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# A delay-fractioning approach to stability analysis of networked control systems with time-varying delay

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*Abstract*— **This paper concerns the establishment of a new stability criterion for networked control systems (NCSs) liable to model uncertainties and time-varying delays. The proposed criterion is an improvement over previous ones, for the employment of a novel delay-fractioning approach and the development of a new Lyapunov–Krasovskii functional (LKF). The analysis incorporates state-of-the-art stability techniques for systems with time-varying delays such as convex optimization technique and piecewise analysis method. Moreover, we consider the derivative character of the NCS' time-varying delay. The analysis is enriched with numerical examples that illustrates the effectiveness of the proposed criterion which outperform state-of-the-art stability criteria in the literature for nominal and uncertain NCSs.**

## I. Introduction

NETWORKED Control Systems (NCSs) refer to a class of control systems whose elements are linked together through a multipurpose shared communication network and the information is exchanged in the form of data packets [1]–[3]. NCSs have many advantages compared to the traditional local control architecture, including lower costs, simple installation and maintenance, and reduced weight [3]. Murray et al [4] identify control over networks as one of the key future directions for control. Nonetheless, the insertion of a multipurpose shared communication network in the control loop unavoidably introduces packet dropouts and different forms of time-delay uncertainty between the elements [2]. Since these delays can degrade the system's performance and even cause instability, there have been a strong research interest in NCS' stability analysis within the control community (see, e.g., [1]–[3], and references therein).

During the last decade, the problem of stability analysis for systems with time-varying delays have been deeply investigated under delay-dependent criteria with different Lyapunov–Krasovskii functionals (LKFs) [5]. Particularly, the employment of Jensen's inequality instead of the cross-terms bounding [6] is a well-established approach that leads to less conservative results. However, this still is a conservative analysis, for the time-varying delay is bounded when considering terms containing not only the delay bounds, but also the delay itself. Instead of bounding the time-varying delay, the convex optimization technique incorporated with the Jensen's inequality proved to be effective in [7]. Further improvements were obtained using similar techniques with different LKFs (see, e.g., [5], [8]–[12]). Recently, new Lyapunov functional candidates inspired on [13] have enriched the stability analysis by extending the delay-fractioning approach from [13] to systems with time-varying delays, see, e.g., [9]–[12].

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Nevertheless, in practice, it is very difficult to obtain an exact mathematical model due to environmental noise or slowly varying parameters. Therefore, the NCSs almost inevitably present some uncertainties [2].

In this context, the present paper proposes a new delay-dependent robust stability criterion for NCSs with model uncertainties and time-varying delays. The method incorporates state-of-the-art techniques for the analysis of systems with time-varying delays, such as Jensen's inequality, Finsler's lemma, and convex optimization technique. The analysis is further enriched with the introduction of new Lyapunov–Krasovskii terms and the employment of the piecewise analysis method. Moreover, a new delay-fractioning approach is proposed to exploit all possible information about the delay's lower bound which leads to further improvements and to overcome the drawbacks of the piecewise analysis method. These methods considerably improve the stability results even for systems with no uncertainties. Numerical examples illustrate the effectiveness of the proposed robust stability criteria which outperform state-of-the-art criteria in the literature for NCSs with and without uncertainties.

## II. Preliminaries

We shall consider an NCS consisting of an LTI plant and a controller module connected through a shared network. All the network communication is performed by the *Sender* and the *Receiver* elements, which are responsible for transmitting and acquiring data packets through the network, respectively.

The modules can either be time-driven or event-driven. Throughout this paper, we assume that the sensor module is *clock-driven* with transmission period $h$. The *Controller* and *Actuator modules* are *event-driven* and start to process a new packet immediately after its arrival. Single packet transmission is assumed, i.e., all data sent or received over the network is assembled together into one network packet and transmitted at the same time. These are standard assumptions for modeling NCSs within the delayed systems framework, which usually yields higher values for the maximum delay's upper bound (see [1]–[3], [14]). Nonetheless, it's important to mention that other approaches, e.g., impulsive systems and sampled-data based ones (see [15], [16]), may be more suitable for asynchronous sampling strategies, specially, when the transfer interval is of the same (or higher) order of the delay,

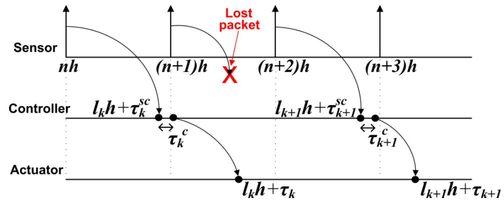


Fig. 1. Time diagram for network-induced delays.

Moreover, the following delays are considered:

- $\tau_k^{sc}$: delay from sensor to controller for the $k$-th packet;
- $\tau_k^{c}$: computation delay for the $k$-th network packet;
- $\tau_k^{ca}$: delay from controller to actuator for the $k$-th packet;
- $\tau_k$: total delay (sensor to actuator) for the $k$-th packet.

Following the data flow's time diagram shown in Figure 1, the sensor module samples data from the plant at instants $nh$, where $h$ is the sampling period and $n \in \mathbb{N}^*$. The integers $\ell_k$, $k \in \mathbb{N}^*$, denote the $n$th sample number which is carried by the $k$th received network packet at the actuator's input.

**Remark 1** If $\{\ell_1,\ell_2,\ldots,\ell_n,\ldots\}=\{1,2,\ldots,n,\ldots\}$, then no packet dropout or disordering occurred in the transmission. However, if the $p$-th sample was lost, then $\nexists q$, $q \in \mathbb{N}^*$, such that $\ell_q = p$. Packet disordering occurs when one packet reaches its destination later than its successors, i.e., $\exists p,q \in \mathbb{N}^*$, $p > q$, such that $\ell_q > \ell_p$. In this case, the old packet, $\ell_p$, is dropped and its data discarded.

