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Detectability and Output Feedback Stabilizability of Nonlinear Networked Control Systems

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Abstract— **This paper addresses problems of detectability and output feedback stabilizability of nonlinear systems with sector-type nonlinearities via limited capacity digital communication channels. The main results are given in terms of Riccati algebraic inequalities.**

I. INTRODUCTION

A standard assumption in the classical control theory is that the data transmission required by the control or state estimation algorithm can be performed with infinite precision. However, due to the growth in communication technology, it is becoming more common to employ digital limited capacity communication networks for exchange of information between system components. The resources available in such systems for communication between sensors, controllers and actuators can be severely limited due to size or cost. This problem may arise when the large number of mobile units need to be controlled remotely by a single controller. Since the radio spectrum is limited, communication constraints are a real concern. For example, the paper [22] shows that the major difficulty in controlling a platoon of autonomous underwater vehicles is the bandwidth limitation on communication between the vehicles. Another classes of examples are offered by complex networked sensor systems containing a very large number of low power sensors and micro-electromechanical systems.

In all these problems, classical optimal control and estimation theory cannot be applied since the control signals and the state information are sent via a limited capacity digital communication channel, hence, the controller or estimator only observes the transmitted sequence of finite-valued symbols. A natural question to ask is how much capacity is needed to achieve a specified control performance or estimation accuracy.

In recent years there has been a significant interest in the problem of control and state estimation via a digital communication channel with bit-rate constraint; e.g., see [23], [1], [9], [2], [10], [11], [21], [4], [3], [7], [12], [13], [8], [6], [5], [18]. The papers [16] considered various state feedback stabilization problems for nonlinear and uncertain systems. The problem of local stabilization for nonlinear networked systems was addressed in [12], [13]. The issue of observability for uncertain linear networked systems was studied in [21], [18].

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In this paper, we consider the problem of observability for nonlinear networked systems with sector-type nonlinearity that is common in absolute stability and robust control theories; see e.g. [14], [17]. Furthermore, we obtain a criterion of stabilizability of a nonlinear networked system. Unlike [16], [12], [13], we consider a more difficult case of output feedback stabilization. Our main results are given in terms of algebraic Riccati inequalities that arise in the theory of robust and H^∞ control; see e.g. [15], [17], [19], [20].

The remainder of the paper is organized as follows. Section II addresses the problem of detectability of a nonlinear system via a digital communication channel. The output feedback stabilizability problem is studied in Section III.

II. DETECTABILITY VIA COMMUNICATION CHANNELS

In this section, we consider a nonlinear continuous-time dynamical system of the form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 f(z(t)); \\ z(t) &= Kx(t); \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $z(t) \in \mathbf{R}^q$ is a linear output, $y(t) \in \mathbf{R}^k$ is the measured output, A, B_1, K and C are given matrices of the corresponding dimensions, $f(z(t)) \in \mathbf{R}^p$ is a given continuous nonlinear vector function. We also assume that initial conditions of the system (1) lie in a known bounded set $\mathcal{X}_0$:

$$x(0) \in \mathcal{X}_0. \tag{2}$$

Notation 2.1: Let $x = [x_1 \; x_2 \; \cdots \; x_n]'$ be a vector from $\mathbf{R}^n$. Then

$$\|x\|_\infty := \max_{j=1,\ldots,n} |x_j|. \tag{3}$$

Furthermore, $\|\cdot\|$ denotes the standard Euclidean vector norm:

$$\|x\| := \sqrt{\sum_{j=1}^{n} x_j^2}.$$

We assume that the vector function $f(\cdot)$ satisfies the following condition:

$$\|f(z_1) - f(z_2)\|^2 \le \|z_1 - z_2\|^2 \quad \forall z_1, z_2. \tag{4}$$

The requirement (4) is a typical sector-type constraint from the absolute stability theory; see e.g. [14], [17]. A simple common example of such constraint is a scalar nonlinearity satisfying conditions $f(0) = 0$ and

$$-1 \le \frac{f(z_1) - f(z_2)}{z_1 - z_2} \le 1 \quad \forall z_1 \ne z_2.$$

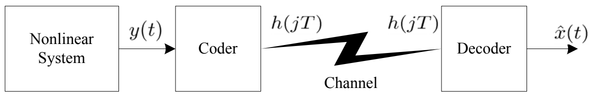

Fig. 1. State estimation via digital communication channel

In our detectability problem, a sensor measures the state $x(t)$ and is connected to the controller that is at the remote location. Moreover, the only way of communicating information from the sensor to that remote location is via a digital communication channel which carries one discrete-valued symbol $h(jT)$ at time jT, selected from a coding alphabet $\mathcal{H}$ of size l. Here $T > 0$ is a given period, and $j = 1, 2, 3, \ldots$.

