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Design of Dynamical Compensators for Matrix Second-order Linear Systems: A Parametric Approach *

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Abstract-**In this paper, the issue of designing a type of dynamical compensators for matrix second-order linear (MSOL) systems is addressed in the matrix second-order framework by using a complete parametric approach. Based on simple and complete parametric solutions of the generalized Sylvester matrix equation of MSOL systems, parametric expressions for the left and right eigenvector matrices of the closed-loop system are obtained. Also a pair of complete parametric expressions for the coefficient matrices of the dynamical compensator is presented. These expressions contain two groups of parameter vectors which represent the degrees of freedom existing in the solution of the closed-loop eigenvectors. The proposed approach provides all the degrees of design freedom, which can be further utilized to achieve additional system specifications. A spring-mass system is utilized to show the effect of the proposed approach.**

Index Terms-***Matrix second-order linear systems, dynamical compensator, eigenstructure assignment, parametric approach.***

I. Introduction

Matrix second-order linear (MSOL) systems arise naturally in a wide range of applications as follows: (1) control of large flexible space structures; (2) earthquake engineering; (3) control of mechanical multi-body systems; (4) stabilization of damped gyroscopic systems; (5) robotics control; (6) vibration control in structural dynamics; (7) linear stability of flows in fluid mechanics; (8) electrical circuit simulation, see e.g. [1-6]. Typically, people always convert MSOL systems to the standard first-order state space forms when performing analysis or design. However, as pointed out in [5] or [7], retaining the model in matrix second order framework has the following advantages: (1) physical insight of the original problem is preserved; (2) system matrix sparsity and structure are preserved; (3) uncertainty structure is preserved; (4) proportional plus derivative feedback can be used directly, entailing easier implementations. Up to date, the existing main problem when keeping the matrix second order framework is that a very few design methods are available, most of them being developed for matrix first order forms. Some attempts to fill up the gap are reported in e.g. [7-10] or more recently in [4-6, 11-13].

This paper proposes a type of dynamical compensators for MSOL systems in the matrix second-order framework through the complete parametric approach. At first, the problem of dynamical compensator design (DCD) for MSOL systems is converted into two equivalent problems, i.e., Problem LESA (Left Eigenstructure Assignment) and Problem RESA (Right Eigenstructure Assignment). Based on the expanded results in [14, 15], i.e., the complete parametric solutions to the generalized Sylvester matrix equation of MSOL systems, parametric expressions for the left and right eigenvector matrices of the closed-loop system and for the coefficient matrices of the dynamical compensator are presented. This parametric approach can provide all the degrees of design freedom that can be further utilized to achieve additional system specifications [16, 17].

This paper is composed of 5 sections. Section II formulates the design problem of the dynamical compensator, while Section III gives the solution to the proposed problem based on some preliminaries. A mass-spring system example is presented in Section IV to demonstrate the effect of the proposed approach. Concluding remarks follow in Section V.

II. Problem Formulation

Consider the following MSOL system

$$\begin{cases} M\ddot{q} + D\dot{q} + Kq = Bu \\ y = Qq + R\dot{q} \end{cases}, \tag{1}$$

where $q \in \Re^n$, $u \in \Re^r$ and $y \in \Re^m$ are the state vector, the input vector and the output vector, respectively. M, D, K, B, Q and R are known matrices of appropriate dimensions. The system (1) satisfies the following assumptions.

Assumption A1: The MSOL system (1) is observable and controllable.

Assumption A2: Matrix B is of full-column rank, and matrix $[Q \quad R]$ is of full-row rank.

Regarding the controllability and observability of system (1), the following results hold.

