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A Feedback-based Sensor Uncertainty Detection Scheme

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Abstract— **In this paper, an adaptive feedback-based stable fault detection scheme is developed for linear time-invariant systems with sensor uncertainties and system parameter uncertainties. A parametric sensor uncertainty model with additive faults and multiplicative faults is introduced. To employ the model-based fault detection method, the sensor dynamics is derived. Based on the newly developed sensor dynamic models, a set of estimation model systems are established to estimate the sensor signals. Unlike most fault detection schemes which operate under the assumption that all system signals remain bounded under sensor uncertainty conditions, the adaptive sensor detection scheme proposed in this paper is integrated with an adaptive feedback controller which is designed to ensure the desired signal boundedness requirement needed for stable sensor uncertainty detection operation. By observing residuals between the sensor signals and the estimation model signals, the sensor uncertainty can be detected and specific uncertainty patterns can be identified. Desired adaptive sensor uncertainty detection performance is demonstrated in the simulation study for a linearized longitudinal aircraft flight control system.**

Keywords: Sensor uncertainty, fault detection, fault-tolerant control, adaptive feedback control.

I. INTRODUCTION

Accuracy of sensor measurement is crucial in safety-critical and mission-critical systems, such as aircrafts, spacecrafts, power plants, and chemical plants. In particular, for the systems with feedback control framework, inaccurate sensor measurements of system signals may cause abnormal performance of designed controllers, which can have severe impacts on the performance of systems. Therefore, it is imperative to detect the malfunctions of sensor for the fault-tolerant control systems to improve the safety and reliability.

Fault detection and diagnosis problems have been studied widely in recent years. One of the common approaches is to use model-based method for fault detection and diagnosis as summarized in [3]. The model-based method uses state observers or parameter estimation models to construct some detector model systems. By comparing residuals between detector output signals and measured system output signals, detection criteria can be derived to quickly and reliably diagnose subtle incipient or abrupt system degradation. The model-based detection technique depends on mathematical models of the system. If the system mathematical model is accurate and known, it can be used to form the basis for analytic redundancy model-based designs that simultaneously diagnose both sensor and actuator faults ([5] and [6]). In the presence of uncertainties, modeling errors can decrease system sensitivity to faults and increase the rate of occurrence of false alarms. [10] proposed a robust design to address the presence of system model uncertainty. In addition, adaptive designs have been presented for the known nominal plant dynamics (e.g., [2], [9], and [11]) and for the plants with unknown parameters (e.g., [1] and [4]).

In this paper, we present a new sensor fault detection methodology for linear systems with parameter uncertainties, which explicitly includes parametric sensor representations of additive faults (sensor bias-uncertainty) and multiplicative faults (sensor scaling). From the sensor uncertainty model and the system dynamic model, we can derive sensor dynamic models [8] which has uncertainties due to unknown system parameters and unknown sensor faults. Based on such sensor dynamic models, a set of estimation models are constructed to detect the sensor uncertainty and identify some specific uncertainty patterns by observing the residuals between the sensor signals and the estimation model signals.

For the model-based design, the construction of detector models requires that the input signals are bounded. In the presence of sensor faults, nominal feedback control designs may deteriorate the performance badly, or even make the closed-loop system unstable. Therefore, in this paper, the sensor uncertainty detection scheme is integrated with a self-stabilization feedback control to compensate the sensor uncertainty and make the system stable. Since the sensors have uncertainties, we cannot directly apply the sensor signals to the feedback controller. Thus, sensor compensators are derived to construct the adaptive feedback controller. With such an adaptive feedback control design, the closed-loop signals including sensor signals are bounded. Then, the sensor detection scheme can be employed by using the bounded sensor signals and system input signals.

Although the adaptive control design can compensate the sensor uncertainty, the detection scheme is also needed to improve the situation awareness of the control personnel and enhance the system safety and reliability by informing the control personnel of a potential system reduction.

This paper is organized as follows. The sensor uncertainty detection problem is formulated in Section II where the sensor uncertainty model is introduced. In Section III, the feedback-based sensor uncertainty detection scheme is developed, where a set of sensor estimation model systems

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are designed based on the sensor dynamic models and the bounded closed-loop system signals ensured by the adaptive feedback control design. A simulation study on an aircraft flight control system with sensor uncertainties is conducted in Section IV to show the desired detection performance.

II. Problem Statement

Consider a single-input and single-output linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \tag{1}$$

where $A \in R^{n\times n}$, $B \in R^{n\times 1}$, and $C \in R^{1\times n}$, are unknown constant parameter matrices, and $x(t) \in R^n$, $u(t) \in R$, and $y(t) \in R$ are system state, input, and output signals. For feedback control designs, construction of the input signal $u(t)$ is based on sensor measurements of the state signal $x(t)$ or the output signal $y(t)$. Performance of the feedback control system can be deteriorated when there are uncertainties in the sensor measurements.

Sensor uncertainty model. For the detection and compensation designs, a sensor uncertainty model is given as

$$z(t) = k_s\varphi(t) + \sum_{i=1}^{q} b_i f_i(t), \tag{2}$$

where $\varphi(t)$ is the actual signal to be measured, which can be the state $x(t)$ or output $y(t)$, $k_s > 0$ and b_i, $i = 1, 2, \ldots, q$, are some unknown constant sensor uncertainty parameters, and $f_i(t)$, $i = 1, 2, \ldots, q$, are known bounded signals with bounded derivative $\dot{f}_i(t)$.

