Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Delay-dependent H_∞ Variable Structure Control of Uncertain Singular Time-delay Systems: An LMI Approach

Min Li, Shuqian Zhu, Jie Sun and Zhaolin Cheng

***Abstract*— A delay-dependent H_∞ variable structure control is presented for uncertain singular time-delay systems. System with mismatched norm-bounded uncertainties and mismatched norm-bounded external disturbances are considered. In terms of linear matrix inequalities (LMIs), we give a delay-dependent sufficient condition for the existence of switching surfaces, on which sliding mode dynamics is regular, impulse free, and internally asymptotically stable and satisfies H_∞ performance. We also give an explicit formula of such linear switching surfaces. A state feedback controller is derived to guarantee system trajectories convergent to the linear switching surface.**

***Index Terms*— Singular time-delay system, mismatched uncertainties, delay-dependent, linear matrix inequality(LMI), switching surface, variable structure control (VSC).**

I. INTRODUCTION

Over the years, much attention has been paid to establishing variable structure control design algorithms. Almost all the published results on variable structure control design are based on the restrictive matching condition on disturbances or uncertainties[1]. VSC research has mostly focused on uncertain systems without time-delay, and their methods are hard to be applied to uncertain time-delay systems[2]-[4]. CHOI(2001) gave a switching surface design method for uncertain delay-free systems with mismatched norm-bounded uncertainties via LMI. Yuanqing Xia(2003) et al. suggested a robust sliding mode control for uncertain time-delay systems with matched disturbances; a delay-independent sufficient condition for the existence of a linear sliding surface is given in terms of LMI. Furthermore, few VSC research on uncertain singular time-delay systems is considered. To the best of our knowledge, it seems that there are no previous results on delay-dependent H_∞ variable structure control. This has motivated our research.

In this paper, we consider the problem of design linear switching surfaces and reaching controller for a class of singular time-delay systems with mismatched norm-bounded uncertainties and mismatched norm-bounded external disturbances. In terms of LMI, we propose a delay-dependent sufficient condition guaranteeing the existence of a linear switching surface such that the sliding mode dynamics restricted to the switching surface is not only regular, impulse free but internally asymptotically stable with disturbance attenuation. A state feedback controller is derived to guarantee system trajectories convergent to the linear switching surface. We also give an explicit formula of such linear switching surfaces, together with a design example.

Most of notations are fairly standard. Among them, I denotes the identity matrix. $\|\cdot\|$ denotes the Euclidean norm for a vector(or the matrix induced norm for a matrix), and inequality signs for matrices are used to express sign-definiteness of symmetric matrices. $diag\{\ldots\}$ denotes the block-diagonal matrix. $\lambda_{min}(\cdot)$ denotes the minimum eigenvalue of a matrix. The symbol $*$ represents blocks that are readily inferred by symmetry in some matrix expressions.

This work was supported by Project 973 of China under Grant G1998020300, National Natural Science Foundation under Grant 30271564.

Min Li is with the College of Electronic Information and Control Engineering, Shandong Institute of Light Industry, Jinan 250100, China lmqbjwendy@sohu.com.

Shuqian Zhu, Zhaolin Cheng are with the School of Mathematics and System Science, Shandong University, Jinan 250100, China.

Jie Sun is with the School of Information Engineering, Shandong University At Weihai, Weihai 264200, China.

II. PRELIMINARIES AND PROBLEM FORMULATION

Consider an uncertain singular time-delay system described by

$$(\Sigma): E\dot{x}(t) = (A+\Delta A)x(t) + (A_d+\Delta A_d)x(t-\tau) + (B+\Delta B)u(t) + (F+\Delta F)w(t) \quad (1)$$

$$z(t) = Cx(t) \quad (2)$$
$$x(t) = \phi(t)$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control input, $z(t) \in \mathbf{R}^p$ is the output. $w \in \mathbf{R}^l$ is the square-integrable external disturbance whose norm is bounded by the known scalar ℓ, i.e., $\| w(t) \| \leq \ell$. τ is a known positive scalar. $\phi(t)$ is a compatible vector valued continuous function. The constant matrices E, A, A_d, B, C and F are of appropriate dimensions, $rankB = m$, $rankE = p$, $0 < p < n$. Assume the uncertainties ΔA, ΔA_d, ΔB and ΔF are time-invariant matrices representing norm-bounded uncertainties and have the following form

$$\| \Delta A \| \leq \rho_1,\ \| \Delta A_d \| \leq \rho_2,\ \| \Delta B \| \leq \rho_3,\ \| \Delta F \| \leq \rho_4 \quad (3)$$

where ρ_i , i = 1, ..., 4, are all known positive constants.

Remark: In (3), we only known the uncertainties' bound. Some papers use uncertainties on e.g. A, A_d as follows

$$\begin{bmatrix} \Delta A & \Delta A_d \end{bmatrix} = DF \begin{bmatrix} E & E_d \end{bmatrix} \quad (4)$$

where D, E and E_d are known constant matrices. F is uncertain matrix satisfying $F^T F \leq I$.

It's obvious that uncertainties(4) can be always expressed as (3) and in reverse it doesn't hold.

Now, let us define the linear switching surface as

$$\Omega = \{x : \sigma(x) = SEx = 0\}$$

where $S \in \mathbf{R}^{m\times n}$. Referring to the previous results, we assume that the switching surface parameter matrix S satisfies the following properties:

P1: $[SB+S\Delta B]$ is nonsingular .

P2: Sliding mode dynamics restricted to the switching surface is regular, impulse free, internally asymptotically stable for all mismatched uncertainties and satisfies $\| T_{zw}(s) \|_\infty \le \gamma$.

