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Stability Analysis for Dynamical Neural Networks with Distributed Delays

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Abstract— **This paper is concerned with stability analysis for a class of dynamical neural networks with distributed delays. The M-matrix theory and new analysis technique are used to obtain new sufficient conditions for the existence, uniqueness and global stability of the equilibrium point of dynamical neural networks with distributed delays. The importance of the new stability criteria in the design and application of dynamical neural networks with distributed delays is due to the fact that they can handle the case when the non-delayed terms can not dominate the delayed terms. The effectiveness of the obtained theoretical results are illustrated by numerical examples.**

I. Introduction

In recent years, dynamical neural networks have attracted considerable attentions because of their potential important applications in areas such as control engineering, signal processing, pattern recognition and associative memories. In these applications, the existence and global stability of the equilibrium plays an important role. This leads to that the model equations of neural networks have been extensively studied (see, e.g. [1]-[3], and the references therein). On the other hand, the integration and communication delays are ubiquitous both in biological and artificial neural networks, which not only deteriorate dynamical performance, but also affect the stability of hardware neural networks by oscillations or unstable phenomena. So studying the problem of stability for delayed neural networks is practically required, and a variety of competing results on the stability of delayed neural networks have been accumulated, see, for example, [4]-[12]. However, most research on delayed neural networks has been restricted to simple cases of discrete delays. Since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing distributed delays. Gopalsamy and He [13] studied the system of integro-differential equations as a model for Hopfield-type neural networks involving distributed time-delays arising from the signal propagation. Some results on the stability of neural networks with distributed delays are given in [14]-[18]. The problem of exponential stability for neural networks with distributed delays is studied in [19]-[21]. All of these methods and results are devoted to the case when non-delayed terms dominate the others.

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In this paper, we investigate a class of neural networks with distributed delays modelled by the following integro-differential equations:

$$\dot{u}_i(t) = -d_i(u_i(t)) + \sum_{j=1}^{n} a_{ij} \int_{-\infty}^{t} K_{ij}(t-s) g_j(u_j(s)) ds + I_i, \qquad i = 1, 2, \ldots, n, \tag{1}$$

where a_{ij} and I_i are constant real numbers. We also make the following assumptions in this paper.

(A_1): d_i is differentiable on $\mathbb{R}$ and $b_i = \inf\{\dot{d}_i(u)\} > 0$, $i = 1, 2, \ldots, n$.

(A_2): There are positive constants k_i such that for any $u,\ v \in \mathbb{R}$,

$$0 \le (g_i(u)-g_i(v))(u-v) \le k_i(u-v)^2,\ i=1,2,\ldots,n.$$

(A_3): The kernel K_{ij} are real valued nonnegative continuous functions defined on $[0, \infty)$ and satisfy

$$\int_0^{\infty} K_{ij}(s) ds = 1, \quad i, j = 1, 2, \ldots, n.$$

For illustration convenience, we introduce some notations. For a scalar b, set

$$b^+ = \max\{b, 0\}, \quad b^- = \max\{-b, 0\}.$$

For a matrix $A = (a_{ij})_{n\times n}$, set

$$\tilde{a}_{ii} = a_{ii}^+, \quad \tilde{a}_{ij} = |a_{ij}|, \quad i \neq j, \ i, j = 1, 2, \ldots, n,$$

and set

$$\tilde{A} = (\tilde{a}_{ij})_{n\times n}, \quad |A| = (|a_{ij}|)_{n\times n}.$$

For a vector $x \in \mathbb{R}^n$, $\|x\|$ denotes a vector norm defined by

$$\|x\| = \max_{1\leq i\leq n} \{|x_i|\}.$$

Let $C = C\left((-\infty, 0], \mathbb{R}\right]$ be the Banach space of continuous functions that map $(-\infty, 0]$ to $\mathbb{R}$. For any $\varphi \in C$, we define

$$\|\varphi\| = \sup_{\theta\leq 0} |\varphi(\theta)|.$$

If $a \in \mathbb{R}$, $\psi \in C((-\infty, a], \mathbb{R})$ and $t \leq a$, then $\psi_t \in C$ is defined by

$$\psi_t(s) = \psi(t + s), \ s \leq 0.$$

Set

$$B = \mathrm{diag}\{b_1, b_2, \ldots, b_n\}, \quad K = \mathrm{diag}\{k_1, k_2, \ldots, k_n\}.$$

Definition 1. A real matrix $A = (a_{ij})_{n\times n}$ is said to be an M-matrix if

$$a_{ij} \leq 0, \quad i, j = 1, 2, \ldots, n, \ i \neq j,$$

and all successive principal minors of A are positive.