Remark 1: The sensor uncertainty modeling problem can be addressed by using redundant sensors. We can use several sensors to measure the same signal $\varphi(t)$, and take the weighted sum of the sensors' output signals $z_i(t)$ as $z(t) = \sum_{i=1}^{m} \alpha_i z_i(t)$, where $\alpha_i > 0, i = 1, \ldots, m$, such that $\sum_{i=1}^{m} \alpha_i = 1$. When there is no uncertainty for all the sensors, the summed sensor signal $z(t)$ is the exact measured signal $\varphi(t)$. When there are some sensor uncertainties, e.g., the $i_1, i_2, \ldots, i_p$th sensors fail and generate some random signals $\bar{z}_i(t)$, the summed sensor signal is $z(t) = \alpha_s\varphi(t) + d_s(t)$, where $\alpha_s = \sum_{i\neq i_1,i_2,\ldots,i_p} \alpha_i$, and $d_s(t) = \sum_{i=i_1,i_2,\ldots,i_p} \alpha_i\bar{z}_i(t)$. Since the indexes $i_1, i_2, \ldots, i_p$ are unknown, we may express $d_s(t) = \sum_{i=1}^{m} \beta_i\bar{z}_i(t)$, where some of β_i are zero (for the unfailed sensors) while others are α_i, and $\bar{z}_i$ is accessible in the bias-uncertainty part of the uncertainty model (2). □

Feedback-based uncertainty detection problem. Based on the sensor uncertainty models (2), we will develop a model-based adaptive diagnosis scheme to detect the modeled sensor uncertainty. More specifically, a set of state sensor detectors and output sensor detectors are designed, which are a set of adaptive estimation models to estimate the unknown parameters in the sensor uncertainty models. By comparing residuals between the detection models and the sensor uncertainty models, detection criteria can be derived to identify and isolate particular uncertainty scenarios.

Feedback uncertainty compensation problem. The construction of detection models requires that the control input signal $u(t)$, the state sensor signal $z_x(t)$, and the output sensor signal $z_y(t)$ are bounded. To ensure the signal boundedness requirement, an adaptive feedback control law will be developed for the system (1) to compensate the sensor uncertainty and make all the closed-loop signals bounded including the signals $u(t)$, $z_x(t)$, and $z_y(t)$.

Since the parameters k_s and b_i, $i = 1, 2, \ldots, q_i$, of the state or output sensor model (2) are unknown, the state signal $x(t)$ or the output signal $y(t)$ cannot be retrieved from the sensor measurement $z(t)$. To overcome this difficulty, we propose to use sensor compensator signals $\hat{x}(t)$ and $\hat{y}(t)$ from the sensor measurement $z(t)$ to construct an adaptive feedback control law $u(t)$, which can make all the closed-loop signals bounded and the plant output signal $y(t)$ track a given reference signal $y_m(t) \in R$ generated from a reference model system

$$y_m(t) = W_m(s)[r](t), \tag{3}$$

with $r(t) \in R$ being a bounded reference input signal and $W_m(s)$ is stable.

Assumptions. To proceed the control and detection scheme designs, for the system (1):

$$y(t) = C(sI - A)^{-1}B[u](t) = \frac{Z(s)}{P(s)}[u](t), \tag{4}$$

where $Z(s) = z_m s^m + \cdots + z_1 s + z_0$ with $z_m \neq 0$ and $P(s)$ is a monic polynomial of degree n, we assume that (A1) $Z(s)$ is a Hurwitz polynomial; (A2) the degree m of $Z(s)$ is known, and $W_m(s) = 1/P_m(s)$ where $P_m(s)$ is a Hurwitz polynomial of degree $n - m$; (A3) the sign of z_m is known; and (A4) (A, B, C) is controllable and observable.

III. Feedback-based Sensor Uncertainty Detection Scheme

In this section, we will present the detailed adaptive feedback-based sensor uncertainty detection design. To develop the sensor uncertainty detection scheme, dynamic models with signals being the state sensor $z_x(t)$ and the output sensor $z_y(t)$ will be given first.

State sensor dynamic model. From the sensor uncertainty model (2), the state sensor signal $z_x(t)$ is expressed as

$$z_x(t) = K_x x(t) + \Theta_{bx}^T f_x(t), \tag{5}$$

where the unknown parameters $K = \mathrm{diag}\{k_{x1}, \ldots, k_{xn}\}$, $\Theta_{bx}^T = \mathrm{diag}\{\theta_{bx1}^T, \ldots, \theta_{bxn}^T\}$ with $\theta_{bxi} = [b_{xi1}, \ldots, b_{xiq_i}]^T$, and the signal $f_x(t) = [f_{x1}^T(t), \ldots, f_{xn}^T(t)]^T$ with $f_{xi}(t) = [f_{xi1}(t), \ldots, f_{xiq_i}(t)]^T$, $i = 1, \ldots, n$. In view of (5) and the system (1), we have the state sensor dynamics as [8]

$$\dot{z}_x(t) = A_z z_x(t) + B_z u(t) + \Theta_z^T f_x(t) + \Theta_{bx}^T \dot{f}_x(t), \tag{6}$$

where the unknown parameter matrices are given as

$$A_z = K_x A K_x^{-1}, B_z = K_x B, \Theta_z^T = -K_x A K_x^{-1}\Theta_{bx}^T. \tag{7}$$

Output sensor parametric model. The output sensor with uncertainties is given as

$$z_y(t) = k_y y(t) + \theta_{by}^T f_y(t) \tag{8}$$

where $\theta_{by} = [b_{y1}, \ldots, b_{yp}]^T$ and $f_y(t) = [f_{y1}, \ldots, f_{yp}]^T$. Operating both sides of the transfer function (4) by a filter $1/\Lambda(s)$, where $\Lambda(s)$ is a chosen stable and monic polynomial of degree n, we can obtain

$$y(t) = \frac{Z(s)}{\Lambda(s)}[u](t) + \frac{\Lambda(s) - P(s)}{\Lambda(s)}[y](t). \qquad (9)$$

