Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Robust Adaptive Fuzzy Output Control for Nonlinear Uncertain Systems

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Abstract— **In this paper, a robust adaptive fuzzy output control scheme for a class of nonlinear systems with uncertainty is proposed. The controller design is based on a novel fuzzy model, which is called the generalized fuzzy hyperbolic model (GFHM) and which does not need the availability of state variables. By designing an observer to estimate the states, the robust adaptive fuzzy output feedback control scheme is realized. Based on the Lyapunov stability theorem, the control system can guarantee that the tracking error converges to a small neighborhood of the origin. Simulation results confirm that the present control algorithms are feasible for practical applications.**

I. Introduction

Based on the universal approximation theorem and by incorporating fuzzy systems into adaptive control schemes, fuzzy adaptive controllers are first proposed by Wang [1], [2], [3]. Afterward, various adaptive fuzzy control approaches for nonlinear systems have been developed [4], [5], [6], [7], [8], [9], [10], [11]. However, since the fuzzy descriptions are imprecise and may be insufficient to achieve the desired accuracy, the approximation error introduced into the feedback loop makes it difficult to guarantee the stability of closed-loop control system [3]. In [7], this problem was solved by sliding mode method, but nonsmooth control input is generated. In general, such discontinuous adaptive control schemes are to be avoided. Another problem in [3], [4], [8] is that bounds on the unknown plant must be known. Generally, this calculation may require an exact model of the plant, which deviates from the purpose of using a model-free technique. However, these adaptive control methods are limited only to systems whose states are assumed to be available for measurement. If system states are not available, which is common in practice, the fuzzy state feedback control algorithms will not work and fuzzy output feedback control by using the estimated states will be required. In [5], [8], [10], the adaptive controller design for nonlinear systems whose states are not available for measurement is discussed, but the number of controller parameters for on-line adjustment is very large.

Recently, Zhang and co-workers proposed a class of fuzzy model, i.e., the fuzzy hyperbolic model (FHM) and the generalized fuzzy hyperbolic model (GFHM) [12], [13], [14]. It is proved that the generalized fuzzy hyperbolic model is a universal approximator and can be used to establish the model for nonlinear systems. There are several good examples using this class of fuzzy model [12], [14]. The purpose of this paper is to develop a robust adaptive control algorithm using the GFHM for nonlinear systems with uncertainties. The Lyapunov stability theorem is used to derive control law and controller parameter update law, which ensure the stability of the closed-loop system and good tracking performance. The present control scheme guarantees that the tracking error converges to a small neighborhood of the origin.

The paper is organized as follows. In Section II, preliminaries for the GFHM are reviewed. In Section III, the GFHM is used to describe a class of nonlinear systems. In Section IV, a state observer is constructed and then the robust adaptive fuzzy output control scheme is developed. In Section V, a simulation example is provided to demonstrate the controller design procedure. Finally, in Section VI, conclusions are provided.

II. Preliminaries

In this section we review some necessary preliminaries for the GFHM.

In the GFHM, there are two types of fuzzy sets, including positive (P_x) and negative (N_x) [13], [14]. The membership functions of P_x and N_x are defined as:

$$\mu_{P_x}(x_z) = e^{-\frac{1}{2}(x_z-k_z)^2}, \quad (1)$$

$$\mu_{N_x}(x_z) = e^{-\frac{1}{2}(x_z+k_z)^2}, \quad (2)$$

where $k_z > 0$ is a constant. We transform the input variable x_z as follows:

$$\bar{x}_i = x_z - d_i, \quad (3)$$

where $i = 1, \cdots, w$ (w is a positive integer) and d_i is a constant. We can see that after the linear transformation of x, the fuzzy sets may cover the whole input space if w is large enough.

Definition 1 ([13]): Given a plant with n input variables $x = [x_1(t), \cdots, x_n(t)]^T$ (where x is any state variable or input variable), and n output variable $\dot{x} = [\dot{x}_1, \cdots \dot{x}_n]^T$, we define

This work was supported by the National Natural Science Foundation of China under Grant 60274017.

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the generalized input variables as follows:

$$\begin{aligned}\bar{x}_1 &= x_1 - d_{11},\\ &\cdots\\ \bar{x}_{w_1} &= x_1 - d_{1w_1},\\ \bar{x}_{w_1+1} &= x_2 - d_{21},\\ &\cdots\\ \bar{x}_m &= x_n - d_{nw_n},\end{aligned}$$

where $m=\sum_{i=1}^{n} w_i$ is the number of generalized input variables, w_z $(z=1,\ldots,n)$ are the numbers to be transformed about x_z, d_{zj} $(z=1,\cdots,n,\ j=1,\cdots,w_z)$ are constants used for transforming x_z. We define the following fuzzy rule base the generalized fuzzy hyperbolic rule base if it satisfies the conditions of:

(1) For each output variable $\dot{x}_l, l=1,\ldots,n$, the corresponding group of fuzzy rules has the following form:

IF (x_1-d_{11}) is $F_{x_{11}}$, …, $(x_1-d_{1w_1})$ is $F_{x_{1w_1}}$, (x_2-d_{21}) is $F_{x_{21}}$, …, $(x_n-d_{nw_n})$ is $F_{x_{nw_n}}$

$$\text{THEN}\ \ \dot{x}_l = c_{F_{11}}+\cdots+c_{F_{1w_1}}+c_{F_{21}}+\cdots+c_{F_{nw_n}}, \tag{4}$$

where $F_{x_{zj}}$ are fuzzy sets of x_z-d_{zj}, which include P_x (positive) and N_x (negative) subsets. $c_{F_{zj}}$ is a constant corresponding to $F_{x_{zj}}$.

