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Wide-Area Damping Control of Large Power Systems Using a Model Reference Approach

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***Abstract*— In this paper we present some initial results on the design of dynamic controllers for electromechanical oscillation damping in large power systems using Synchronized Phasor Measurements. Our approach consists of three steps, namely - 1. *Model Reduction*, where phasor data are used to identify second-order models of the oscillation clusters of the system, 2. *Aggregate Control*, where state-feedback controllers are designed to achieve a desired closed-loop transient response between every pair of clusters, and finally 3. *Control Inversion*, where the aggregate control design is distributed and tuned to actual realistic controllers at the generator terminals until the inter-area responses of the full-order power system matches the respective inter-machine responses of the reduced-order system. Although a general optimization framework is needed to formulate these three steps for any n-area power system, we specifically show that model reference control (MRC) can be an excellent choice to solve this damping problem when the power system consists of two dominant areas, or equivalently one dominant interarea mode.**

***Index Terms*— Power systems, damping control, model reduction, swing equation, model reference control.**

I. Introduction

Model-based control of multi-machine power systems has seen a rich history of nearly fifty years, addressing the fundamental issues of mathematical modeling, stability and robustness of large power networks with a natural progress towards more advanced concepts of component-level dynamics. Over the past few years, however, following the US Northeast Blackout of 2003, the research mindset of power system control engineers has steered more towards measurement-based control designs. The relevance of this interest has been particularly facilitated by the recent outburst of power system measurement and instrumentation facilities in the form of the Wide-Area Measurement System (WAMS) technology [1]. Sophisticated digital recording devices called Phasor Measurement Units or PMUs (shown in Figure 1) are currently being installed in accelerating proportions at different points in the North American grid. Concerted efforts are being made to develop nationwide 'early warning' mechanisms using PMU measurements that will enable power system operators to take timely actions against blackouts and other widespread contingencies. Excellent visualization tools, for example, in the form of Real Time Dynamics Monitoring System (RT-DMS) and US-Wide Frequency Monitoring Network (FNET) [2] are currently being deployed across various corners of the US grid using voltage, current and frequency phasors, complimented by data analysis methods and software platforms such as Dynamic System Identification (DSI), Prony analysis and Mode Meter , Hilbert-Huang transforms [3] and phasor-based state estimation.

The majority of research done so far in the Synchrophasor community in the United States, however, pertains only to ideas of *monitoring* and *observation*. No rigorous research has yet been done to investigate how Synchrophasors, beyond simply monitoring, can also be used for autonomous, wide-area *damping control* of power system oscillations. In this paper we address this pertinent problem, and present some initial results inspired from the model-reference adaptive control (MRAC) literature, applied to simplistic yet practically relevant topologies of multi-area power systems with a particular focus on two-area systems. The main idea behind our design is a so-called, novel *control inversion* framework which allows PMU-based linear/nonlinear control designs, developed for reduced-order power systems, to be inverted (or, equivalently *distributed*) to local controllers in actual higher-order systems via suitable optimization methods. The approach, in general, consists of three precise steps, namely:

1. *Model Reduction/Dynamic Equivalencing* - where PMU data are used to identify equivalent models of the oscillation clusters of the entire power system based on the differences in their coupling strengths; for example, Figure 1 shows a 6-machine 8-line power system, where the electrical reactances or the edge weights of lines $\{1, 5, 6, 7, 8\}$ are assumed to be significantly smaller compared to those of lines 2, 3 and 4, separating the entire network into three coherent clusters or areas, and forcing the system to evolve as an *equivalent* 3-machine system over a slow time-scale treating all the machines inside Area 1 to be aggregated into one *hypothetical* equivalent machine. The first step in our design involves identification of these equivalent machine models using PMU data available from the terminal buses of each area. Detailed derivations of these measurement-based equivalencing methods have been presented in our recent work [4], and, therefore, will not be our focus in this paper. Our objective is to design controllers for damping the oscillations between these areas, for which we will simply assume that the area models are available to us by prior identification methods available from [4].