We assume the existence of constants $\tau_{min}$ and $\tau_{max}$,

$$(\ell_{k+1}-\ell_k)h + \tau_{k+1} \le \tau_{max},$$
$$\tau_{min} \le \tau_k, \qquad \forall k \in \mathbb{N}^*.$$

where $\tau_{min}$ and $\tau_{max}$ denote, respectively, the lower and the upper bounds of the total network-induced delay, involving both transmission delays and packet dropouts.

The LTI plant has a state space model of the form

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$
$$y(t) = Cx(t), \tag{2}$$

where $x(t) \in \mathbb{R}^{r_x}$ is the plant's state vector, $u(t) \in \mathbb{R}^{r_u}$ and $y(t) \in \mathbb{R}^{r_y}$ are the plant's input and output vectors, respectively. The matrices $A$, $B$ and $C$ are considered not exactly known, but belonging to bounded sets: $A \in \mathscr{A} \subset \mathbb{R}^{r_x \times r_x}$, $B \in \mathscr{B} \subset \mathbb{R}^{r_x \times r_u}$ and $C \in \mathscr{C} \subset \mathbb{R}^{r_y \times r_x}$.

Considering the total communication delay from sensor to actuator, including the computation delay, $\tau_k = \tau_k^{sc} + \tau_k^{c} + \tau_k^{ca}$, and considering a non-fragile state-feedback control law with a gain matrix $K$ not exactly known, but belonging to a bounded set $\mathscr{K} \subset \mathbb{R}^{r_u \times r_y}$, the resulting control law can be described as

$$u(t) = y_c(\ell_k h + \tau_k^{sc} + \tau_k^{c}) = KCx(\ell_k h), \quad t \in [\ell_k h + \tau_k, \ell_{k+1} h + \tau_{k+1}), \quad \forall\, k \in \mathbb{N}^*, \tag{3}$$

where $y_c$ is the controller's output.

From a straightforward combination of (1)-(3), the closed-loop system can be described as

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d(t)), & t > 0; \\ x(t) = \rho(t), & t \in [-\tau_{max},\ 0], \end{cases} \tag{4}$$

$t \in [\ell_k h + \tau_k, \ell_{k+1} h + \tau_{k+1})$, $\forall\, k \in \mathbb{N}^*$, where $\rho(t)$ is a given function which describes the state's initial condition, $A_d = BKC$; and the function $d(t) = t - \ell_k h$ denotes the time-varying delay that satisfies

$$\tau_{min} \le d(t) \le \tau_{max}, \tag{5}$$

where $0 \le \tau_{min} \le \tau_{max}$ are constants. Moreover, it's noteworthy that function $d(t)$ is piecewise linear with derivative $\dot{d}(t) = 1$ for $t \ne \ell_k h + \tau_k$. Therefore, the time-varying delay $d(t)$ is discontinuous at the interrupt points $t = \ell_k h + \tau_k$ $\forall\, k \in \mathbb{N}^*$.

Finally, taking the parameter uncertainties into consideration, the closed-loop NCS (4) can be rewritten as:

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t-d(t)), \quad t \in [t_k, t_{k+1}), \quad \forall k \in \mathbb{N}^* \tag{6}$$

where $t_k = \ell_k h + \tau_k$.

The parameter uncertainties $\Delta A$ and $\Delta A_d$ are time-varying matrices with appropriate dimensions, which are defined as follows:

$$\begin{bmatrix} \Delta A & \Delta A_d \end{bmatrix} = H\Delta(t) \begin{bmatrix} \Xi_A & \Xi_{Ad} \end{bmatrix} \tag{7}$$

where $H$, $\Xi_A$ and $\Xi_{Ad}$ are known constant matrices with appropriate dimensions and $\Delta(t)$ represents an unknown time-varying matrix, which is Lebesque measurable in $t$ and satisfies $\Delta(t)^T \Delta(t) \le I$.

Throughout this paper, the following results will be useful to derive conditions for the establishment of a new delay-depedent stability criterion for system (6).

**Lemma 1 ([17])** *For given scalars $r_1$, $r_2$ and matrix $M \in \mathbb{R}^{m \times m}$ such that $(r_2 - r_1) \ge 0$ and $M > 0$, and any vectorial function $x : [r_1, r_2] \longrightarrow \mathbb{R}^m$, we have:*

$$(r_2 - r_1)\int_{r_1}^{r_2} x^T(s) M x(s) ds \ge \left(\int_{r_1}^{r_2} x(s) ds\right)^T M \left(\int_{r_1}^{r_2} x(s) ds\right).$$

**Lemma 2 ([18])** *Given matrices $M = M^T \in \mathbb{R}^{m \times m}$, $B \in \mathbb{R}^{r \times m}$, the following statement*

$$x^T M x > 0 \ \Leftrightarrow M + FB + B^T F^T > 0,$$

*holds for some $F \in \mathbb{R}^{m \times r}$ and any $x \in \mathbb{R}^m \backslash \{0\}$ such that $Bx = 0$.*

## III. Stability Analysis

This section presents the main results of this paper. To establish a new delay-dependent stability criterion for NCSs, we first consider the delay range $[\tau_{min}, \tau_{max}]$. Similarly to [10], [11], we divide this interval into two equally spaced subintervals: $[\tau_1, \tau_2]$ and $[\tau_2, \tau_3]$, where $\tau_1 = \tau_{min}$, $\tau_3 = \tau_{max}$, and $\tau_2 = \dfrac{\tau_{max} + \tau_{min}}{2}$. Therefore, the linear delayed system (6) can be written as

$$\begin{cases} \dot{x}(t) = Ax(t) + \chi_{[\tau_1,\tau_2]}(d(t)) A_d x(t-d(t)) \\ \qquad + \left(1 - \chi_{[\tau_1,\tau_2]}(d(t))\right) A_d x(t-d(t)) & t > 0 \\ x(t) = \rho(t), \qquad t \in [-\tau_{max},\ 0] \end{cases} \tag{8}$$

where $\chi_{[\tau_1,\tau_2]} : \mathbb{R} \to \{0,1\}$ is the characteristic function of $[\tau_1,\tau_2]$, i.e., $\chi_{[\tau_1,\tau_2]}(d(t)) = 1$ if $d(t) \in [\tau_1,\tau_2]$ and $\chi_{[\tau_1,\tau_2]}(d(t)) = 0$, otherwise.