(2) The constant $c_{F_{zj}}$ $(z=1,\ldots,n,\ j=1,\ldots,w_z)$ in the "THEN" part correspond to $F_{x_{zj}}$ in the "IF" part, that is, if there is $F_{x_{zj}}$ in the "IF" part, $c_{F_{zj}}$ must appear in the "THEN" part. Otherwise, $c_{F_{zj}}$ does not appear in the "THEN" part.

(3) There are 2^m fuzzy rules for each output variable in the rule base, where $m=\sum_{i=1}^{n} w_i$, that is, all the possible P_x and N_x combinations of input variables in the "IF" part and all the linear combinations of constants in the "THEN" part.

Lemma 1 ([13]): For a multi-input dynamic system, $x=[x_1(t),\cdots,x_n(t)]^T$ is the state variable vector, $u=[u_1,\cdots,u_p]^T$ is the input variable vector. If we define the generalized fuzzy hyperbolic rule base and generalized input variables as Definition 1, and define the membership functions of the generalized input variables P_x and N_x as (1) and (2), then we can derive the following model:

$$\begin{aligned}\dot{x}_l &= \sum_{i=1}^{m}\frac{c_{P_{x_i}}e^{k_{x_i}\bar{x}_i}+c_{N_{x_i}}e^{-k_{x_i}\bar{x}_i}}{e^{k_{x_i}\bar{x}_i}+e^{-k_{x_i}\bar{x}_i}}+\sum_{j=1}^{q}\frac{c_{P_{u_j}}e^{k_{u_j}\bar{u}_j}+c_{N_{u_j}}e^{-k_{u_j}\bar{u}_j}}{e^{k_{u_j}\bar{u}_j}+e^{-k_{u_j}\bar{u}_j}}\\ &= \frac{1}{2}\sum_{i=1}^{m}(c_{Px_i}+c_{Nx_i})+\frac{1}{2}\sum_{i=1}^{m}(c_{Px_i}-c_{Nx_i})\frac{e^{k_{x_i}\bar{x}_i}-e^{-k_{x_i}\bar{x}_i}}{e^{k_{x_i}\bar{x}_i}+e^{-k_{x_i}\bar{x}_i}}\\ &\quad+\frac{1}{2}\sum_{j}^{q}(c_{Pu_j}+c_{N_{u_j}})+\frac{1}{2}\sum_{j=1}^{q}(c_{P_{u_j}}-c_{N_{u_j}})\frac{e^{k_{u_j}\bar{u}_j}+e^{-k_{u_j}\bar{u}_j}}{e^{k_{u_j}\bar{u}_j}+e^{-k_{u_j}\bar{u}_j}}\\ &= A_0+A_1\tanh(K_x\bar{x})+B\tanh(K_u\bar{u})\\ &= F(x),\end{aligned} \tag{5}$$

where $A_0=\frac{1}{2}\sum_{i=1}^{m}(c_{P_{x_i}}+c_{N_{x_i}})+\frac{1}{2}\sum_{j=1}^{q}(c_{P_{u_j}}+c_{N_{u_j}})$; $A_1=[a_1,\ldots,a_m]$, $a_i=\frac{1}{2}(c_{Px_i}-c_{Nx_i})$; $B=[b_1,\ldots,b_q]$, $b_j=\frac{1}{2}(c_{Pu_j}-c_{Nu_j})$; $\bar{u}_j$ $(j=1,\ldots,q,\ q=\sum_{l=1}^{p} r_l)$ is the generalized input variable after the linear transformation of u_l $(l=1,\ldots,p)$; $\tanh(K_x\bar{x})$ and $\tanh(K_u\bar{u})$ are defined by $\tanh(K_x\bar{x})=[\tanh(k_1\bar{x}_1),\cdots,\tanh(k_m\bar{x}_m)]^T$ and $\tanh(K_u\bar{u})=[\tanh(k_{u_1}\bar{u}_1),\cdots,\tanh(k_{u_q}\bar{u}_q)]^T$, respectively; and $K_x=\mathrm{diag}[k_{x_1},\ldots,k_{x_m}]$, $K_u=\mathrm{diag}[k_{u_1},\ldots,k_{u_q}]$. We call (5) the generalized fuzzy hyperbolic model (GFHM).

Let Y be the space composed of all the functions having the form of the right-hand side of (5). Then we have the following conclusion.

Lemma 2 ([13]): For any given real continuous $g(x)$ on the compact set $U\subset R^n$ and any arbitrary $\varepsilon>0$, there exists an $F(x)\in Y$ such that

$$\sup_{x\in U}|g(x)-F(x)|<\varepsilon.$$

Remark 1: There are some distinguishing characteristics about the GFHM:

1) The GFHM is a nonlinear model in nature. Unlike the Takagi-Sugeno (T-S) fuzzy model, which is a combination of local linear models, the GFHM is a global nonlinear model.

2) The GFHM can be proved to be a universal approximator.

3) The GFHM is a fuzzy model that can easily be derived from known linguistic information.

4) The GFHM is equivalent to a series expansion of fuzzy hyperbolic basis functions, $[1,\tanh(k_{x_1}\bar{x}_1),\cdots,\tanh(k_{x_m}\bar{x}_m),\tanh(k_{u_1}\bar{u}_1),\cdots,\tanh(k_{u_q}\bar{u}_q)]^T$. This basis function expansion is linear in its adjustable parameters; therefore, we can use the least squares algorithm to determine the parameters.