2. *Aggregate Control* - where state-feedback controllers are designed to achieve a desired closed-loop transient response between every pair of clusters in the reduced-order system. For the system in Figure 1 this would mean that a controller,

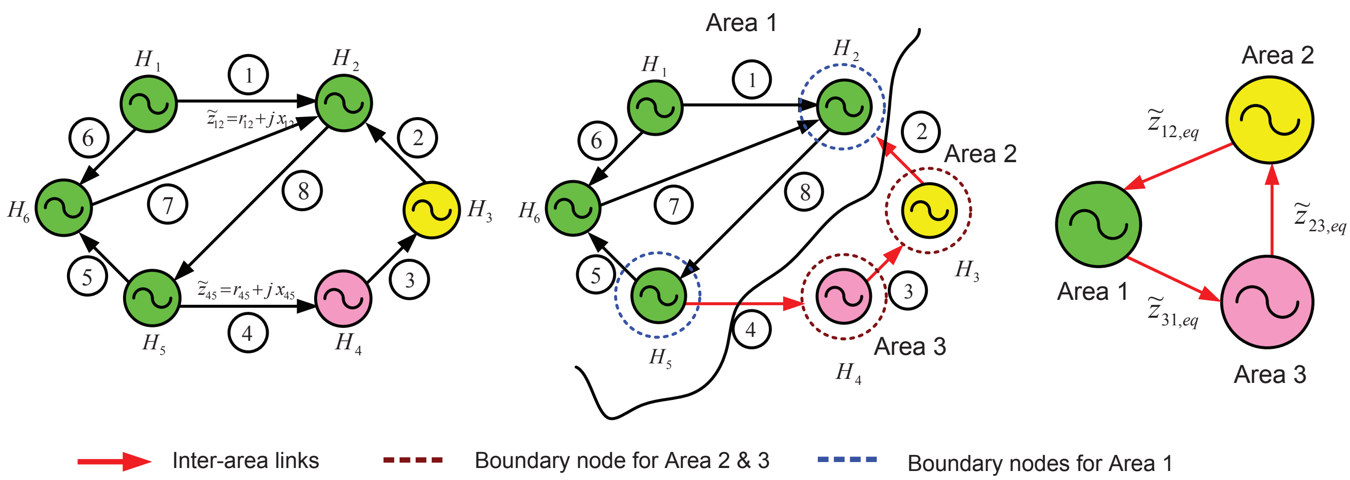

Fig. 1. Network of 6 generator nodes and 8 tie-line edges

either centralized or distributed, is designed for the three equivalent machines to achieve a desired dynamic response of the *aggregated* state variables (technically referred to as the *inter-area* state responses), and finally

3. *Control Inversion*- where the aggregate control design is distributed and tuned back to actual realistic controllers at the generator terminals until the inter-area responses of the full-order power system matches the respective inter-machine responses of the reduced-order system. For the 6-machine system of Figure 1 this would mean that the controller designed for the equivalent machine of Area 1 will be mapped back to a set of controllers for the four machines contained in this area, and tuned until the slow modes of this full-order system exhibit similar transient response as the respective closed-loop states in Step 2.

Although a general optimization framework is needed to formulate these three steps for a n-area power system, we specifically show in this paper that model reference control (MRC) can be an excellent choice to solve this damping problem when the power system consists of two dominant areas, or equivalently one dominant interarea mode.

II. Problem Formulation for n-Area Systems

Consider a network of electrical oscillators with n generators (nodes) connected to each other through m tie-lines (edges) with $m \leq n(n-1)/2$, forming a connected graph with cardinality (n, m), such that atmost one edge exists between any two nodes. This may also be thought of as a power system although we use the word 'power' with reservation as in a real power system generators are not necessarily connected directly but via intermediate buses due to which the network Laplacian becomes extremely complicated, especially for large networks. To avoid this difficulty and in the interest of the specific application discussed in this paper, we restrict our discussion to networks where each dynamic element i.e., a generator is directly connected to its neighbors. An example of such a network consisting of $n = 6$ generators and $m = 8$ tie-lines is shown in Figure 1. The arrows along each edge denote the direction of effective power flow. Let the internal voltage phasor of the i^{th} machine be denoted as

$$\tilde{E}_i = E_i \angle \delta_i, \quad i = 1, 2, \ldots, n \tag{1}$$

where, following synchronous machine theory [5], E_i is constant, δ_i is the angular position of the generator rotor, and $E_i \angle \delta_i$ denotes the polar representation $E_i \varepsilon^{j\delta_i}$ ($j = \sqrt{-1}$). The transmission line connecting the p^{th} and the q^{th} machines is assumed to have an impedance

$$\tilde{z}_{pq} = r_{pq} + jx_{pq} \tag{2}$$

where 'r' denotes the resistive part and 'x' denotes the reactive part. Here $p \in \{1, 2, ..., n\}$ and $q \in \mathcal{N}_p$ where $\mathcal{N}_p$ is the set of nodes to which the p^{th} node is connected. It follows that the total number of tuples formed by pairing p and q is m. For the rest of the paper we will denote the edge connecting the p^{th} and the q^{th} node by e_{pq}. Equation (2) can also be regarded as a complex *weight* of an edge in the network, and implies that $\tilde{z}_{pq} = \tilde{z}_{qp}$. If two nodes do not share a connection then the impedance corresponding to that non-existing edge is infinite (i.e., open circuit), or equivalently,

$$\tilde{y}_{pq} = \frac{1}{\tilde{z}_{pq}} = \frac{1}{r_{pq} + jx_{pq}} = 0 \quad \forall q \notin \mathcal{N}_p \tag{3}$$

where $\tilde{y}_{pq}$ is the admittance of e_{pq}. The mechanical inertia of the i^{th} machine is denoted as H_i.