This analysis, known as piecewise analysis method, concerns the establishment of different LMIs conditions for each subinterval, reducing considerably the conservativeness which arises from the LMI analysis of the interval $[\tau_{min}, \tau_{max}]$. The contributions of this analysis turn out more significant as the delay interval grows. On the other hand, when the interval $[\tau_{min}, \tau_{max}]$ is reduced, i.e., when $\tau_{min} \to \tau_{max}$, the benefits that arise from the piecewise analysis are substantially shortened. In this context, we propose a new delay-fractioning approach which overcomes the piecewise analysis drawback for large values of $\tau_{min}$. The focus of this new strategy is to further exploit the information of the delay lower bound through the partitioning of the delay interval $[0, \tau_{min}]$ into $\eta > 0$ equally spaced subintervals. The straightforward consequence of this analysis is the introduction of new $\tau_{min}$-dependent auxiliary delayed states:

$$x\left(t - i\frac{\tau_1}{\eta}\right), \qquad \forall i \in \{0, \ldots, \eta\}. \tag{9}$$

The proposed robust stability analysis, with $\eta$ subintervals, is based on the Lyapunov–Krasovskii functional candidate

$$V(t) = \sum_{i=1}^{6} V_i(t), \tag{10}$$

where

$$V_1(t) = x^T(t)Px(t),$$

$$V_2(t) = \int_{t-\frac{1}{2}d(t)}^{t} x^T(s)Q_1x(s)ds,$$

$$V_3(t) = \int_{t-\tau_2}^{t-\tau_1} \begin{bmatrix} x(s) \\ x(s-\tau_2+\tau_1) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-\tau_2+\tau_1) \end{bmatrix} ds,$$

$$V_4(t) = \int_{t-\frac{1}{\eta}\tau_1}^{t} \begin{bmatrix} x(s-\frac{0}{\eta}\tau_1) \\ \vdots \\ x(s-\frac{\eta-1}{\eta}\tau_1) \end{bmatrix}^T \begin{bmatrix} M_{11} & \ldots & M_{1\eta} \\ \vdots & \ddots & \vdots \\ * & \ldots & M_{\eta\eta} \end{bmatrix} \begin{bmatrix} x(s-\frac{0}{\eta}\tau_1) \\ \vdots \\ x(s-\frac{\eta-1}{\eta}\tau_1) \end{bmatrix} ds,$$

$$V_5(t) = \sum_{k=1}^{\eta} \left(\frac{\tau_1}{\eta}\right) \int_{-\frac{k}{\eta}\tau_1}^{-\frac{k-1}{\eta}\tau_1} \int_{t+\beta}^{t} \dot{x}^T(s)S_k\dot{x}(s)dsd\beta,$$

$$V_6(t) = (\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\beta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\beta + (\tau_3-\tau_2)\int_{-\tau_3}^{-\tau_2}\int_{t+\beta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\beta.$$

One can note that if the conditions

$$P>0,\ Q_1\geq 0,\ \ Z_1>0,\ Z_2>0,\ \ S_j>0,\ j=\{1,\ldots,\eta\},$$

$$N = \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix} \geq 0, \text{ e } \ M = \begin{bmatrix} M_{11} & \ldots & M_{1\eta} \\ \vdots & \ddots & \vdots \\ * & \ldots & M_{\eta\eta} \end{bmatrix} \geq 0, \tag{11}$$

are satisfied, than the positiveness of (10) is assured.

In the following, we propose a novel delay-dependent robust stability criterion for uncertain NCSs (6).

**Theorem 1** *For given scalars $\tau_{min}$, $\tau_{max}$, and $\eta$, such that $0 \leq \tau_{min} \leq \tau_{max}$ and $\eta > 1$, the uncertain NCS (6) with time-varying delay satisfying (5) and parameter uncertainties described in (7) is robust asymptotically stable if there exist scalars $\varepsilon_{1i}>0$ and $\varepsilon_{2i}>0$, $i=\{1,2\}$, and matrices $P$, $Q_1$, $S_j$, $j=\{1,\ldots,\eta\}$, $Z_1$, $Z_2$, $N$ e $M$ with appropriate dimensions, satisfying (11) and free-weighting matrices $F_1 \in \mathbb{R}^{7r_x \times 3r_x}$ and $F_2 \in \mathbb{R}^{7r_x \times 3r_x}$ such that the following LMIs hold:*

$$\Omega_{11} < 0; \qquad \Omega_{12} < 0; \qquad \Omega_{21} < 0; \qquad \Omega_{22} < 0; \tag{12}$$

*where*

$$\Omega_{1m} = \begin{bmatrix} \left(\Psi^{(1)}+F_1B_1+(F_1B_1)^T\right) & (\tau_2-\tau_1)F_1\Gamma_m & \Theta(\eta) & F_1\Gamma_3H & \varepsilon_{1m}\Gamma_\Xi^T \\ * & -(\tau_2-\tau_1)^2Z_1 & 0 & 0 & 0 \\ * & * & \Phi(\eta) & 0 & 0 \\ * & * & * & -\varepsilon_{1m}I & 0 \\ * & * & * & * & -\varepsilon_{1m}I \end{bmatrix},$$

$$\Omega_{2m} = \begin{bmatrix} \left(\Psi^{(2)}+F_2B_2+(F_2B_2)^T\right) & (\tau_3-\tau_2)F_2\Gamma_m & \Theta(\eta) & F_2\Gamma_3H & \varepsilon_{2m}\Gamma_\Xi^T \\ * & -(\tau_3-\tau_2)^2Z_2 & 0 & 0 & 0 \\ * & * & \Phi(\eta) & 0 & 0 \\ * & * & * & -\varepsilon_{2m}I & 0 \\ * & * & * & * & -\varepsilon_{2m}I \end{bmatrix},$$