In this paper, we will design a robust adaptive fuzzy controller based on the GFHM in the form of (5).

III. Description of Nonlinear Systems with Uncertainties

Consider the following MIMO nonlinear systems:

$$\begin{aligned}\dot{x}_{11} &= x_{12},\cdots,\dot{x}_{1(r_1-1)}=x_{1r_1},\\ \dot{x}_{1r_1} &= f_1(X)+g_{11}(X)u_1+\cdots+g_{1m}(X)u_m+d_1,\\ &\ \ \vdots\\ \dot{x}_{m1} &= x_{m2},\cdots,\dot{x}_{m(r_m-1)}=x_{mr_m},\\ \dot{x}_{mr_m} &= f_m(X)+g_{m1}(X)u_1+\cdots+g_{mm}(X)u_m+d_m,\\ y_1 &= x_{11},\cdots,y_m=x_{m1},\end{aligned} \tag{6}$$

where $X=[x_{11},\cdots,x_{1r_1},x_{21},\cdots,x_{2r_2},\cdots,x_{m1},\cdots,x_{mr_m}]^T\in R^n$ is the state vector, $y=[y_1,\cdots,y_m]^T\in R^m$ is the output vector, $u=[u_1,\cdots,u_m]^T\in R^m$ is the input vector, $[r_1,\cdots,r_m]$ is the relative order of the system and $r_1+\cdots+r_m=n$. $f(X)=[f_1(X),\cdots,f_m(X)]^T$ is unknown continuous function vector, $G(X)=\begin{bmatrix} g_{11}(X) & \cdots & g_{1m}(X)\\ \vdots & \vdots & \vdots\\ g_{m1}(X) & \cdots & g_{mm}(X)\end{bmatrix}$ is control gain matrix, $g_{ij}(X)$ is an unknown continuous function, and $d=[d_1,\cdots,d_m]^T\in R^m$ is the bounded uncertainty including external disturbance, unmodeled dynamics and measurement noise.

Expressing (6) in the following form

$$\begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} f_1(X) \\ \vdots \\ f_m(X) \end{bmatrix} + G(X) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}, \quad (7)$$

the system (7) can be reduced to m multiple-input-single-output subsystems. Then, the i-th subsystem can be expressed as

$$\begin{aligned} y_i^{(r_i)} &= f_i(X) + \sum_{j=1}^{m} g_{ij}(X)u_j + d_i \\ &= f_i(X) + g_{ii}(X)u_i + d_{ui}, \end{aligned} \quad (8)$$

where $d_{ui} = \sum_{j=1, j\neq i}^{m} g_{ij}(X)u_j + d_i$.

Next, we derive the fuzzy model in the form of (5) using partial knowledge about the system. If we define the generalized fuzzy hyperbolic rule base and the generalized input variables as in Definition 1, then we can derive the following model:

$$y_i^{(r_i)} = \dot{x}_{ir_i} = A_{0i} + A_{1i}\tanh(K_x\bar{X}) + B_i\tanh(K_{u_i}\bar{u}_i) + \varepsilon_i + d_{ui}, \quad (9)$$

where $A_{0i} \in R^{1\times 1}$, $A_{1i} \in R^{1\times p}$, $B_i \in R^{1\times q}$, $\bar{X} = [\bar{X}_1, \cdots, \bar{X}_p]^T$ is the generalized state variable vector, $\bar{X}_i$ $(i = 1, \ldots, p, p = \sum_{i=1}^{n} w_i)$ is the generalized state variable after the linear transformation of X_z $(z = 1, \ldots, n)$, $\bar{u}_i = u_i - d_{ui}$ is the generalized input variable after the linear transformation of u_i, $\tanh(K_x\bar{X}) = \left[\tanh(k_{x_1}\bar{X}_1) \ \cdots \ \tanh(k_{x_p}\bar{X}_p)\right]^T$, $\tanh(K_u\bar{u}_i) = \left[\tanh(k_{u_1}\bar{u}_1) \ \cdots \ \tanh(k_{u_q}\bar{u}_q)\right]^T$, and ε_i is the model error.

We assume that the control u_i is bounded. Since the variables of real physical systems are always bounded, such an assumption seems reasonable. After the linearization of (9) in $\bar{u}_i$ we get the following form:

$$\begin{aligned} y_i^{(r_i)} &= A_{0i} + A_{1i}\tanh(K_x\bar{X}) + B_iK_{u_i}(u_i - d_{u_i}) + \Delta_i + \varepsilon_i + d_{ui} \\ &= \bar{A}_{0i} + A_{1i}\tanh(K_x\bar{X}) + b_iu_i + d_{si}, \end{aligned} \quad (10)$$

where $\bar{A}_{0i} = A_{0i} - B_iK_{u_i}d_{u_i}$, $b_i = B_iK_u$, Δ_i is the linearized bias, and $d_{si} = \Delta_i + \varepsilon_i + d_{ui}$ is a combining uncertainty. Since the control u_i is bounded, the error Δ_i is also bounded. Thus, d_{si} is a bounded uncertainty. Let $\bar{A}_i = [\bar{A}_{0i}, A_{1i}]$, $\bar{f}_i(X) = [1, \tanh(k_{x_1}\bar{X}_1), \cdots, \tanh(k_{x_p}\bar{X}_p)]^T$. Then (10) is rewritten in the following form

$$y_i^{(r_i)} = \dot{x}_{ir_i} = \bar{A}_i\bar{f}_i(X) + b_iu_i + d_{si}. \quad (11)$$