Substituting $y(t) = \frac{1}{k_y} z_y(t) - \frac{\theta_{by}^T}{k_y} f_y(t)$ in (9), we have

$$z_y(t) = \theta_u^T \phi_u(t) + \theta_z^T \phi_z(t) + \theta_{bf}^T f_y(t) + \sum_{i=1}^{p} \theta_i^T \phi_i(t), \qquad (10)$$

where the signals are

$$\phi_u(t) = [\frac{1}{\Lambda(s)}[u](t), \frac{s}{\Lambda(s)}[u](t), \ldots, \frac{s^m}{\Lambda(s)}[u](t)]^T,$$
$$\phi_z(t) = [\frac{1}{\Lambda(s)}[z_y](t), \frac{s}{\Lambda(s)}[z_y](t), \ldots, \frac{s^{(n-1)}}{\Lambda(s)}[z_y](t)]^T,$$
$$\phi_i(t) = [\frac{1}{\Lambda(s)}[f_{yi}](t), \frac{s}{\Lambda(s)}[f_{yi}](t), \ldots, \frac{s^{(n-1)}}{\Lambda(s)}[f_{yi}](t)]^T,$$

for $i = 1, 2, \ldots, p$, and θ_u, θ_z, and θ_i, $i = 1, 2, \ldots, p$ are the corresponding unknown parameters.

Signal boundedness. The uncertainty detector models will be developed based on the dynamic models (6) and (10) with bounded signals. To ensure the signal boundedness requirement, an adaptive control design will be applied.

A. *Self-stabilization Feedback Control Design*

Since the state sensors have uncertainties, a state sensor compensator will be used to constructed the controller.

State sensor compensator. From the state sensor uncertainty model (5), the state signal $x(t)$ can be retrieved as

$$x(t) = [x_1(t), \ldots, x_n(t)]^T = \Theta_x^{*T} \psi_x(t), \qquad (11)$$

where the accessible signal $\psi_x(t)$ is given as $\psi_x(t) = [\psi_{x1}^T(t), \ldots, \psi_{xn}^T(t)]^T$ with $\psi_{xi}(t) = [z_{xi}(t), f_{xi1}(t), \ldots, f_{xiq_i}(t)]^T$ and the unknown parameter Θ_x^* is given as $\Theta_x^{*T} = \text{diag}\{\theta_{x1}^{*T}, \theta_{x2}^{*T}, \ldots, \theta_{xn}^{*T}\}$ with θ_{xi}^*, $i = 1, \ldots, n$ being the corresponding unknown parameters. Since the parameter Θ_x^* is unknown, the actual signal $x(t)$ is not accessible. Thus, a sensor compensator is introduced:

$$\hat{x}(t) = [\hat{x}_1(t), \ldots, \hat{x}_n(t)]^T = \Theta_x^T(t) \psi_x(t), \qquad (12)$$

where $\Theta_x^T(t) = \text{diag}\{\theta_{x1}^T(t), \theta_{x2}^T(t), \ldots, \theta_{xn}^T(t)\}$ is the adaptively updated estimate of the unknown parameter Θ_x^{*T}.

Controller structure.We choose the controller $u(t)$ as

$$u(t) = K_x^T(t) \psi_x(t) + k_2(t) r(t), \qquad (13)$$

where $K_x^T(t) = K_1^T(t) \Theta_x^T(t)$ and $k_2(t)$ are the estimates of the unknown nominal parameters $K_x^{*T} = K_1^{*T} \Theta_x^{*T}$ and k_2^*. The nominal parameters K_1^* and k_2^* satisfy the following plant-model matching equations:

$$\det(sI - A - BK_1^{*T}) = \frac{Z(s)}{z_m W_m(s)}, k_2^{*-1} = z_m. \qquad (14)$$

Closed-loop system. Substituting the controller signal (13) in the plant (1) and applying the state signal model (11), we have the closed-loop system as

$$\dot{x} = (A + BK_1^{*T})x + Bk_2^* r + B\widetilde{\Theta}^T \omega, \;\; y = Cx, \qquad (15)$$

where $\widetilde{\Theta}(t) = \Theta(t) - \Theta^*, \Theta(t) = [K_x^T(t), k_2(t)]^T, \Theta^* = [K_x^{*T}, k_2^*]^T$, and $\omega(t) = [\psi_x^T(t), r^T(t)]^T$. From the matching equation (14), if $\Theta(t) = \Theta^*$, we have $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. That is, the controller (13) with nominal parameter Θ^* can make the output track the reference output. However, Θ^* is unknown, we need to apply the controller (13) with adaptively updated parameter $\Theta(t)$.

To develop an adaptive law for the parameter $\Theta(t)$, the output tracking error information is needed. Since the output sensors have uncertainties, we cannot obtain the exact output signal $y(t)$. An output sensor compensator $\hat{y}(t)$ is introduced to estimate the output signal $y(t)$.

Output sensor compensator. From the output sensor uncertainty model (8), we retrieve that

$$y(t) = \theta_y^{*T} \psi_y(t), \qquad (16)$$

where $\theta_y^* = [\theta_{ky}^*, \theta_{by1}^*, \ldots, \theta_{byp}^*]^T$ and $\psi_y(t) = [z_y(t), f_{y1}(t), \ldots, f_{yp}(t)]^T$ with $\theta_{ky}^* = 1/k_y$ and $\theta_{byj}^* = -b_{yj}/k_y$, $j = 1, 2, \ldots, p_i$ being unknown constant parameters. Then, the output compensation signal $\hat{y}(t)$ is given as

$$\hat{y}(t) = \theta_y^T(t) \psi_y(t), \qquad (17)$$

where $\theta_y(t)$ is an estimate of the unknown parameter θ_y^*.