The dynamic electro-mechanical model of the i^{th} generator, neglecting damping, can be written as [5]

$$\dot{\delta}_i = \omega_i - \omega_s \tag{4}$$

$$2H_i\dot{\omega}_i = P_{mi} - \sum_{k \in \mathcal{N}_i} \left(\frac{E_i^2 r_{ik} - E_i E_k p_{ik} \cos(\delta_{ik} + \alpha_{ik})}{p_{ik}^2} \right) \tag{5}$$

where $\delta_{ik} = \delta_i - \delta_k$, $\omega_s = 120\pi$ is the synchronous speed for a 60 Hz system, ω_i is the rotor angular velocity, P_{mi} is the mechanical power input, $p_{ik} = \sqrt{r_{ik}^2 + x_{ik}^2}$ and $\alpha_{ik} = \tan^{-1}(x_{ik}/r_{ik})$. All quantities are in per unit except for the phase angles which are in radians. We assume that the network structure is known, i.e., the set $\mathcal{N}_i$ for all $i = 1, 2, \ldots, n$ in (4)-(5) is known.

We linearize (4)-(5) about an initial equilibrium $(\delta_{i0}, 0)$ where $0 < \delta_{i0} < 90°$ for all $i = 1, 2, \ldots, n$, and denote the perturbed state variables as

$$\Delta\delta = \text{col}(\Delta\delta_1, \Delta\delta_2, \ldots, \Delta\delta_n) \quad (6)$$

$$\Delta\omega = \text{col}(\Delta\omega_1, \Delta\omega_2, \ldots, \Delta\omega_n). \quad (7)$$

We assume that the control input u enters the system (note: the network graph is connected, by assumption) through the j^{th} node, $j \in \{1, 2, \ldots, n\}$.

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ \mathcal{M}^{-1}\mathcal{L} & 0 \end{bmatrix}}_{A} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \mathcal{E}_j \end{bmatrix}}_{B} u \quad (8)$$

where I is the n-dimensional identity matrix, $\mathcal{E}_j$ is the j^{th} unit vector with all elements zero except the j^{th} element which is 1, $\mathcal{M} = \text{diag}(M_1, M_2, \ldots, M_n)$, M_i is the inertia of the i^{th} generator, and $\mathcal{L}$ is the $n \times n$ Laplacian matrix with elements:

$$\mathcal{L}_{ii} = -\sum_{k\in\mathcal{N}_i} \frac{E_i E_k}{p_{ik}} \sin(\delta_{i0} - \delta_{k0} + \alpha_{ik}), \quad (9)$$

$$\mathcal{L}_{ik} = \frac{E_i E_k}{p_{ik}} \sin(\delta_{i0} - \delta_{k0} + \alpha_{ik}), \quad k \in \mathcal{N}_i, \quad (10)$$

$$\mathcal{L}_{ik} = 0, \quad \text{otherwise} \quad (11)$$

for $i = 1, 2, \ldots, n$. It follows that if $M_i = M_j, \forall(i, j)$, then $\mathcal{L} = \mathcal{L}^T$. In general, however, each machine will have distinct inertia as a result of which the symmetry property does not hold. We, therefore, refer to $\mathcal{M}^{-1}\mathcal{L}$ as the unsymmteric Laplacian matrix for the linearized swing model. It is obvious from (8) that the coupling strengths p_{ik} of the links are contained in this matrix, and will decide the separation of areas depending on the differences in the strengths. The *inter-area* coupling strengths between any pair of areas can then be used to reduce the full-order network into a dynamic equivalent system of n-equivalent machines using the parameter identification methods outlined in [4].

