2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Moving horizon estimator for measurement delay compensation in model predictive control schemes

Felipe Valencia*, José David López*, Alejandro Márquez*, Jairo José Espinosa*
*School of Mechatronics, Universidad Nacional de Colombia. Medellín, Colombia
Email: fvalenc@unal.edu.co, jodlopezhi@unal.edu.co, amarque@unal.edu.co, jairo.espinosa@ieee.org

***Abstract*—This paper deals with the problem of the loss of performance of the pair observer-controller, when measurements have a delay due to communication over networks. Here we consider the case where the estimation of the states is carried out using a moving horizon estimator (MHE), the control actions are computed by using a centralized model predictive controller (MPC), and the delay varies randomly and is n times the sampling time ($n \in \mathbb{N}$). In order to tackle the loss of performance associated with the pair MHE-MPC, an MHE with variable structure is proposed. The resulting pair MHE-MPC was tested using the four tank process as a test bed showing an improvement on the performance.**

***Index Terms*—Variable delay, Model Predictive Controller, Moving Horizon Estimator**

I. INTRODUCTION

When measurements are transmitted from networked sensors to the control system using communication networks, a variable transport delay appears due to communication problems such as congestion, noisy environments, error correction sequences, variable routing paths, etc. These delays can cause deterioration on the control system performance. In some cases these variable communication paths can cause that data measured at time instant k are received after data measured at $k+d$, with $d > 0$; it means that the data arrive to the controller not only with delay but also in an incorrect sequence.

Dealing with such challenging situation demands the use of a state observer capable of accommodating large data sequences received at a non regular basis, in order to estimate correctly the states despite of the delays. This problem can be seen in a similar way as inferential sensors are used in chemical applications, where variable delayed lab samples are used together with regular samples to reconciliate model based predictions. Nowadays this problem has been treated with modified versions of Kalman filters [1], [2], but in this work we show how the Moving Horizon Estimation (MHE) observer can be modified to accommodate such irregular sampling, and even improve the performance of the Kalman filters.

The MHE is a well known model based technique used to estimate the states and parameters of a wide variety of plants. It performs a nonlinear optimization to estimate the system states including constraints on its formulation [3]–[5]. This estimation technique has shown excellent results when used in non-linear processes, where restricted control actions and constrained states are enforced for guaranteeing stability [6], [7].

The finite moving horizon of the MHE is a fixed-size window observer that only takes into account the last N time instants. The window size must guarantee enough information to reconstruct the states. The size of the window is chosen accordingly to the dynamics of the plant, roughly around the settling time.

In order to deal with the variable transport delay conditions, we propose to organize the incoming data from the sensors into a stack assuming that each package with a measurement includes the time stamp of the sample. Then the MHE estimates the states with the available organized data at each time instant, by updating the state covariance matrix penalizing only the estimations where are data available. Finally it is assumed that states are only modified if evidence coming from the sensor is telling so. In order to verify the proposed methodology, a quadruple tank process is implemented in simulation with and without delay conditions on the state measurements. As result, the performance of a pair MHE-MPC was evaluated with and without the proposed variable structure MHE, and compared with a estimation technique based on Kalman filtering.

This paper is organized as follows: In Section II the problem statement is presented, in Section III the simulation results are shown, and in Section IV the conclusions and future work are presented.

II. PROBLEM STATEMENT

The Moving Horizon Estimation (MHE) strategy was developed as a pair with the Model Predictive Control (MPC). Despite the similarities among MPC and MHE, the MPC was successfully developed and exploited in process industry, whereas MHE theory remained to be a topic of study in academia with few industrial applications [8].

The MHE technique is based on a quadratic estimation problem using a moving, fixed-size estimation window. The fixed-size window is needed to bind the computational effort for solving an otherwise infinite sized problem. This is the main difference of MHE with the batch estimation problem (or full information estimator) [8]–[10]. After the window size

This work presents the results of the project "Procesamiento de Señales - Localización de fuentes", financially supported by ARTICA research center for excellence, Universidad Nacional de Colombia and COLCIENCIAS, and by the European 7th Framework STREP project "Hierarchical and distributed model predictive control for large scale systems (HD-MPC)", contract number INFSO-ICT-223854.