Then, we will give some preliminary results about the nominal unforced disturbance-free system of (Σ)

$$\begin{array}{l} E\dot{x}(t) = Ax(t) + A_d x(t-\tau) \\ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{array} \tag{5}$$

Lemma 1[6]: The system (5) is regular, impulse free and asymptotically stable if there exist appropriate matrices $Q > 0$, $Z > 0$, $R \ge 0$ and general matrices P, Y satisfying the LMIs:

$$PE = E^T P^T \ge 0 \tag{6a}$$

$$\left[\begin{array}{cc} R & Y \\ Y^T & E^T Z E \end{array}\right] \ge 0 \tag{6b}$$

$$\left[\begin{array}{cc} \Theta + \tau A^T Z A & PA_d - Y + \tau A^T Z A_d \\ A_d^T P^T - Y^T + \tau A_d^T Z A & -Q + \tau A_d^T Z A_d \end{array}\right] < 0 \tag{6c}$$

where $\Theta = A^T P^T + PA + Q + \tau R + Y + Y^T$.

Proof: See the appendix.

The nominal unforced system of (Σ) can be written as

$$\begin{array}{l} E\dot{x}(t) = Ax(t) + A_d x(t-\tau) + Fw(t) \\ z(t) = Cx(t) \\ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{array} \tag{7}$$

Lemma 2: For given $\gamma > 0$, the system (7) is regular, impulse free, internally asymptotically stable and satisfies $\| T_{zw}(s) \|_\infty \le \gamma$, if there exist appropriate matrices $Q > 0$, $Z > 0$, $R \ge 0$ and general matrices P, Y satisfying the LMIs:

$$PE = E^T P^T \ge 0 \tag{8a}$$

$$\left[\begin{array}{cc} R & Y \\ Y^T & E^T Z E \end{array}\right] \ge 0 \quad , \quad \tau R + Y + Y^T \ge 0 \tag{8b}$$

$$\left[\begin{array}{ccccc} \Theta + C^T C & PA_d - Y & \tau A^T Z & PA_d & PF \\ * & -Q & \tau A_d^T Z & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -Q & 0 \\ * & * & * & * & -\gamma^2 I \end{array}\right] < 0 \tag{8c}.$$

Proof : By Lemma 1, it is obvious that the system (7) is regular, impulse free, internally asymptotically stable.

Now, we will show $\| T_{zw}(s) \|_\infty \le \gamma$, and $T_{zw}(s) = C(sE - (A + A_d e^{-s\tau}))^{-1} F$.

Applying the define of H_∞ norm, one can see that $\| T_{zw}(s) \|_\infty \le \gamma$ holds if $\gamma^2 I - T_{zw}^T(-j\omega) T_{zw}(j\omega) \ge 0$. Using Schur complement, (8c) is equivalent to

$$\begin{array}{l} A^T P^T + PA + Q + \tau R + Y + Y^T + \gamma^{-2} PFF^T P^T + C^T C \\ \quad + PA_d Q^{-1} A_d^T P^T + \Upsilon \; < \; 0 \end{array} \tag{9}$$

where $\Upsilon = (PA_d - Y + \tau A^T Z A_d)(Q - \tau A_d^T Z A_d)^{-1}$
$(PA_d - Y + \tau A^T Z A_d)^T + \tau A^T Z A.$

Rearranging (9) and noting that $PE = E^T P^T$, one can get

$$\begin{array}{l} C^T C < (-j\omega E - A - A_d e^{j\omega\tau})^T P^T + P(j\omega E - A - A_d e^{-j\omega\tau}) \\ \quad - \gamma^{-2} PFF^T P^T - \Upsilon - \Gamma - (\tau R + Y + Y^T) \end{array} \tag{10}$$

where $\Gamma = (A_d^T P^T e^{-j\omega\tau} - Q)^T Q^{-1} (A_d^T P^T e^{j\omega\tau} - Q)$.

Letting $\chi(j\omega) = (j\omega E - A - A_d e^{-j\omega\tau})^{-1}$, noting that $T_{zw}(j\omega) = C\chi(j\omega)F$, pre-multiplying (10) by $(\chi(-j\omega)F)^T$, post-multiplying by $\chi(j\omega)F$ and rearranging, the above inequality reduces to

$$\begin{array}{l} T_{zw}^T(-j\omega) T_{zw}(j\omega) \le F^T P^T \chi(j\omega) F + (F^T P^T \chi(-j\omega) F)^T \\ \quad - \gamma^{-2} (F^T P^T \chi(-j\omega) F)^T (F^T P^T \chi(j\omega) F) \\ \quad - (\chi(-j\omega) F)^T (\Upsilon + \Gamma + \tau R + Y + Y^T)(\chi(j\omega) F) \end{array}$$

Inequalities (8) implies that $\Upsilon + \Gamma + \tau R + Y + Y^T \ge 0$, so one can obtain

$$\begin{array}{l} \gamma^2 I - T_{zw}^T(-j\omega) T_{zw}(j\omega) \ge \gamma^2 I - (F^T P^T \chi(-j\omega) F)^T \\ \quad - F^T P^T \chi(j\omega) F + \gamma^{-2} (F^T P^T \chi(-j\omega) F)^T (F^T P^T \chi(j\omega) F) \\ \quad = (\gamma I - \gamma^{-1} F^T P^T \chi(-j\omega) F)^T (\gamma I - \gamma^{-1} F^T P^T \chi(j\omega) F) \ge 0 \end{array}$$

This completes the proof. □

Lemma 3: For any real matrices E and Y with appropriate dimensions the following inequality holds

$$\begin{array}{l} E^T W^{-1} E + Y^T W Y + E^T Y + Y^T E \\ \quad = (E + WY)^T W^{-1} (E + WY) \ge 0 \end{array} \tag{11}$$

where W is a symmetric matrix such that $W > 0$.

III. MAIN RESULTS

By using the equivalent control method[7], we get the equivalent control of the system (Σ)

$$\begin{array}{l} u_{eq}(t) = -(SB + S\Delta B)^{-1} S[(A + \Delta A)x(t) \\ \quad + (A_d + \Delta A_d)x(t-\tau) + (F + \Delta F)w(t)] \end{array}$$

then the sliding mode dynamics is given by

$$\begin{array}{l} E\dot{x}(t) = [I - (B + \Delta B)(SB + S\Delta B)^{-1} S][(A + \Delta A)x(t) \\ \quad + (A_d + \Delta A_d)x(t-\tau) + (F + \Delta F)w(t)] \\ z(t) = Cx(t) \\ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{array}$$

or equivalently,

$$\begin{array}{l} E\dot{x}(t) = (I - \tilde{S})(A + \Delta A)x(t) + (I - \tilde{S})(A_d + \Delta A_d)x(t-\tau) \\ \quad + (I - \tilde{S})(F + \Delta F)w(t) \\ z(t) = Cx(t) \\ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{array} \tag{12}$$

where $\tilde{S} = (B + \Delta B)(I + S_o \Delta B)^{-1} S_o$, $S_o = (SB)^{-1} S$.