A more concrete example is given by the 3-area power system shown in Figure 2, where Area 1 and 3 consist of two coherent machines each while Area 2 consists of three coherent machines, and, therefore, reduced to a three-machine equivalent interconnected through an equivalent graph.[1] Since our wide-area control method is based on shaping the closed-loop response of the reduced-order system followed by control inversion, this means that our first task would be to design a distributed excitation control system for each of the three equivalent machines G_{A1}, G_{A2} and G_{A3}, using, for example, classical linear state-feedback designs. None of these controllers, however, can be implemented in practice as G_{A1}, G_{A2} and G_{A3} do not exist physically. Therefore, keeping the aggregate design as a reference, we next need to design individual excitation for each generator $G_1, G_2, ..., G_7$ in the 13-bus system of Figure 2, and tune them optimally until the interarea responses for the 13-bus system coincide with the closed-loop state responses of the 3-area system. Our approach for this control distribution are based on various optimization methods, inspired by different engineering applications. For example, one potential direction of our analysis, as described next, will be to distribute the feedback gains of the aggregate controllers via continuous functions (for nonlinear control design) or averaging coefficients (for linear control design), and then to achieve interarea performance matching through optimal tuning of the controller parameters. Starting from the reduced-order system as in Figure 2, we consider the swing model of the j^{th} equivalent generator

$$\dot{\delta}_j = \omega_j, \;\; M_j\dot{\omega}_j = P_{mj} - D_j\omega_j - P_{ej}. \quad (12)$$

We assume that the measurements available for feedback for this j^{th} aggregate generator, and their corresponding feedback gains are:

$$(y_1^{j_1}, k_{j11}), (y_2^{j_1}, k_{j21}), ..., (y_1^{j_2}, k_{j12}), ..., (y_m^{j_n}, k_{jmn})$$

where $y_i^{j_p}$ denotes the i^{th} measured variable by the PMU located at the j_p^{th} bus. The choices for $y_{j_p}^i$, for example, can be voltage and current magnitudes, voltage and current phase angles, bus frequency, and the active power (calculated from the voltage and current phasors) measured by the PMU at Bus j_p. The index j_p corresponds to a bus that is possibly located in close neighborhood of this j^{th} generator in the reduced network. Assigning the mechanical power input $u_j = P_{mj}$ as the control input, an state/output feedback design can then be of the form:

$$u_j = f(y_1^{j_1}(t), y_2^{j_1}(t), ..., y_m^{j_n}(t), k_{j11}, k_{j21}, ..., k_{jmn}), \quad (13)$$

where $f(\cdot)$ is a smooth function producing a desired closed-loop inter-machine transient response. However, for implementation in the actual 13-bus system of Figure 2(a), u_j needs to be distributed to each local machine belonging to the j^{th} area. A plausible way of achieving this would be, for example, to construct nonlinear functions $\rho(\cdot)$ mapping each of the feedback gains $(k_{j11}, k_{j21}, ..., k_{jmn})$ to each individual machine in the area. The symbol l denotes the total number of machines in the j^{th} area. Stacking the functions $\rho_{mn}^l(\cdot)$ and the gains k_{jmn} for all (j, l, m, n) into vectors $\mathcal{R}$ and $\mathcal{K}$, respectively, the problem that we must, therefore, solve is:

$$\min_{\mathcal{R}(\mathcal{K})} ||x_{ij}(t, \mathcal{R}(\mathcal{K})) - \bar{x}_{ij}(t, \mathcal{K})||_2 \quad \text{st. } \mathcal{K} \in \mathcal{K}^* \quad (14)$$

for all (i, j), and for all $t \geq t^* \geq 0$, where: x_{ij} is the interarea state response (phase or frequency) between i^{th} and j^{th} areas in the full-order system, $\bar{x}_{ij}$ is the *designed* inter-machine state response (phase or frequency, respectively) between i^{th} and j^{th} machines in the reduced-order system, and $\mathcal{K}^*$ denotes a constraint set for the feedback gains specifying

[1]Construction of these equivalent connectivity structures is yet another significant aspect of the model reduction step, and is currently being investigated by us using stochastic graphical models.

