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Robust H_∞ Fuzzy Control Design for Markovian Jump Systems

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Abstract— **This paper addresses the problem of stabilizing a class of nonlinear systems subject to Markovian jump parameters using a robust stochastic fuzzy controller with H_∞ performance. The class of jump nonlinear systems is described by a fuzzy model composed of two levels, a crisp level which represents the jumps and a fuzzy level which represents the system nonlinearities. Considering the approximation error between the fuzzy model and the jump nonlinear system as norm-bounded uncertainties we develop a systematic technique to obtain a robust stochastic fuzzy controller which guarantees the $\mathcal{L}_2$ gain of the closed-loop system in respect to external inputs to be equal to or less than a prescribed value. A simulation example on an electrical power system in co-generation scheme is presented to illustrate the approach.**

Index Terms— **Fuzzy system models, H_∞ controller, Robust control, Markovian jump systems, stabilizing controller.**

I. INTRODUCTION

The class of systems with Markovian jump parameters can be used to model a variety of physical systems. This class of systems has two components in the state vector. The first one varies continuously and is referred to as the continuous state of the system. The second one varies discretely and is referred to as the mode assumed by the system. This class of systems can be used to represent complex real systems, which may experience abrupt changes in their structure and parameters caused by phenomena such as component failures or repairs, changing of subsystem interconnections and abrupt environmental disturbances.

Using a linear representation of the Markovian jump systems, stochastic stability and stabilizability problems were addressed, for example in [1]. Lately, considerable attention has been paid to the robust control, robust stochastic stability and stabilizability of jump linear uncertain systems, for example in [2]. In general, the system uncertainties appear as norm-bounded sets, which facilitate the extension of the deterministic robust and optimal control techniques to the Markovian jump linear systems. To the best of our knowledge, the problem of stochastic stability and stochastic stabilizability of nonlinear systems have not been drawn much attention yet. The control of a class of nonlinear systems with Markovian jump parameters was considered in [3] wherein the problem is formulated in terms of dynamic programming. Recently, the H_∞ control problem for a class of linear systems with Markovian jump parameters and unknown nonlinearities was addressed in [4] wherein the robust stochastic stabilizability and disturbance attenuation are considered. In this approach, the unknown nonlinearities are modeled as norm-bounded uncertainties satisfying the matching condition. However, if nonlinearity is not distinguished from uncertainty, the results obtained are in general conservatives.

The fuzzy-model-based control techniques have been successful used to stabilize nonlinear systems [5], [6]. In general, the nonlinear dynamics are represented by a Takagi-Sugeno (TS) fuzzy system which is described by fuzzy IF-THEN rules representing local input-output relations of the nonlinear system. The fuzzy control system design is based on the so-called parallel distributed compensation (PDC) scheme which establishes that a linear control is designed for each local linear system. The overall controller is a fuzzy blending of all local linear controllers, which is nonlinear in general. Recently, a Markovian jump fuzzy control system technique was introduced in [7], [8]. In this approach, a class of Markovian jump nonlinear systems is represented by a fuzzy system model with two levels of structure, a crisp level which describes the jumps of the Markov process and a fuzzy level which describes the system nonlinearities. Using this fuzzy system, the control design is formulated in terms of the stochastic stabilizability concept, which can be efficiently solved using convex optimization techniques with LMI's.

Our goal is to include in the Markovian jump fuzzy model the approximation error between the fuzzy model and the nonlinear system. In practice, the effect of the approximation error can deteriorate the stability and control performance of nonlinear control systems. Then, using stochastic stability, we develop a systematic technique for designing a robust fuzzy control which guarantees the $\mathcal{L}_2$ gain of the closed loop system in respect to bounded disturbances to to be less or equal to a prescribed value. The remainder of the paper is organized as follows. Section II describes the fuzzy modeling and the problem statement. Section III establishes the main results on the robust stochastic fuzzy-model-based control design. We then present in Section IV the simulation results for an electrical power system in co-generation scheme. In Section V, we conclude with some remarks.