Define $X_i = [x_{i1}, \cdots, x_{ir_i}]^T$ and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{r_i\times r_i}$$

$$B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_i\times 1} \quad \text{and} \quad C_i = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{r_i\times 1.}$$

Then (11) is equivalent to the following system

$$\begin{aligned} \dot{X}_i &= A_iX_i + B_i[\bar{A}_i\bar{f}_i(X) + b_iu_i + d_{si}], \\ y_i &= C_i^TX_i, \quad i = 1, \cdots, m. \end{aligned} \quad (12)$$

From the above, we can obtain the fuzzy model using prior knowledge about the plant such that we can incorporate the information into controller design later.

IV. Robust Adaptive Fuzzy Output Feedback Control

For nonlinear systems with uncertainties in the form of (12), the control objective is to determine a control scheme and adaptive law for the parameters such that the output vector $y(t) = [y_1(t), \cdots, y_m(t)]^T$ tracks a given bounded reference signal vector $y_M(t) = [y_{M1}(t), \cdots, y_{Mm}(t)]^T$.

For i-th subsystem, if $d_{si} = 0$ and all states of the system are available for measurement, we can take $e_i = y_{Mi} - y_i = y_{Mi} - x_i$, $E_i = [e_i, \dot{e}_i, \cdots, e_i^{(r_i-1)}]^T$ and $K_{ci} = [k_{ir_i}, \cdots, k_{i1}]^T \in R^{r_i}$ such that all roots of the polynomial $h_i(s) = s^{r_i} + k_{ir_i}s^{r_i-1} + \cdots + k_{i1}$ are in the open left-half complex plane, and choose the control law as

$$u_i = \frac{1}{b_i}[-\bar{A}_i\bar{f}_i(X) + y_{Mi}^{(r_i)} + K_{ci}^TE_i], \quad (13)$$

where u_i is the so-called certainty equivalent controller [2].

Consider the system in the form of (12) whose states are not available and only the system output $y_i(t)$ is available, which is common in practice, the above controller could not be realized. Therefore, an observer is designed to estimate states and the error, and then fuzzy output feedback control is developed.

Defined $\hat{x}_i$ and $\hat{e}_i = y_{Mi} - \hat{x}_{i1}$ as the estimate of x_i and e_i, the observation error $\tilde{e}_i = e_i - \hat{e}_i$, $\hat{E}_i = [\hat{e}_i, \dot{\hat{e}}_i, \cdots, \hat{e}_i^{(r_i-1)}]^T$, the observation error vector $\tilde{E}_i = [\tilde{e}_i, \dot{\tilde{e}}_i, \cdots, \tilde{e}_i^{(r_i-1)}]^T$, $Y_{Mi} = [y_{Mi}, \cdots, y_{Mi}^{(r_i-1)}]^T$, and the state estimate vector $\hat{X}_i = Y_{Mi} - \hat{E}_i = [\hat{x}_{i1}, \cdots, \hat{x}_{ir_i}]^T$. The fuzzy controller is designed as

$$u_i = u_{ci}(\hat{X}) - \frac{1}{b_i}u_{si} - \frac{1}{b_i}u_{ai}, \quad (14)$$

where

$$u_{ci}(\hat{X}) = \frac{1}{b_i}[-\bar{A}_i\bar{f}_i(\hat{X}) + y_{Mi}^{(r_i)} + K_{ci}^T\hat{E}_i], \quad (15)$$

and u_{si} and u_{ai} will be defined later.

Substituting (14) into (12) results in

$$\dot{E}_i = A_iE_i - B_iK_{ci}^T\hat{E}_i + B_iu_{si} + B_iu_{ai} + B_iD_{si}, \quad (16)$$

where $D_{si} = \bar{A}_i\bar{f}_i(\hat{X}) - \bar{A}_i\bar{f}_i(X) - d_{si}$ is a bounded uncertainty. Design the observer as follows

$$\begin{aligned} \dot{\hat{E}}_i &= A_i\hat{E}_i - B_iK_{ci}^T\hat{E}_i + K_{0i}(e_i - \hat{e}_i), \\ \hat{e}_i &= C_i^T\hat{E}_i, \end{aligned} \quad (17)$$

where $K_{0i}^T = [k_{ir_i}^0, k_{i(r_i-1)}^0, \cdots, k_{i1}^0]$ is the state observer gain vector. Subtracting (17) from (16) results in

$$\begin{aligned} \dot{\tilde{E}}_i &= (A_i - K_{0i}C_i^T)\tilde{E}_i + B_i(u_{si} + u_{ai} + D_{si}), \\ \tilde{e}_i &= C_i^T\tilde{E}_i. \end{aligned} \quad (18)$$

Since only the output $\tilde{e}_i$ in (18) is assumed to be measurable, we use the strictly-positive-real (SPR) Lyapunov design approach to analyze the stability of (18).