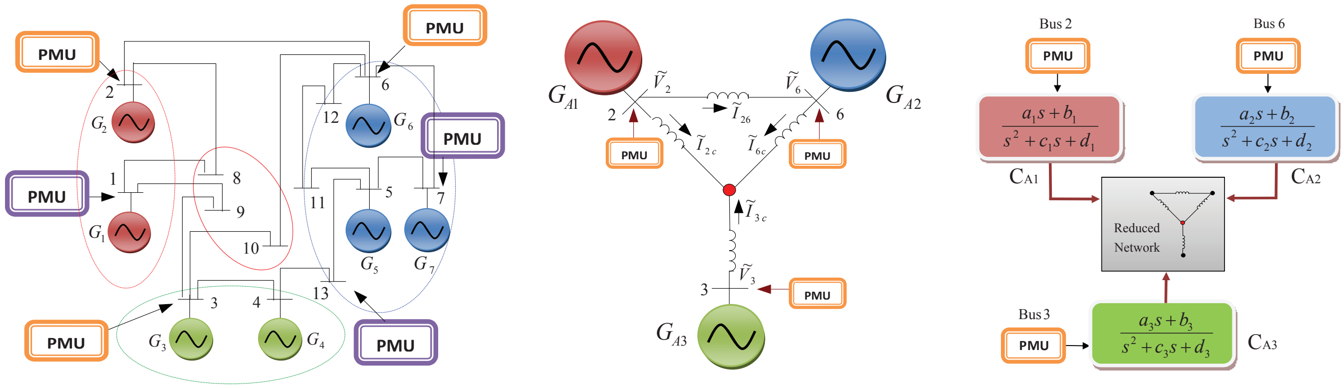

Fig. 2. Network of 7 generator nodes divided into 3 coherent areas

their allowable upper and lower bounds. Equivalently, we need to search for an optimal function set $\mathcal{R}^*(\mathcal{K})$ such that

$$||x_{ij}(t,\mathcal{R}^*(\mathcal{K})) - \bar{x}_{ij}(t,\mathcal{K})||_2 \leq \epsilon \tag{15}$$

for time $t > t^* > 0$, $\mathcal{K} \in \mathcal{K}^*$, where $\epsilon > 0$ is a chosen tolerance for the performance matching between the two responses.

However, when there are multiple inter-area modes in a system, a fundamental concern for solving the optimization problem (15) is the so-called non-uniqueness of the equivalent model with respect to the choice of interarea modes. This means that when the phase angle response at any bus in the power system consists of more than one interarea mode, then a specific mode needs to be extracted from the signal via modal decomposition methods such as Prony analysis [6], Eigenvalue Realization Algorithm [4], etc., and the impulse response of this extracted mode needs to be used for identifying the equivalent model of the system. However, if a different interarea mode is extracted and used for this identification, then there is no guarantee that model will match with that obtained for the first mode. The mathematical justification behind this is that the transfer function of the full-order power system can be written in the pole residue form as

$$G(s) = \sum_{i\in\mathcal{N}_l} \frac{\sigma_i s + \mu_i}{s^2 + \gamma_i s + \pi_i} + \sum_{j,k\in\mathcal{I}_j\times\mathcal{I}_k} \frac{\sigma_{jk} s + \mu_{jk}}{s^2 + \gamma_{jk} s + \pi_{jk}} \tag{16}$$

where $\mathcal{N}_l$ is the set of local modes and $\mathcal{N}_i \times N_j$ is the set of interarea modes operating between the i^{th} and the j^{th} cluster of the system. Since the residue parameter set (σ_{jk}, μ_{jk}) for different pairs of (i, j) are not equal, this implies that the participation of the different interarea modes on the measured signal, as quantified by their participation factors, are different, and, hence, the equivalent model computed based on different pairs of clusters are disparate as well. In terms of our inversion problem, this implies that the optimization (15) needs to be performed for each equivalent interarea model, with an obvious challenge being whether the design of a single controller that can damp all interarea modes at their corresponding frequencies, is feasible or not. While the problem needs more introspection, in this paper we bypass this obstacle by presenting our control designs for a simplistic and yet highly important class of power system, namely a two-area system, i.e., a system with a single dominant interarea mode.

III. Model Reference Control for Two-area Systems

When the full-order transfer function (16) consists of only one interarea mode, then there is only one participation factor present in any measured state from this mode, as a result of which the reduced-order system has a unique topology, and, therefore, admits for a feasible control inversion. In fact, given the simple Laplacian structure of such systems, as already indicated in Section I, Model Reference Control can be used as a helpful tool for the inversion problem instead of using the generic optimization framework proposed in Section II. Recalling the swing equation (8), the input and output matrices for the linearized model of the two-area system can be written as $B = \text{col}(0, \mathcal{E})$, $C = [C_1\ 0]$, where $\mathcal{E}$ is the unit vector with entry 1 at the index corresponding to the control input [2], and $C_1 \in \mathbb{R}^{1\times(n_1+n_2)}$ has all zero entries except for the i^{th} and the j^{th} entries, each belonging to one distinct area, which are $+1$ and -1 (or, vice versa), where i and j are the indices of the nodes whose phase angle difference is being regulated, and n_1 and n_2 are, respectively, the number of nodes in Area 1 and 2. Denoting $\bar{A} = \mathcal{M}^{-1}\mathcal{L}$, it can be easily shown that for this system,