Consider the LMIs

$$\left[\begin{array}{cc} X & I \\ I & d_1 I \end{array}\right] > 0 \quad , \quad X < d_2 I \tag{13a}$$

$$d_3 = \lambda_B - d_2 - d_1 \rho_3^2 \lambda_B > 0 \tag{13b}$$

where $\lambda_B = \lambda_{min}(B^T B)$. Then one can establish the following theorem to guarantee P1.

Theorem 1: Suppose that the LMIs (13) have a solution vector (X, d_1, d_2) for given λ_B and ρ_3, then there exists a linear switching surface parameter matrix S satisfying

P1, and by using a solution matrix X to (13), S can be parameterized as

$$\sigma(x) = SEx = LB^T X^{-1} Ex,\ S = LB^T X^{-1} \qquad (14)$$

where L is any nonsingular $m \times m$ matrix.

Proof: It is similar to Theorem 1 of CHOI (2001), and omitted here.

Now, we will give a delay-dependent sufficient condition to guarantee P2.

Consider the following LMIs

$$\begin{bmatrix} R & Y \\ Y^T & E^T ZE \end{bmatrix} \geq 0 \quad , \quad \tau R + Y + Y^T \geq 0 \qquad (15a)$$

$$\begin{bmatrix} J & \pi \\ \pi^T & \tilde{J} \end{bmatrix} < 0 \quad , \quad M > 0 \qquad (15b)$$

where

$$J = \begin{bmatrix} J_1 & PA_d - Y & \tau A^T Z & PF & PA_d \\ * & J_2 & \tau A_d^T Z & 0 & 0 \\ * & * & J_3 & 0 & 0 \\ * & * & * & J_4 & 0 \\ * & * & * & * & J_5 \end{bmatrix}$$

$$\begin{aligned}
\pi &= diag[\pi_1, (1+\tau)\pi_2, \tau\pi_3, \pi_4, \pi_2] \\
\tilde{J} &= diag[J_6, (1+\tau)J_7, \tau J_8, J_7, J_7] \\
J_1 &= \Theta + \lambda(1+\tau)\rho_1^2 I + C^T C \\
J_2 &= -Q + \lambda(1+\tau)\rho_2^2 I \\
J_3 &= -\tau Z \\
J_4 &= -\gamma^2 I + \lambda \rho_4^2 I \\
J_5 &= -Q + \lambda \rho_2^2 I \\
J_6 &= diag[-\frac{1}{4}\lambda I, -\frac{1}{8}d_3 I, -\frac{1}{8}d_3 I, (1+\tau)J_7] \\
J_7 &= diag[-I, -I, -I, -I] \\
J_8 &= diag[-\frac{1}{2}\lambda I, -\frac{1}{4}d_3 I, -\frac{1}{4}d_3 I] \\
\pi_1 &= \begin{bmatrix} P & PB & \rho_3 P & (1+\tau)\tilde{\pi}_1 \end{bmatrix} \\
\tilde{\pi}_1 &= \begin{bmatrix} d_1 A^T & d_1\rho_1 I & d_2 A^T & d_2 \rho_1 I \end{bmatrix} \\
\pi_2 &= \begin{bmatrix} d_1 A_d^T & d_1\rho_2 I & d_2 A_d^T & d_2\rho_2 I \end{bmatrix} \\
\pi_3 &= \begin{bmatrix} Z & ZB & \rho_3 Z \end{bmatrix} \\
\pi_4 &= \begin{bmatrix} d_1 F^T & d_1\rho_4 I & d_2 F^T & d_2\rho_4 I \end{bmatrix}
\end{aligned}$$

where $P = E^T M + N\Phi^T$, $\Phi \in \mathbf{R}^{n\times(n-p)}$ satisfying $E^T\Phi = 0$ and rank$\Phi = n - p$. d_1, d_2, λ are all scalars.

Theorem 2: Suppose that LMIs (13a),(15) have solutions $X > 0, M > 0, Q > 0, R \geq 0, Z > 0, d_1 > 0, d_2 > 0, \lambda > 0$ and general matrices N, Y, for given τ and γ, then there exists a parameter matrix S satisfying P1-2.

Proof: By theorem 1, we only have to show that the sliding mode dynamics (12) is regular, impulse free, internally asymptotically stable, and satisfies $\| T_{zw}(s) \|_\infty \leq \gamma$ for all mismatched uncertainties.

By Schur complement, one can see that Theorem 2 holds if the following inequalities holds

$$PE = E^T P^T \geq 0 \qquad (16a)$$

$$\begin{bmatrix} R & Y \\ Y^T & E^T ZE \end{bmatrix} \geq 0 \quad , \quad \tau R + Y + Y^T \geq 0 \qquad (16b)$$

$$\begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \\ * & -Q & K_6 & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -Q \end{bmatrix} < 0 \qquad (16c)$$

where

$$\begin{aligned}
K_1 &= \tilde{K}_1 + Q + \tau R + Y + Y^T + C^T C \\
\tilde{K}_1 &= (A+\Delta A)^T (I - \tilde{S})^T P^T + P(I - \tilde{S})(A + \Delta A) \\
K_2 &= P(I - \tilde{S})(A_d + \Delta A_d) - Y \\
K_3 &= \tau(A + \Delta A)^T (I - \tilde{S})^T Z \\
K_4 &= P(I - \tilde{S})(F + \Delta F) \\
K_5 &= P(I - \tilde{S})(A_d + \Delta A_d) \\
K_6 &= \tau(A_d + \Delta A_d)^T (I - \tilde{S})^T Z
\end{aligned}$$