In [14], Chen et al. have proved that under assumptions (A_1)-(A_3), if

$$B - |A|K \text{ is an } M\text{-matrix}, \tag{2}$$

then system (1) has a unique equilibrium which is globally sable.

The purpose of this paper is to present some new sufficient conditions for the existence, uniqueness and global stability of the equilibrium point of system (1). In Section II, we will prove that the condition (2) for the existence and uniqueness of system (1) can be weakened to be that "$B - \tilde{A}K$ is an M-matrix". Then, in Section III, we will prove that if there exists a nonnegative diagonal matrix L such that $B - \tilde{A}K - L$ is an M-matrix and a_{ii}^- satisfies a certain constraint condition for each $i \in \{1, 2, \ldots, n\}$, then the equilibrium of system (1) can also be globally stable. Moreover, the corresponding results in [14] are extended and improved. It should be mentioned that the idea of this paper is quite different from that used in [14]-[21] since we make use of the fact that the nonlinear delayed terms can ensure stability [23]. Therefore, our results can be applied to the case when the non-delayed terms can not dominate the others. Some examples are presented in Section IV to illustrate the effectiveness of our results. Finally, some conclusions are drawn in Section V.

II. Existence and Uniqueness of the Equilibrium

As usual, a vector $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}} \in \mathbb{R}^n$ is said to be an equilibrium point of system (1) if it satisfies

$$-d(u^*) + Ag(u^*) + I = 0,$$

where $I = (I_1, I_2, \ldots, I_n)^{\mathrm{T}}$. To solve the above equation, we need some preliminaries.

Lemma 1 [22]. If $H : \mathbb{R}^n \to \mathbb{R}^n$ is continuous, and satisfies the following conditions:

$i)$ H is injective on $\mathbb{R}^n$,

$ii)$ $\lim_{\|u\|\to\infty} \|H(u)\| = \infty$,

then $H(u)$ is a homeomorphism of $\mathbb{R}^n$.

Now we give one important result in which the condition (2) is relaxed.

Theorem 1. If assumptions (A_1)-(A_3) are satisfied and $B - \tilde{A}K$ is an M-matrix, then system (1) has a unique equilibrium point u^*.

Proof. Since $B - \tilde{A}K$ is an M-matrix, there exist $r_i > 0$, $i = 1, 2, \ldots, n$, such that

$$-b_i r_i + \sum_{j=1}^{n} r_j \tilde{a}_{ji} k_i < 0.$$

Set

$$\alpha = \min_{1\leq i\leq n} \left\{ b_i - \sum_{j=1}^{n} r_j r_i^{-1} \tilde{a}_{ji} k_i \right\},$$

then $\alpha > 0$. Let

$$R_0 = \mathrm{diag}\left\{r_1, r_2, \ldots, r_n\right\}$$

and

$$G(u) = -d(u) + Ag(u) + I.$$

It is known that if $G(u)$ is a homeomorphism on $\mathbb{R}^n$, then there exists a unique point u^* such that $G(u^*)=0$. Hence, it suffices to prove that $G(u)$ is a homeomorphism on $\mathbb{R}^n$. To this end, we set

$$H(u)=R_0G(R_0^{-1}u)=(H_1(u),H_2(u),\ldots,H_n(u))^{\mathrm{T}}.$$

We first assert that $H(u)$ is injective on $\mathbb{R}^n$. If it were not true, there exist $u,\ v\in\mathbb{R}^n$, $u\neq v$, such that $H(u)=H(v)$. Notice that

$$H_i(u)=-r_id_i\left(r_i^{-1}u_i\right)+\sum_{j=1}^{n}r_ia_{ij}g_j\left(r_j^{-1}u_j\right)+r_iI_i.$$