*for $m \in \{1,2\}$, and*

$$\Gamma_1 = \begin{bmatrix} 0 & I & 0 \end{bmatrix}^T, \qquad \Gamma_2 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}^T, \qquad \Gamma_3 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T,$$

$$\Gamma_\Xi = \begin{bmatrix} \Xi_A & \Xi_{Ad} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & I & 0 & 0 \\ A & A_d & -I & 0 & 0 & 0 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & I & 0 & 0 & -I & 0 & 0 \\ 0 & -I & 0 & 0 & 0 & I & 0 \\ A & A_d & -I & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Theta(\eta) = \begin{bmatrix} (M_{12}+S_1) & M_{13} & \ldots & M_{1(\eta-1)} & M_{1\eta} \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \\ -M_{1\eta}^T & -M_{2\eta}^T & \ldots & -M_{(\eta-2)\eta}^T & S_\eta - M_{(\eta-1)\eta}^T \\ 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix},$$

$$\Phi(\eta) = \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1(\eta-2)} & \phi_{1(\eta-1)} \\ * & \phi_{22} & \cdots & \phi_{2(\eta-2)} & \phi_{2(\eta-1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & \phi_{(\eta-2)(\eta-2)} & \phi_{(\eta-2)(\eta-1)} \\ * & * & \cdots & * & \phi_{(\eta-1)(\eta-1)} \end{bmatrix},$$

*with*

$$\phi_{ij} = \begin{cases} M_{(i+1)(i+1)} - M_{ii} - \left(S_i + S_{(i+1)}\right), & \text{if } i=j, \\ M_{(i+1)(j+1)} - M_{ij} + S_j, & \text{if } |i-j|=1, \\ M_{(i+1)(j+1)} - M_{ij}, & \text{otherwise,} \end{cases}$$

*where $i,j \in \{1,\ldots,(\eta-1)\}$, and*

$$\Psi^{(1)} = \begin{bmatrix} \Psi_{11} & 0 & P & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & N_{12} & 0 & 0 \\ * & * & * & * & N_{22}-N_{11}-Z_2 & -N_{12}+Z_2 & 0 \\ * & * & * & * & * & -N_{22}-Z_2 & 0 \\ * & * & * & * & * & * & -\frac{1}{2}Q_1 \end{bmatrix},$$

$$\Psi^{(2)} = \begin{bmatrix} \Psi_{11} & 0 & P & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44}-Z_1 & N_{12}+Z_1 & 0 & 0 \\ * & * & * & * & N_{22}-N_{11}-Z_1 & -N_{12} & 0 \\ * & * & * & * & * & -N_{22} & 0 \\ * & * & * & * & * & * & -\frac{1}{2}Q_1 \end{bmatrix},$$

*with*

$$\Psi_{11} = +M_{11} - S_1 + Q_1,$$

$$\Psi_{33} = \sum_{k=1}^{\eta}\left(\frac{\tau_1}{\eta}\right)^2 S_k + (\tau_2-\tau_1)^2 Z_1 + (\tau_3-\tau_2)^2 Z_2,$$

$$\Psi_{44} = N_{11} - M_{\eta\eta} - S_{\eta}. \quad (13)$$

*Proof:* Firstly, we shall consider the first subinterval, where $d(t) < \tau_2$. Taking the time derivative of the Lyapunov functional (10) with respect to $t \in [t_k, t_{k+1}), \forall k \in \mathbb{N}^*$ along the trajectory of (6) yields

$$\dot{V}_1(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t),$$

$$\dot{V}_2(t) = x^T(t)Q_1x(t) - (1/2)\, x^T\left(t - \frac{d(t)}{2}\right)Q_1 x\left(t - \frac{d(t)}{2}\right),$$

$$\dot{V}_3(t) = \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix} \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} - \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_3) \end{bmatrix}^T \begin{bmatrix} N_{11} & N_{12} \\ N_{12}^T & N_{22} \end{bmatrix} \begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_3) \end{bmatrix},$$

$$\dot{V}_4(t) = \begin{bmatrix} x(t - \frac{0}{\eta}\tau_1) \\ \vdots \\ x(t - \frac{\eta-1}{\eta}\tau_1) \end{bmatrix}^T \begin{bmatrix} M_{11} & \dots & M_{1\eta} \\ \vdots & \ddots & \vdots \\ * & \dots & M_{\eta\eta} \end{bmatrix} \begin{bmatrix} x(t - \frac{0}{\eta}\tau_1) \\ \vdots \\ x(t - \frac{\eta-1}{\eta}\tau_1) \end{bmatrix}$$
$$- \begin{bmatrix} x(t - \frac{1}{\eta}\tau_1) \\ \vdots \\ x(t - \frac{\eta}{\eta}\tau_1) \end{bmatrix}^T \begin{bmatrix} M_{11} & \dots & M_{1\eta} \\ \vdots & \ddots & \vdots \\ * & \dots & M_{\eta\eta} \end{bmatrix} \begin{bmatrix} x(t - \frac{1}{\eta}\tau_1) \\ \vdots \\ x(t - \frac{\eta}{\eta}\tau_1) \end{bmatrix},$$

$$\dot{V}_5(t) = \sum_{k=1}^{\eta} \left(\frac{\tau_1}{\eta}\right) \left( \frac{\tau_1}{\eta} \dot{x}^T(t) S_k \dot{x}(t) - \int_{t-\frac{k}{\eta}\tau_1}^{t-\frac{k-1}{\eta}\tau_1} \dot{x}^T(s) S_k \dot{x}(s) ds \right),$$

$$\dot{V}_6(t) = \dot{x}^T(t)\left[(\tau_2-\tau_1)^2 Z_1 + (\tau_3-\tau_2)^2 Z_2\right]\dot{x}(t)$$
$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) Z_1 \dot{x}(s) ds - (\tau_3-\tau_2)\int_{t-\tau_3}^{t-\tau_2} \dot{x}^T(s) Z_2 \dot{x}(s) ds, \quad (14)$$