For convenience, let that E_i is column vector and has five blocks and only the *ith* block is not zero matrix but identity matrix, i=1, …, 5.

$$\begin{aligned}
W := &\begin{bmatrix} K_1 & K_2 & K_3 & K_4 & K_5 \\ * & -Q & K_6 & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -Q \end{bmatrix} \\
= &\begin{bmatrix} \Theta + C^T C & PA_d - Y & \tau A^T Z & PF & PA_d \\ * & -Q & \tau A_d^T Z & 0 & 0 \\ * & * & -\tau Z & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -Q \end{bmatrix} \\
&+ E_1(\Delta A)^T P^T E_1^T + E_1 P \Delta A E_1^T \\
&- E_1(A+\Delta A)^T \tilde{S}^T P^T E_1^T - E_1 P\tilde{S}(A+\Delta A)E_1^T \\
&+ E_1[P\Delta A_d - P\tilde{S}(A_d + \Delta A_d)]E_2^T \\
&+ E_2[P\Delta A_d - P\tilde{S}(A_d + \Delta A_d)]^T E_1^T \\
&+ E_1[Z\Delta A - Z\tilde{S}(A + \Delta A)]^T E_3^T \\
&+ E_3[Z\Delta A - Z\tilde{S}(A+\Delta A)]E_1^T \\
&+ E_2[Z\Delta A_d - Z\tilde{S}(A_d + \Delta A_d)]^T E_3^T \\
&+ E_3[Z\Delta A_d - Z\tilde{S}(A_d + \Delta A_d)]E_2^T \\
&+ E_1[P\Delta F - P\tilde{S}(F + \Delta F)]E_4^T \\
&+ E_4[P\Delta F - P\tilde{S}(F+\Delta F)]^T E_1^T \\
&+ E_1[P\Delta A_d - P\tilde{S}(A_d + \Delta A_d)]E_5^T \\
&+ E_5[P\Delta A_d - P\tilde{S}(A_d + \Delta A_d)]^T E_1^T
\end{aligned} \qquad (17)$$

From Lemma 3 and condition (3), one can obtain that

$$\begin{aligned}
(A+\Delta A)^T (A + \Delta A) &= A^T A + A^T \Delta A + (\Delta A)^T A + (\Delta A)^T \Delta A \\
&\leq 2(A^T A + \rho_1^2 I)
\end{aligned}$$

similarly, one can get

$$\begin{aligned}
(A_d + \Delta A_d)^T (A_d + \Delta A_d) &\leq 2(A_d^T A_d + \rho_2^2 I) \\
(B + \Delta B)(B + \Delta B)^T &\leq 2(BB^T + \rho_3^2 I)
\end{aligned} \qquad (18)$$

$$(F+\Delta F)^T(F+\Delta F) \le 2(F^TF+\rho_4^2 I)$$

Note that (17), it follows from the above that

$$E_1(\Delta A)^T P^T E_1^T + E_1 P\Delta A E_1^T \le E_1(\varepsilon_1\rho_1^2 I + \varepsilon_1^{-1}PP^T)E_1^T \quad (19)$$

and

$$\begin{aligned}&-E_1(A+\Delta A)^T\tilde{S}^T P^T E_1^T - E_1 P\tilde{S}(A+\Delta A)E_1^T \\ &\quad \le E_1[2\varepsilon_2(A^TA+\rho_1^2 I)+\varepsilon_2^{-1}P\tilde{S}\tilde{S}^TP^T]E_1^T\end{aligned} \quad (20)$$

and

$$\begin{aligned}&E_1[P\Delta A_d - P\tilde{S}(A_d+\Delta A_d)]E_2^T \\ &\quad + E_2[P\Delta A_d - P\tilde{S}(A_d+\Delta A_d)]^T E_1^T \\ &\le E_1 P\Delta A_d E_2^T + E_2\Delta A_d^T P^T E_1^T \\ &\quad - E_1 P\tilde{S}(A_d+\Delta A_d)E_2^T \\ &\quad - E_2(A_d+\Delta A_d)^T\tilde{S}^T P^T E_1^T \\ &\le E_1^T(\varepsilon_3^{-1}PP^T+\varepsilon_4^{-1}P\tilde{S}\tilde{S}^TP^T)E_1^T \\ &\quad + E_2[\varepsilon_3\rho_2^2 I + 2\varepsilon_4(A_d^TA_d+\rho_2^2 I)]E_2^T\end{aligned} \quad (21)$$

similarly, one can get

$$\begin{aligned}&E_1[Z\Delta A - Z\tilde{S}(A+\Delta A)]^T E_3^T \\ &\quad + E_3[Z\Delta A - Z\tilde{S}(A+\Delta A)]E_1^T \\ &\le E_1[\varepsilon_5\rho_1^2 I + 2\varepsilon_6(A^TA+\rho_1^2 I)]E_1^T \\ &\quad + E_3(\varepsilon_5^{-1}Z^2+\varepsilon_6^{-1}Z\tilde{S}\tilde{S}^TZ)E_3^T\end{aligned} \quad (22)$$

and

$$\begin{aligned}&E_2[Z\Delta A_d - Z\tilde{S}(A_d+\Delta A_d)]^T E_3^T \\ &\quad + E_3[Z\Delta A_d - Z\tilde{S}(A_d+\Delta A_d)]E_2^T \\ &\le E_2[\varepsilon_7\rho_2^2 I + 2\varepsilon_8(A_d^TA_d+\rho_2^2 I)]E_2^T \\ &\quad + E_3(\varepsilon_7^{-1}Z^2+\varepsilon_8^{-1}Z\tilde{S}\tilde{S}^TZ)E_3^T\end{aligned} \quad (23)$$