Throughout the paper, $M > 0$ means that M is a positive definite matrix of appropriate dimensions, $\mathbf{I}$ and $\mathbf{0}$ denote the identity and zero matrices of appropriate dimensions, respectively, $\sum_{j<k}^{R}$ denotes, for instance for $R = 3$, $\sum_{j<k}^{3} a_{jk} \Leftrightarrow a_{12} + a_{13} + a_{23}$, $E[\cdot]$ denotes the expectancy operator with respect to some probability measure $\Pr$, $\mathcal{L}_2[0, T]$ stands for the space of square integrable vector functions over the

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interval $[0,T]$, $\|\cdot\|$ refers to either the Euclidean vector norm or the matrix norm which is the operator norm induced by the standard vector norm, $\|\cdot\|_2$ stands for the norm in $\mathcal{L}_2[0,T]$, $E\|\cdot\|_2$ denotes the norm in $\mathcal{L}_2((\Omega,\mathcal{F},\Pr),[0,T])$ and $(\Omega,\mathcal{F},\Pr)$ is a probability space.

II. Problem Statement

We consider in this paper a class of Markovian jump nonlinear systems (MJNLS) defined in the probability space $\mathcal{L}_2((\Omega,\mathcal{F},\Pr),[0,T])$ which is described by the following differential equation

$$\dot{x} = f(x,r) + g(x,r)u + w;\ x(0) = x_0;\ r(0) = r_0 \qquad (1)$$

where $x \in \mathbb{R}^n$ is the system state vector, $u \in \mathbb{R}^m$ is the control input vector, $w \in \mathbb{R}^n$ is a bounded external disturbance belonging to $\mathcal{L}_2[0,\infty)$, r is a continuous-time Markovian process taking values in a finite state space denoted by $\mathbb{S} = \{1,2,...,N\}$ which determines the mode the system is, $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$, with $f(0,\cdot) = 0$ and $g(0,\cdot) = 0$ are smooth nonlinear functions with respect to the first argument, x_0 and r_0 are the initial values of the state and the mode at time $t = 0$, respectively.

The evolution of the stochastic process $\{r, t \geq 0\}$ that determines the mode of the system at each time t is assumed to be described by the following transition probability

$$\Pr\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j \\ 1 - \pi_i\Delta + o(\Delta), & i = j \end{cases} \qquad (2)$$

where $\Delta > 0$, $\lim_{\Delta\to 0} o(\Delta)\Delta^{-1} = 0$, $\pi_{ij} \geq 0$ is the probability rate between modes i and j, for $i \neq j$; $i,j \in \mathbb{S}$ and $\forall i \in \mathbb{S}$, $\pi_i = -\pi_{ii} = \sum_{j=1,j\neq i}^{N} \pi_{ij}$. The matrix $\Pi = [\pi_{ij}]_{i,j=1,2,...,N}$ is called transition rate matrix. We assume that the Markov process r has initial distribution $\mu = (\mu_1, \mu_2, \ldots, \mu_N)$, $\mu_i = \Pr\{r_0 = i\}$.

Let $z \in \mathbb{R}$ be a mode indicator variable, M_i a crisp set and N_{ijk} a fuzzy set. The MJNLS (1)-(2) is then represented by the Markovian jump fuzzy system as in [7], [8]. In a similar way as in the PDC scheme, this fuzzy system is structured in an upper and lower levels defined by the crisp sets M_i associated to the jumps of the Markov process indicated by z and by the fuzzy sets N_{ijk} associated to the nonlinearities in the state vector x. Thus, the ith mode assumed by system (1)-(2) is given by

Mode i :

If z is M_i

Then

Rule j :

If x_1 is N_{ij1} and $\ldots$ and x_n is N_{ijn}

Then $\dot{x} = A_{ij}x + B_{ij}u + w$

$i \in \mathbb{S};\ j = 1,2,\ldots,R \qquad (3)$

where A_{ij} and B_{ij} are matrices of appropriate dimensions which describe local linear representations of the nonlinear system in the vicinity of chosen operation points, R is the number of inference rules in each mode and variables $x_1,\ldots,x_n$ are components of the state vector x. In this fuzzy system, x and z are known as premise variables. Usually, the premise variables may be functions of state variables, external disturbances, and/or time.