We expand the integral terms in $\dot{V}(t)$ considering $\chi = 1$ and taking the fact that $\int_{t-\tau_2}^{t-\tau_1} f(s)ds = \int_{t-d(t)}^{t-\tau_1} f(s)ds + \int_{t-\tau_2}^{t-d(t)} f(s)ds$. Therefore, from Jensen's inequality (Lemma 1) we have

$$\dot{V}_5(t) \le \sum_{k=1}^{\eta} \left( \left(\frac{\tau_1}{\eta}\right)^2 \dot{x}^T(t) S_k \dot{x}(t) - \left[x\left(t - \frac{k-1}{\eta}\tau_1\right) - x\left(t - \frac{k}{\eta}\tau_1\right)\right]^T S_k \right.$$
$$\left. \times \left[x\left(t - \frac{k-1}{\eta}\tau_1\right) - x\left(t - \frac{k}{\eta}\tau_1\right)\right] \right), \quad (15)$$

$$\dot{V}_6(t)|_{d(t)<\tau_2} \le \dot{x}^T(t)\left((\tau_2-\tau_1)^2 Z_1 + (\tau_3-\tau_2)^2 Z_2\right)\dot{x}(t)$$
$$-\gamma_{1d}^T(\tau_2-\tau_1)(d(t)-\tau_1)Z_1\gamma_{1d} - \gamma_{d2}^T(\tau_2-\tau_1)(\tau_2-d(t))Z_1\gamma_{d2}$$
$$-[x(t-\tau_2)-x(t-\tau_3)]^T Z_2 [x(t-\tau_2)-x(t-\tau_3)]. \quad (16)$$

where $\gamma_{1d}$, $\gamma_{d2}$ are defined by

$$\gamma_{1d} := \frac{1}{d(t)-\tau_1}\int_{t-d(t)}^{t-\tau_1} \dot{x}(s)ds \quad \text{and} \quad \gamma_{d2} := \frac{1}{\tau_2-d(t)}\int_{t-\tau_2}^{t-d(t)} \dot{x}(s)ds,$$

with $\lim_{d(t)\to\tau_1}\gamma_{1d} = \dot{x}(t-\tau_1)$, and $\lim_{d(t)\to\tau_2}\gamma_{d2} = \dot{x}(t-\tau_2)$.

Suppose now we denote

$$\zeta_1^T(t) := \begin{bmatrix} \zeta_x^T & \gamma_{1d}^T & \gamma_{d2}^T & \zeta_\delta^T \end{bmatrix} \in \mathbb{R}^{(8+\eta)r},$$

with

$$\zeta_x^T := \left[x^T(t) \;\; x^T(t-d(t)) \;\; \dot{x}^T(t) \;\; x^T(t-\tau_1) \;\; x^T(t-\tau_2) \;\; x^T(t-\tau_3) \;\; x^T\left(t - \frac{d(t)}{2}\right)\right],$$
$$\zeta_\delta^T := \left[x(t - \tfrac{1}{\eta}\tau_1) \quad \dots \quad x(t - \tfrac{\eta-1}{\eta}\tau_1)\right]. \quad (17)$$

Then a straightforward combination of (14)-(16) yields

$$\dot{V}(t)|_{d(t)<\tau_2} \le \zeta_1^T(t)\left(\Omega|_{d(t)<\tau_2}\right)\zeta_1(t), \quad (18)$$

$t \in [t_k, t_{k+1}), \forall k \in \mathbb{N}^*$, where

$$\Omega|_{d(t)<\tau_2} = \begin{bmatrix} \Psi^{(1)} & 0 & \Theta(\eta) \\ * & \Lambda^{(1)} & 0 \\ * & * & \Phi(\eta) \end{bmatrix} \in \mathbb{R}^{(8+\eta)r_x \times (8+\eta)r_x},$$

$$\Lambda^{(1)} = \begin{bmatrix} -(d(t)-\tau_1)(\tau_2-\tau_1)Z_1 & 0 \\ 0 & -(\tau_2-d(t))(\tau_2-\tau_1)Z_1 \end{bmatrix},$$

and $\Psi^{(1)}$, $\Theta(\eta)$, and $\Phi(\eta)$ are defined in (13).

Furthermore, we introduce $B_1 = \begin{bmatrix} B_{11} & B_{12}(d(t)) & 0 \end{bmatrix} \in \mathbb{R}^{3r \times (8+\eta)r_x}$ and $\widetilde{F}_1 = \begin{bmatrix} F_1^T & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{(8+\eta)r_x \times 3r_x}$, where $F_1$ is a $7r_x \times 3r_x$ free-weighting matrix, and

$$B_{11} = \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & -I & 0 & 0 & I & 0 & 0 \\ A+\Delta A & A_d+\Delta A_d & -I & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} (d(t)-\tau_1)I & 0 \\ 0 & (\tau_2-d(t))I \\ 0 & 0 \end{bmatrix}.$$

Taking the fact that $\zeta_1(t)$ belongs to the kernel of $B_1$, i.e., $B_1\zeta_1 = 0$ and applying Finsler's lemma (Lemma 2), it follows that the right side of (18) is negative definite if $\Omega_1 < 0$ holds, where

$$\Omega_1 = \Omega|_{d(t)<\tau_2} + \widetilde{F}_1 B_1 + B_1^T \widetilde{F}_1^T$$
$$= \begin{bmatrix} \Psi^{(1)} + F_1 B_{11} + B_{11}^T F_1^T & F_1 B_{12}(d(t)) & \Theta(\eta) \\ * & \Lambda^{(1)} & 0 \\ * & * & \Phi(\eta) \end{bmatrix}. \quad (19)$$