978-1-61284-799-3/11/$26.00 ©2011 IEEE

is filled, when a new measurement is available the oldest one is discarded using the concept of window shifting.

The main advantage of MHE in comparison with other estimation schemes (like the Kalman Filter) is the straightforward constraint handling inside the optimization problem, and the possibility to propose a cost function. However, given that the MHE is a limited memory filter, stability and convergence issues arise. A review on latest developments on MHE procedures was published by García and Espinosa in [11].

Despite of the advantages of the MHE, when the measurement delay is not properly handled the performance of the estimation may fall. Consequently, a procedure or method to handle the delay in this type of estimator should be developed. Below, the MHE is introduced and a procedure for tackling the problem of the delay in the measurements of the states is presented.

A. Moving horizon estimator

Assume a system modeled by the following nonlinear difference equation:

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) + w(k) \\ y(k) &= h(x(k), u(k)) + v(k) \end{aligned} \tag{1}$$

where some constraints are imposed over the state variables, disturbances, and measurement noise as follows:

$$x \in \mathbb{X}, \quad w \in \mathbb{W}, \quad \text{and } v \in \mathbb{V} \tag{2}$$

with $x(k)$ and $y(k)$ the states and output at k sample respectively, $w(k)$ is the disturbance or model uncertainty, $v(k)$ is the measurement noise, and $\mathbb{X}$, $\mathbb{W}$, $\mathbb{V}$ are the feasible sets of the states, of the disturbances, and of the measurement noise, respectively. Also, $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ with $g(\cdot, 0) = 0$, and $h : \mathbb{R}^n \to \mathbb{R}^p$. Finally it is assumed that $\mathbb{X}$ and $\mathbb{W}$ are closed with $0 \in \mathbb{W}$.

A linear large-scale constrained system generating the measurement sequence $\{y(k)\}$ can be derived from a linearization around each operating point of (1) as:

$$\begin{aligned} \hat{x}(k+1) &= A(k)\hat{x}(k) + B(k)u(k) + G(k)w(k) \\ \hat{y}(k) &= C(k)\hat{x}(k) + v(k) \end{aligned} \tag{3}$$

where for simplicity $\hat{x}(k) \in \mathbb{R}^n$ and $w(k) \in \mathbb{R}^w$ are the linearized states and uncertainty respectively, $v(k) \in \mathbb{R}^p$ is the linearized measurement noise, and $u(k) \in \mathbb{R}^m$ denotes the system input. Moreover, those variables are constrained as it is shown in (2). Thus, the estimation of all the states in (3) can be formulated as an MHE problem as follows:

$$\Phi_k^* = \min_{x_0, \{w_j\}_{j=0}^{k-1}} \Phi_k(x_0, \{w_j\}_{j=0}^{k-1}) \tag{4}$$

where x_0 is the initial state. The problem is subject to the following constraints:

$$x_j \in \mathbb{X} \text{ for } j = 0, \dots, k, \; w_j \in \mathbb{W} \text{ for } j = 0, \dots, k-1,$$

in which the cost function is of the form:

$$\Phi_k(x_0, \{w_j\}_{j=0}^{k-1}) \triangleq \sum_{j=0}^{k-1} \|y(j) - \hat{y}(j)\|_Q^2 + \|w(j)\|_R^2 \tag{5}$$

As the problem (4) gets more information as time goes, the optimization becomes intractable because the computational complexity increases at least as a linear function of time, making difficult its treatment on-line. In order to avoid this problem, a fixed dimension optimal problem by a moving horizon approximation is proposed in [5], [8], [10], [12]. With this approach, the cost function (5) can be rewritten as

$$\Phi_T(x_{T-N}, \{w_k\}_{k=T-N}^{T-1}) = \sum_{k=T-N}^{T-1} \|y(k) - \hat{y}(k)\|_Q^2 + \|w(k)\|_R^2 \tag{6}$$

where N is the horizon of the MHE. Considering (6) as a cost function in the original MHE problem, the complexity of the MHE increases at least as a linear function of time until the horizon N is reached. When the horizon N is reached the complexity of the MHE problem remains constant.