By assumptions (A_1) and (A_2), we get

$$\begin{aligned}0 &= (H_i(u)-H_i(v))\,\mathrm{sgn}\,(u_i-v_i)\\ &\leq -b_i\left|u_i-v_i\right|+\sum_{j=1}^{n}r_ir_j^{-1}\tilde{a}_{ij}k_j\left|u_j-v_j\right|,\end{aligned}$$

which yields

$$\begin{aligned}0 &\leq -\sum_{i=1}^{n}b_i\left|u_i-v_i\right|+\sum_{i=1}^{n}\sum_{j=1}^{n}r_ir_j^{-1}\tilde{a}_{ij}k_j\left|u_j-v_j\right|\\ &= -\sum_{i=1}^{n}b_i\left|u_i-v_i\right|+\sum_{j=1}^{n}\sum_{i=1}^{n}r_ir_j^{-1}\tilde{a}_{ij}k_j\left|u_j-v_j\right|\\ &= -\sum_{i=1}^{n}b_i\left|u_i-v_i\right|+\sum_{i=1}^{n}\sum_{j=1}^{n}r_jr_i^{-1}\tilde{a}_{ji}k_i\left|u_i-v_i\right|\\ &= -\sum_{i=1}^{n}\left\{b_i-\sum_{j=1}^{n}r_jr_i^{-1}\tilde{a}_{ji}k_i\right\}\left|u_i-v_i\right|\\ &\leq -\alpha\sum_{i=1}^{n}\left|u_i-v_i\right|.\end{aligned}$$

This is a contradiction. So, the map H is injective on $\mathbb{R}^n$. Moreover, from the above inequality, we have

$$\sum_{i=1}^{n}(H_i(u)-H_i(0))\,\mathrm{sgn}u_i\leq-\alpha\sum_{i=1}^{n}\left|u_i\right|.$$

Letting $\|u\|\to\infty$, we get $\|H(u)\|\to\infty$. By Lemma 1, $H(u)$ is a homeomorphism on $\mathbb{R}^n$. Since R_0 is nonsingular, it follows that $G(u)$ is a homeomorphism on $\mathbb{R}^n$. The proof is complete. ■

III. Global Stability of the Equilibrium

In this section, we will present some new sufficient conditions for the global stability of the equilibrium point of the neural network (1). We first give a lemma, but its proof is omitted due to limited space.

Lemma 2. For neural network (1), suppose that assumptions (A_1)-(A_3) are satisfied. Set

$$c_i=\sup\{\dot{d}_i(u)\},\ i=1,2,\ldots,n.$$

If there exists a nonnegative diagonal matrix

$$L=\mathrm{diag}\left\{l_1,l_2,\ldots,l_n\right\}$$

such that $B-\tilde{A}K-L$ is an M-matrix, and for each $i\in\{1,2,\ldots,n\}$, either $a_{ii}^{-}k_i\leq l_i$ or

$$\begin{aligned}&a_{ii}^{-}k_i\left\{\int_0^{+\infty}sK_{ii}(s)ds+\int_0^{h_{i_1}}\left(h_{i_1}-s\right)K_{ii}(s)ds\right\}\\ &\quad\leq 1-(2b_i-l_i)h_{i_1},\end{aligned}\tag{3}$$

where

$$h_{i_1}=\frac{b_i-l_i}{b_i\left(2b_i+a_{ii}^{-}k_i-l_i\right)},$$

then system (1) has a unique equilibrium point, which is uniformly stable.

With Lemma 2, we are now able to establish the global stability of the equilibrium point of the neural network (1).

Theorem 2. For neural network (1), suppose that assumptions (A_1)-(A_3) are satisfied. Set $c_i=\sup\{\dot{d}_i(u)\},\ i=1,2,\ldots,n$. If there exists a nonnegative diagonal matrix $L=\mathrm{diag}\left\{l_1,l_2,\ldots,l_n\right\}$ such that $B-\tilde{A}K-L$ is an M-matrix, and for each $i\in\{1,2,\ldots,n\}$, either $a_{ii}^{-}k_i\leq l_i$ or condition (3) holds then system (1) has a unique equilibrium point, which is globally asymptotically stable.