The overall MJFS is inferred by a fuzzy blending of subsystems, which are selected according to the mode assumed by the Markov process, that is

$$\dot{x} = \sum_{i=1}^{N} m_i(z)\left[\sum_{j=1}^{R} n_{ij}(x)(A_{ij}x + B_{ij}u)\right] + w \qquad (4)$$

where $m_i(z)$ is the mode indicator which yields $m_i(z) = 1$ when $r = i$, that is, $z \in M_i$ and $m_i(z) = 0$ otherwise, and $n_{ij}(x)$ are normalized membership functions given by

$$n_{ij}(x) = \frac{\prod_{k=1}^{n} N_{ijk}(x_k)}{\sum_{l=1}^{R}\prod_{k=1}^{n} N_{ilk}(x_k)} \qquad (5)$$

with $N_{ijk}(x) \in [0,1]$ the grade of membership of x_k, $k = 1,2,\ldots,n$ in the fuzzy set N_{ijk}. In addition, considering the fact that in (5) $N_{ijk}(x_k) \geq 0$, $j = 1,2,\ldots,R$, we have $n_{ij}(x) \geq 0$ and $\sum_{j=1}^{R} n_{ij}(x) = 1$.

We consider a state feedback fuzzy controller which shares the same structure of the MJFS (3).Thus, the overall fuzzy controller is given by

$$u = -\sum_{i=1}^{N}\sum_{j=1}^{R} m_i(z)n_{ij}(x)K_{ij}x \qquad (6)$$

where $K_{ij} \in \mathbb{R}^{m\times n}$ are the local feedback gain vectors. Substituting (6) in (4), it results

$$\dot{x} = \sum_{i=1}^{N} m_i(z)\left[\sum_{j=1}^{R}\sum_{k=1}^{R} n_{ij}(x)n_{ik}(x)(A_{ij} - B_{ij}K_{ik})\right]x + w. \qquad (7)$$

Now, using the fact that

$$\sum_{j=1}^{R}\sum_{k=1}^{R} n_{ij}(x)n_{ik}(x) = \sum_{j=1}^{R} n_{ij}^2(x) + 2\sum_{j<k}^{R} n_{ij}(x)n_{ik}(x)$$

and $\sum_{j=1}^{R} n_{ij}(x) = 1$, system (1)-(2) can be described by the uncertain fuzzy system (8), where Δf and Δg are the approximating errors defined in (9) and (10), respectively (see the next page).

Using the fact that $m_i(z) = 1$ when $z \in M_i$, that is, $r = i$, we suppose there exist bounding matrices ΔA_{ij} and ΔB_{ij}, $j = 1,2,\ldots,R$ for each mode i such that

$$\|\Delta f\| \leq \left\|\sum_{j=1}^{R} n_{ij}(x)\Delta A_{ij}x\right\| \qquad (11)$$

$$\|\Delta g\| \leq \left\|\sum_{j=1}^{R}\sum_{k=1}^{R} n_{ij}(x)n_{ik}(x)\Delta B_{ij}K_{ik}x\right\| \qquad (12)$$

for all trajectory x, which can be described as [9]

$$\Delta A_{ij} = \delta_{ij}A_{\mathbf{p}i} \qquad (13)$$

$$\dot{x} = \sum_{i=1}^{N} m_i(z) \left[\sum_{j=1}^{R} n_{ij}^2(x) \left(A_{ij} - B_{ij} K_{ij} \right) + 2 \sum_{j<k}^{R} n_{ij}(x) n_{ik}(x) \left(\frac{A_{ij} - B_{ij} K_{ik} + A_{ik} - B_{ik} K_{ij}}{2} \right) \right] x + \Delta f + \Delta g + w \tag{8}$$

$$\Delta f := \sum_{i=1}^{N} m_i(z) \left[\sum_{j=1}^{R} n_{ij}(x)(f(x,r) - A_{ij} x) \right] \tag{9}$$

$$\Delta g := -\sum_{i=1}^{N} m_i(z) \left[\sum_{j=1}^{R} \sum_{k=1}^{R} n_{ij}(x) n_{ik}(x)(g(x,r) - B_{ij}) K_{ik} x \right]. \tag{10}$$

$$\Delta B_{ij} = \eta_{ij} B_{\mathbf{p}i} \tag{14}$$

where $\|\delta_{ij}\| \leq 1$ and $\|\eta_{ij}\| \leq 1$, $j = 1, 2, \ldots, R$ and $A_{\mathbf{p}i}$ and $B_{\mathbf{p}i}$ are bounding matrices of appropriate structure and dimensions which can be chosen according to the system nonlinearity.

Let $x(t, x_0, r_0, u)$ denote the solution of the MJFS (8) with fuzzy control law (6) under the initial conditions x_0 and r_0.

