transmission rate. These methods are deterministic and their robustness to noise remains to be investigated.

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