First, (18) can be rewritten as

$$\tilde{e}_i = H_i(s)[u_{si} + u_{ai} + D_{si}], \quad (19)$$

where $H_i(s) = C_i^T[sI - (A_i - K_{0i}C_i^T)]^{-1}B_i$ is a known stable transfer function. In order to employ the SPR-Lyapunov design approach, (19) can be written as

$$\tilde{e}_i = H_i(s)L_i(s)[u_{ssi} + u_{aai} + D_{ssi}], \quad (20)$$

where $u_{ssi} = L_i^{-1}(s)u_{si}$, $u_{aai} = L_i^{-1}(s)u_{ai}$, $D_{ssi} = L_i^{-1}(s)D_{si}$ and $L_i(s)$ is chosen so that $L_i^{-1}(s)$ is a proper stable transfer function and $H_i(s)L_i(s)$ is a proper SPR transfer function. Let

$$L_i(s) = s^{m_i} + b_{i1}s^{m_i-1} + \cdots + b_{im_i} \ (m_i = r_i - 1).$$

The state-space realization of (20) can be written as

$$\begin{aligned} \dot{\tilde{E}}_{si} &= A_{si}\tilde{E}_{si} + B_{si}[u_{ssi} + u_{aai} + D_{ssi}], \quad (21) \\ \tilde{e}_i &= C_{si}^T\tilde{E}_{si}, \end{aligned}$$

where

$$\begin{aligned} \tilde{E}_{si} &= [\tilde{e}_i, \dot{\tilde{e}}_i, \cdots, \tilde{e}_i^{(r_i-1)}]^T, A_{si} = A_i - K_{0i}C_i^T, \\ B_{si} &= [1, b_{i1}, \cdots, b_{im_i}]^T, C_{si} = [1, 0, \cdots, 0]^T. \end{aligned}$$

In order to design the robust control law, a GFHM is employed to approximate the uncertainty in this paper. From Lemma 2, we may conclude that there exists a fuzzy system (5) that can be used to approximate the uncertain bounded function D_{ssi}. Hence, we can obtain the following bounded function:

$$|D_{ssi}| \le \sigma_i^T \left|\xi_i(\hat{X})\right| + \varepsilon_i, \quad (22)$$

where $\xi_i(\hat{X}) = [1, \xi_{i1}(\hat{X}), \cdots, \xi_{ik}(\hat{X})]^T = [1, \tanh(k_{d_1}\bar{\hat{X}}_1), \cdots, \tanh(k_{d_k}\bar{\hat{X}}_k)]^T$ is the unknown fuzzy hyperbolic base function vector, $\sigma_i = [\sigma_{i0}, \sigma_{i1}, \cdots, \sigma_{ik}]^T$ is the weight parameter vector of fuzzy system. ε_i is a parameter for respecting approximating accuracy. Suppose that σ_i and ε_i are unknown, the above bounded function D_{ssi} can be rewritten in the following form:

$$|D_{ssi}| \le \theta_i^T \psi_i(\hat{X}), \quad (23)$$

where $\psi_i(\hat{X}) = [1, 1, \left|\xi_{i1}(\hat{X})\right|, \cdots, \left|\xi_{ik}(\hat{X})\right|]^T$ is a known vector and $\theta_i = [\varepsilon_i, \sigma_{i0}, \cdots, \sigma_{ik}]^T$ is an unknown vector.

Assumption 1: For the given positive definite matrices Q_{i1} and Q_{i2}, there exists positive definite matrix solutions P_{i1} and P_{i2}, respectively, for the matrix equations (16) and (21)

$$(A_i - B_iK_{ci}^T)^TP_{i1} + P_{i1}(A_i - B_iK_{ci}^T) = -Q_{i1}, \quad (24)$$

$$A_{si}^TP_{i2} + P_{i2}A_{si} = -Q_{i2}, \quad (25)$$

$$P_{i2}B_{si} = C_{si}.$$

From (25), we know that $\tilde{E}_{si}^TP_{i2}B_{si} = C_{si}^T\tilde{E}_{si} = \tilde{e}_i$, while $\tilde{e}_i = y_{Mi} - y_i - \hat{e}_i$ is available for feedback control. We design the compensation control terms u_{ssi}, u_{aai} and the parameter update law as follows:

$$\begin{aligned} u_{aai} &= -K_{0i}^TP_{i1}\hat{E}_i, \quad (26) \\ u_{ssi} &= -\hat{\theta}_i^T\psi_i(\hat{X})\tanh(\hat{\theta}_i^T\psi_i(\hat{X})B_{si}^TP_{i2}\tilde{E}_{si}/\varepsilon_{di}) \\ &= -\hat{\theta}_i^T\psi_i(\hat{X})\tanh(\hat{\theta}_i^T\psi_i(\hat{X})\tilde{e}_i/\varepsilon_{di}), \quad (27) \\ \dot{\hat{\theta}}_i &= -\lambda_i(\tilde{\theta}_i + \theta_i) + r_i\psi_i(\hat{X})\left\|B_{si}^TP_{i2}\tilde{E}_{si}\right\| \\ &= -\lambda_i(\tilde{\theta}_i + \theta_i) + r_i\psi_i(\hat{X})\left\|\tilde{e}_i\right\|, \quad (28) \end{aligned}$$

where $\lambda_i \in (0, \infty)$, $r_i = \mathrm{diag}[r_{i1}, r_{i2}, \cdots, r_{il}]$, $r_{il} \in (0, \infty)$, l is the dimension of θ_i, $\hat{\theta}_i = \theta_i + \tilde{\theta}_i$, $\hat{\theta}_i$ is an estimate of θ_i. λ_i, r_i, ε_{di} are parameters determined by the designer.

Following the preceding consideration, we obtain the following theorem.