$$CA^pB = 0, \quad p \geq 0 \text{ is even} \tag{17}$$

$$CA^{(2r+1)}B = C_1\bar{A}^r\mathcal{E}, \quad r = 0, 1, 2, \ldots \tag{18}$$

Furthermore, given that our objective is to track and control the inter-area oscillations, we consider the observed output as the phase angle difference between the boundary nodes, and arrange the states as

$$\delta = \text{col}(\delta_{11}, \delta_{12}, \ldots, \delta_{1n_1}, \delta_{21}, \delta_{22} \ldots, \delta_{2n_2}) \tag{19}$$

$$\omega = \text{col}(\omega_{11}, \omega_{12}, \ldots, \omega_{1n_1}, \omega_{21}, \omega_{22}, \ldots, \omega_{2n_2}) \tag{20}$$

where δ_{ij} and ω_{ij} denote the phase and machine speed for the j^{th} machine in the i^{th} area, the total number of

[2]For simplicity we consider only a scalar control input although the MRC design can be easily extended to multiple control inputs as well

machines in Area 1 and 2 are n_1 and n_2, respectively, and the index pair (n_1, n_2) correspond to the boundary nodes of the two respective areas. To ensure persistency of excitation we apply the input at any of the boundary nodes so that it the control effect will have maximum participation in the inter-area mode [4], [7]. For example, considering the input at the boundary node of Area 1, we get

$$C = [C_1 \ 0], \ C_1 = [0\,0...1\,0\,0\,...-1], \ B = \mathrm{col}(0, \mathcal{E}_{n_1}) \quad (21)$$

where the non-zero entries of C_1 are at the n_1^{th} and n_2^{th} positions, and $\mathcal{E}_{n_1}$ is the unit vector with all entries zero except the n_1^{th} entry which is 1. Rearranging (19)-(20) as

$$\begin{aligned} \delta_a &\triangleq \mathrm{col}(\delta_{11}, \delta_{12}, ..., \delta_{1n_1}), \ \delta_b \triangleq \mathrm{col}(\delta_{21}, \delta_{22} ..., \delta_{2n_2}) \\ \omega_a &= \triangleq \mathrm{col}(\omega_{11}, \omega_{12}, ..., \omega_{1n_1}, \omega_b \triangleq \mathrm{col}(\omega_{21}, \omega_{22}, ..., \omega_{2n_2}) \end{aligned}$$

the state equation then takes the form

$$\begin{bmatrix} \dot{\delta}_a \\ \dot{\delta}_b \\ \hline \dot{\omega}_a \\ \dot{\omega}_b \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ \mathcal{L} & 0 \end{bmatrix}}_{A} \begin{bmatrix} \delta_a \\ \delta_b \\ \hline \omega_a \\ \omega_b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \hline \beta\mathcal{E}_{n1} \\ 0 \end{bmatrix} u \quad (22)$$

where $\beta = M_{n_1}^{-1} > 0$ i.e., the reciprocal of the inertia constant of the machine located at the boundary node of Area 1, and the unsymmetric Laplacian matrix $\mathcal{L}$ is the of the form

$$\begin{aligned} \mathcal{L} &= \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ \mathcal{L}_3 & \mathcal{L}_4 \end{bmatrix}, \mathcal{L}_2 = \begin{bmatrix} 0 & 0 \\ 0 & \gamma_{n_1,n_2} \end{bmatrix}_{n_1 \times (n_1+n_2)} \quad (23) \\ \mathcal{L}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & \gamma_{n_2,n_1} \end{bmatrix}_{n_2 \times (n_1+n_2)} \quad (24) \end{aligned}$$

with $\gamma_{n_1,n_2} \in \mathbb{R}^{1\times n_2}$, $\gamma_{n_2,n_1} \in \mathbb{R}^{1\times n_1}$ given as

$$\begin{aligned} \gamma_{n_1,n_2} &= \mathrm{col}(0, 0, ..., \frac{E_{n_1}E_{n_2}}{M_{n_1}\, x_{n_1,n_2}} \cos(\delta_{n_1 0} - \delta_{n_2 0}))^T \\ \gamma_{n_2,n_1} &= \mathrm{col}(0, 0, ..., \frac{E_{n_1}E_{n_2}}{M_{n_2}\, x_{n_1,n_2}} \cos(\delta_{n_1 0} - \delta_{n_2 0}))^T. \end{aligned}$$