This restricted number l of codewords $h(jT)$ is determined by the transmission data rate of the channel. For example, if μ is the number of bits that our channel can transmit at any time instant, then $l = 2^\mu$ is the number of admissible codewords. We assume that the channel is a perfect noiseless channel and there is no time delay.

We consider the problem of estimation of the state $x(t)$ via a digital communication channel with a bit-rate constraint. Our state estimator consists of two components. The first component is developed at the measurement location by taking the measured output $y(\cdot)$ and coding to the codeword $h(jT)$. This component will be called "coder". Then the codeword $h(jT)$ is transmitted via a limited capacity communication channel to the second component which is called "decoder". The second component developed at the remote location takes the codeword $h(jT)$ and produces the estimated state $\hat{x}(t)$. This situation is illustrated in Figure 1.

The coder and the decoder are of the following form:

Coder:

$$h(jT) = \mathcal{F}_j \left(y(\cdot)|_0^{jT} \right); \tag{5}$$

Decoder:

$$\hat{x}(t)|_{jT}^{(j+1)T} = \mathcal{G}_j \left(h_T, h_{2T}, ..., h_{(j-1)T}, h_{jT} \right) \tag{6}$$

Here $j = 1, 2, 3, \ldots$.

Definition 2.1: The system (1), is said to be detectable via a digital communication channel of capacity l if there exists a coder-decoder pair (5), (6) with a coding alphabet of size l such that

$$\lim_{t\to\infty} \|x(t) - \hat{x}(t)\|_\infty = 0 \tag{7}$$

for any solution of (1), (2), (4). A coder-decoder pair (5), (6) satisfying condition (7) is called detecting.

A. Preliminary lemmas

We will consider the following pair of Riccati algebraic inequalities

$$(A - \alpha I)'X + X(A - \alpha I) + K'K + XB_1B_1'X < 0, \tag{8}$$

$$YA + A'Y + YB_1B_1'Y + K'K - \alpha_1 C'C < 0 \tag{9}$$

where I is the identity square matrix, $\alpha > 0$ and $\alpha_1 > 0$ are given numbers.

In this subsection, we prove two preliminary lemmas.

Lemma 2.1: Suppose that for some $\alpha > 0$ there exists a solution $X > 0$ of the Riccati inequality (8). Then there exists a time $T_0 > 0$ such that for any $T \geq T_0$ and any two solutions $x_1(\cdot), x_2(\cdot)$ of the system (1), (2), (4) the following inequality holds:

$$\|x_1(t+T) - x_2(t+T)\|_\infty \leq e^{\alpha T} \|x_1(t) - x_2(t)\|_\infty \tag{10}$$

for all $t \geq 0$.

Proof of Lemma 2.1 Let

$$\tilde{x}(t) := e^{-\alpha t}(x_1(t) - x_2(t));$$
$$\phi(t) := e^{-\alpha t}(f(Kx_1(t)) - f(Kx_2(t))). \tag{11}$$

Then $\tilde{x}(\cdot), \phi(\cdot)$ obviously satisfy the equation

$$\begin{aligned} \dot{\tilde{x}}(t) &= (A - \alpha I)\tilde{x}(t) + B_1\phi(t); \\ \tilde{z}(t) &= K\tilde{x}(t), \end{aligned} \tag{12}$$

and the constraint

$$\|\phi(t)\|^2 \leq \|\tilde{z}(t)\|^2. \tag{13}$$

Then according to Strict Bounded Real Lemma (see e.g. Lemma 3.1.2 of [17] the system (12), (13) is quadratically stable. This implies that there exists a time $T_0 > 0$ such that for any $T \geq T_0$ and any solution $\tilde{x}(\cdot)$ of the system (12), (13) the following inequality holds:

$$\|\tilde{x}(t+T)\|_\infty \leq \|\tilde{x}(t)\|_\infty \quad \forall t \geq 0. \tag{14}$$

The property (14) and (11) immediately imply (10). This completes the proof of Lemma 2.1.