Theorem 1: Assume that the nonlinear system (6) satisfies Assumption 1. Then the robust adaptive fuzzy tracking controller described by the control laws (14), (26) and (27) with the parameter adaptation law (28) guarantees that for given any $\rho > \max\{\sqrt{\frac{\bar{\varepsilon}_1}{\mu_1}}, \cdots, \sqrt{\frac{\bar{\varepsilon}_m}{\mu_m}}\}$, there exists $T(\rho)$ such that for all $t > T$, $e_M \le \rho$, where $e_M = \max\{|e_1(t)|, \cdots, |e_m(t)|\}$,

$$\mu_i = \frac{1}{2}\min\left\{\frac{\lambda_{\min}(Q_{i1})}{\lambda_{\max}(P_{i1})}, \frac{\lambda_{\min}(Q_{i2})}{\lambda_{\max}(P_{i2})}, \lambda_i\right\},$$

$$\bar{\varepsilon}_i = \kappa\varepsilon_{di} + \frac{\lambda_i}{2r_{i\min}}\left\|\theta_i\right\|,$$

$r_{i\min} = \min\{r_{i1}, \cdots, r_{il}\}$.

Proof: For i-th subsystem, choose the following Lyapunov function candidate:

$$\begin{aligned} V_i &= \hat{E}_i^TP_{i1}\hat{E}_i + \tilde{E}_{si}^TP_{i2}\tilde{E}_{si} + \frac{1}{r_i}\tilde{\theta}_i^T\tilde{\theta}_i \\ &= z_i^T\bar{P}_iz_i = \bar{V}_i(z_i, t), \quad (29) \end{aligned}$$

where $z_i = [\hat{E}_i^T, \tilde{E}_{si}^T, \tilde{\theta}_i^T]^T$ and $\bar{P}_i = \mathrm{diag}[P_{i1}, P_{i2}, \frac{1}{r_i}]$.

The derivative of V_i along the trajectory of the system is given by

$$\begin{aligned} \dot{V}_i =& -\hat{E}_i^TQ_{i1}\hat{E}_i - \tilde{E}_{si}^TQ_{i2}\tilde{E}_{si} + 2\hat{E}_i^TP_{i1}(K_{0i}C_i^T)\tilde{E}_{si} \\ & + 2\tilde{E}_{si}^TP_{i2}B_{si}u_{aai} + 2\tilde{E}_{si}^TP_{i2}B_{si}u_{ssi} + 2\tilde{E}_{si}^TP_{i2}B_{si}D_{ssi} + \frac{2}{r_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i \\ \le& -\hat{E}_i^TQ_{i1}\hat{E}_i - \tilde{E}_{si}^TQ_{i2}\tilde{E}_{si} + 2\theta_i^T\psi_i(\hat{X})\left\|B_{si}^TP_{i2}\tilde{E}_{si}\right\| \\ & - 2\hat{\theta}_i^T\psi_i(\hat{X})B_{si}^TP_{i2}\tilde{E}_{si}\tanh(\hat{\theta}_i^T\psi_i(\hat{X})B_{si}^TP_{i2}\tilde{E}_{si}/\varepsilon_{di}) \\ & + \frac{2}{r_i}\tilde{\theta}_i^T\dot{\hat{\theta}}_i \\ =& -\hat{E}_i^TQ_{i1}\hat{E}_i - \tilde{E}_{si}^TQ_{i2}\tilde{E}_{si} + 2[\hat{\theta}_i^T\psi_i(\hat{X})\left\|B_{si}^TP_{i2}\tilde{E}_{si}\right\| \\ & - \hat{\theta}_i^T\psi_i(\hat{X})B_{si}^TP_{i2}\tilde{E}_{si}\tanh(\hat{\theta}_i^T\psi_i(\hat{X})B_{si}^TP_{i2}\tilde{E}_{si}/\varepsilon_{di})] \\ & + 2\tilde{\theta}_i^T\left[\frac{1}{r_i}\dot{\hat{\theta}}_i - \psi_i(\hat{X})\left\|B_{si}^TP_{i2}\tilde{E}_{si}\right\|\right]. \quad (30) \end{aligned}$$

It can be shown that the following inequality holds for any $\varepsilon_{di} > 0$ and for any $\eta \in R$

$$0 \le |\eta| - \eta\tanh\left(\frac{\eta}{\varepsilon_{di}}\right) \le \kappa\varepsilon_{di}, \quad (31)$$

where κ is a constant that satisfies $\kappa = \exp(-(\kappa+1))$, i.e., $\kappa = 0.2785$ [9].

Setting $\eta = \hat{\theta}_i^T \psi_i(\hat{X}) B_{si}^T P_{i2} \tilde{E}_{si}$, applying (31) and substitute (28) into (30), we have

$$\dot{V}_i \le -\hat{E}_i^T Q_{i1} \hat{E}_i - \tilde{E}_{si}^T Q_{i2} \tilde{E}_{si} + 2\kappa\varepsilon_{di} - 2\frac{\lambda_i}{r_i}\tilde{\theta}_i^T(\tilde{\theta}_i + \theta_i). \quad (32)$$

From $\frac{1}{2}(\tilde{\theta}_i+\theta_i)(\tilde{\theta}_i+\theta_i) \ge 0$,we can obtain

$$\tilde{\theta}_i^T\tilde{\theta}_i + \tilde{\theta}_i^T\theta_i \ge \frac{1}{2}(\tilde{\theta}_i^T\tilde{\theta}_i - \theta_i^T\theta_i). \quad (33)$$