B. Moving horizon estimator with variable structure

In Subsection II-A, the MHE problem was introduced. In this subsection it is assumed that the measurements of the states arrive once they are taken. However, in real applications there are delays associated with the communication network used to transmit the data from the sensors to the MHE; this may affect the performance of the estimator. Figure 1 shows a block diagram considering the delay on the states measurements transmission. If there exists a delay $d(k)$ on

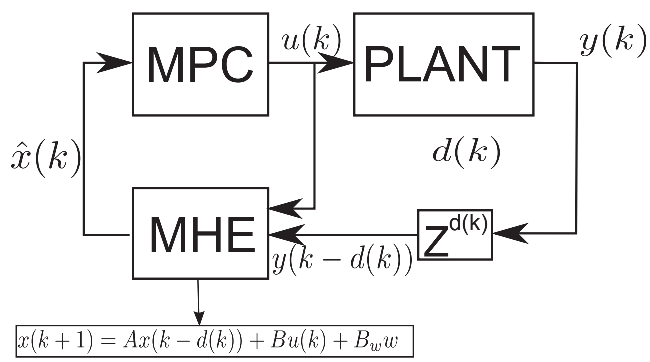

Fig. 1: Block diagram of a control system considering the delay on the states measurements transmission.

the measurement of the states the estimation is made based on (3), which does not represent the dynamical behavior of the system (1), especially if the delay varies randomly because it is possible that future measurements of the states arrive before previous ones. Then, the estimator may not be able to calculate the real value of the states, affecting the performance of the pair MHE-controller, and thus of the entire system.

Therefore, it is necessary to make a correction of this problem for guaranteeing that the estimator calculates the appropriate value of the states. With this purpose, a variant of the MHE is presented on Subsection II-A. It consists on using a time variant weighting matrix Q, for computing the term $\|y(k) - \hat{y}(k)\|_Q^2$ in (6).

Consider a system of Figure 1. Let $\widetilde{y}(k)$ denote the sequence of available measurements at time step k. Let $\widetilde{d}(k)$ denote the sequence containing the delay associated to each measurement in $\widetilde{y}(k)$. The sequence $\widetilde{y}(k)$ does not contains all the information on the window of the MHE, because some data may have not arrived yet due to the delay.

Assume that the delay of each measurement belonging to $\widetilde{y}(k)$ is known (this is possible by using tags for identifying the sending time of each measurement). Also, assume that the delays are randomly distributed, and that $\frac{d(k)}{T_{\mathrm{s}}} \in \mathbb{N}$, with T_{s} the sample time. Then the real position of each measurement in $\widetilde{y}(k)$ can be identified and organized accordingly to the real positions of the measurements. Also, with the sequence of delays it is possible to identify which blocks of the weighting matrix Q should be set to 0 (or neglected), because there is not data available to compare the estimated and the measured values. With this approach the MHE problem becomes a variable structure problem, in which the length of the sequence of available measurements and the blocks different of 0 of the weighting matrix Q depends on the time step k.

So, the expression for computing the estimated output $\bar{y}(k)$ becomes

$$\bar{y}(k) = \begin{bmatrix} \Gamma^{N-d_1} \\ \vdots \\ \Gamma^{N-d_n} \end{bmatrix} x(k-N) + \begin{bmatrix} \Xi^{N-d_1} \\ \vdots \\ \Xi^{N-d_n} \end{bmatrix} \widetilde{u}(k) \qquad (7)$$

where $\{d_1, \ldots, d_n\}$ is the sequence of the delays, $\Gamma^a = CA^a$, $\Xi^a = [CA^{a-1}B, \ldots, CB]$, $\widetilde{u}(k) = [u^T(k-N), \ldots, u^T(k-1)]^T$. Hence the cost function (6) becomes:

$$\Phi_T(x(k-N)) = \|\tilde{y}(k) - \bar{y}(k)\|_Q^2 + \|\tilde{w}(k)\|_R^2 \qquad (8)$$

where $\tilde{w}(k)$, denote the disturbance vector at time step k.