Lemma 1 [8]: System (1) is observable if and only if

$$\text{rank}\begin{bmatrix} \lambda R + Q \\ \lambda^2 M + D\lambda + K \end{bmatrix} = n, \quad \forall \lambda \in C \tag{2}$$

and is controllable if and only if

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$$\text{rank}\begin{bmatrix}\lambda^2 M + D\lambda + K & B\end{bmatrix} = n,\ \forall \lambda \in C \quad (3)$$

Applying the following dynamic compensator

$$\begin{cases}\ddot{z} + F_2\dot{z} + F_1 z = H_1 y + H_2\dot{y} \\ u = N_2\dot{z} + N_1 z + M_1 y + M_2\dot{y}\end{cases}, \quad (4)$$

to system (1), obtains the closed-loop system

$$\begin{bmatrix}M - BM_2R & 0 \\ -H_2R & I\end{bmatrix}\ddot{X} + \begin{bmatrix}D - BM_1R - BM_2Q & -BN_2 \\ -H_1R - H_2Q & F_2\end{bmatrix}\dot{X} + \begin{bmatrix}K - BM_1Q & -BN_1 \\ -H_1Q & F_1\end{bmatrix}X = 0 \quad (5)$$

where $z \in \Re^p$ and p are the state vector and the order of dynamic compensator (4), respectively; matrices F_1, F_2, H_1, H_2, N_1, N_2, M_1 and M_2 are of appropriate dimensions; $X = [q^T \quad z^T]^T \in R^{n+p}$.

Then the problem to be studied in this paper can be stated as follows.

Problem DCD: Given system (1) under the assumptions A1 and A2, find matrices F_1, F_2, H_1, H_2, N_1, N_2, M_1 and M_2 to make the close-loop system (5) stable.

III. Main Results

A. Preliminaries

Note that system (5) is equivalent to the following system

$$\begin{cases}A_0\ddot{X} + A_1\dot{X} + A_2X = B_1U \\ Y = C_1\dot{X} + C_2X\end{cases} \quad (6)$$

under the output feedback law

$$U = K_1Y + K_2\dot{Y}, \quad (7)$$

where

$$A_0 = \begin{bmatrix}M & 0 \\ 0 & I_p\end{bmatrix},\ A_1 = \begin{bmatrix}D & 0 \\ 0 & 0\end{bmatrix},\ A_2 = \begin{bmatrix}K & 0 \\ 0 & 0\end{bmatrix}, \quad (8a)$$

$$B_1 = \begin{bmatrix}B & 0 \\ 0 & I_p\end{bmatrix},\ C_1 = \begin{bmatrix}R & 0 \\ 0 & 0\end{bmatrix},\ C_2 = \begin{bmatrix}Q & 0 \\ 0 & I_p\end{bmatrix}, \quad (8b)$$

$$K_1 = \begin{bmatrix}M_1 & N_1 \\ H_1 & -F_1\end{bmatrix},\ K_2 = \begin{bmatrix}M_2 & N_2 \\ H_2 & -F_2\end{bmatrix}. \quad (8c)$$

Clearly according to Lemma 1 and the representations in (8a) and (8b), we can easily conclude that system (6) is observable if and only if system (1) is observable, and system (6) is controllable if and only if system (1) is controllable.

For system (1), there exist a set of real coefficient polynomial matrices $N(s) \in \Re^{n\times m}[s]$, $D(s) \in \Re^{m\times m}[s]$, $H(s) \in \Re^{n\times r}[s]$ and $L(s) \in \Re^{r\times r}[s]$ to make the following two right factorizations hold:

$$(s^2M^T + sD^T + K^T)^{-1}(R^Ts + Q^T) = N(s)D^{-1}(s) \quad (9)$$

and

$$(s^2M + sD + K)^{-1}B = H(s)L^{-1}(s). \quad (10)$$

Denote

$$\widetilde{N}(s) = \begin{bmatrix}0 & N(s) \\ I_p & 0\end{bmatrix},\ \widetilde{D}(s) = \begin{bmatrix}0 & D(s) \\ s^2I_p & 0\end{bmatrix}, \quad (11a)$$

$$\widetilde{H}(s) = \begin{bmatrix}0 & H(s) \\ I_p & 0\end{bmatrix} \text{ and } \widetilde{L}(s) = \begin{bmatrix}0 & L(s) \\ s^2I_p & 0\end{bmatrix}, \quad (11b)$$

we can easily obtain the following lemma.