and

$$\begin{aligned}&E_1[P\Delta F - P\tilde{S}(F+\Delta F)]E_4^T \\ &\quad + E_4[P\Delta F - P\tilde{S}(F+\Delta F)]^T E_1^T \\ &\le E_1(\varepsilon_9^{-1}PP^T+\varepsilon_{10}^{-1}P\tilde{S}\tilde{S}^TP^T)E_1^T \\ &\quad + E_4[\varepsilon_9\rho_4^2 I + 2\varepsilon_{10}(F^TF+\rho_4^2 I)]E_4^T\end{aligned} \quad (24)$$

and

$$\begin{aligned}&E_1[P\Delta A_d - P\tilde{S}(A_d+\Delta A_d)]E_5^T \\ &\quad + E_5[P\Delta A_d - P\tilde{S}(A_d+\Delta A_d)]^T E_1^T \\ &\le E_1(\varepsilon_{11}^{-1}PP^T+\varepsilon_{12}^{-1}P\tilde{S}\tilde{S}^TP^T)E_1^T \\ &\quad + E_5[\varepsilon_{11}\rho_2^2 I + 2\varepsilon_{12}(A_d^TA_d+\rho_2^2 I)]E_5^T\end{aligned} \quad (25)$$

Schur complement implies that (13a) is equivalent to

$$0 < \frac{1}{d_1}I < X < d_2 I \quad (26)$$

By using (26) and $B^TB > \lambda_B I > 0$, one can see that

$$\begin{aligned}S_oS_o^T &= (B^TX^{-1}B)^{-1}B^TX^{-2}B(B^TX^{-1}B)^{-1} \\ &< d_1(B^TX^{-1}B)^{-1} < d_1d_2(B^TB)^{-1} < \frac{d_1d_2}{\lambda_B}I\end{aligned} \quad (27)$$

By using Lemma 3 with $Y^T = S_o$, $E = \Delta B$ and $W = X$, one can show that the following inequality holds

$$\begin{aligned}S_o\Delta B + (\Delta B)^TS_o^T &\ge -S_oXS_o^T - (\Delta B)^TX^{-1}\Delta B \\ &\ge -(B^TX^{-1}B)^{-1} - d_1(\Delta B)^T\Delta B \\ &\ge -d_2(B^TB)^{-1} - d_1\rho_3^2 I \ge -\frac{d_2}{\lambda_B}I - d_1\rho_3^2 I\end{aligned} \quad (28)$$

Since the above inequality implies

$$\begin{aligned}&(I+S_o\Delta B)^{-1}(I+S_o\Delta B)^{-T} \le [I+(\Delta B)^TS_o^T+S_o\Delta B]^{-1} \\ &\quad \le [I-\frac{d_2}{\lambda_B}I - d_1\rho_3^2 I]^{-1} = \frac{\lambda_B}{d_3}I > 0\end{aligned} \quad (29)$$

It follows from (18),(27),(29) that

$$\tilde{S}\tilde{S}^T = (B+\Delta B)(I+S_o\Delta B)^{-1}S_oS_o^T(I+S_o\Delta B)^{-T}(B+\Delta B)^T$$

and $2d_1d_2 \le d_1^2+d_2^2$, one can get

$$\left.\begin{aligned}P\tilde{S}\tilde{S}^TP^T &\le \frac{d_1^2+d_2^2}{d_3}P(BB^T+\rho_3^2 I)P^T \\ Z\tilde{S}\tilde{S}^TZ &\le \frac{d_1^2+d_2^2}{d_3}Z(BB^T+\rho_3^2 I)Z\end{aligned}\right\} \quad (30)$$

For simplicity, define $\varepsilon_{2n} = \frac{1}{2}(d_1^2+d_2^2)$, $\varepsilon_{2n-1} = \lambda$, $n = 1,\ldots,6$. So (17) from (19)-(25) and (30) can be expressed as

$$W \le \begin{bmatrix} H_1 & PA_d - Y & \tau A^TZ & PF & PA_d \\ * & H_2 & \tau A_d^TZ & 0 & 0 \\ * & * & H_3 & 0 & 0 \\ * & * & * & H_4 & 0 \\ * & * & * & * & H_5 \end{bmatrix}$$

where

$$\begin{aligned}H_1 &= A^TP^T + PA + (\varepsilon_1+\tau\varepsilon_5)\rho_1^2 I + (\varepsilon_1^{-1}+\varepsilon_3^{-1}+\varepsilon_9^{-1}+\varepsilon_{11}^{-1})PP^T \\ &\quad + 2(\varepsilon_2+\tau\varepsilon_6)(A^TA+\rho_1^2 I) + Q + \tau R + Y + Y^T + C^TC \\ &\quad + (\varepsilon_2^{-1}+\varepsilon_4^{-1}+\varepsilon_{10}^{-1}+\varepsilon_{12}^{-1})\frac{d_1^2+d_2^2}{d_3}P(BB^T+\rho_3^2 I)P^T \\ &= A^TP^T + PA + \lambda(1+\tau)\rho_1^2 I + 4\lambda^{-1}PP^T \\ &\quad + (1+\tau)(d_1^2+d_2^2)(A^TA+\rho_1^2 I) \\ &\quad + \frac{8}{d_3}P(BB^T+\rho_3^2 I)P^T + Q + \tau R + Y + Y^T + C^TC \\ H_2 &= -Q + (\varepsilon_3+\tau\varepsilon_7)\rho_2^2 I + 2(\varepsilon_4+\tau\varepsilon_8)(A_d^TA_d+\rho_2^2 I) \\ &= -Q + \lambda(1+\tau)\rho_2^2 I + (1+\tau)(d_1^2+d_2^2)(A_d^TA_d+\rho_2^2 I) \\ H_3 &= -\tau Z + \tau(\varepsilon_5^{-1}+\varepsilon_7^{-1})Z^2 \\ &\quad + \frac{d_1^2+d_2^2}{d_3}\tau(\varepsilon_6^{-1}+\varepsilon_8^{-1})Z(BB^T+\rho_3^2 I)Z \\ &= -\tau Z + 2\lambda^{-1}\tau Z^2 + \frac{4\tau}{d_3}Z(BB^T+\rho_3^2 I)Z \\ H_4 &= -\gamma^2 I + \varepsilon_9\rho_4^2 I + 2\varepsilon_{10}(F^TF+\rho_4^2 I) \\ &= -\gamma^2 I + \lambda\rho_4^2 I + (d_1^2+d_2^2)(F^TF+\rho_4^2 I) \\ H_5 &= -Q + \varepsilon_{11}\rho_2^2 I + 2\varepsilon_{12}(A_d^TA_d+\rho_2^2 I) \\ &= -Q + \lambda\rho_2^2 I + (d_1^2+d_2^2)(A_d^TA_d+\rho_2^2 I)\end{aligned}$$