In order to implement the proposed MHE, the following steps are suggested:

1) Given the sequence of measurements $\{\tilde{y}(k-N), \ldots, \tilde{y}(k-1)\}$, and the sequence of delays $\{d(k-N), \ldots, d(k-1)\}$, arrange the vector of measurements, where each measurement position is given by $\frac{d(k-l)}{T_{\mathrm{s}}}$, with T_{s} the sample time.
2) With the arranged vector of measurements identify which inputs do not have data, the blocks of Q corresponding to these inputs should be set to 0 (or neglected).
3) Estimate the states accordingly to the MHE (see section II-A).
4) After computing the estimated value of the states, send them to the controller and go back to step 1.

Note that the matrix Q varies when new data is received, but its dimension remains constant. If a measurement arrives at time stamp k and it belongs to the window N, then the blocks of the matrix Q associated to that sample time change. The matrix Q is also updated at each time stamp by moving its blocks to their corresponding position accordingly to the window N.

On the MHE design the convergence of the estimator is not guaranteed, the MHE is based on a finite estimation horizon that reduces the computational cost of the optimization problem, but can fall in instabilities. In this case, in order to include the cost out of the estimation window N, there is included an arriving cost given by $y^T(k)Py(k)$, with $k > 0$. In [12] the full demonstration of the MHE convergence is presented.

In the following section the simulation results are presented.

III. Simulation results

In this section we compared the performance of the pair MHE-MPC on a four tank process with and without considering the proposed variable structure in the MHE, when a random delay on the measurements of the states is considered. First, we performed a simulation without delay in order to set a reference behavior. Then, we added a random delay on the states measurements to show the loss of performance of the system when a fixed MHE structure is used. Finally, the proposed MHE with variable structure was implemented on the same random delay conditions for allowing a comparison with the fixed structure MHE. The performance of the pair MHE-MPC on the three cases was determined with a time-variant reference value of the controllable variables.

A. System description: the four plant process

The four-tank plant is designed for testing control techniques using industrial instrumentation and control systems. The plant consists on a hydraulic process of four interconnected tanks inspired by the educational quadruple-tank process proposed by [13]. A schematic diagram of the process is shown on Figure 2. The target in the system shown on Figure 2 is to regulate the level of the tanks 1 and 2, by modifying the flows q_{a} and q_{b} feeding the tanks. In this case we considered as manipulated variables the flows q_{a} and q_{b}, as controlled variables the levels h_1 and h_2, and as estimated variables the levels h_3 and h_4.

From the mass balance and the Bernoulli flow equation, the first principle model for the process is:

$$\begin{aligned} \frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 q_{\mathrm{a}}}{A_1} \\ \frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 q_{\mathrm{b}}}{A_2} \\ \frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)q_{\mathrm{b}}}{A_3} \\ \frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)q_{\mathrm{a}}}{A_4} \end{aligned} \qquad (9)$$

where A_i is the cross-section area, a_i is the cross-section area of the outlet hole, and h_i is the level of the tank i, $i = 1, \ldots, 4$. The parameters $\gamma_1, \ \gamma_2 \in [0 \ 1]$ are set prior to the experiment. The flow to tank 1 is $\gamma_1 q_{\mathrm{a}}$ and the flow to tank 4 is $(1-\gamma_1)q_{\mathrm{a}}$ (similarly for tanks 2 and 3). The acceleration of gravity is denoted by g. For the control test presented in this work, the plant parameters are shown in Table I. The Linearized model at an operating point given