Definition 1: The MJFS (8) with $w \equiv 0$ is said to be stochastically stabilizable if, for all initial conditions (x_0, r_0) and admissible uncertainties, there exists a fuzzy control law (6) satisfying

$$E \left[\int_0^\infty x(t, x_0, r_0, u)^T x(t, x_0, r_0, u) \; dt | x_0, r_0 \right] \leq x_0^T M x_0 \tag{15}$$

for some matrix $M = M^T, \; M > 0$ of appropriate dimensions.

Under the above definition, stochastic stabilizability of the Markovian jump fuzzy system means that there exists a feedback fuzzy control law that asymptotically drives the state x from any given initial condition (x_0, r_0) to the origin in the mean square sense, which implies the asymptotic stability of the closed-loop system.

The problem here is the design of feedback gain vectors $K_{ij}, \; i \in \mathbb{S}, \; j = 1, 2, \ldots, R$ for the fuzzy control law (6) that stochastically stabilizes MJFS (8) and attenuates the effect of external disturbance w. The attenuation of the external disturbance effect is formulated in terms of the system H_∞ norm in the stochastic sense.

Definition 2: Consider the MJFS (8). Let γ be a positive scalar, then the mapping from w to x is said to have $\mathcal{L}_2((\Omega, \mathcal{F}, \Pr), [0, T])$ gain less than or equal to γ, if for all initial state $x_0, \; x_0 < \infty$, initial mode r_0 and fuzzy control law (6), the following constraint

$$E \left[\int_0^T x^T S(r) x \; dt \right] - \gamma^2 \int_0^T w^T w \; dt < V(x_0, r_0) - E[V(x(T), r(T))] \tag{16}$$

holds for all admissible uncertainties, with $S(r) = S(r)^T, \; S(r) > 0$ a set of weighting matrices of appropriate dimensions which can be chosen to yield the desired performance.

Let $T_{\zeta w}$ denote the system transfer matrix from w to the regulated output $\zeta := S(r)^{1/2} x$. Then, the H_∞ norm of $T_{\zeta w}$ is

$$\|T_{\zeta w}\|_\infty = sup_{w \in \mathcal{L}_2[0,T]} \frac{E \|\zeta\|_2}{\|w\|_2}. \tag{17}$$

Hence, (16) implies $\|T_{\zeta w}\|_\infty < \gamma$ and the problem of γ disturbance attenuation corresponds to the suboptimal H_∞ control problem.

III. Robust Fuzzy Control

In this section, we present a sufficient condition for the robust stochastic stabilization of the MJFS (8) using stochastic stability and a systematic fuzzy control design both formulated in the context LMI's.

Proposition 1: Consider the MJFS (8). Let x_0 and r_0 be the initial state vector and the initial mode, respectively. The closed-loop system with the fuzzy control law (6) has the H_∞ performance (16) for all admissible uncertainties if there exists a set of matrices $X_i = X_i^T, \; X_i > 0$ of appropriate dimensions satisfying $\forall i \in \mathbb{S}$

$$\begin{bmatrix} \hat{T}_{ij} & Z_i \\ Z_i^T & -W_i \end{bmatrix} < 0; \; j = 1, \ldots, R \tag{18}$$

$$\begin{bmatrix} \hat{U}_{ijk} & Z_i \\ Z_i^T & -W_i \end{bmatrix} < 0; \; j < k; \; j, k = 1, \ldots, R \tag{19}$$

with $\hat{T}_{ij}, \hat{U}_{ijk}, Z_i$ and W_i given in (20), (21), (22), and (23), respectively (see the next page).

The proof of Proposition 1 is derivative from theory of Markovian jump linear systems and it can be found in [10].

Remark 1: For bounded but nonvanishing disturbance w, that is, $\|w\| \leq w_{bd}$, it can be shown using a standard Lyapunov extension [11] that the solution of the closed-loop MJFS (8) is ultimately bounded in the mean square sense whenever $\|x\| \geq \gamma w_{bd} / \sqrt{c_1}$, $c_1 = \lambda_{min}(S_i), \; i \in \mathbb{S}$, and that the control performance H_∞ given by (16) is guaranteed.

Using Proposition 1, we formulate the following optimization problem to the stochastic fuzzy control design with H_∞ performance.