Lemma 2: Given system (1) under Assumption A1, the polynomial matrices $\widetilde{N}(s)$, $\widetilde{D}(s)$, $\widetilde{H}(s)$ and $\widetilde{L}(s)$ meet the following right factorizations:

$$(s^2A_0^T + sA_1^T + A_2^T)^{-1}(C_1^Ts + C_2^T) = \widetilde{N}(s)\widetilde{D}^{-1}(s), \quad (12)$$

$$(s^2A_0 + sA_1 + A_2)^{-1}B_1 = \widetilde{H}(s)\widetilde{L}^{-1}(s). \quad (13)$$

Denote $\overline{A}_0 = A_0 - B_1K_2C_1$, $\overline{A}_1 = A_1 - B_1K_1C_1 - B_1K_2C_2$, and $\overline{A}_2 = A_2 - B_1K_1C_2$. The closed-loop system (5) can be written as

$$\overline{A}_0\ddot{X} + \overline{A}_1\dot{X} + \overline{A}_2X = 0.$$

Under the following constraint

Constraint C1: $\det \overline{A}_0 \neq 0$,

the above equation can also be rewritten in the first-order state space form as

$$\dot{\widetilde{Z}} = A\widetilde{Z}, \quad (14)$$

where

$$\widetilde{Z} = \begin{bmatrix}X \\ \dot{X}\end{bmatrix},\ A = \begin{bmatrix}0 & I \\ -\overline{A}_0^{-1}\overline{A}_2 & -\overline{A}_0^{-1}\overline{A}_1\end{bmatrix}. \quad (15)$$

Note that the eigenvalues of system (14) are the same as those of the close-loop system (5) and recall the fact that a non-defective matrix possesses eigenvalues which are less insensitive to the parameter perturbations in the matrix, we here require that the matrix A to be non-defective, that is, the Jordan form of the matrix A possesses a diagonal form:

$$\Lambda = \text{diag}(s_1, s_2, \ldots, s_{2n_1}), \quad (16)$$

where $s_i, i = 1,2,\ldots,2n_1$, are clearly the eigenvalues of the matrix A; $n_1 = n + p$.

Lemma 3: Let A and Λ be given by (15) and (16), respectively. Then there exist matrices $T_1, T_2 \in C^{n_1\times 2n_1}$ satisfying

$$[T_1^T \quad T_2^T]A = \Lambda[T_1^T \quad T_2^T] \quad (17)$$

if and only if

$$\Lambda^2\overline{T}^TA_0 + \Lambda\overline{T}^TA_1 + \overline{T}^TA_2 - \Lambda Z_o^TC_1 - Z_o^TC_2 = 0 \quad (18)$$

and

$$T_1^T = -\Lambda^{-1}\overline{T}^T(A_2 - B_1K_1C_2), \quad (19)$$

where

$$\overline{T}^T = T_2^T\overline{A}_0^{-1},\ Z_o^T = \overline{T}^TB_1K_1 + \Lambda\overline{T}^TB_1K_2. \quad (20)$$

Proof: Since equation (17) can be divided into the following equations

$$\begin{cases} -T_2^T\overline{A}_0^{-1}\overline{A}_2 = \Lambda T_1^T & (21) \\ T_1^T - T_2^T\overline{A}_0^{-1}\overline{A}_1 = \Lambda T_2^T & (22) \end{cases}.$$

Pre-multiplying (22) by Λ and substituting (21) into the obtained equation, yields

$$\Lambda^2 T_2^T + \Lambda T_2^T\overline{A}_0^{-1}\overline{A}_1 + T_2^T\overline{A}_0^{-1}\overline{A}_2 = 0\,. \quad (23)$$

Substituting the representations of $\overline{A}_1$ and $\overline{A}_2$ into (23), we can obtain (18) after some manipulation by using (20). In addition, Pre-multiplying (21) by Λ^{-1}, the equation (19) is easily obtained.