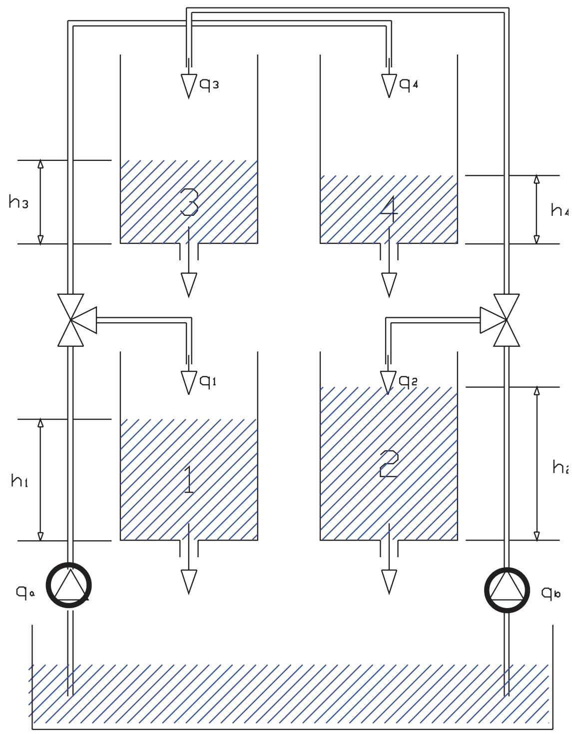

Fig. 2: Four tank process used for validation of the proposed variable structure MHE

TABLE I: Parameters used for the simulation of the four tank system

Parameter	Units	Value
$h_{1\max}$	[m]	1.36
$h_{2\max}$	[m]	1.36
$h_{3\max}$	[m]	1.30
$h_{4\max}$	[m]	1.30
$h_{1\min}$	[m]	0.20
$h_{2\min}$	[m]	0.20
$h_{3\min}$	[m]	0.20
$h_{4\min}$	[m]	0.20
$q_{a\max}$	[m^3/h]	3.26
$q_{b\max}$	[m^3/h]	4.00
$q_{a\min}$	[m^3/h]	0.00
$q_{b\min}$	[m^3/h]	0.00
a_1	[m^2]	$1.310 * 10^{-4}$
a_2	[m^2]	$1.507 * 10^{-4}$
a_3	[m^2]	$9.267 * 10^{-5}$
a_4	[m^2]	$8.816 * 10^{-5}$
A_1	[m^2]	0.06
A_2	[m^2]	0.06
A_3	[m^2]	0.06
A_4	[m^2]	0.06
γ_1		0.3
γ_2		0.4
q_{a0}	[m^3/h]	1.63
q_{b0}	[m^3/h]	2.00
h_{10}	[m]	0.6487
h_{20}	[m]	0.6639
h_{30}	[m]	0.6498
h_{40}	[m]	0.6592

by the equilibrium levels and flows is shown in Table I, and by defining the deviation variables $x_i = h_i - h_{i0},\ u_j = q_j - q_{j0},\ i \in \{1,2,3,4\},\ j \in \{a,b\}$, the resultant continuous-time linear model is:

$$\frac{dx(t)}{dt} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0 \\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A_4}{A_2\tau_4} \\ 0 & 0 & \frac{-1}{\tau_3} & 0 \\ 0 & 0 & 0 & \frac{-1}{\tau_4} \end{bmatrix} x(t) + \dots$$

$$\dots \begin{bmatrix} \frac{\gamma_1}{A_1} & 0 \\ 0 & \frac{\gamma_1}{A_2} \\ 0 & \frac{(1-\gamma_2)}{A_3} \\ \frac{(1-\gamma_1)}{A_4} & 0 \end{bmatrix} u(t) \tag{10}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

where $\tau_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{i0}}{g}} \geq 0,\ i \in \{1,2,3,4\}$ are the time constants of the tank i. For the parameters chosen the linear system shows four real stable poles and two non-minimum phase multivariable zeros.