Now consider the following state estimator that will be a part of our proposed coder:

$$\dot{\tilde{x}}(t) = (A - GC)\tilde{x}(t) + Gy(t) + B_1 f(\tilde{z}(t));$$
$$\tilde{z}(t) = K\tilde{x}(t), \quad \tilde{x}(0) = 0. \tag{15}$$

Furthermore, we introduce the gain G by

$$G := \frac{\alpha_1}{2} Y^{-1} C' \tag{16}$$

where $Y > 0$ is a solution of (9).

Lemma 2.2: Suppose that for some $\alpha_1 > 0$ there exists a solution $Y > 0$ of the Riccati inequality (9). Then there exists a time $T_0 > 0$ and a constant $\alpha_0 > 0$ such that

$$\|x(t+T) - \tilde{x}(t+T)\|_\infty \leq e^{-\alpha_0 T} \|x(t) - \tilde{x}(t)\|_\infty \tag{17}$$

for any $t \geq 0, T > T_0$ and any solution of (1), (2), (4), (15), (16).

Proof of Lemma 2.2 Let

$$\xi(t) := x(t) - \tilde{x}(t); \quad \zeta(t) := K\xi(t);$$
$$\phi(t) := f(z(t)) - f(\tilde{z}(t)). \tag{18}$$

Then $\xi(\cdot), \phi(\cdot)$ obviously satisfy the equation

$$\dot{\xi}(t) = (A - GC)\xi(t) + B_1\phi(t) \tag{19}$$

and the constraint

$$\|\phi(t)\|^2 \leq \|\zeta(t)\|^2. \tag{20}$$

Since $Y > 0$ is a solution of (9) and G is defined by (16), the matrix Y is a also positive-definite solution of the Riccati inequality

$$(A - GC)'Y + Y(A - GC) + K^{'}K + YB_1B_1^{'}Y < 0.$$

Therefore, according to Strict Bounded Real Lemma (see e.g. Lemma 3.1.2 of [17] the system (18), (19), (20) is quadratically stable. Now the statement of the lemma immediately follows from quadratic stability. This completes the proof of Lemma 2.2.

B. Uniform state quantization

Our proposed coder-decoder uses uniform quantization of the states $\tilde{x}$ of the system (15) in which the same number of bits is used to quantize each state variable.

To quantize the state space of the estimator (15), let $a > 0$ be a given constant and consider the set:

$$\mathcal{B}_a := \{x \in \mathbf{R}^n : \|x\|_\infty \leq a\}.$$

We propose to quantize the state space of the system (15) by dividing the set $\mathcal{B}_a$ into q^n hypercubes where q is a specified integer. Indeed, for each $i \in \{1, 2, \ldots, n\}$, we divide the corresponding component of the vector $\tilde{x}_i$ into q intervals as follows:

$$\begin{aligned} I_1^i(a) &:= \left\{\tilde{x}_i : -a \leq \tilde{x}_i < -a + \frac{2a}{q}\right\}; \\ I_2^i(a) &:= \left\{\tilde{x}_i : -a + \frac{2a}{q} \leq \tilde{x}_i < -a + \frac{4a}{q}\right\}; \\ &\vdots \\ I_q^i(a) &:= \left\{\tilde{x}_i : a - \frac{2a}{q} \leq \tilde{x}_i \leq a\right\}. \end{aligned} \tag{21}$$

Then for any $\tilde{x} \in \mathcal{B}_a$, there exist unique integers $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, q\}$ such that

$$\tilde{x} \in I_{i_1}^1(a) \times I_{i_2}^2(a) \times \ldots \times I_{i_n}^n(a).$$

Also, corresponding to the integers $i_1, i_2, \ldots, i_n$, we define the vector

$$\eta(i_1, i_2, \ldots, i_n) := \begin{bmatrix} -a + a(2i_1 - 1)/q \\ -a + a(2i_2 - 1)/q \\ \vdots \\ -a + a(2i_n - 1)/q \end{bmatrix}. \tag{22}$$

This vector is the center of the hypercube $I_{i_1}^1(a) \times I_{i_2}^2(a) \times \ldots \times I_{i_n}^n(a)$ containing the original point $\tilde{x}$.