Also we can give the following lemma which states the relation of the Jordan form and the corresponding right eigenvector matrix $V = [V_1^T \quad V_2^T]^T \in C^{2n_1\times 2n_1}$ of system (14).

Lemma 4: Let A and Λ be given by (15) and (16), respectively. Then there exist matrices $V_1, V_2 \in C^{n_1\times 2n_1}$ satisfying

$$A\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}\Lambda \quad (24)$$

if and only if

$$A_0V_1\Lambda^2 + A_1V_1\Lambda + A_2V_1 - B_1Z_c = 0 \quad (25)$$

and

$$V_2 = V_1\Lambda\,, \quad (26)$$

where

$$Z_c = K_2(C_1V_1\Lambda + C_2V_1)\Lambda + K_1(C_1V_1\Lambda + C_2V_1)\,. \quad (27)$$

The proof is omitted.

The above two lemmas state that the Jordan matrix of A is Λ if and only if there exists $\overline{T} \in C^{n_1\times 2n_1}$ or $V_1 \in C^{n_1\times 2n_1}$ satisfying (18) or (25), respectively. In the two cases the corresponding left and right eigenvector matrix of A are given by

$$T_L^T = [T_1^T \quad T_2^T] = [-\Lambda^{-1}\overline{T}^T\overline{A}_2 \quad \overline{T}^T\overline{A}_0] \quad (28)$$

and

$$V_R = [V_1^T \quad \Lambda^T V_1^T]^T\,. \quad (29)$$

Clearly, equation (25) and and the following equation

$$A_0^T\overline{T}\Lambda^{2T} + A_1^T\overline{T}\Lambda^T + A_2^T\overline{T} - C_1^TZ_o\Lambda^T - C_2^TZ_o = 0 \quad (30)$$

become the type of generalized Sylvester matrix equations investigated in [14,15] when $A_0 = 0$ and $C_1 = 0$. Due to this fact, the equations (25) and (30) are called the second-order generalized Sylvester matrix equations. Therefore, Problem DCD can be converted into the following two problems.

Problem LESA (Left Eigenstructure Assignment): Given system (6) and a group of distinct conjugate complex numbers $s_i, i = 1,2,\ldots,2n_1$, determine output feedback gain matrices K_1, K_2 and left eigenvector matrix T_L such that equation (18) and $\det T_L \neq 0$ hold simultaneously.

Problem RESA (Right Eigenstructure Assignment): Given system (6) and a group of distinct conjugate complex numbers $s_i, i = 1,2,\ldots,2n_1$, determine output feedback gain matrices K_1, K_2 and right eigenvector matrix V_R such that equation (25) and $\det V_R \neq 0$ hold simultaneously.

B. Solution to Problem LESA

Before solving Problem LEAS, we need to give the following lemma.

Lemma 5: Let $\widetilde{N}(s) \in \Re^{n_1\times m_1}[s]$ and $\widetilde{D}(s) \in \Re^{m_1\times m_1}[s]$ be a pair of polynomial matrices satisfying the right factorization (12). Then for equation (30), matrices $\overline{T}$ and Z_o are given by

$$\overline{T} = [\widetilde{N}(s_1)f_1 \quad \widetilde{N}(s_2)f_2 \quad \cdots \quad \widetilde{N}(s_{2n_1})f_{2n_1}] \quad (31)$$

and

$$Z_o = [\widetilde{D}(s_1)f_1 \quad \widetilde{D}(s_2)f_2 \quad \cdots \quad \widetilde{D}(s_{2n_1})f_{2n_1}]\,, \quad (32)$$

where $m_1 = m + p$, $n_1 = n + p$; $f_i \in C^{m_1}$, $i = 1,2,\ldots,2n_1$, are a set of parameter vectors.