Compensation tracking error. We introduce a compensation output tracking error signal as

$$\hat{e}(t) = \hat{y}(t) - y_m(t) = e(t) + (\hat{y}(t) - y(t)), \qquad (18)$$

where $e(t) = y(t) - y_m(t)$ is the actual output tracking error.

In view of the closed-loop system (15), the reference system (3), and the matching condition (14), we have

$$\hat{e}(t) = \rho^* W_m(s)[\widetilde{\Theta}^T \omega](t) + \tilde{\theta}_y^T(t) \psi_y(t), \qquad (19)$$

where $\rho^* = z_m$ and $\tilde{\theta}_y(t) = \theta_y(t) - \theta_y^*$.

Estimation error. We introduce an estimation error as

$$\hat{\epsilon}(t) = \hat{e}(t) + \rho(t)\xi(t), \qquad (20)$$

where $\rho(t)$ is an estimate of the unknown ρ^* and

$$\xi(t) = \Theta^T(t)\zeta(t) - W_m(s)[\Theta^T \omega](t), \qquad (21)$$

with $\zeta(t) = W_m(s)[\omega](t)$. Substituting (19) in (20), we have

$$\hat{\epsilon}(t) = \rho^* \widetilde{\Theta}^T(t)\zeta(t) + \tilde{\rho}(t)\xi(t) + \tilde{\theta}_y^T(t)\psi_y(t), \qquad (22)$$

where $\tilde{\rho}(t) = \rho(t) - \rho^*$.

Adaptive laws. With the estimation error model (22), adaptive laws for $\Theta(t)$, $\rho(t)$, and $\theta_y(t)$ are chosen as

$$\dot{\Theta}(t) = -\frac{\Gamma \text{sign}(z_m)\zeta(t)\hat{\epsilon}(t)}{m^2(t)}, \qquad (23)$$

$$\dot{\rho}(t) = -\frac{\gamma \xi(t)\hat{\epsilon}(t)}{m^2(t)}, \qquad (24)$$

$$\dot{\theta}_y(t) = -\frac{\Gamma_y \psi_y(t)\hat{\epsilon}(t)}{m^2(t)}, \qquad (25)$$

where $\hat{\epsilon}(t)$ is computed from (20), in which $\hat{e}(t)$ is computed from $\hat{e}(t) = \Theta_y^T(t)\psi_y(t) - y_m(t)$, $\Gamma = \Gamma^T > 0$ and $\Gamma_y = \Gamma_y^T > 0$ are adaptation gain matrices, and

$$m(t) = (1 + \xi^2(t) + \zeta^T(t)\zeta(t) + \psi_y^T(t)\psi_y(t))^{1/2}$$

is a standard normalization signal.

Theorem 1: *The sensor uncertainty compensation scheme with the control law (13) updated by the adaptive laws (23)–(25), when applied to the plant (1), guarantees the closed-loop signal boundedness and asymptotic compensation output tracking:* $\lim_{t\to\infty}(\hat{y}(t) - y_m(t)) = 0$.

The first step of the proof of this theorem is to express a filtered version of the plant output compensator $\hat{y}(t)$ in a feedback framework which has a small gain due to the L^2 properties of $\dot{\Theta}(t)$, $\dot{\rho}(t)$, $\dot{\theta}_y(t)$, and $\frac{\hat{\epsilon}(t)}{m(t)}$. This step leads to the closed-loop signal boundedness. The asymptotic tracking property follows from the complete parametrization of the error equation (20), the L^2 properties, and the signal boundedness of the closed-loop system. The convergence of the actual tracking error $e(t) = y(t) - y_m(t)$ to zero is under investigation and it may need some additional conditions in the adaptive control system, as similar to an adaptive observer case where the adaptive state estimation error converges to zero under some persistent excitation conditions in the case when (A, B) are unknown.

B. Sensor Uncertainty Detection Design

Since the adaptive state feedback controller (13) with the adaptive laws (23)–(25) can ensure the boundedness of the closed-loop signals, the bounded control input signal $u(t)$, state sensor signal $z_x(t)$, and output sensor signal $z_y(t)$ are used to construct detector models. By observing residuals between the sensor signals ($z_x(t)$ or $z_y(t)$) and the corresponding detector model signals, we can determine whether there exist sensor uncertainties.

1) ***State sensor uncertainty detection scheme:*** Based on the state sensor dynamic model (6), we start with design and analysis of a benchmark detection model system which will be used to develop a bank of detector model systems for different sensor uncertainty patterns.

Total uncertainty sensor estimation model. We introduce an estimation model to estimate the unknown parameters in sensor dynamic model (6), which is given as [8]

$$\dot{z}_m = A_m z_m + (\widehat{A}_z - A_m)z_x + \widehat{B}_z u + \widehat{\Theta}_z^T f_x + \widehat{\Theta}_{bx}^T \dot{f}_x, \quad (26)$$

where A_m is a chosen stable matrix, and $\widehat{A}_z(t)$, $\widehat{B}_z(t)$, $\widehat{\Theta}_z(t)$, and $\widehat{\Theta}_{bx}(t)$ are the adaptive estimates of A_z, B_z, Θ_z, and Θ_{bx} in the total uncertainty dynamic model (6). From (6) and (26), we obtain the error dynamic system as

$$\dot{e}_x = A_m e_x + \widetilde{A}_z z_x + \widetilde{B}_z u + \widetilde{\Theta}_z^T f_x + \widetilde{\Theta}_{bx}^T \dot{f}_x, \quad (27)$$