Consider now the terms that arise from $\Omega_1$ for $d(t) \to \tau_1$ and $d(t) \to \tau_2$, respectively. It is straightforward to conclude that

$$\zeta_1^T(t)\Omega_1\zeta_1(t) = \frac{\tau_2 - d(t)}{\tau_2 - \tau_1}\zeta_{11}^T(t)\Omega_1|_{d(t)\to\tau_1}\zeta_{11}(t)$$
$$+ \frac{d(t)-\tau_1}{\tau_2-\tau_1}\zeta_{12}^T(t)\Omega_1|_{d(t)\to\tau_2}\zeta_{12}(t), \quad t \in [t_k, t_{k+1}), \forall k \in \mathbb{N}^*$$

where $\zeta_{11}^T(t) := \begin{bmatrix} \zeta_x^T & \gamma_{d2}^T & \zeta_\delta^T \end{bmatrix}$, $\zeta_{12}^T(t) := \begin{bmatrix} \zeta_x^T & \gamma_{1d}^T & \zeta_\delta^T \end{bmatrix}$, $\zeta_x$ and $\zeta_\delta$ are defined in (17). Therefore, considering the convexity properties of $\Omega_1$, one can conclude that $\zeta_1^T(t)\Omega_1\zeta_1(t)$ is negative definite only if the vertices ($\Omega_1|_{d(t)\to\tau_1}$ and $\Omega_1|_{d(t)\to\tau_2}$) are.

Furthermore, to eliminate the time-varying matrix $\Delta(t)$ from $\Omega_1|_{d(t)\to\tau_1}$ and $\Omega_1|_{d(t)\to\tau_2}$, we use the definition of $\Delta A$ and $\Delta A_d$ from (7) and rewrite $B_{11}$ as

$$B_{11} = B_1 + \Gamma_3\begin{bmatrix} \Delta A & \Delta A_d & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = B_1 + \Gamma_3 H \Delta(t) \Gamma_\Xi, \quad (20)$$

where $B_1$, $\Gamma_3$, and $\Gamma_\Xi$ are defined in (13). Then, the matrices $\Omega_1|_{d(t)\to\tau_m}$, for $m = \{1,2\}$, may be rewritten as

$$\Omega_1|_{d(t)\to\tau_m} = \begin{bmatrix} \Psi^{(1)} + F_1 G_1 + G_1^T F_1^T & (\tau_2-\tau_1)F_1\Gamma_m & \Theta(\eta) \\ * & (\tau_2-\tau_1)^2 Z_1 & 0 \\ * & * & \Phi(\eta) \end{bmatrix} + \alpha_1^T\Delta(t)\beta + \beta^T\Delta^T(t)\alpha_1, \quad (21)$$

where $\alpha_1 = \begin{bmatrix} (F_1\Gamma_3 H)^T & 0 & 0 \end{bmatrix}^T$ and $\beta = \begin{bmatrix} \Gamma_\Xi & 0 & 0 \end{bmatrix}$.

Then it follows from applying Lemma 3 in [2] that $\Omega_1|_{d(t)\to\tau_m}$ holds for $m = \{1,2\}$ only if there exists scalars $\varepsilon_{11} > 0$ and $\varepsilon_{12} > 0$ such that

$$\begin{bmatrix} \Psi^{(1)} + F_1 G_1 + G_1^T F_1^T & (\tau_2-\tau_1)F_1\Gamma_m & \Theta(\eta) \\ * & (\tau_2-\tau_1)^2 Z_1 & 0 \\ * & * & \Phi(\eta) \end{bmatrix} + \frac{1}{\varepsilon_{1m}}\alpha_1\alpha_1^T + \varepsilon_{1m}\beta^T\beta < 0$$

holds for $m \in \{1,2\}$. Moreover, taking the Schur's complement, we have the matrices $\Omega_{11}$ and $\Omega_{12}$ as described in (13). Therefore, $\Omega_1$ is negative definite only if $\Omega_{11}$ and $\Omega_{12}$ are.

We shall now consider the case where $\tau_2 < d(t) \le \tau_3$. Using the same arguments of the former case, we derive analogous results. Taking the time derivative of the Lyapunov functional candidate previously obtained in (14) and considering $\chi = 0$, we apply the Lemma 1 to $\dot{V}_6(t)$ in (14), which yields

$$\dot{V}_6|_{d(t)>\tau_2} = \dot{x}^T(t)\left[(\tau_2-\tau_1)^2 Z_1 + (\tau_3-\tau_2)^2 Z_2\right]\dot{x}(t) - [x(t-\tau_1) - x(t-\tau_2)]^T$$
$$\times Z_1[x(t-\tau_1) - x(t-\tau_2)] - \xi_{2d}^T(t)\left[(\tau_3-\tau_2)(d(t)-\tau_2)Z_2\right]\xi_{2d}(t)$$
$$- \xi_{d3}^T(t)\left[(\tau_3-\tau_2)(\tau_3-d(t))Z_2\right]\xi_{d3}(t), \quad (22)$$

where $\gamma_{2d}$ and $\gamma_{d3}$ are defined by

$$\gamma_{2d}:=\frac{1}{d(t)-\tau_2}\int_{t-d(t)}^{t-\tau_2}\dot{x}(s)ds \quad \text{and} \quad \gamma_{d3}:=\frac{1}{\tau_3-d(t)}\int_{t-\tau_3}^{t-d(t)}\dot{x}(s)ds,$$

with $\lim_{d(t)\to\tau_2}\gamma_{2d}=\dot{x}(t-\tau_2)$ and $\lim_{d(t)\to\tau_3}\gamma_{d3}=\dot{x}(t-\tau_3)$. Analogously to the first case, we now denote

$$\zeta_2^T(t):=\begin{bmatrix}\zeta_x^T & \gamma_{2d}^T & \gamma_{d3}^T & \zeta_\delta^T\end{bmatrix}\in\mathbb{R}^{(8+\eta)r},$$