Proof: Denote

$$\overline{T} = [t_1 \quad t_2 \quad \cdots \quad t_{2n_1}]\,, \quad (33)$$

$$Z_o = [z_1 \quad z_2 \quad \cdots \quad z_{2n_1}]\,, \quad (34)$$

then we can convert the second-order generalized Sylvester matrix equation (30) into the following column form

$$(s_i^2A_0^T + s_iA_1^T + A_2^T)t_i - (s_iC_1^T + C_2^T)z_i = 0\,, \quad (35)$$

It follows from (12) that

$$(s_i^2A_0^T + s_iA_1^T + A_2^T)\widetilde{N}(s_i) - (s_iC_1^T + C_2^T)\widetilde{D}(s_i) = 0\,,$$
$$i = 1,2,\cdots,2n_1\,.$$

Let

$$\begin{bmatrix} t_i \\ z_i \end{bmatrix} = \begin{bmatrix} \widetilde{N}(s_i) \\ \widetilde{D}(s_i) \end{bmatrix} f_i\,,\ i = 1,2,\cdots,2n_1\,, \quad (36)$$

Using (35) and (36), yields

$$\begin{aligned} &(s_i^2A_0^T + s_iA_1^T + A_2^T)t_i - (s_iC_1^T + C_2^T)z_i \\ &= [(s_i^2A_0^T + s_iA_1^T + A_2^T)\widetilde{N}(s_i) - (s_iC_1^T + C_2^T)\widetilde{D}(s_i)]f_i \\ &= 0, \quad i = 1,2,\ldots,2n_1 \end{aligned}$$

The matrices $\overline{T}$ and Z_o given by (33), (34) and (36) satisfy the second-order generalized Sylvester matrix equation (30) for all $f_i \in C^{m_1}$, $i = 1,2,\cdots,2n_1$.

Based on the above lemma, we can obtain the solution to Problem LESA.

Theorem 1: Let system (6) be observable and controllable, and $\widetilde{N}(s) \in \Re^{n_1\times m_1}[s]$ and $\widetilde{D}(s) \in \Re^{m_1\times m_1}[s]$ be a pair of polynomial matrices satisfying the right factorization (12).

1) The control matrices K_1 and K_2 can be obtained if and only if there exist a group of parameters $f_i \in C^{m_1}$, $i = 1,2,\ldots,2n_1$, satisfying Constraint C1 and the following constraints:

   Constraint C2a: $f_i = \bar{f}_j$ if $s_i = \bar{s}_j$,

   Constraint C3a: $\det T_L \neq 0$,

Constraint C4$_a$: $rank[T_{LN}^T \quad Z_o^T] = 2r_1$,

where $m_1 = m + p$, $n_1 = n + p$, $r_1 = r + p$, and

$$T_{LN} = \begin{bmatrix} B_1^T \widetilde{N}(s_1) f_1 & \cdots & B_1^T \widetilde{N}(s_{2n_1}) f_{2n_1} \\ s_1 B_1^T \widetilde{N}(s_1) f_1 & \cdots & s_{2n_1} B_1^T \widetilde{N}(s_{2n_1}) f_{2n_1} \end{bmatrix}.$$

2) When the above conditions are met, all the solutions to the control matrices K_1 and K_2 are given by

$$[K_1^T \quad K_2^T]^T = (T_{LN} T_{LN}^T)^{-1} T_{LN} Z_o^T \tag{37}$$

and matrix Z_o is given by (32). The corresponding left eigenvector matrix is

$$T_L^T = [T_1^T \quad T_2^T] = [-\Lambda^{-1} \overline{T}^T \overline{A}_2 \quad \overline{T}^T \overline{A}_0] = F(s_i, f_i), \quad i = 1, 2, \cdots, 2n_1 \tag{38}$$

where $F(*)$ represents a nonlinear function matrix with respect to the variable $*$.

By using Lemma 5 and (20), we can easily conclude the results of Theorem 1.

C. Solution to Problem RESA

Before solving Problem REAS, we need to give the following lemma.