Problem 1: Find a set of matrices $X_i = X_i^T, \; X_i > 0$ and a set of matrices Y_{ij} of appropriate dimensions to minimize γ^2 subject to $X_i > 0$, (18) and (19) $\forall i \in \mathbb{S}$.

$$\hat{T}_{ij} := \begin{bmatrix} T_{ij}+2\mathbf{I} & 1 & X_i & B_{\mathbf{p}i}Y_{i1} & \cdots & B_{\mathbf{p}i}Y_{iR} \\ 1 & -\gamma^2\mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ X_i & \mathbf{0} & -(S_i+A_{\mathbf{p}i}^T A_{\mathbf{p}i})^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ (B_{\mathbf{p}i}Y_{i1})^T & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (B_{\mathbf{p}i}Y_{iR})^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{I} \end{bmatrix} \quad (20)$$

$$\hat{U}_{ijk} := \begin{bmatrix} U_{ijk}+2\mathbf{I} & 1 & X_i & B_{\mathbf{p}i}Y_{i1} & \cdots & B_{\mathbf{p}i}Y_{iR} \\ 1 & -\gamma^2\mathbf{I} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ X_i & \mathbf{0} & -(S_i+A_{\mathbf{p}i}^T A_{\mathbf{p}i})^{-1} & \mathbf{0} & \cdots & \mathbf{0} \\ (B_{\mathbf{p}i}Y_{i1})^T & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (B_{\mathbf{p}i}Y_{iR})^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & -\mathbf{I} \end{bmatrix} \quad (21)$$

$$Z_i := \begin{bmatrix} \pi_{i1}^{1/2}X_i & \dots & \pi_{ii-1}^{1/2}X_i & \pi_{ii+1}^{1/2}X_i & \dots & \pi_{iN}^{1/2}X_i \end{bmatrix} \quad (22)$$

$$W_i := diag\left\{ X_1 \ \dots \ X_{i-1} \ X_{i+1} \ \dots \ X_N \right\} \quad (23)$$

$$T_{ij} := X_i A_{ij}^T + A_{ij}X_i - Y_{ij}^T B_{ij}^T - B_{ij}Y_{ij} - \pi_i X_i \quad (24)$$

$$U_{ijk} := X_i A_{ij}^T + A_{ij}X_i - Y_{ik}^T B_{ij}^T - B_{ij}Y_{ik} + X_i A_{ik}^T + A_{ik}X_i - Y_{ij}^T B_{ik}^T - B_{ik}Y_{ij} - \pi_i X_i \quad (25)$$

If Problem 1 is feasible, we can obtain the state feedback gain vectors as $K_{ij} = Y_{ij}X_i^{-1}$, $i \in \mathbb{S}$, $j = 1, 2, \dots, R$.

Remark 2: In order to consider the stochastic stabilization for the MJNLS in case the equilibrium point is not the origin, that is, $(x, u) \neq 0$, one should perform a change of coordinates to make the origin the equilibrium before designing the fuzzy control (6) using Problem 1.

IV. Simulation Results

To illustrate the usefulness of the robust fuzzy control approach, the stabilization of an electrical power system in co-generation scheme [12] is considered. The dynamics of the power system are given by

$$\dot{\delta} = \omega_0\, \omega \quad (26)$$

$$\dot{\omega} = \frac{1}{2H}\left[P_m - E_q^{'} I_q(r)\right] + w \quad (27)$$

$$\dot{E}_q^{'} = \frac{1}{\tau_{do}^{'}}\left[E_{fd} - E_q^{'} + (X_d - X_d^{'})I_d(r)\right] \quad (28)$$

$$\dot{E}_{fd} = \frac{1}{T_e}\left[K_e(V_{ref} - |V_t| + V_s) - E_{fd}\right] \quad (29)$$

where $|V_t| = \left[(E_q^{'} + X_d^{'}I_d)^2 + (X_d^{'}I_q)^2\right]^{1/2}$, with δ the power angle [rad]; ω the rotor speed [rad/s]; ω_0 the synchronous machine speed [rad/s]; P_m the mechanical input power [p.u.]; $E_q^{'}$ the transient EMF in the quadrature axis [p.u.]; E_{fd} the equivalent EMF in the excitation coil [p.u.]; I_q the quadrature axis current [p.u.]; I_d the direct axis current [p.u.]; H the inertia constant [p.u.]; $\tau_{do}^{'}$ the direct axis transient open circuit time constant [s]; T_e the regulator time constant [s]; K_e the regulator gain; V_{ref} the regulator reference voltage [p.u.]; V_t the terminal voltage [p.u.]; V_s the power system stabilizer voltage [p.u.]; w the external disturbance [p.u.]; $X_d^{'}$ the direct axis transient reactance [p.u.] and X_d the direct axis reactance [p.u.].