where $\zeta_x$ and $\zeta_\delta$ are defined in (17). Then, combining (14), (15), and (22) yields

$$\dot{V}(t)|_{d(t)>\tau_2} \leq \zeta_2^T(t)\left(\Omega|_{d(t)>\tau_2}\right)\zeta_2(t), \tag{23}$$

where

$$\Omega|_{d(t)>\tau_2}=\begin{bmatrix}\Psi^{(2)} & 0 & \Theta(\eta)\\ * & \Lambda^{(2)} & 0\\ * & * & \Phi(\eta)\end{bmatrix}\in\mathbb{R}^{(8+\eta)r_x\times(8+\eta)r_x},$$

$$\Lambda^{(2)}=\begin{bmatrix}-(d(t)-\tau_2)(\tau_3-\tau_2)Z_2 & 0\\ 0 & -(\tau_3-d(t))(\tau_3-\tau_2)Z_2\end{bmatrix},$$

and $\Psi^{(2)}$, $\Theta(\eta)$, and $\Phi(\eta)$ are defined in (13).

Suppose now we define $B_2=\begin{bmatrix}B_{21} & B_{22}(d(t)) & 0\end{bmatrix}\in\mathbb{R}^{3r\times(8+\eta)r}$ and $\widetilde{F}_2=\begin{bmatrix}F_2^T & 0 & 0\end{bmatrix}^T\in\mathbb{R}^{(8+\eta)r\times 3r}$, where $F_2\in\mathbb{R}^{7r\times 3r}$ is a free-weighting matrix, and

$$B_{21}=\begin{bmatrix}0 & I & 0 & 0 & -I & 0 & 0\\ 0 & -I & 0 & 0 & 0 & I & 0\\ A+\Delta A & A_d+\Delta A_d & -I & 0 & 0 & 0 & 0\end{bmatrix},\ B_{22}=\begin{bmatrix}(d(t)-\tau_2)I & 0\\ 0 & (\tau_3-d(t))I\\ 0 & 0\end{bmatrix}.$$

Therefore, using the fact that $B_2\zeta_2=0$ and applying Lemma 2 to the right side of (23), one conclude that $\zeta_2^T(t)\left(\Omega|_{d(t)>\tau_2}\right)\zeta_2(t)$ is negative definite only if $\Omega_2<0$ holds, where

$$\Omega_2=\begin{bmatrix}\Psi^{(2)}+F_2B_{21}+B_{21}^TF_2^T & F_2B_{22}(d(t)) & \Theta(\eta)\\ * & \Lambda^{(2)} & 0\\ * & * & \Phi(\eta)\end{bmatrix}.$$

Finally, we consider the terms that arise from $\Omega_2$ for $d(t)\to\tau_2$ and $d(t)\to\tau_3$, respectively. It is easy now to conclude that

$$\zeta_2^T(t)\Omega_2\zeta_2(t)=\frac{\tau_3-d(t)}{\tau_3-\tau_2}\zeta_{21}^T(t)\Omega_2|_{d(t)\to\tau_2}\zeta_{21}(t)+\frac{d(t)-\tau_2}{\tau_3-\tau_2}\zeta_{22}^T(t)\Omega_2|_{d(t)\to\tau_3}\zeta_{22}(t),\quad t\in[t_k,t_{k+1}),\forall k\in\mathbb{N}^*$$

where $\zeta_{21}^T(t):=\begin{bmatrix}\zeta_x^T & \gamma_{d3}^T & \zeta_\delta^T\end{bmatrix}$, $\zeta_{22}^T(t):=\begin{bmatrix}\zeta_x^T & \gamma_{2d}^T & \zeta_\delta^T\end{bmatrix}$, $\zeta_x$, and $\zeta_\delta$ are defined in (17). Then, from the convexity of $\zeta_2^T(t)\Omega_2\zeta_2(t)$, it is sufficient to verify the feasibility for $d(t)\to\tau_2$ and for $d(t)\to\tau_3$. Moreover, using exactly the same arguments presented in the investigation of the first subinterval, we can eliminate the time-varying matrix $\Delta(t)$ from the analysis. The analysis is straightforward and will be omitted for brevity. Thus, one can easily conclude that the analysis yields the same matrices $\Omega_{21}$ and $\Omega_{22}$, previously defined in (13).

We are now ready to complete the proof by establishing conditions that guarantee the negativeness of the Lyapunov functional's derivative. For the first case where $d(t)\neq\tau_2$, it is easy to check that, for $t\in[t_k,t_{k+1}),\forall k\in\mathbb{N}^*$,

$$\dot{V}(t)|_{d(t)\neq\tau_2}\leq\chi_{[\tau_1,\tau_2]}(d(t))\zeta_1^T(t)\Omega_1\zeta_1(t)+\left(1-\chi_{[\tau_1,\tau_2]}(d(t))\right)\zeta_2^T(t)\Omega_2\zeta_2(t).$$

For the second case where $d(t)=\tau_2$, due to the properties of the Lyapunov functional, one can conclude that

$$\dot{V}(t)|_{d(t)=\tau_2}\leq\max\left\{\zeta_1^T(t)\Omega_1\zeta_1(t),\ \zeta_2^T(t)\Omega_2\zeta_2(t)\right\},$$

for $t\in[t_k,t_{k+1}),\forall k\in\mathbb{N}^*$. Therefore, it is straightforward to conclude that if the conditions in (12) are fulfilled, then we guarantee that $\dot{V}(t)$ is negative definite for $t\in[t_k,t_{k+1}),\forall k\in\mathbb{N}^*$. Moreover, from the Remark 2 one can note that $V(t_{k+1}^-)\geq V(t_{k+1}^+)$ for all $k\in\mathbb{N}^*$. In other words, the LKF candidate (10) decreases monotonously at interrupted points $t=t_k$, $\forall k\in\mathbb{N}^*$. Consequently, the uncertain NCS is asymptotically stable, which concludes the proof. ■