Lemma 6: Let $\widetilde{H}(s) \in \Re^{n_1 \times r_1}[s]$ and $\widetilde{L}(s) \in \Re^{r_1 \times r_1}[s]$ be a pair of polynomial matrices satisfying the right factorization (13). Then for equation (25) matrices V_1 and Z_c are given by

$$V_1 = [\widetilde{H}(s_1) f_1 \quad \widetilde{H}(s_2) f_2 \quad \cdots \quad \widetilde{H}(s_{2n_1}) f_{2n_1}] \tag{39}$$

and

$$Z_c = [\widetilde{L}(s_1) f_1 \quad \widetilde{L}(s_2) f_2 \quad \cdots \quad \widetilde{L}(s_{2n_1}) f_{2n_1}]. \tag{40}$$

Theorem 2: Let system (6) be observable and controllable, and $\widetilde{H}(s) \in \Re^{n_1 \times r_1}[s]$ and $\widetilde{L}(s) \in \Re^{r_1 \times r_1}[s]$ be a set of polynomial matrices satisfying the right factorization (13).

1) The control matrices K_1 and K_2 can be obtained if and only if there exist a group of parameters $g_i \in C^{r_1}$, $i = 1, 2, \ldots, 2n_1$, satisfying Constraint C1 and the following constraints:

Constraint C2$_b$: $g_i = \overline{g}_j$ if $s_i = \overline{s}_j$,

Constraint C3$_b$: $\det V_R \neq 0$,

Constraint C4$_b$: $rank \begin{bmatrix} T_{RH} \\ Z_c \end{bmatrix} = 2m_1$,

where $m_1 = m + p$, $n_1 = n + p$, $r_1 = r + p$, and

$$T_{RH} = \begin{bmatrix} (s_1^2 C_1 + s_1 C_2) \widetilde{H}(s_1) g_1 & \cdots & (s_{2n_1}^2 C_1 + s_{2n_1} C_2) \widetilde{H}(s_{2n_1}) g_{2n_1} \\ (s_1 C_1 + C_2) \widetilde{H}(s_1) g_1 & \cdots & (s_{2n_1} C_1 + C_2) \widetilde{H}(s_{2n_1}) g_{2n_1} \end{bmatrix}.$$

2) When the above conditions are met, all the solutions to the control matrices K_1 and K_2 are given by

$$[K_2 \quad K_1] = Z_c T_{RH}^T (T_{RH} T_{RH}^T)^{-1} \tag{41}$$

and matrix Z_c is given by (41). The corresponding right eigenvector matrix is

$$V_R = \begin{bmatrix} V_1 \\ V_1 \Lambda \end{bmatrix} = \begin{bmatrix} \widetilde{H}(s_1) g_1 & \cdots & \widetilde{H}(s_{2n_1}) g_{2n_1} \\ s_1 \widetilde{H}(s_1) g_1 & \cdots & s_{2n} \widetilde{H}(s_{2n_1}) g_{2n_1} \end{bmatrix}. \tag{42}$$

The proof is omitted due to paper length.

Remark 1: The free parameter vectors f_i and g_i, $i = 1, 2, \ldots, 2n$, represent the degrees of freedom in the eigenstructure assignment design, and can be sought to meet certain desired system performances.

Remark 2: Regarding solutions to the right factorization (22) or (32), several methods have been given in [18]. Numerical methods can be found in [19].

IV. An Example

Consider a simple linear dynamical system in the form of (1) with the coefficient matrices

$$M = \text{diag}(1, 1, 1),$$

$$D = \begin{bmatrix} 2.5 & -0.5 & 0 \\ -0.5 & 2.5 & -2 \\ 0 & -2 & 2 \end{bmatrix}, \quad K = \begin{bmatrix} 10 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

It is verified according to Lemma 1 that this system is observable and controllable. In this example we design a compensator of two-dimension, i.e., $p = 2$ by using the solution to LESA in order to demonstrate the effect of the proposed approach.