One can obtain that (16c) holds if the following inequality satisfies

$$\begin{bmatrix} H_1 & PA_d - Y & \tau A^T Z & PF & PA_d \\ * & H_2 & \tau A_d^T Z & 0 & 0 \\ * & * & H_3 & 0 & 0 \\ * & * & * & H_4 & 0 \\ * & * & * & * & H_5 \end{bmatrix} < 0 \qquad (31)$$

Since $P = E^T M + N\Phi^T$, $E^T\Phi = 0$, $M > 0$, so we get

$$PE = (E^T M + N\Phi^T)E = E^T ME \geq 0$$

$$E^T P^T = E^T(ME + \Phi N) = E^T ME \geq 0$$

Because (31) is equivalent to (15b) and (31) guarantee (16c), this completes the proof. □

Now, let the switched feedback control strategy be given as

$$\begin{aligned} u(t) &= -(SB)^{-1}SAx(t) - (SB)^{-1}SA_d x(t-\tau) \\ &\quad - p(x(t))\frac{(SB)^{-1}\sigma}{\| (SB)^{-1}\sigma \|}, \qquad \| \sigma \| \neq 0 \end{aligned} \qquad (32)$$

where

$$\begin{aligned} p(x(t)) = & \; 2\{\varepsilon + \| S_o \| \cdot [\rho_1 \| x(t) \| + \rho_2 \| x(t-\tau) \| \\ & + \rho_3 \| S_o Ax(t) + S_o A_d x(t-\tau) \|] \\ & + \ell \| S_o F \| + \rho_4 \ell \| S_o \| \} \end{aligned} \qquad (33)$$

and ε is a positive scalar.

Theorem 3: Consider the closed-loop control system of the system (1)-(2) with the control (32), and assume that the LMIs (13a),(15) have solutions and that the linear switching surface is given by (14). Then, every solution trajectory of the system (1)-(2) is directed towards the switching surface $\sigma(x) = 0$ in limited time, and once the trajectory hits the switching surface it will remain on the surface for all subsequent time.

Proof : we will show that the control law (32) with σ of (14) drives the system trajectory onto the linear switching surface, but also keeps it there for all subsequent time, i. e. the reaching condition $\sigma^T W_o \dot{\sigma} < 0$ is satisfied where W_o is a positive-definite matrix. If LMIs (13a), (15) have a solution vector $(X, M, N, Q, R, Y, Z, d_1, d_2, \lambda)$ and the linear switching surface is given by (14). Then using (1) and (32), one can obtain

$$\begin{aligned} \sigma^T (SB)^{-T}(SB)^{-1}\dot{\sigma} &= \sigma^T (SB)^{-T}[S_o\Delta Ax(t) + S_o\Delta A_d x(t-\tau) \\ &\quad - S_o\Delta BS_o Ax(t) - S_o\Delta BS_o A_d x(t-\tau) + S_o(F+\Delta F)w(t)] \\ &\quad - \frac{p(x(t))}{\| (SB)^{-1}\sigma \|}\sigma^T (SB)^{-T}(I + S_o\Delta B)(SB)^{-1}\sigma \\ &\leq \| (SB)^{-1}\sigma \| \cdot \{\| S_o \| \cdot [\rho_1 \| x(t) \| + \rho_2 \| x(t-\tau) \| \\ &\quad + \rho_3 \| S_o Ax(t) + S_o A_d x(t-\tau) \|] + \ell \| S_o F \| + \rho_4 \ell \| S_o \|\} \\ &\quad - \frac{p(x(t))}{\| (SB)^{-1}\sigma \|}\sigma^T (SB)^{-T}(I + S_o\Delta B)(SB)^{-1}\sigma \end{aligned}$$

By using Lemma 3 with $Y^T = \sigma^T (SB)^{-T} S_o$, $E = \Delta B(SB)^{-1}\sigma$ and $W = X$, one can show that

$$\begin{aligned} -\sigma^T (SB)^{-T} S_o \Delta B(SB)^{-1}\sigma &\leq \frac{1}{2}\sigma^T (SB)^{-T} S_o X S_o^T (SB)^{-1}\sigma \\ &\quad + \frac{1}{2}\sigma^T (SB)^{-T}(\Delta B)^T X^{-1}\Delta B(SB)^{-1}\sigma \\ &\leq \frac{1}{2}\sigma^T (SB)^{-T}[(B^T X^{-1}B)^{-1} + \rho_3^2 \| X^{-1} \| I](SB)^{-1}\sigma \end{aligned}$$

Therefore, one can get

$$\begin{aligned} \sigma^T (SB)^{-T}(SB)^{-1}\dot{\sigma} &\leq \| (SB)^{-1}\sigma \| \cdot \{\| S_o \| \cdot [\rho_1 \| x(t) \| + \rho_2 \| x(t-\tau) \| \\ &\quad + \rho_3 \| S_o Ax(t) + S_o A_d x(t-\tau) \|] \\ &\quad + \ell \| S_o F \| + \rho_4 \ell \| S_o \|\} \\ &\quad - \frac{p(x(t))}{2 \| (SB)^{-1}\sigma \|}\sigma^T (SB)^{-T}[2I - (B^T X^{-1}B)^{-1} \\ &\quad - \rho_3^2 \| X^{-1} \| I](SB)^{-1}\sigma \end{aligned}$$

Because as shown in the following

$$\begin{aligned} 2I - (B^T X^{-1}B)^{-1} - \rho_3^2 \| X^{-1} \| I &\geq (2 - d_1\rho_3^2)I - d_2(B^T B)^{-1} \\ &\geq (2 - d_1\rho_3^2)I - \frac{d_2}{\lambda_B}I = I + \frac{d_3}{\lambda_B}I > 0 \end{aligned}$$