Note the regions $I_{i_1}^1(a) \times I_{i_2}^2(a) \times \ldots \times I_{i_n}^n(a)$ partition the region $\mathcal{B}_a$ into q^n regions; e.g., for $n = 2$ and $q = 3$, the region $\mathcal{B}_a$ would be divided into nine regions.

In our proposed coder-decoder, each one of these regions will be assigned a codeword and the coder will transmit the codeword corresponding to the current state of the system (15) $\tilde{x}(jT)$. The transmitted codeword will correspond to the integers $i_1, i_2, \ldots, i_n$.

The above quantization of the state space of the system (15) depends on the scaling parameter $a > 0$, In our proposed coder-decoder, this parameter will be the quantization scaling $a(jT)$ where $j = 1, 2, \ldots$.

We now suppose that assumptions of Lemmas 2.1 and 2.2 are satisfied. Then let $T > 0$ be a time such that conditions (10) and (17) hold. Furthermore, introduce

$$\begin{aligned} m_0 := \sup_{x_0 \in X_0} \|x_0\|_\infty; \quad a(T) := (e^{\alpha T} + e^{-\alpha_0 T})m_0; \\ a(jT) := e^{\alpha T}\frac{a((j-1)T)}{q} + \\ (e^{(\alpha - \alpha_0(j-1))T} + e^{-\alpha_0 jT})m_0 \quad \forall j = 2, 3, \ldots. \end{aligned} \tag{23}$$

Introduce now our proposed coder-decoder:
Coder:

$$h(jT) = \{i_1, i_2, \ldots, i_n\} \tag{24}$$

for $(\tilde{x}(jT) - \hat{x}(jT - 0)) \in I_{i_1}^1(a(jT)) \times I_{i_2}^2(a(jT)) \times \ldots \times I_{i_n}^n(a(jT)) \subset \mathcal{B}_{a(jT)}$.
Decoder:

$$\begin{aligned} \hat{x}(0) = 0; \quad \hat{z}(t) = K\hat{x}(t), \\ \dot{\hat{x}}(t) = A\hat{x}(t) + B_1 f(\hat{z}(t)) \quad \forall t \neq jT; \\ \hat{x}(jT) = \hat{x}(jT - 0) + \eta(i_1, i_2, \ldots, i_n) \\ \text{for } h(jT) = \{i_1, i_2, \ldots, i_n\} \quad \forall j = 1, 2, \ldots. \end{aligned} \tag{25}$$

Here $\nu(t - 0)$ denotes the limit of the function $\nu(\cdot)$ at the point t from the left, i.e.,

$$\nu(t - 0) := \lim_{\epsilon > 0, \epsilon \to 0} \nu(t - \epsilon).$$

Notice that our decoder is described by a differential equation with jumps.

Also notice that the equations (25) are a part of the both coder and decoder, and it follows immediately from (10), (17) and initial condition $\tilde{x}(0) = 0$ that

$$\tilde{x}(T) \in \mathcal{B}_{a(T)}; \quad (\tilde{x}(jT) - \hat{x}(jT - 0)) \in \mathcal{B}_{a(jT)} \tag{26}$$

for all $j = 2, 3, \ldots$ and for any solution of (1), (2), (4).

The main requirement for our coding-decoding scheme is as follows:

$$q > e^{\alpha T}. \tag{27}$$

Now we are in a position to present the main result of this section.

Theorem 2.1: Suppose that for some $\alpha > 0$ there exists a solution $X > 0$ of the Riccati inequality (8) and for some $\alpha_1 > 0$ there exists a solution $Y > 0$ of the Riccati inequality (9). Furthermore, suppose that for some $T > 0$ satisfying conditions (10), (17) and some positive integer q the inequality (27) holds. Then the coder-decoder (15), (16), (23), (24), (25) is detecting for the system (1), (2), (4).