Therefore, we get

$$\dot{V}_i \le -\hat{E}_i^T Q_{i1} \hat{E}_i - \tilde{E}_{si}^T Q_{i2} \tilde{E}_{si} + \frac{\lambda_i}{r_i}\theta_i^T\theta_i + 2\kappa\varepsilon_{di} - \frac{\lambda_i}{r_i}\tilde{\theta}_i^T\tilde{\theta}_i. \quad (34)$$

Furthermore, let

$$\bar{Q}_i = \text{diag}\left[Q_{i1}, Q_{i2}, \frac{\lambda_i}{r_i}\right].$$

It yields

$$\dot{\bar{V}}_i \le -z_i^T \bar{Q}_i z_i + 2\bar{\varepsilon}_i.$$

Substituting the parameters given in Theorem 1 into the above expression, we get

$$\dot{\bar{V}}_i \le -2\mu\bar{V}_i + 2\bar{\varepsilon}_i = -2\mu\left(\bar{V}_i - \frac{\bar{\varepsilon}_i}{\mu_i}\right) \quad (35)$$

Now, if we let $\frac{\bar{\varepsilon}_i}{\mu_i} > 0$, then (35) satisfies

$$0 \le V_i(t) \le \frac{\bar{\varepsilon}_i}{\mu_i} + \left(V_i(0) - \frac{\bar{\varepsilon}_i}{\mu_i}\right)\exp(-2\mu_i t). \quad (36)$$

Therefore, for i-th subsystem there exists $T(\rho_i)$, $\rho_i > \sqrt{\frac{\bar{\varepsilon}_i}{\mu_i}}$, such that for all $t \ge T$, we have $|e_i(t)| \le \rho_i$.

Furthermore, for the system (6) we obtain that given any $\rho > \max\left\{\sqrt{\frac{\bar{\varepsilon}_1}{\mu_1}}, \cdots, \sqrt{\frac{\bar{\varepsilon}_m}{\mu_m}}\right\}$, there exists $T(\rho)$ such that for all $t \ge T$ we have $e_M \le \rho$. This completes the proof.

V. Simulation Example

To illustrate the effectiveness of the present control scheme, we consider the control of the nonlinear system of [15], which is descried by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1 + x_2^2 + x_3 \\ x_1 + 2x_2 + 3x_3x_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3u_1 + u_2 \\ u_1 + 2(2+0.5\sin(x_1))u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ e^{-t}\sin t \\ e^{-t}\sin t \end{bmatrix} \quad (37)$$

where $y_1 = x_1$ and $y_2 = x_3$.

Rewrite (37) in the following form:

$$\begin{bmatrix} \ddot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2^2 + x_3 \\ x_1 + 2x_2 + 3x_3x_1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 1 & 4+\sin(x_1) \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} e^{-t}\sin t \\ e^{-t}\sin t \end{bmatrix} \quad (38)$$

where the relative order of the system is $[r_1 \ r_2\ ] = [2 \quad 1]$. The tracking reference signals are:

$$y_{M1} = 2\sin(0.5t + 0.5), \quad y_{M2} = \sin(t)$$

First, we derive the GFHM in the form of (12) from the prior knowledge about the system. By the linguistic knowledge, we can obtain the following fuzzy rules:

R^1: IF $x_1 - d_1$ is P_{x_1}, $x_2 - d_2$ is P_{x_2}, $x_3 - d_3$ is P_{x_3}, u_1 is P_{u_1}, and u_2 is P_{u_2},

THEN $\dot{x}_2 = c_{P_{x_{11}}} + c_{P_{x_{12}}} + c_{P_{x_{13}}} + c_{P_{u_{11}}} + c_{P_{u_{12}}}$,

$\vdots$

R^{32}: IF $x_1 - d_1$ is N_{x_1}, $x_2 - d_2$ is N_{x_2}, $x_3 - d_3$ is N_{x_3}, u_1 is N_{u_1}, and u_2 is N_{u_2},

THEN $\dot{x}_2 = c_{N_{x_{11}}} + c_{N_{x_{12}}} + c_{N_{x_{13}}} + c_{N_{u_{11}}} + c_{N_{u_{12}}}$,

R^{33}: IF $x_1 - d_1$ is P_{x_1}, $x_2 - d_2$ is P_{x_2}, $x_3 - d_3$ is P_{x_3}, u_2 is P_{u_2}, and u_2 is P_{u_2},

THEN $\dot{x}_3 = c_{P_{x_{21}}} + c_{P_{x_{22}}} + c_{P_{x_{23}}} + c_{P_{u_{21}}} + c_{P_{u_{22}}}$,

$\vdots$

R^{64}: IF $x_1 - d_1$ is N_{x_1}, $x_2 - d_2$ is N_{x_2}, $x_3 - d_3$ is N_{x_3}, u_2 is $N_{u_{12}}$, and u_2 is $N_{u_{22}}$,

THEN $\dot{x}_3 = c_{N_{x_{21}}} + c_{N_{x_2}} + c_{N_{x_{23}}} + c_{N_{u_{21}}} + c_{N_{u_{22}}}$,

where the membership functions of the fuzzy sets are in the form of (1) and (2), $K_x = \text{diag}[0.6, 0.6, 0.6]$, $K_u = \text{diag}[1, 1]$, $d_1 = d_2 = d_3 = 0$, $A_{01} = 0.0064, A_{02} = 0.0028$, $A_{11} = [0.027, 1.8395, 0.0051]$, $A_{21} = [-0.0128, 0.1065, 0.0070]$, $B_1 = [0.56, 1]$, and $B_2 = [1, 0.6]$. By the above fuzzy rules we derive the GFHM in the form of (12), where $b_1 = 0.56$ and $b_2 = 0.6$.