Proof. Let $x(t)=u(t)-u^*$. System (1) can be written as

$$\begin{aligned}\dot{x}_i(t)=-\bar{d}_i(x_i(t))+\sum_{j=1}^{n}a_{ij}\int_{-\infty}^{t}K_{ij}(t-s)f_j(x_j(s))ds,\\ i=1,2,\ldots,n,\end{aligned}\tag{4}$$

By Lemma 2, it suffices to prove that any solution of system (4) tends to zero as $t\to+\infty$. We only consider the case that condition (3) holds, since the proof for the case of $a_{ii}^{-}k_i\leq l_i$ is similar. Set

$$\gamma_i=\sum_{j=1}^{n}\tilde{a}_{ij}k_jr_jr_i^{-1},$$

then $\gamma_i<b_i-l_i$. Set

$$\alpha_{i_1}=c_i+a_{ii}^{-}k_i+\gamma_i,$$

then

$$\frac{\gamma_i}{b_i \alpha_{i_1}} < h_{i_1}.$$

Since the function

$$\int_0^x (x-s) K_{ii}(s) ds$$

is nondecreasing with respect to x on $[0, +\infty)$, it follows that

$$\int_0^{\frac{\gamma_i}{b_i \alpha_{i_1}}} \left(\frac{\gamma_i}{b_i \alpha_{i_1}} - s \right) K_{ii}(s) ds \leq \int_0^{h_{i_1}} (h_{i_1} - s) K_{ii}(s) ds.$$

On the other hand, the function

$$1 - \frac{x(c_i + x)}{b_i \left(c_i + a_{ii}^- k_i + x \right)}$$

is strictly decreasing with respect to x on $[0, +\infty)$. So,

$$1 - \frac{(b_i - l_i)(b_i + c_i - l_i)}{b_i \left(b_i + c_i + a_{ii}^- k_i - l_i \right)} < 1 - \frac{\gamma_i (c_i + \gamma_i)}{b_i \alpha_{i_1}}.$$

Therefore, condition (3) implies that

$$a_{ii}^- k_i \left\{ \int_0^{+\infty} s K_{ii}(s) ds + \int_0^{\frac{\gamma_i}{b_i \alpha_{i_1}}} \left(\frac{\gamma_i}{b_i \alpha_{i_1}} - s \right) K_{ii}(s) ds \right\} < 1 - \frac{\gamma_i (c_i + \gamma_i)}{b_i \alpha_{i_1}},$$

which is equivalent to

$$a_{ii}^- k_i \left\{ \int_{\frac{\gamma_i}{b_i \alpha_{i_1}}}^{+\infty} \left(s - \frac{\gamma_i}{b_i \alpha_{i_1}} \right) K_{ii}(s) ds \right\} < 1 - \frac{\gamma_i}{b_i}. \quad (5)$$

Let

$$\beta_0 = \max_{1 \leq i \leq n} \sup_{-\infty < t < +\infty} |x_i(t)|$$

and

$$\beta_i = \limsup_{t \to +\infty} |x_i(t)|.$$

From Lemma 2, we have

$$0 \leq \beta_i < +\infty, \; i = 0, 1, 2, \ldots, n.$$

We assume that

$$\beta_{i_0} / r_{i_0} = \max_{1 \leq i \leq n} \{ \beta_i / r_i \}$$

for some $i_0 \in \{1, 2, \ldots, n\}$. It suffices to prove that $\beta_{i_0} = 0$. By the contrary, we will have $\beta_{i_0} > 0$. For simplicity, we write i_0 by i. Set

$$\bar{h}_{i_1}(\sigma) = \frac{\gamma_i \beta_i + \mu_i \sigma}{b_i \left(\alpha_{i_1} \beta_i + \alpha_{i_2} \sigma \right)}, \quad \varepsilon_i(\sigma) = \frac{\gamma_i \beta_i + \mu_i \sigma}{b_i},$$

and

$$\bar{h}_{i_2}(\sigma) = \frac{\beta_i + \sigma + \varepsilon_i(\sigma)}{\alpha_{i_1} \beta_i + \alpha_{i_2} \sigma},$$

where

$$\alpha_{i_2} = c_i + \mu_i, \quad \mu_i = (\beta_0 + 1) \sum_{j=1}^n |a_{ij}| \, k_j.$$