The right polynomial matrices $N(s)$ and $D(s)$ satisfying (22) can be easily solved as

$$N(s) = \text{diag}(s, s, s)$$

and

$$D(s) = \begin{bmatrix} -(s^2 + 2.5s + 10) & 0.5s + 5 & 0 \\ 0 & 2s + 20 & -(s^2 + 2s + 20) \\ (0.5s + 5)s & -s(s^2 + 2.5s + 25) & (2s + 20)s \end{bmatrix}.$$

Let $f_i = [f_{i1} \quad f_{i2} \quad f_{i3} \quad f_{i4} \quad f_{i5}]^T$, $i = 1, 2, \cdots, 10$, Thus according to (11a), we have

$$\widetilde{N}(s_i) f_i = [s_i f_{i3} \quad s_i f_{i4} \quad s_i f_{i5} \quad f_{i1} \quad f_{i2}]^T,$$

$$\widetilde{D}(s_i) f_i = \begin{bmatrix} -(s_i^2 + 2.5s_i + 10) f_{i1} + (0.5s_i + 5) f_{i2} \\ (2s_i + 20) f_{i2} - (s_i^2 + 2s_i + 20) f_{i3} - (s_i^2 + 2.5s_i + 25) s_i f_{i2} \\ + (2s_i + 20) s_i f_{i3} + (0.5s_i + 5) s_i f_{i1} \\ s_i^2 f_{i1} \\ s_i^2 f_{i2} \end{bmatrix}.$$

According to (37), the parametric expression of the gain matrices K_1 and K_2 is obtained. Specially selecting the parameters

$$s_{1,2} = -1 \pm \mathrm{i}, \; s_{3,4} = -2 \pm \mathrm{i}, \; s_{5,6} = -1.5 \pm 2\mathrm{i},$$

$$s_{7,8} = -0.5 \pm 1.5\mathrm{i}, \; s_{9,10} = -3 \pm \mathrm{i}, \; f_{1,2} = [1 \quad -1 \quad 1 \quad 1 \quad 0]^T,$$

$f_{3,4} = [-1 \quad -1 \quad 1 \quad 1 \quad -1]^T$, $f_{5,6} = [1 \quad 0 \quad 1 \quad 2 \quad 1]^T$,

$f_{7,8} = [1 \quad 0 \quad 1 \quad 0 \quad 1]^T$, $f_{9,10} = [1 \quad 0 \quad 1 \quad -1 \quad -1]^T$,

then we have the coefficient matrices of dynamical compensator in the form (4):

$$H_1 = \begin{bmatrix} 10.3448 & 23.4483 & 16.3793 \\ 14.3966 & 8.9655 & 15.0862 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 9.7414 & 23.4138 & 11.7780 \\ 8.3448 & 9.4483 & 10.2543 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 8.6207 & 2.5862 \\ 7.4138 & 4.2241 \end{bmatrix},\ F_2 = \begin{bmatrix} 7.5345 & 2.0603 \\ 3.6207 & 3.0862 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 1.3621 & 11.6207 & 0.1358 \\ 2.7414 & -3.5862 & -19.9095 \\ 1.0862 & 5.8621 & 18.0948 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -2.7155 & -5.6552 & -2.1121 \\ -4.4224 & -10.7241 & -6.9397 \\ 3.2414 & 8.4138 & 1.4655 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} -3.0517 & -0.7155 \\ -1.0345 & -0.3103 \\ 0.3448 & 0.1034 \end{bmatrix},\ N_2 = \begin{bmatrix} 0.6379 & 0.4914 \\ 1.8103 & 0.5431 \\ -1.0345 & -0.3103 \end{bmatrix}.$$

V. Concluding Remarks

This paper designs the type of the dynamical compensators for MSOL systems in second-order framework by using the complete parametric approach. Parametric expressions for the left and right closed-loop eigenvector matrices are given on the basis of the simple and complete parametric solutions of the generalized Sylvester matrix equation of MSOL systems. Based on it, a pair of the complete expressions for the coefficient matrices of the dynamical compensator is presented. Finally, the example shows the effect of the proposed approach.

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