Proof of Theorem 2.1 Condition (27) implies that

$$\lim_{j \to \infty} a(jT) = 0.$$

Hence

$$\lim_{j \to \infty} (\eta(i_1, i_2, \ldots, i_n) - (\hat{x}(jT) - \tilde{x}(jT))) = 0$$

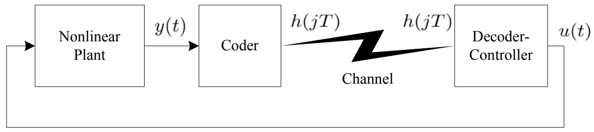

Fig. 2. Stabilization via digital communication channel

where $h(jT) = \{i_1, i_2, \dots, i_n\}$. This implies that

$$\lim_{j\to\infty} (\hat{x}(jT) - \tilde{x}(jT)) = 0$$

¿From this and Lemma 2.2 we obtain that

$$\lim_{j\to\infty} (x(jT) - \hat{x}(jT)) = 0$$

for any solution of the system (1), (2), (4). Detectability now immediately follows from this and equations (25). This completes the proof of Theorem 2.1.

III. STABILIZATION VIA COMMUNICATION CHANNELS

In this section, we consider a nonlinear continuous-time dynamical system of the form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 f(z(t)) + B_2 u(t); \\ z(t) &= Kx(t); \\ y(t) &= Cx(t) \end{aligned} \qquad (28)$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control input, $z(t) \in \mathbf{R}^q$ is a linear output, $y(t) \in \mathbf{R}^k$ is the measured output, A, B_1, B_2, K and C are given matrices of the corresponding dimensions, $f(z(t)) \in \mathbf{R}^p$ is a given continuous nonlinear vector function.

We also assume that initial conditions of the system (28) lie in a known bounded set $\mathcal{X}_0$ (2). Furthermore, we suppose that the nonlinearity $f(z)$ satisfies the constraint (4).

We consider the problem of output feedback stabilization of the nonlinear system (28), (2), (4) via a digital communication channel with a bit-rate constraint. Our controller consists of two components. The first component is developed at the measurement location by taking the measured output $y(\cdot)$ and coding to the codeword $h(jT)$. This component will be called "coder". Then the codeword $h(jT)$ is transmitted via a limited capacity communication channel to the second component which is called "decoder-controller". The second component developed at a remote location takes the codeword $h(jT)$ and produces the control input $u(t)$ where $t \in [jT, (j+1)T)$. This situation is illustrated in Figure 2.

The coder and the decoder are of the following form:
Coder:

$$h(jT) = \mathcal{F}_j\left(y(\cdot)|_0^{jT}\right); \qquad (29)$$

Decoder-Controller:

$$u(t)|_{jT}^{(j+1)T} = \mathcal{G}_j\left(h_T, h_{2T}, ..., h_{(j-1)T}, h_{jT}\right). \qquad (30)$$

Here $j = 1, 2, 3, \dots$.

Definition 3.1: The system (28), is said to be stabilizable via a digital communication channel of capacity l if there exists a coder-decoder-controller pair (29), (30) with a coding alphabet of size l such that

$$\lim_{t\to\infty} \|x(t)\|_\infty = 0; \qquad \lim_{t\to\infty} \|u(t)\|_\infty = 0 \qquad (31)$$

for any solution of the closed-loop system (28), (2), (4). A coder-decoder pair (29), (30) satisfying condition (31) is called stabilizing.

We will need the following Riccati algebraic inequality

$$A'R + RA + K'K + R\left(B_1B_1' - \alpha_2 B_2 B_2'\right)R < 0 \qquad (32)$$

and a related state feedback controller

$$u(t) = -\frac{\alpha_2}{2}B_2'Rx(t). \qquad (33)$$

Lemma 3.1: Suppose that for some $\alpha_2 > 0$ there exists a solution $R > 0$ of the Riccati inequality (32). Then the closed-loop system (28), (4), (33) is globally asymptotically stable, i.e.,

$$\lim_{t\to\infty} \|x(t)\|_\infty = 0. \qquad (34)$$

Proof of Lemma 3.1 The system (28), (33) can be re-written as

$$\dot{x}(t) = (A - \frac{\alpha_2}{2}B_2B_2'R)x(t) + B_1\phi(t) \qquad (35)$$

and the constraint (20). Since $R > 0$ is a solution of (32), it is also a solution of of the Riccati inequality

$$\begin{aligned} (A - \frac{\alpha_2}{2}B_2B_2'R)'R + R(A - \frac{\alpha_2}{2}B_2B_2'R) \\ +K'K + RB_1B_1'R < 0. \end{aligned}$$

Therefore, according to Strict Bounded Real Lemma (see e.g. Lemma 3.1.2 of [17] the system (35), (33), (20) is quadratically stable. Now the statement of the lemma immediately follows from quadratic stability. This completes the proof of Lemma 3.1.