where $e_x(t) = z_m(t) - z_x(t)$, $\widetilde{A}_z(t) = \widehat{A}_z(t) - A_z$, $\widetilde{B}_z(t) = \widehat{B}_z(t) - B_z$, $\widetilde{\Theta}_z(t) = \widehat{\Theta}_z(t) - \Theta_z$, $\widetilde{\Theta}_{bx}(t) = \widehat{\Theta}_{bx}(t) - \Theta_{bx}$. Then, the adaptive laws are chosen as

$$\dot{\widehat{A}}_z = -\Gamma_1 P e_x z_x^T, \quad \dot{\widehat{B}}_z = -\Gamma_2 P e_x u, \quad (28)$$

$$\dot{\widehat{\Theta}}_z^T = -\Gamma_3 P e_x f_x^T, \quad \dot{\widehat{\Theta}}_{bx}^T = -\Gamma_4 P e_x \dot{f}_x^T, \quad (29)$$

where $\Gamma_i = \Gamma_i^T > 0, i = 1, 2, 3, 4$, $P = P^T > 0$ satisfying $PA_m + A_m^T P = -Q$ with $Q = Q^T > 0$.

Since the proposed adaptive control law (13) ensures that $z_x(t)$ and $u(t)$ are bounded, we have

Proposition 1: *Given that the signals $z_x(t)$, $u(t)$, $f_x(t)$, and $\dot{f}_x(t)$ are bounded, the proposed estimation model system (26) with the adaptive laws (28) and (29) ensures that $z_m(t)$, $\widehat{A}_z(t)$, $\widehat{B}_z(t)$, $\widehat{\Theta}_z(t)$, and $\widehat{\Theta}_{bx}(t)$ are bounded, and* $\lim_{t\to\infty} e_x(t) = \lim_{t\to\infty}(z_m(t) - z_x(t)) = 0$, *when $z_x(t)$ is of the total sensor uncertainty pattern (5).*

The proof of this result is standard. Consider a positive definite function

$$V = e_x^T P e_x + \mathrm{tr}[\widetilde{A}_z^T \Gamma_1^{-1} \widetilde{A}_z] + [\widetilde{B}_z^T \Gamma_2^{-1} \widetilde{B}_z] + \mathrm{tr}[\widetilde{\Theta}_z \Gamma_3^{-1} \widetilde{\Theta}_z^T] + \mathrm{tr}[\widetilde{\Theta}_{bx} \Gamma_4^{-1} \widetilde{\Theta}_{bx}^T]. \quad (30)$$

From the adaptive laws (28) and (29), we obtain its time-derivative as

$$\dot{V} = -e_x^T(t) Q e_x(t) \leq 0. \quad (31)$$

Then, the properties in Proposition 1 can be derived.

The above estimation model is designed for the general uncertainty signal $\Theta_{bx}^T f_x(t)$ in the state sensor model (5). For a specific situation, some of the terms $\theta_{bxi}^T f_{xi}(t)$ may be not in the sensor uncertainty model (5), that is the corresponding parameter $\theta_{bxi} = 0$. To identify the specific sensor uncertainty patterns, some partial sensor uncertainty estimation model system will be designed. A special one is for the case when no sensor bias uncertainty is present, i.e. $\Theta_{bx}^T f_x(t) = 0$ in the sensor uncertainty model (5).

Bias-uncertainty free sensor estimation model. The sensor dynamic model without bias-uncertainties is given as

$$\dot{z}_x(t) = A_z z_x(t) + B_z u(t). \quad (32)$$

Based on (32), we design a bias-uncertainty free sensor estimation model [8]:

$$\dot{z}_m(t) = A_m z_m(t) + (\widehat{A}_z(t) - A_m)z_x(t) + \widehat{B}_z(t)u(t), \quad (33)$$

where $\widehat{A}_z(t)$ and $\widehat{B}_z(t)$ are updated from the adaptive laws in (28). This estimation model has similar properties to that in Proposition 1, in particular, $\lim_{t\to\infty}(z_m(t) - z_x(t)) = 0$, for the bias-uncertainty free sensor dynamic model (32). On the other hand, when the sensor has bias-uncertainties such as the model (6), the tracking property may not hold, that is $\lim_{t\to\infty}(z_m(t) - z_x(t)) \neq 0$.

Therefore, the bias-uncertainty free sensor output estimation model can be used to detect the sensor bias-uncertainties. The detection criterion is that if $z_m(t)$ from the bias-uncertainty free estimation model (33) cannot track the state sensor signal $z_x(t)$, the state sensors have bias-uncertainties.

To identify which state sensor has bias-uncertainty, we need to design a bank of uncertainty-specific sensor output estimation model systems.

Uncertainty-specific sensor estimation models. The bias-uncertainty model in (5):

$$\Theta_{bx}^T f_x(t) = [\theta_{bx1}^T f_{x1}(t), \theta_{bx2}^T f_{x2}(t), \ldots, \theta_{bxn}^T f_{xn}(t)]^T \quad (34)$$

contains all possible cases of sensor uncertainties with $\theta_{bxi} = 0$ or not. To identify and isolate some uncertainty patterns where a part of the state sensors do not have bias-uncertainty, i.e. $\theta_{bxi} = 0$, for $i = j_1, j_2, \ldots, j_k$ with $\{j_1, j_2, \ldots, j_k\} \subset \{1, 2, \ldots, n\}$, while others have bias-uncertainty, we need to construct some estimation models to ensure that the error $e_x(t) = z_m(t) - z_x(t)$ converges to zero only when $z_x(t)$ is of the specific uncertainty patterns.