In this example, the currents I_q and I_d are obtained considering the effects of random abrupt variations in the load-bus of the power system which results

$$I_q(r) = k_{1q}(r)cos\delta + k_{2q}(r)sin\delta + k_{3q}(r)E_q^{'} \quad (30)$$

$$I_d(r) = k_{1d}(r)cos\delta + k_{2d}(r)sin\delta + k_{3d}(r)E_q^{'} \quad (31)$$

where k_{1q}, k_{2q}, k_{3q}, k_{1d}, k_{2d} and k_{3d} are parameters which vary according to the equivalent load Z_{eq} in the load-bus. The operating conditions of the power system according to load variations is modeled as a Markov chain with three different modes, that is, $S = \{1, 2, 3\}$, which correspond to the possible combinations between loads $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$, with R_1 and R_2 the resistances in p.u. and X_1 and X_2 the reactances in p.u. as follows

- mode 1: only load Z_1 ($Z_{eq} = Z_1$),
- mode 2: only load Z_2 ($Z_{eq} = Z_2$),
- mode 3: both loads Z_1 and Z_2 $\left(Z_{eq} = \frac{Z_1 Z_2}{Z1+Z_2}\right)$.

We assume that the system under nominal operating conditions is described by mode 3. However, during a period of time, the dynamics of the power system changes according to a Markov process r with initial distribution and transition rate matrix

$$\mu = (0.30, 0.20, 0.50)$$

and

$$\Pi = \begin{bmatrix} -1.80 & 0.60 & 1.20 \\ 0.90 & -1.80 & 0.90 \\ 0.72 & 1.08 & -1.80 \end{bmatrix}.$$

The numerical values of the physical parameters adopted are: $\omega_0 = 377$, $H = 5$, $P_m = 1$, $\tau_{do}^{'} = 6$, $T_e = 0.01$, $K_e = 100$, $X_d^{'} = 0.32$, $X_d = 1.60$, $V_{ref} = 1.05$, $R_1 = 7.96$, $R_2 = 1.77$, $X_1 = 5.30$ and $X_2 = 4.3$. Table I shows the numerical values of parameters k_{1q}, k_{2q}, k_{3q}, k_{1d}, k_{2d} and k_{3d} computed for each operating mode.

TABLE I

POWER SYSTEM PARAMETERS FOR THE DIFFERENT OPERATING CONDITIONS.

r	k_{1q}	k_{2q}	k_{3q}	k_{1d}	k_{2d}	k_{3d}
1	0.009	1.381	0.011	1.381	-0.009	-1.423
2	0.013	1.368	0.016	1.368	-0.013	-1.440
3	0.021	1.341	0.026	1.341	-0.021	-1.472

In the nominal operating conditions of the power system, the power angle is kept in the range $\delta \cong 35° \pm 10\%$ which assures a real power in the range $P = 1.0 \pm 10\%$ p.u. and a reactive power $Q = 0.5 \pm 10\%$. The external disturbance $w = 10^{-2} \times sin(2\pi 20t)$ is used to produce variations about $\pm 10\%$ in both the mechanical input power P_m and the rotor speed of the generator ω during a period of time that are due to possible torsional oscillations occuring in the system structure [13].

Let $x = [\delta,\ \omega,\ E_q^{'},\ E_{fd}]^T$ be the vector of state variables and $u = V_s$ be the input vector. System (26)-(29) presents the following equilibrium point $x_e = [0.61\ \ 0\ \ 1.22\ \ 2.14]^T$ and $u_e = 0$. As mentioned before, it is necessary to perform a change of coordinates to bring the equilibrium of the power system to the origin. For this purpose, we adopt $\xi = x - x_e$ as the new system coordinate and obtain

$$\dot{\xi} = f(\xi + x_e, r) + Bu + \bar{w} \tag{32}$$

where $f(\xi + x_e, r) := [f_1, f_2, f_3, f_4]^T$, with $f_1, \ldots, f_4$ given in (33)-(36), respectively (see the next page), $B := [0, 0, 0, \frac{K_e}{T_e}]^T$, $\bar{w} := [0, w, 0, 0]$. For $\delta \approx 35° \pm 20\%$, two local linear representations ($R = 2$) can be obtained around of the following linearization points $\bar{x}$