The RHS of the above expressions follow straight from (5) with all line resistances assumed to be negligible compared to the reactances. The expressions E_{n_j}, M_{n_j} and $\delta_{n_j 0}$ respectively denote the generator voltage, machine inertia and phase angle at pre-disturbance equilibrium for the j^{th} boundary node, $j = 1, 2$. x_{n_1,n_2} denotes the reactance or the edge weight of the transmission lines joining the two areas connecting node n_1 with node n_2. Considering (19)-(24), after a few calculations it can be easily shown that (17)-(18) simple reduces to

$$CB = CAB = CA^2B := 0, \ CA^3B \neq 0 \quad (25)$$

implying that the relative-degree of the two-area system with the chosen input-output pair is $n^* = 2$. However, we must recall that $\mathcal{L}$ contains both local and interarea couplings implying that the output measurement will contain the modal effects due to $(n_1 + n_2 - 2)$ local modes and one interarea mode of oscillation, say denoted as ν_a. Since our goal is to dampen the interarea mode, the output $y = \delta_{n_1} - \delta_{n_2}$ must be passed through a band-pass filter (BPF) with bandwidth frequency set to ν_a. Typically such values are learnt apriori by power system operators from offline modal analysis [7]. If the BPF is designed as a relative-degree zero Butterworth filter , then the open-loop transfer function of the two-area system retains $n^* = 2$.

The state-space model of the two-machine equivalent of the two-area system, on the other hand, is given as

$$\underbrace{\begin{bmatrix} \dot{\bar{\delta}}_a \\ \dot{\bar{\delta}}_b \\ \hline \dot{\bar{\omega}}_a \\ \dot{\bar{\omega}}_b \end{bmatrix}}_{\bar{x}} = \begin{bmatrix} 0 & I_{2\times 2} \\ \bar{\mathcal{L}} & 0 \end{bmatrix}_{4\times 4} \begin{bmatrix} \bar{\delta}_a \\ \bar{\delta}_b \\ \hline \bar{\omega}_a \\ \bar{\omega}_b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \hline \beta \\ 0 \end{bmatrix} \bar{u} \quad (26)$$

$$\bar{y} = = [1 \ -1; 0\ 0]\,\bar{x}, \quad \mathcal{L} = \begin{bmatrix} -\bar{c}_{ab} & \bar{c}_{ab} \\ \bar{c}_{ab} & \bar{c}_{ab} \end{bmatrix} (27)$$

with $\bar{c}_{ab} \triangleq (E_aE_b \cos(\delta_{a0} - \delta_{b0})/x_{ab}$. The bar sign denotes that the corresponding variables are 'equivalents' while subscripts a and b refer to Area 1 and 2, respectively. From (26)-(27) it is clear that the reduced order system has relative degree $\bar{n}^* = 2$ as well.

Summarizing the foregoing analysis, we write the reduced-order, equivalent system as a reference model in the form

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u}, \ \ \bar{y} = \bar{C}\bar{x} \quad (28)$$

where a state feedback design of the form $\bar{u} = K\bar{x}(t)) \triangleq r(t)$ can be applied to achieve a desired transient response $\bar{y}^*(t)$ of the output[3]. The actual full-order system model, however, has the form

$$\dot{x} = Ax + Bu, \ \ y = Cx, \ \ y_m(t) = G(s)[y](t) \quad (29)$$

where $G(s)$ is a relative-degree zero filter transfer function with Hurwitz zero dynamics [**?**]. The control objective, therefore, is to design the input u via state feedback such that all closed loop signals are bounded and the plant output $y_m(t)$ tracks $\bar{y}(t)$ asymptotically over time. The following four assumptions hold true for our system:

(A1): Denoting

$$G_p(s) \triangleq k_p \frac{Z_p(s)}{P_p(s)} = G(s)[C(sI - A)^{-1}Bu](t) \quad (30)$$

the polynomial $Z(s)$ is stable.

(A2): The degree $n = 4$ for $P_p(s)$ is known and fixed.

(A3): Sign of the high frequency gain k_p is known.

(A4): Relative degree of $G_m(s) \triangleq [\bar{C}(sI - \bar{A})^{-1}\bar{B}r](t)$ is same as that of $G_p(s)$, both being equal to 2.

The natural choice for designing the input $u(t)$, therefore, can be a model-reference controller with the following structure [8]:

$$u(t) = \theta_1^T\vartheta_1(t) + \theta_2^T\vartheta_2(t) + \theta_{20}y_m(t) + \theta_3 r(t) \quad (31)$$

[3]Please refer to [4] for the reconstruction of states in the reduced model that will the proposed state feedback design.