It is easy to see that $\bar{h}_{i_1}(\sigma), \; \varepsilon_i(\sigma), \; \bar{h}_{i_2}(\sigma)$ are continuous functions of σ on $[0, +\infty)$. Moreover,

$$\bar{h}_{i_1}(0) = \frac{\gamma_i}{b_i \alpha_{i1}} < h_{i1}, \quad \varepsilon_i(0) = \frac{\gamma_i}{b_i} \beta_i,$$

$$\bar{h}_{i_2}(0) = \frac{b_i + \gamma_i}{b_i \alpha_{i1}} < h_{i2}.$$

By (5) and the continuity of $\bar{h}_{i_1}(\sigma), \; \varepsilon_i(\sigma)$ on $[0, +\infty)$, there exists $\sigma_1 > 0$ such that for any $\sigma \in [0, \sigma_1)$, the following inequality holds:

$$(\beta_i + \sigma) a_{ii}^- k_i \int_{\bar{h}_{i_1}(\sigma)}^{+\infty} \left(s - \bar{h}_{i_1} \right) K_{ii}(s) ds < \beta_i - \sigma - \varepsilon_i(\sigma).$$

Set

$$\sigma_0 = \min \left\{ \sigma_1, \frac{b_i - \gamma_i}{b_i + \mu_i} \beta_i \right\}.$$

For any given $\sigma \in (0, \sigma_0)$, in the sequel, we will write $\bar{h}_{i_1}(\sigma), \; \varepsilon_i(\sigma), \; \bar{h}_{i_2}(\sigma)$ by $\bar{h}_{i_1}, \; \varepsilon_i, \; \bar{h}_{i_2}$, respectively. By assumption (A_3), there exits $T_1 > \bar{h}_{i_2}$ such that

$$\sup_{1 \leq j \leq n} \int_{T_1}^{+\infty} K_{ij}(s) ds < \sigma, \quad \text{and}$$
$$|x_j(t)| < \beta_j + \sigma, \; t \geq T_1, \quad j = 1, 2, \ldots, n.$$

Since

$$\limsup_{t \to +\infty} |x_i(t)| = \beta_i,$$

there exists a sequence $\{t_m\}$, $t_m \to +\infty$, such that

$$|x_i(t_m)| \to \beta_i.$$

Consider the following two cases:

Case A. $\dot{x}_i(t)$ is eventually sign-definite.

Case B. $\dot{x}_i(t)$ is oscillatory.

In both cases a contradiction can be deduced (the details are omitted due to limited space). This completes the proof. ■

It is easy to see that

$$\int_0^{h_{i_1}} (h_{i_1} - s) K_{ii}(s) ds \leq h_{i_1}.$$

Hence, we have the following result.

Corollary 1. For neural network (1), suppose that assumptions (A_1)-(A_3) are satisfied. If there exists a nonnegative diagonal matrix $L = \mathrm{diag}\{l_1, l_2, \ldots, l_n\}$ such that $B - \tilde{A}K - L$ is an M-matrix, and

$$a_{ii}^{-} k_i \min\left\{1, b_i \int_0^{+\infty} sK_{ii}(s)ds\right\} \leq l_i, \tag{6}$$

then system (1) has a unique equilibrium point, which is globally asymptotically stable.

Note that if $l_i = a_{ii}^{-} k_i,\ i = 1, 2, \ldots, n$ are chosen, Corollary 1 implies the following criterion.

Corollary 2. For neural network (1), suppose that assumptions (A_1)-(A_3) are satisfied. If $B - |A|K$ is an M-matrix, then system (1) has a unique equilibrium point, which is globally asymptotically stable.

Remark 1. Corollary 2 is also obtained in [14] by using Lyapunov functional method. Therefore, our results extend and improve the corresponding ones in [14].