Then, we will give an illustrative design to isolate the uncertainty pattern where $\theta_{bx2} \neq 0$, $\theta_{bx4} \neq 0$, and $\theta_{bxi} = 0$, for $i = 1, 3, 5, 6, \ldots, n$, i.e. only the 2nd and the 4th sensors have bias-uncertainties. For this particular uncertain sensor signal $z_x(t)$, we have

$$\Theta_{bx}^T f_x(t) = [0, \theta_{bx2}^T f_{x2}(t), 0, \theta_{bx4}^T f_{x4}(t), 0, \ldots, 0]^T. \quad (35)$$

Then the state sensor dynamics (6) becomes to be

$$\begin{aligned}\dot{z}_x(t) = A_z z_x(t) + B_z u(t) + \Theta_{z2}^T f_{x2}(t) + \Theta_{z4}^T f_{x4}(t) \\ +[0, \theta_{bx2}^T \dot{f}_{x2}(t), 0, \theta_{bx4}^T \dot{f}_{x4}(t), 0, \ldots, 0]^T, \end{aligned} \quad (36)$$

where $\Theta_{z2}^T = a_2 \theta_{bx2}^T$ with a_2 being the second column of $-K_x A K_x^{-1}$ and $\Theta_{z4}^T = a_4 \theta_{bx4}^T$ with a_4 being the fourth column of $-K_x A K_x^{-1}$. Based on (36), we choose the estimation model as [8]

$$\begin{aligned}\dot{z}_m = A_m z_m + (\widehat{A}_z - A_m) z_x + \widehat{B}_z u + \widehat{\Theta}_{z2}^T f_{x2} + \widehat{\Theta}_{z4}^T f_{x4} \\ +[0, \hat{\theta}_{bx2}^T \dot{f}_{x2}, 0, \hat{\theta}_{bx4}^T \dot{f}_{x4}, 0, \ldots, 0]^T. \end{aligned} \quad (37)$$

We obtain the error dynamics from (36) and (37) as

$$\begin{aligned}\dot{e}_x = A_m e_x + \widetilde{A}_z z_x + \widetilde{B}_z u + \widetilde{\Theta}_{z2}^T f_{x2} + \widetilde{\Theta}_{z4}^T f_{x4} \\ +[0, \tilde{\theta}_{bx2}^T \dot{f}_{x2}, 0, \tilde{\theta}_{bx4}^T \dot{f}_{x4}, 0, \ldots, 0]^T, \end{aligned} \quad (38)$$

where $\widetilde{\Theta}_{zi} = \widehat{\Theta}_{zi} - \Theta_{zi}$ and $\tilde{\theta}_{bxi} = \hat{\theta}_{bxi} - \theta_{bxi}$, for $i = 2, 4$.

The adaptive laws for $\widehat{A}_z$ and $\widehat{B}_z$ are chosen as (28), and the adaptive laws for $\widehat{\Theta}_{zi}$ and $\hat{\theta}_{bxi}$ $(i = 2, 4)$ are chosen as

$$\dot{\widehat{\Theta}}_{z2}^T = -\Gamma_5 P e_x f_{x2}^T, \quad \dot{\widehat{\Theta}}_{z4}^T = -\Gamma_6 P e_x f_{x4}^T, \quad (39)$$

$$\dot{\hat{\theta}}_{bx2}^T = -\Gamma_7 p_2^T e_x \dot{f}_{x2}^T, \quad \dot{\hat{\theta}}_{bx4}^T = -\Gamma_8 p_4^T e_x \dot{f}_{x4}^T, \quad (40)$$

where $\Gamma_i = \Gamma_i^T > 0, i = 5, 6, 7, 8$, $P = P^T > 0$ satisfying $P A_m + A_m^T P = -Q$ with $Q = Q^T > 0$, and $p_i, i = 2, 4$, is the ith column of P.

Similar with Proposition 1, the residual converges to 0: $\lim_{t\to\infty}(z_m - z_x) = 0$ when the bias-uncertainty pattern is $\Theta_{bx}^T f_x(t) = [0, \theta_{bx2}^T f_{x2}(t), 0, \theta_{bx4}^T f_{x4}(t), 0, \ldots, 0]^T$. Then, we have the following detection criteria.

If the residual $e_x = z_m - z_x$ converges to zero, where z_m is from the estimation model (37), we can obtain that there exist three possible patterns for the sensor $z_x = [z_{x1}, z_{x2}, \ldots, z_{xn}]^T$: (1) all the state sensors z_{xi} do not have the bias-uncertainty terms; (2) either z_{x2} has the bias-uncertainty term $\theta_{bx2}^T f_{x2}$ or z_{x4} has the bias-uncertainty term $\theta_{bx4}^T f_{x4}$; (3) both z_{x2} and z_{x4} have bias-uncertainty terms. To further isolate the sensor uncertainty pattern, we need to observe the residuals e_x obtained from the bias-uncertainty free estimation model (33) and the estimation models corresponding to the sensor uncertainty patterns: $\Theta_{bx}^T f_x = [0, \theta_{bx2}^T f_{x2}, 0, \ldots, 0]^T$ and $\Theta_{bx}^T f_x(t) = [0, 0, 0, \theta_{bx4}^T f_{x4}, 0, \ldots, 0]^T$ respectively. The isolation of particular uncertainty patterns will be shown in the simulation.

*2) **Output sensor uncertainty detection scheme**:* Since the system only has one output signal, we only detect whether the output sensor has bias-uncertainty. Based on the output sensor model (10), we have the following design.