where, θ's are constant parameters, and

$$\vartheta_1(t) = \frac{q(s)}{\Lambda(s)}[u](t),\ \vartheta_2(t) = \frac{q(s)}{\Lambda(s)}[y](t), \quad (32)$$
$$q(s) = [1,\ s,\ s^2], \vartheta_1, \vartheta_2 \in \mathbb{R}^3, \vartheta_{20} \in \mathbb{R} \quad (33)$$

and $\Lambda(s)$ is a Hurwitz polynomial of degree 3. Following standard MRC theory, the two filter variables $\vartheta_1(t)$ and $\vartheta_2(t)$, in this case, can be designed simply as

$$\dot{\vartheta}_1(t) = A_\lambda \vartheta_1(t) + B_\lambda u(t) \quad (34)$$
$$\dot{\vartheta}_2(t) = A_\lambda \vartheta_2(t) + B_\lambda y(t) \quad (35)$$
$$A_\lambda = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\lambda_0 & -\lambda_1 & -\lambda_2 \end{bmatrix},\ B_\lambda = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (36)$$

where $\Lambda(s) := \lambda_0 + \lambda_1 s + \lambda_2 s^2$. The control parameters in (31) can then be solved for from Diophantyne's equation as in Lemma 5.1 in [8] which states that constants θ_1, θ_2, θ_{20} and θ_3 exist such that

$$\theta_1^T q(s) P_p(s) + (\theta_2^T q(s) + \theta_{20}\Lambda(s)) k_p Z_p(s) = \Lambda(s)(P_p(s) - k_p \theta_3 Z_p(s) P_m(s)) \quad (37)$$

where $P_m(s)$ is the characteristic polynomial of the reference model (28).

As an example, we consider a three-machine cyclic power system, where two generators G_1 and G_2 are connected by a strong link, and, therefore, form one aggregate area, while both machines are connected to a distance generator G_3 via long and, hence, weak tie-lines. The separation of time-scales due to the difference in coupling strengths arises due to the three line reactances and machine inertias, which, for this example, we consider as (all in per unit): $M_1 = 1$, $M_2 = 5$, $M_3 = 10$, $x_{13} = x_{23} = 100\,x_{12}$. The three modes (i.e., eigenvalues of the Laplacian matrix, i.e., the state matrix for the double integrator dynamics) of the linearized swing equation are given as $\lambda_1 = 0$, $\lambda_1 = 3.407$ and $\lambda_1 = 0.2307$, clearly indicating the DC mode, the local mode and the interarea mode, respectively. We, therefore, design a 2^{nd} order Butterworth BPF with bandwidth frequency $\omega_s = 0.2307$ (or, alternately with $\omega_s = 3.4$ followed by subtracting the filter output from the original signal), and pass the phase angle difference between G_3 and G_1 through this filter to extract the interarea component. Figure 3 shows the asymptotic convergence as well as transient matching of the full-order system response with the reference model over time $t = 0$ to $t = 3$ seconds for three different values of λ_3 fixing $\lambda_1 = \lambda_2 = 1$. Finer matching can be achieved by an iterative choice of the λ's in (36).

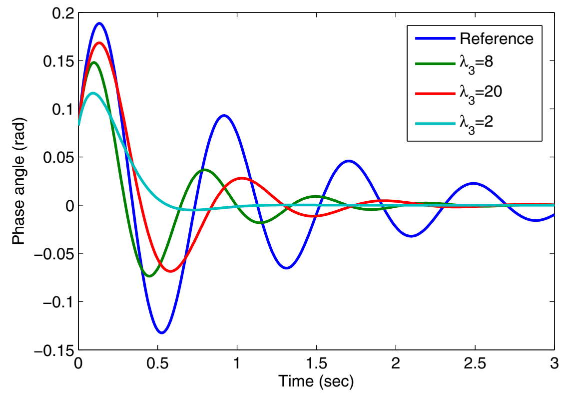

Fig. 3. Interarea damping control via MRC

IV. Conclusions

In this paper we presented a set of initial results on the problem of damping interarea oscillations using model-reference state-feedback control designs. The approach consists of model reduction of large power systems into coherent clusters using Synchrophasors, and designing linear state-feedback controllers to achieve a desired damping between the clusters. The final step, thereafter, is to treat the closed-loop response of the reduced system as a reference and employ model-reference control to track this reference for the full-order system. A natural question that may arise is on our proposed approach of *model reduction* preceding control design, i.e., why should one reduce a given network into clusters first, and then invert its closed-loop responses to the actual system? The answer follows from the size and complexity of any realistic power system. The WECC, for example, has roughly 2,000 generators, 11,000 transmission lines, 6,000 transformers, and 6,500 loads. Designing a distributed control mechanism to shape a desired set of *interarea* responses starting from this entire model and using PMU measurements from arbitrary locations would be practically intractable. We believe that the proposed *detour* of reducing such large systems into simpler chunks (even if approximately), and then redistributing their control efforts would give the problem a much more well-defined and less chaotic formulation.

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