One can conclude that the reaching condition is satisfied as

$$\sigma^T (SB)^{-T}(SB)^{-1}\dot{\sigma} \leq -\varepsilon \| (SB)^{-1}\sigma \| < 0$$

After all, it is known that there exists a sliding mode, i.e., the trajectory can reach the surface $(SB)^{-1}\sigma = 0$ in limited time[7]. Since the surface $\sigma = 0$ and the surface $(SB)^{-1}\sigma = 0$ is identical, the trajectory can reach the surface $\sigma = 0$ in limited time. This completes the proof. □

IV. NUMERICAL EXAMPLE

Consider the uncertain singular time-delay systems (Σ) of the form with

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 2 \\ 0 \\ 2 \\ -2 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -100 & 2 & -2 & 0 & 0 \\ -200 & -201 & 0 & 0 & 240 \\ 100 & 0 & 100 & 0 & 0 \\ -25 & 0 & 0 & 0 & -100 \\ 0 & 80 & -10 & 0 & 100 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 10 & 6 & -1 & 0 & 0 \\ 0 & 10 & -5 & 0 & 3 \\ 0 & 5 & -1 & 0 & 0 \\ -2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0.5 \end{bmatrix}, \; F = \begin{bmatrix} 0 \\ 0 \\ -0.54 \\ 0 \\ 0 \end{bmatrix},$$

$$C^T = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, \; \Phi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$\rho_1 = 0, \; \rho_2 = 0, \; \rho_3 = 0.01$$

$$\rho_4 = 0.01, \; \gamma = 2, \; \tau = 0.1, \; L = 1$$

According to Theorem 2, the following feasible solution can be obtained

$$X = \begin{bmatrix} 1.0113 & 0 & 0.0000 & 0.0000 & 0 \\ 0 & 1.0113 & 0 & 0 & 0 \\ 0.0000 & 0 & 1.0113 & -0.0000 & 0 \\ 0.0000 & 0 & -0.0000 & 1.0113 & 0 \\ 0 & 0 & 0 & 0 & 1.0113 \end{bmatrix}$$

so we get

$$S = \begin{bmatrix} 1.9777 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (34)$$

V. CONCLUSION

In this paper, the problem of design linear switching surfaces and reaching controller has been considered for a class of singular time-delay systems with mismatched norm-bounded uncertainties and mismatched external disturbances. Based on linear matrix inequalities(LMI), we propose a delay-dependent sufficient condition guaranteeing the existence of a linear switching surface such that the sliding mode dynamics restricted to the switching surface is regular, impulse free, internally asymptotically stable with disturbance attenuation. A state feedback controller is given to guarantee system trajectories convergent to the linear switching surface in finite time. We also give an explicit formula of such linear switching surfaces, together with a numerical example.

APPENDIX

Proof of Lemma 1:

The following notations are needed:

$\bar{S} := \{\phi(t) \mid \phi(t) \in C_{n,\tau}, \phi(t)$ is the compatible initial function of system (5), and there exists a unique continuous solution of system (5) on $[0,+\infty)$ for $\phi(t)\}$,

$B(0,\delta) := \{\phi(t) \mid \phi(t) \in C_{n,\tau}, \| \phi \|_c \le \delta, \delta > 0\}$.

1. Prove the system (5) is regular and impulse free.

From $0 < \text{rank} E = p < n$, there exist nonsingular matrices M,N such that

$$\bar{E} := MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{Z} := M^{-T}ZM^{-1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix} \quad (35a)$$

$$\bar{A} := MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \ \bar{A}_d := MA_dN = \begin{bmatrix} A_{\tau 11} & A_{\tau 12} \\ A_{\tau 21} & A_{\tau 22} \end{bmatrix} \quad (35b)$$

$$\bar{P} := N^T PM^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \ \bar{Q} := N^T QN = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \quad (35c)$$

$$\bar{X} := N^T XN = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \ \bar{Y} := N^T YN = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \quad (35d)$$

Substituting (35) into (6), we get

$$P_{21} = 0, \ P_{11} > 0, \ \begin{bmatrix} X_{11} & X_{12} & Y_{11} & 0 \\ * & X_{22} & Y_{21} & 0 \\ * & * & Z_{11} & 0 \\ * & * & * & 0 \end{bmatrix} \ge 0 \quad (36)$$

$$\begin{bmatrix} P_{22}A_{22} + A_{22}^T P_{22}^T + Q_{22} & P_{22}A_{\tau 22} \\ * & -Q_{22} \end{bmatrix} < 0 \quad (37)$$

which implies the system (5) is regular and impulse free. Hence, there exist nonsingular matrices $\breve{M},\breve{N}$ such that (E,A) r. s. e. to the Weierstrass standard form $(\breve{E},\breve{A})$. For simplicity, $\breve{M},\breve{N}$ are still denoted by M,N, $\breve{E},\breve{A}$ by $\bar{E},\bar{A}$ and then other notations in (36) are still used. So

$$\bar{E} := MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}, \ \bar{A} := MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-p} \end{bmatrix} \quad (38)$$

Under coordinate transformation of $x(t) = Ny(t) = N \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$, system (5) is equivalently transformed into:

$$\begin{cases} \dot{y}_1(t) = A_1 y_1(t) + A_{\tau 11} y_1(t-\tau) + A_{\tau 12} y_2(t-\tau), \\ 0 = y_2(t) + A_{\tau 21} y_1(t-\tau) + A_{\tau 22} y_2(t-\tau), \\ y(t) = N^{-1}\phi(t) = \psi(t), \quad t \in [-\tau, 0] \end{cases} \quad (39)$$

Obviously, the asymptotical stability of the zero solution to system (39) is equivalent to that of the system (5).