Total uncertainty estimation model. We introduce an estimation model system to estimate the parameters in (10):

$$\hat{z}_y = \hat{\theta}_u^T \phi_u + \hat{\theta}_z^T \phi_z + \hat{\theta}_{bf}^T f_y + \sum_{i=1}^{p} \hat{\theta}_i^T \phi_i, \quad (41)$$

where $\hat{\theta}_u(t)$, $\hat{\theta}_z(t)$, $\hat{\theta}_{bf}(t)$, and $\hat{\theta}_i(t)$, $i = 1, \ldots, p$ are the estimates of corresponding unknown parameters. Then, the estimation error $e_y(t) = \hat{z}_y(t) - z_y(t)$ is obtained as

$$e_y = \tilde{\theta}_u^T \phi_u + \tilde{\theta}_z^T \phi_z + \tilde{\theta}_{bf}^T f_y + \sum_{i=1}^{p} \tilde{\theta}_i^T \phi_i, \quad (42)$$

where $\tilde{\theta}_u(t)$, $\tilde{\theta}_z(t)$, $\tilde{\theta}_{bf}(t)$, and $\tilde{\theta}_i(t)$ for $i = 1, 2, \ldots, p$, are the parameter errors. We choose the adaptive laws as

$$\dot{\hat{\theta}}_u = -\frac{\Gamma_u \phi_u e_y}{m^2}, \quad \dot{\hat{\theta}}_z = -\frac{\Gamma_z \phi_z e_y}{m^2}, \quad (43)$$

$$\dot{\hat{\theta}}_{bf} = -\frac{\Gamma_{bf} \phi_{bf} e_y}{m^2(t)}, \quad \dot{\hat{\theta}}_i = -\frac{\Gamma_{fi} \phi_i e_y}{m^2}, \quad (44)$$

where Γ_u, Γ_z, Γ_{bf}, and $\Gamma_{fi}, i = 1, 2, \ldots, p$, are positive definite and symmetric gain matrices, and

$$m(t) = (1 + \phi_u^T \phi_u + \phi_z^T \phi_z + \phi_{bf}^T \phi_{bf} + \sum_{i=1}^{p} \phi_i^T \phi_i)^{1/2}.$$

Proposition 2: *Given that the signals $z_y(t)$, $u(t)$, $f_y(t)$, and $\dot{f}_y(t)$ are bounded, the proposed estimation model system (41) with the adaptive laws (43)–(44) ensures that* $\lim_{t\to\infty} e_y(t) = \lim_{t\to\infty}(\hat{z}_y(t) - z_y(t)) = 0$, *when $z_y(t)$ is of the total sensor uncertainty pattern (10).*

Bias-uncertainty free sensor estimation model. The above estimation model (41) is based on the total sensor uncertainty pattern (10). To detect the no bias-uncertainty case, i.e. $z_y(t) = k_y y(t)$, we build the estimation model as

$$\hat{z}_y(t) = \hat{\theta}_u^T(t) \phi_u(t) + \hat{\theta}_z^T(t) \phi_z(t), \quad (45)$$

where $\hat{\theta}_u(t)$ and $\hat{\theta}_z(t)$ are updated from the adaptive laws in (43). This estimation model has similar properties to that in Proposition 2, which is $\lim_{t\to\infty}(\hat{z}_y(t) - z_y(t)) = 0$, for the case when $\theta_{by}^T f_y(t) = 0$ in the sensor model (8). When there is bias-uncertainty, the tracking property may not hold, that is $\lim_{t\to\infty}(\hat{z}_y(t) - z_y(t)) \neq 0$. Thus, the detection criterion is that if $\hat{z}_y(t)$ from the bias-uncertainty free estimation model (45) cannot track the output sensor signal $z_y(t)$, the output sensor has bias-uncertainty.

IV. Simulation Study

In this simulation study, we consider a linearized aircraft longitudinal dynamic model with sensor uncertainties and system parameter uncertainties.

Longitudinal aircraft model. The linearized aircraft longitudinal model can be described as (1) with state and input variables: $x = [u_b, w_b, q_b, \theta]^T$ and $u = d_e$, where u_b and w_b are the x-axis and z-axis velocity components of the body-axis frame whose units are ft/sec, q_b is y-axis angular velocity component of the body-axis frame whose unit is rad/sec, θ is the Euler pitch angle whose unit is radian, and d_e is the elevator angular position whose unit is degree. In this study, we choose the pitch angle $\theta(t)$ as the output signal $y(t)$.

Sensor uncertainty. The sensors in the simulation are

$$\begin{aligned} z_{x1}(t) = x_1(t), \quad z_{x2}(t) = k_{x2}x_2(t) + b_{x1}\sin(t), \\ z_{x3}(t) = x_3(t), \quad z_{x4}(t) = k_{x4}x_4(t) + b_{x4}\sin(2t), \end{aligned} \quad (46)$$

where only the 2nd and the 4th state sensors have bias-uncertainties. Since the state signal $x_4 = \theta$ is the output signal, the sensor z_{x4} is also used as an output sensor.

Adaptive feedback control. The reference model is chosen as $y_m(t) = \frac{1}{(s+1)^2}[r](t)$. We apply the adaptive controller (13) to the longitudinal aircraft model (1) with sensor uncertainties (46) to ensure the closed-loop signal boundedness. Then, we can use the sensor estimation models to detect and identify the sensor uncertainties.