To compare our results with the existing ones, apply Corollary 1 to the one-dimensional neural network with distributed delay of the form

$$\dot{x}(t) = -d(x(t)) + a \int_{-\infty}^{t} K(t-s) g(x(s)) ds + I,\ x \in \mathbb{R} \tag{7}$$

where d is differentiable on $\mathbb{R}$, $b = \inf\{\dot{d}(x)\} > 0$, K is a nonnegative continuous function on $[0, \infty)$ satisfying

$$\int_0^{+\infty} K(s)ds = 1,$$

and there exists a constant $k > 0$ such that

$$0 \leq (g(u) - g(v))(u - v) \leq k(u - v)^2$$

for any $u,\ v \in \mathbb{R}$. It is straightforward to deduce the following corollary.

Corollary 3. For neural network (7), suppose that $|a|k < b$, or

$$a < 0, \text{ and } |a|k \int_0^{+\infty} sK(s)ds < 1. \tag{8}$$

Then system (7) has a unique equilibrium point, which is globally asymptotically stable.

Remark 2. From the results in [14]-[16], the condition for global stability of system (7) is $|a|k < b$. This condition requires that the non-delayed term of the neural network dominates the delayed term, i.e., the delayed term must be small enough not to destroy the stability caused by the non-delayed term. However, the new stability condition (8) shows that when the non-delayed term can not dominate the delayed term, a nonlinear retarded term with infinite delay can also ensure stability. This feature is attractive in the design and application of neural networks with distributed delays.

IV. Illustrative Examples

Example 1. Consider the one-dimensional neural network of the form

$$\dot{x}(t) = -0.5x(t) + a \int_{-\infty}^{t} \frac{2}{1 + (t-s)^3} \tanh(x(s)) ds. \tag{9}$$

In this example,

$$b = 0.5, \quad K(t) = \frac{2}{1 + t^3}, \quad f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

One can verify that

$$\int_0^{+\infty} K(s)ds = 1, \quad \int_0^{+\infty} sK(s)ds = 1, \quad k = 1.$$

Applying Corollary 3, we obtain that when $0 \leq a < 0.5$ or $-1 < a \leq 0$, neural network (9) is globally asymptotically stable. It is noted that $|a| > b$ when $-1 < a < -0.5$, however, neural network (9) is still stable.

Example 2. Consider the two-dimensional neural network of the form

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + 0.8 \int_{-\infty}^{t} K_1(t-s) f(x_1(s)) ds \\ &\quad +0.5 \int_{-\infty}^{t} K_1(t-s) f(x_2(s)) ds \\ \dot{x}_2(t) &= -x_2(t) + 0.1 \int_{-\infty}^{t} K_1(t-s) f(x_1(s)) ds \\ &\quad -2 \int_{-\infty}^{t} K_2(t-s) f(x_2(s)) ds, \end{aligned} \tag{10}$$

where

$$K_1(t) = \frac{2}{\pi(1 + t^2)}, \quad K_2(t) = 3\exp(-3t),$$
$$f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

One can verify that

$$\int_0^{+\infty} K_1(t)dt = \int_0^{+\infty} K_2(t)dt = 1,$$
$$\int_0^{+\infty} tK_2(t)dt = \frac{1}{3}, \quad k_1 = k_2 = 1.$$

Therefore,

$$B - |A|K = \begin{bmatrix} 0.2 & -0.5 \\ -0.1 & -1 \end{bmatrix},$$
$$B - \tilde{A}K = \begin{bmatrix} 0.2 & -0.5 \\ -0.1 & 1 \end{bmatrix}.$$

Since $B - |A|K$ is not an M-matrix, the results in [14] can not be applied to this example. However, by choosing $L = \mathrm{diag}\{0, 0.74\}$, it is easy to see that

$$B - \tilde{A}K - L = \begin{bmatrix} 0.2 & -0.5 \\ -0.1 & 0.26 \end{bmatrix}$$

is an M-matrix and condition (6) is satisfied. Thus, by Corollary 1, neural network (10) is globally asymptotically stable.

V. Conclusions

In this paper, we have studied a model of neural networks with distributed delays. It may have applications in processing of motion related phenomena such as moving images and associative memories. We have established simple, checkable conditions for global stability. It has been shown that these new stability criteria are applicable to the case when the non-delayed terms can not dominate the delayed terms, thus being of great importance in the design and application of dynamical neural networks with distributed delays.

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