Now consider the following state estimator that will be a part of our proposed coder:

$$\begin{aligned} \dot{\tilde{x}}(t) = (A - GC)\tilde{x}(t) + Gy(t) \\ +B_1 f(\tilde{z}(t)) + B_2 u(t); \\ \tilde{z}(t) = K\tilde{x}(t), \quad \tilde{x}(0) = 0, \end{aligned} \qquad (36)$$

where the gain G is introduced by (16).

We now suppose that assumptions of Lemmas 2.1 and 2.2 are satisfied. Then let $T > 0$ be a time such that conditions (10) and (17) hold. Furthermore, introduce $a(jT)$ by (23).

Introduce now our proposed coder-decoder-controller:
Coder:

$$h(jT) = \{i_1, i_2, \dots, i_n\} \qquad (37)$$

for $(\tilde{x}(jT) - \hat{x}(jT - 0)) \in I^1_{i_1}(a(jT)) \times I^2_{i_2}(a(jT)) \times \dots \times I^n_{i_n}(a(jT)) \subset \mathcal{B}_{a(jT)}$;

Decoder-Controller:

$$\begin{aligned}
\hat{x}(0) = 0; \quad \hat{z}(t) = K\hat{x}(t); \\
\dot{\hat{x}}(t) = A\hat{x}(t) + B_1 f(\hat{z}(t)) + B_2 u(t) \quad \forall t \neq jT; \\
\hat{x}(jT) = \hat{x}(jT-0) + \eta(i_1, i_2, \ldots, i_n) \\
\text{for } h(jT) = \{i_1, i_2, \ldots, i_n\} \quad \forall j = 1, 2, \ldots; \\
u(t) = -\frac{\alpha_2}{2} B_2^{'} R\hat{x}(t).
\end{aligned} \tag{38}$$

Similar to the coder-decoder proposed for the detectability problem in Section II, the equations (38) are a part of the both coder and decoder-controller. It then follows immediately from (10), (17) and initial condition $\tilde{x}(0) = 0$ that

$$\tilde{x}(T) \in \mathcal{B}_{a(T)}; \quad (\tilde{x}(jT) - \hat{x}(jT-0)) \in \mathcal{B}_{a(jT)} \tag{39}$$

for all $j = 2, 3, \ldots$ and for any solution of (28), (2), (4).

Now we are in a position to present the main result of this section.

Theorem 3.1: Suppose that for some $\alpha > 0$ there exists a solution $X > 0$ of the Riccati inequality (8), for some $\alpha_1 > 0$ there exists a solution $Y > 0$ of the Riccati inequality (9) and for some $\alpha_2 > 0$ there exists a solution $R > 0$ of the Riccati inequality (32). Furthermore, suppose that for some $T > 0$ satisfying conditions (10), (17) and some positive integer q the inequality (27) holds. Then the coder-decoder-controller (36), (16), (23), (37), (38) is stabilizing for the system (28), (2), (4).

Proof of Theorem 3.1 Condition (27) implies that $\lim_{j\to\infty} a(jT) = 0$. Hence

$$\lim_{j\to\infty} (\eta(i_1, i_2, \ldots, i_n) - (\tilde{x}(jT) - \hat{x}(jT))) = 0$$

where $h(jT) = \{i_1, i_2, \ldots, i_n\}$. This implies that

$$\lim_{j\to\infty} (\hat{x}(jT) - \tilde{x}(jT)) = 0$$

¿From this and Lemma 2.2 we obtain that

$$\lim_{j\to\infty} (x(jT) - \hat{x}(jT)) = 0$$

for any solution of the system (28), (2), (4). This and Lemma 3.1 implies stability of the closed-loop system. This completes the proof of Theorem 3.1.

Remark 3.1: Our future research will be focused on extensions of the results of this paper to the case of noisy discrete channels using techniques of [6], [5].

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