Simulation results of uncertainty detection. Since the state sensor for pitch angle θ is also used as the output sensor, in this simulation study, we only consider the state sensor uncertainty detection problem. To detect the sensor uncertainties and identify that only the 2nd and the 4th sensors have bias-uncertainties, we observe the following four sensor estimation models:

(i) bias-uncertainty free sensor estimation model (33);
(ii) uncertainty-specific sensor estimation model (37) for the case when $\Theta_{bx}^T f_x(t) = [0, \theta_{bx2}^T f_{x2}(t), 0, \theta_{bx4}^T f_{x4}(t)]^T$;
(iii) uncertainty-specific sensor estimation model (37) for the case when $\Theta_{bx}^T f_x(t) = [0, \theta_{bx2}^T f_{x2}(t), 0, 0]^T$;
(iv) uncertainty-specific sensor estimation model (37) for the case when $\Theta_{bx}^T f_x(t) = [0, 0, 0, \theta_{bx4}^T f_{x4}(t)]^T$.

Fig. 1 shows the residual norms $||e_x(t)||$ obtained from the four estimation models. From Fig. 1, we can see that the residual from the model (i) (bias-uncertainty free model) does not converge to 0, which means that there have sensor uncertainties. Since the residual from the model (ii) converges to 0, we can conclude that the uncertainties are in the 2nd or the 4th sensors. To further identify the pattern, we need to check the residuals from (iii) and (iv). If one of the residuals converges to 0, we can have that only one of the sensors has uncertainty. But, from Fig. 1, both residuals do not converge to 0. Thus, it can be concluded that both sensors have uncertainty.

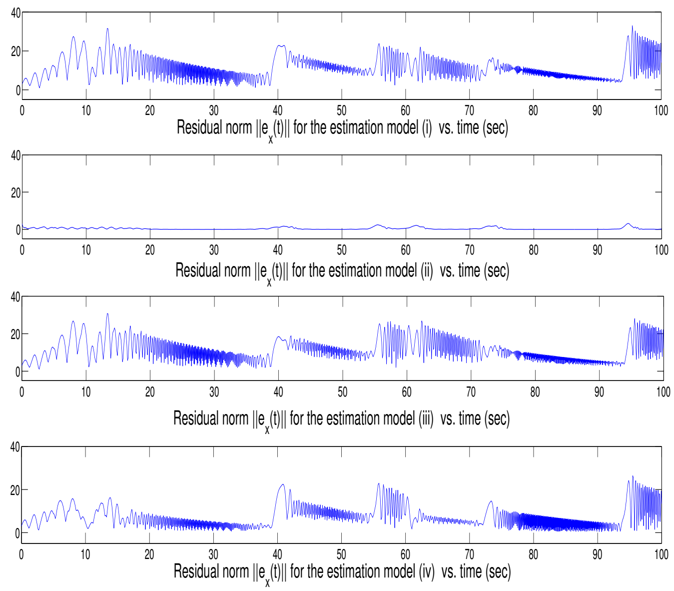

Fig. 1. Residuals $||e_x(t)||$ for the estimation models (i)–(iv)

V. CONCLUSIONS

This paper addressed the design, analysis and evaluation of the adaptive feedback-based sensor uncertainty detection scheme. The sensor dynamics has been derived based on the parametric sensor uncertainty model, and a set of sensor estimation model systems have been constructed to estimate the uncertain sensor signals. To ensure the signal boundedness requirement of the estimation model, the adaptive feedback control design has been applied to the system with the sensor uncertainty. By comparing the estimation model signals with the sensor signals, we can determine whether there are sensor uncertainties or not, and the uncertainty patterns can also be identified. The simulation study of the linearized longitudinal aircraft system showed the effectiveness of the proposed feedback-based sensor uncertainty design.

REFERENCES

[1] Chen, W. and M. Saif, "Adaptive sensor fault detection and isolation in uncertain systems," *Proc. of the ACC*, 2007, pp. 3240–3245.
[2] Demetriou, M., "Robust adaptive techniques for sensor fault detection and diagnosis," *Proc. of the 37th IEEE Conf. on Decision and Control*, 1998, pp.1143–1148.
[3] Isermann, R. and P. Ballé, "Trends in the application of model-based fault detection and diagnosis of technical processes," *Control Engineering Practice*, Vol. 5, No. 5, pp. 709-719, 1997.
[4] Jiang, B., M. Staroswiecki, and V. Cocquempot, "Fault diagnosis based on adaptive observer for a class of nonlinear systems with unknonw parameters," *International Journal of Control*, Vol. 77, No. 4, pp. 415–426, 2004.
[5] Richards, N., T. Summers, J. Monaco, J. Burkholder, and D. Ward, "Integrated FDI and Outer-loop control for inflight failure accommodation and upset recovery," *Proc. of the AIAA GNC Conference*, Paper No. AIAA-2005-5935, August 2005.
[6] Summers, T., J. Burkholder, J. Wadley, and D. Hopper, "Integrated FDI enhancements for inflight failure accommodation and upset recovery," *Proc. of the AIAA GNC Conference*, Paper No. AIAA-2006-6550, August 2006.
[7] Tao, G., *Adaptive Control Design and Analysis*, John Wiley and Sons, New York, 2003.
[8] Tao, G. and J. O. Burkholder, "Adaptive detection of sensor uncertainties and failures," *Proceedings of 2009 AIAA Guidance, Navigation and Control Conference*, Paper AIAA-2009-5889, Chicago, IL, August 2009.
[9] Wang, H., Z.J. Huang, and S. Daley, "On the use of adaptive updating rules for actuator and sensor fault diagnosis," *Automatica,* Vol. 33, No. 2, pp. 217–225, 1997.
[10] Xiong, Y. and M. Saif, "Robust fault detection and isolation via diagnostic observer," *International Journal of Robust and Nonlinear Control*, Vol. 10, No. 14, pp. 1175–1192, 2000.
[11] Zhang, X., T. Parisini, and M. M. Polycarpou, "Sensor bias fault isolation in a class of nonlinear systems," *IEEE Trans. on Automatic Control*, Vol. 50, No. 3, pp. 370–376, 2005.