- mode 1: $\bar{x}_{R=1} = [0.486\ \ 0\ \ 1.492\ \ 2.652]^T$ and $\bar{x}_{R=2} = [0.730\ \ 0\ \ 1.065\ \ 1.694]^T$
- mode 2: $\bar{x}_{R=1} = [0.486\ \ 0\ \ 1.480\ \ 2.669]^T$ and $\bar{x}_{R=2} = [0.730\ \ 0\ \ 1.065\ \ 1.734]^T$
- mode 3: $\bar{x}_{R=1} = [0.486\ \ 0\ \ 1.461\ \ 2.710]^T$ and $\bar{x}_{R=2} = [0.730\ \ 0\ \ 1.066\ \ 1.814]^T$.

The mode indicator membership functions $m_i(.)$, $i = 1, 2, 3$ are crisp functions described before. In this example, the premise variable z is determined by monitoring the load activation instants. The normalized membership functions $n_{ij}(.)$, $j = 1, 2$ describe the range of the state variables x_1 and x_3 in each mode and are obtained from gaussian membership functions available in the Fuzzy Logic Toolbox of MATLAB. A suitable range for the state variables can be determined by constraining x_1 in the interval $[27.9°,\ 41.80°]$.

Let the initial conditions be $r(0) = 3$ and $x(0) = [0.61\ \ -0.001\ \ 1.22\ \ 2.14]^T$. The following design specifications are considered in the stabilization of the power system: 1) maintain the system operating in its equilibrium mode, that is, mode 3; 2) improve the damping of the electromechanical oscillations of the power system; 3) maintain the terminal voltage of the generator in the range of $\pm 10\%$ of its nominal value $|V_t| = 1.03$ p.u.

The robust fuzzy control design considers the approximating errors Δf and Δg between the nonlinear system and the fuzzy system model. In case of the power system $\Delta g = 0$ and Δf has the following bounding matrices

- mode 1:
$$A_{\mathbf{p}1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0001 & 0 & 0.0001 & 0 \\ 0.0002 & 0 & 0.0005 & 0.0002 \\ 0.2425 & 0 & 4.1086 & 0.0881 \end{bmatrix};$$
- mode 2:
$$A_{\mathbf{p}2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0001 & 0 & 0.0001 & 0 \\ 0.0002 & 0 & 0.0005 & 0.0002 \\ 0.2623 & 0 & 4.2869 & 0.0928 \end{bmatrix};$$
- mode 3:
$$A_{\mathbf{p}3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.0001 & 0 & 0.0001 & 0 \\ 0.0002 & 0 & 0.0005 & 0.0002 \\ 0.3012 & 0 & 4.6341 & 0.1022 \end{bmatrix}.$$

A procedure to obtain the above bounding matrices can be found in [10]. The weighting matrices S_i, $i = 1, 2, 3$ are chosen as $S_1 = S_2 = S_3 = \text{diag}[20,\ 20,\ 0.009,\ 0.009]$. Therefore, for (A_{ij}, B_{ij}, Π), $A_{\mathbf{p}i}$ and S_i, $i = 1, 2, 3$, $j = 1, 2$, we obtain the feedback gains for the stabilization of the SMIB power system (32) by solving the LMI's in Problem 1. Table II presents the control design results.

TABLE II

CONTROL DESIGN RESULTS.

r	Resulting matrices
1	$K_{11} = [0.019\ \ -20.324\ \ -0.119\ \ 0.005]$ $K_{12} = [0.271\ \ 7.065\ \ -0.301\ \ -0.005]$
2	$K_{21} = [0.024\ \ -19.375\ \ -0.120\ \ -0.005]$ $K_{22} = [0.267\ \ 6.6794\ \ -0.294\ \ -0.005]$
3	$K_{31} = [0.031\ \ -17.641\ \ -0.119\ \ -0.005]$ $K_{32} = [0.258\ \ 5.9605\ \ -0.281\ \ -0.005]$
	L_2-gain: $\gamma^2 = 0.5$

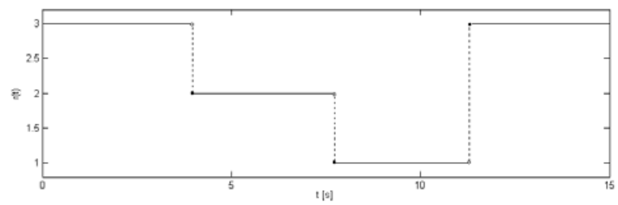

Fig. 1. Load variations during a period of the operation of the power system.