Assume that the states of the system are not available and only the system output y is available. The fuzzy output feedback controller is constructed. Select $K_{c1} = [\ 100 \quad 10\ ]^T$, $K_{01} = [\ 135 \quad 701\ ]^T$, $L_1(s) = s+5, L_2(s) = 1$, $B_{s1} = \begin{bmatrix} 1 & 5 \end{bmatrix}^T$, $B_{s2} = [1]$, $Q_{11} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, $Q_{12} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, $K_{c2} = 120$, $K_{02} = 100$, $Q_{21} = 20$, $Q_{22} = 200$. By solving the matrix equation (24) and (25), we obtain positive definite matrices as follows $P_{11} = \begin{bmatrix} 51 & 0.05 \\ 0.05 & 0.505 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 26 & -5 \\ -5 & 1 \end{bmatrix}$, $P_{21} = 1/12$, $P_{22} = [1]$. Let $\varepsilon_{d1} = 0.5, \lambda_1 = 0.2$, $\varepsilon_{d2} = 0.8, \lambda_2 = 0.4$, $r_1 = r_2 = \text{diag}[0.1, 0.1, 0.1, 0.1, 0.1]$, $\psi_1(x) = [1, 1, |\tanh(0.8x_1)|, |\tanh(0.8x_2)|, |\tanh(0.8x_3)|]^T$, $\psi_2(x) = [1, 1, |\tanh(x_1)|, |\tanh(x_2)|, |\tanh(x_3)|]^T$ and the initial values are chosen as $X(0) = [0.25, 0, 0]^T, \hat{X}(0) = [0.15, 0, 0]^T$. The result of simulation is illustrated in Figures 1–4. Figure 1 shows the trajectories of the states x_1 (dash-dot line), desired output y_{M1} (solid line). Figure 2 shows the trajectories of the states x_3 (dash-dot line), desired output y_{M2} (solid line). The trajectories of the state x_1 and its estimate $\hat{x}_1$ are depicted in Figures 3. Also, the control input u_1 and u_2 are shown in Figure 4.

It is clear that the observed states converge rapidly to the real ones. The simulation results also demonstrate that the present controller provides good tracking performance and generates smooth control input.

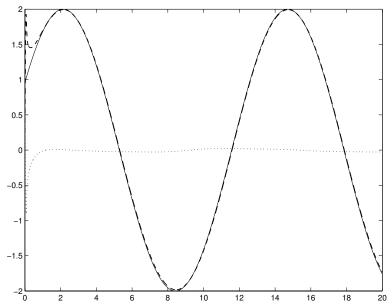

Fig. 1. Trajectories of the states x_1 (dash-dot line), desired output y_{M1} (solid line), and tracking error e_1 (dotted line)

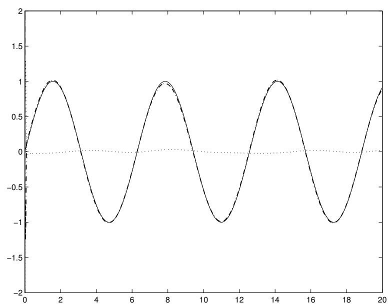

Fig. 2. Trajectories of the states x_3 (dash-dot line), desired output y_{M2} (solid line) and tracking error e_2 (dotted line)

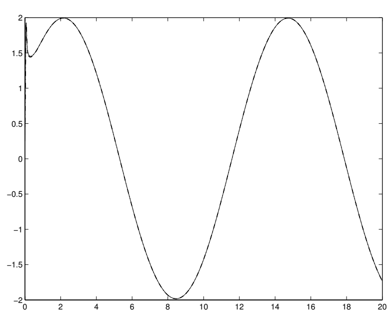

Fig. 3. State x_1 (solid line) and its estimate $\hat{x}_1$ (dash-dot line)

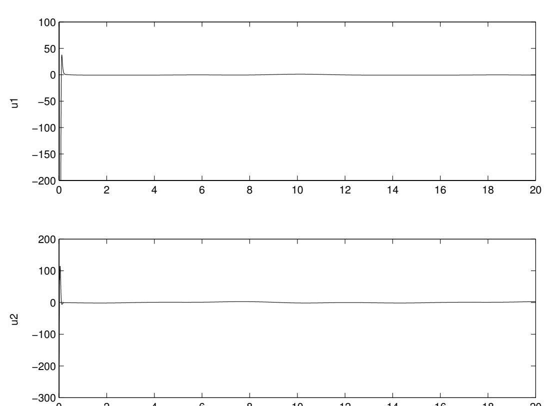

Fig. 4. Trajectories of the control input u_1 (in the upper figure) and u_2 (in the lower figure)

VI. CONCLUSIONS

In this paper, for MIMO nonlinear systems with uncertainties a robust adaptive state feedback control algorithm based on the GFHM is developed. The present control scheme guarantees that the tracking error of the closed-loop system converges to a small neighborhood of origin. The main advantage of the present control law is that the human knowledge about the plant under control is used to design the controller and the present control scheme is a smooth control with no chattering phenomena. Also, in every subsystem there is only one parameter vector to be adjusted on-line in the adaptive mechanism; thus, the on-line computing load is light. Simulation example demonstrates that the present controller provides good tracking performance and generates smooth control inputs.

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