2. Prove the asymptotical stability of the zero solution to the system (39). Introduce the following Lemma:

Lemma [9]: If there exists a continuous functional $V(y_t): C_{n,\tau} \to R$ and continuous nondecreasing functions $u,v,w : R^+ \to R^+$, with $u(0) = v(0) = 0$, $u(s) > 0, v(s) > 0, \forall s > 0, V(y_t)$ satisfies:

c1) $u(\| y_1(t) \|^2) \le V(y_t) \le v(\| y_t \|_c^2), \ t \ge \tau$

c2) $D^+(V(y_t)) \le -w(\| y_1(t) \|^2), \ t \ge \tau$

where $y_t := y(t+\theta), \ \theta \in [-2\tau, 0], t \ge \tau$. Then the first p-dimension vector of the zero solution to system (39) is stable, i. e., for any $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$, such that $\| y_1(t) \| \le \varepsilon, t \ge 0$ when the initial function $\psi(t) \in B(0,\delta(\varepsilon)) \cap \bar{S}$.

Furthermore, if $w(s) > 0$ for $s > 0$, and there exist constant scalars l_0, m_0, such that $\| \dot{y}_1(t) \| \le m_0, t \ge 0$ when $\| y_1(t) \| \le l_0, t \ge 0$, then the first p-dimension vector of the zero solution to system (39) is asymptotically stable, i. e., i) the the first p-dimension vector of the zero solution is stable; ii) there exists a sufficiently small scalar $\delta_0 > 0$, such that $\lim_{t\to\infty} y_1(t) = 0$ when the initial function $\psi(t) \in B(0,\delta_0) \cap \bar{S}$.

Define Lyapunov-Krasovskii functional:

$$V(y_t) = y^T(t)\bar{P}\bar{E}y(t) + \int_{t-\tau}^{t} y^T(s)\bar{Q}y(s)ds + \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{y}^T(\alpha)\bar{E}^T\bar{Z}\bar{E}\dot{y}(\alpha)d\alpha d\beta, \ t \ge \tau.$$

Then we can deduce that $V(y_t)$ satisfies (c1) and (c2). By the above Lemma, we have that the first p-dimension vector of the zero solution to system (39) is stable.

Next to prove the first p-dimension vector of the zero solution to system (39) is asymptotically stable. From (37), we have

$$A_{\tau 22}^T Q_{22} A_{\tau 22} - Q_{22} < 0, \ \| A_{\tau 22}^k \| \le \beta\alpha^k \quad (40)$$

where β,α are scalars, $\alpha \in (0,1)$ and $\beta > 1$. From (39), we get

$$y_2(t) = (-A_{\tau 22})^k y_2(t-k\tau) - \sum_{i=1}^{k} (-A_{\tau 22})^{i-1} A_{\tau 21} y_1(t-i\tau)$$
$$(k-1)\tau \le t < k\tau, \ k = 1,2,\ldots \quad (41)$$

Recall the first p-dimension vector of the zero solution to system (39) is stable, so for any $\varepsilon > 0$, there exists $0 < \delta(\varepsilon) < \varepsilon$, such that

$$\| y_1(t) \| \le \varepsilon, \ t \ge -\tau \quad (42)$$

when the initial function $\psi(t) \in B(0,\delta(\varepsilon)) \cap \bar{S}$. Furthermore, $\| y_2(\theta) \| = \| \psi_2(\theta) \| \le \delta(\varepsilon) \le \varepsilon, \theta \in [-\tau, 0]$, it follows that

$$\| y_2(t) \| \le \beta(1 + \tfrac{1}{1-\alpha} \| A_{\tau 21} \|)\varepsilon, \ t \ge -\tau. \quad (43)$$

Thus from (42), (43) and (39), we get $\| \dot{y}_1(t) \| \le \bar{\beta}\varepsilon, \ t \ge 0$. $\bar{\beta}$ is a scalar. So then using the above Lemma gives that the first p-dimension vector of the zero solution to system (39) is asymptotically stable.

Finally, by (40) -(43), we will prove that the zero solution of system (39) is asymptotically stable. It completes the proof. □

REFERENCES

[1] Kyung-Soo, k. , "Designing robust sliding hyperplanes for parametric uncertain systems: a Riccati approach", *Automatica*, 1999, No. 11, pp. 1041-1047.

[2] Richardo H. C., Takahashi, and Pedro L. D. Peres, "H_2 Guaranteed Cost-Switching Surface Design or Sliding Modes with Nonmatching Disterbances", *IEEE Trans. Automat. Contr.*, 1999, Vol. 44, pp. 2214-2218.

[3] Richardo H. C., Takahashi, and Pedro L. D. Peres, "H_2 Guaranteed Cost-Switching Surface Design or Sliding Modes with Nonmatching Disterbances", *IEEE Trans. Automat. Contr.*, 1999, Vol. 44, pp. 2214-2218.

[4] CHOI, H. H., "An LMI-Based Switching Surface Design Method for a Class of Mismatched Uncertain Systems", *IEEE Transactions on Automatic Control*, 2003, Vol. 48, pp. 1634-1638.

[5] CHOI, H. H. , "Variable structure control of dynamcal systems with mismatched norm-bounded uncertainties : an LMI approach", *Int.J. Control*, Vol. 74, 2001, pp. 1324-1334.

[6] Shuqian, Z. and Zhaolin C. ,"Delay-dependent Robust Stability Criterion and Robust Stabilization for Uncertain Singular Time-delay Systems", *Proceedings of the 2005 American Control Conference*, Portland, Oregon, June 8-10, 2005.

[7] UTKIN, V. I. , "Variable structure systems with sliding modes", *IEEE Transactions on Automatic Control*, 1977, Vol. AC-22, pp. 212-222.

[8] Yuanqing Xia and Yingmin Jia, "Robust Sliding-Mode Control for Uncertain Time-Delay Systems: An LMI Approach", *IEEE Trans. Automat. Contr.*, 2003, Vol. 48, pp.1086-1092.

[9] J. Feng, S. Zhu and Z. Cheng, "Guaranteed cost control of linear uncertain singular time-delay systems," *in Proc. 41st IEEE Conf. Decision Control*, pp. 1802-1807, Las Vegas, Nenasa USA, 2002.