With μ and Π as previously defined, the software provided in [14] is used to simulate transitions among operating modes which are given by the Markovian jump process r. Figure 1 shows the load variations during a period of operation of the power system. In Figures 2 and 3 we compare the results obtained with the robust fuzzy control to the results obtained with a classical power system stabilizer which was tuned according to [15].

V. CONCLUDING REMARKS

In this paper a fuzzy control design with H_∞ performance for Markovian jump systems is developed. The feedback

$$f_1 = \omega_0 \xi_2 \quad (33)$$

$$f_2 = \frac{1}{2H}\left[P_m - (\xi_3 + x_{e3})(k_{1q}(r)cos(\xi_1 + x_{e1}) + k_{2q}(r)sin(\xi_1 + x_{e1}) + k_{3q}(r)(\xi_3 + x_{e3}))\right] \quad (34)$$

$$f_3 = \frac{1}{\tau'_{do}}\left[(\xi_4 + x_{e4}) - (\xi_3 + x_{e3}) + (X_d - X'_d)\right.$$
$$\left.(k_{1d}(r)cos(\xi_1 + x_{e1}) + k_{2d}(r)sin(\xi_1 + x_{e1}) + k_{3d}(r)(\xi_3 + x_{e3}))\right] \quad (35)$$

$$f_4 = \frac{1}{T_e}\left[K_e(V_{ref} - |V_t|) - (\xi_4 + x_{e4})\right] \quad (36)$$

$$|V_t| := \left\{\left[(x_3 + x_{e3}) + X'_d\,(k_{1d}(r)cos(\xi_1 + x_{e1}) + k_{2d}(r)sin(x_1 + x_{e1}) + k_{3d}(r)(x_3 + x_{e3}))\right]^2\right.$$
$$\left. + \left[X'_d\,(k_{1q}(r)cos(x_1 + x_{e1}) + k_{2q}(r)sin(x_1 + x_{e1}) + k_{3q}(r)(x_3 + x_{e3}))\right]^2\right\}^{1/2}. \quad (37)$$

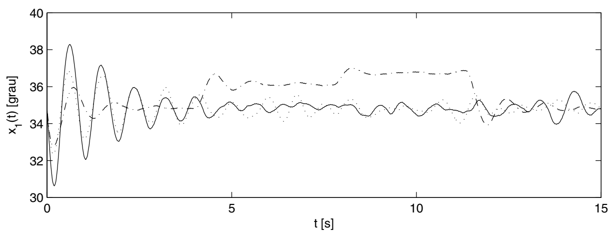

Fig. 2. State variable x_1: '—' power system with robust fuzzy control, '−·' power system with classical stabilizer and '· · ·' fuzzy system with robust fuzzy control.

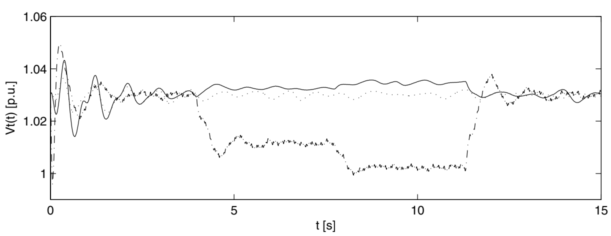

Fig. 3. Terminal voltage V_t: '—' power system with robust fuzzy control, '−·' power system with classical stabilizer and '· · ·' fuzzy system with robust fuzzy control.

nonlinear system achieves a minimum $\mathcal{L}_2$ gain for disturbance attenuation. The proposed approach is appropriated for practical control design for nonlinear systems with external disturbance and can be solved very efficiently by convex optimization techniques with the aid of the LMI Control Toolbox of MATLAB. The advantage of the fuzzy-model-based control design is that we can consider a more refined description of the parameter variations in the nonlinear system. Because of that, we can provide less conservative conditions for stability in the stochastic sense. Simulation results show the usefulness of the proposed approach to the stabilization of an electrical power system in co-generation scheme which is a complex real system subject to random abrupt variations in its dynamics.

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