TABLE I

ADMISSIBLE VALUES OF $\tau_{max}$ FOR VARIOUS $\tau_{min}$ (EXAMPLE 1)

| Method $\diagdown \tau_{min}$ | | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Shao [5] | | 1.874 | 2.505 | 3.259 | 4.074 | – | – |
| Zhang et al. [3] | | 1.918 | 2.533 | 3.274 | 4.079 | – | – |
| Orihuela et al. [9] | | 2.169 | 2.646 | 3.322 | 4.091 | – | – |
| Sun et al. [19] | | – | 2.567 | 3.341 | 4.169 | 5.028 | – |
| Fridman et al. [10] | Thm 1 | 2.169 | 2.646 | 3.321 | 4.090 | – | – |
| | Thm 2 | 2.120 | 2.724 | 3.458 | 4.257 | 5.097 | – |
| **Theorem 1** | $\eta$=1 | 2.169 | 2.646 | 3.321 | 4.090 | – | – |
| | $\eta$=2 | 2.217 | 2.751 | 3.462 | 4.258 | 5.098 | – |
| | $\eta$=6 | 2.229 | 2.777 | 3.497 | 4.298 | 5.143 | 6.014 |
| | $\eta$=12 | **2.230** | **2.779** | **3.501** | **4.302** | **5.148** | **6.020** |

TABLE II

ALLOWABLE $\tau_{max}$ FOR VARIOUS $\tau_{min}$ (EXAMPLE 2)

| Method $\diagdown \tau_{min}$ | | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Shao [5] | | 1,617 | 2,480 | 3,389 | 4,325 | 5,277 |
| Sun et al. [19] | | 1,620 | 2,488 | 3,403 | 4,342 | 5,297 |
| **Theorem 1** | $\eta$=1 | 1,792 | 2,609 s | 3,490 | 4,406 | 5,345 |
| | $\eta$=2 | 1,797 | 2,624 s | 3,514 | 4,437 | 5,380 |
| | $\eta$=6 | 1,798 | 2,628 s | 3,520 | 4,444 | 5,388 |
| | $\eta$=12 | **1,798** | **2,628** | **3,521** | **4,445** | **5,390** |

**Remark 2** As stressed in Section II, the time-varying delay is piecewise differentiable with $\dot{d}(t)=1$, except at interrupted points $t=t_k$, $\forall k\in\mathbb{N}^*$. This character is rarely exploited, since it requires the introduction of non-continuous terms in the LKF [14]. Nonetheless, we present a new term in the LKF which exploits the delay derivative information. The employment of this non-continuous term is only possible, for $V_2(t)$ monotonically decreases at interrupted points.

## IV. NUMERICAL EXAMPLES

**Example 1** Consider the NCS (6) with no uncertainties and

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\quad \Delta A=0,\ \Delta A_d=0.$$

For various values of $\tau_{min}$, the maximum values of $\tau_{max}$ which maintain the system's asymptotic stability are listed in Table I. From Table I, one can also see that the partitioning of the delay interval $[0,\tau_{min}]$ into $\eta$ subintervals proposed in this paper considerably improves the results. The results from Theorem 1 for $\eta\geq 2$ are considerably less conservative than previous published results. Especially, when $\tau_{min}=6$, the obtained $\tau_{max}$ using Theorem 1 with $\eta=12$ is $6.020s$ while previous stability criteria are not feasible.

TABLE III

MAXIMUM VALUE OF $\tau_{max}$ FOR $\tau_{min}$=0 (EXAMPLE 3)

| Wu et al. [20] | Jing et al. [21] | He et al. [22] | Qian et al. [23] | Park & Ko [7] | Theorem 1 $\eta=1$ |
|---|---|---|---|---|---|
| 0.242 | 0.242 | 0.336 | 0.379 | 0.397 | **0.464** |

**Example 2** Consider the following NCS described by

$$A=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix},\ A_d=\begin{bmatrix}0 & 0\\ -1 & 1\end{bmatrix},\quad \Delta A=0,\ \ \Delta A_d=0.$$

For various $\tau_{min}$, the results from different criteria in the literature are listed in table II. From the table, it is clear that for any choice of $\eta$, our results are less conservative than the ones obtained by previous criteria in the literature.

**Example 3** Consider the following uncertain NCS (6) with

$$A=\begin{bmatrix}-0.5 & -2\\ 1 & -1\end{bmatrix},\ A_d=\begin{bmatrix}-0.5 & -1\\ 0 & 0.6\end{bmatrix},\ D=I,\ \ E_A=E_{A_d}=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}.$$

In Table III, we compare the results of Theorem 1 ($\eta=1$) with those in [7], [20]–[23] for $\tau_{min}=0$. From the table, it is clear that our results, even for $\eta=1$, are considerably less conservative than those in previous criteria in the literature. The improvements over [7] are as high as 17%.

**Remark 3** These are benchmark examples widely employed for comparison in the literature. Nonetheless, the proposed method with $\eta>1$ should also yields better results when applied to different systems, specially for larger values of $\tau_{min}$. Also, it should be mentioned that the previous results presented in the examples represent state-of-the-art methods and were directly obtained from the corresponding papers.

## V. CONCLUSIONS

This work's main result concern the establishment of a new stability criterion for NCSs liable to model uncertainties, packet dropouts and uncertain time-varying delays. The conservativeness of the analysis is considerably reduced with the employment of a new delay-fractioning approach, which allows further exploitation of the delay's lower bound information, and the development of a new LKF that incorporates state-of-the-art stability techniques for systems with time-varying delays. Although this paper's main contribution concerns the analysis of uncertain NCSs, our criteria, when applied to nominal systems, also yields less conservative results than previous methods in the literature. The analysis is enriched with numerical examples that illustrates the effectiveness of our criteria which outperform state-of-the-art stability criteria in the literature.

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