With the purpose of applying the proposed MHE, the model (10) was discretized with a sample time $T_s = 5$ s. The resulting model was also used for prediction in the MPC.

B. Simulation Results

For validation purposes, three different operation conditions were simulated in order to compare the performance of the proposed approach with the classical MHE structure:

1. The measurements of the states were taken without delay.
2. The measurements of the states were taken with delay and a fixed structure MHE was implemented.
3. The measurements of the states were taken with delay and the proposed MHE was implemented.

In these three cases the prediction horizon for the MPC was 90 sample times, and the horizon for the MHE was 200 sample times for covering the transient response of the system. For the cases 2 and 3, the delay was considered normally distributed with mean $\mu = 12$ times the sample time, and variance $\sigma^2 = 12$.

The values used for the estimation matrices were $Q = 0.01 I_1$ and $R = 0.01 I_2$. These values were selected after preliminary tests. Figures 3 and 4 show the behavior of the four-tank process when there was no delay in the measurement of the states. Figure 3 shows how the estimated states are coherent with the real variables. Figure 4 shows that the pair MHE-MPC is able to lead the controlled variables of the system to their desired values, and that the control inputs have small amplitude changes before stabilizing. It is possible to conclude that the values of the states given by the MHE converge to their real values without delay. On Figure 3, note that after the convergence of the MHE (and despite the changes on the reference values of the controllable variables) the values of the states estimated by the MHE are the same than their real values. But, if a time delay is included in the measurements of the states, the performance of the system decreases, as shown on Figure 5. On Figure 5 is shown that

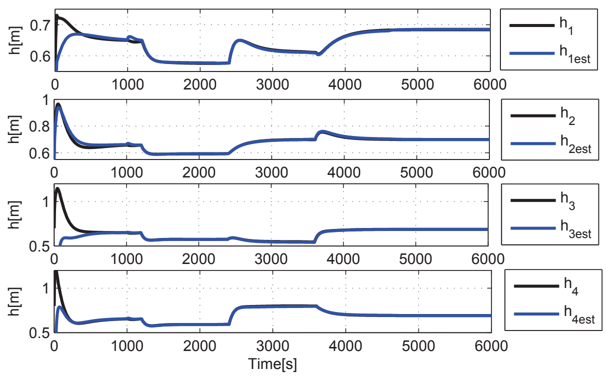

Fig. 3: Evolution of the real and estimated levels. Without delay both the estimated values for controlled tanks h_1 and h_2 (top) and the feeding tanks h_3 and h_4 (bottom) converge to their real values

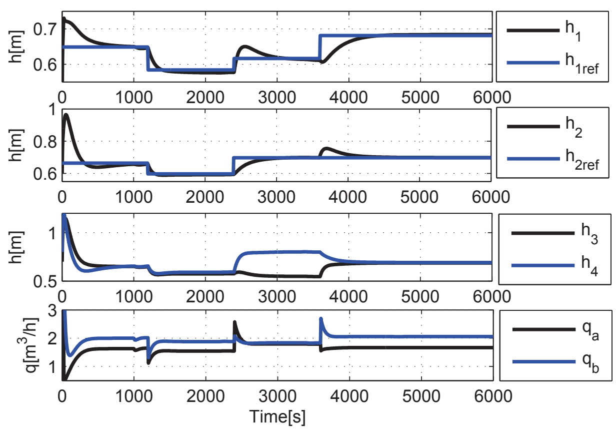

Fig. 4: Evolution of the controlled states corresponding to the levels h_1 and h_2, and their reference values (firs two panels), and of the levels h_3 and h_4 (third panel). On bottom it is shown the evolution of the control inputs q_{a} and q_{b}.

despite of the convergence of the MHE, the pair MHE-MPC is not able to lead the controllable variables to their set-point, because of an oscillatory behavior induced by the delay.

In order to avoid the effects of the delay on the system (which are displayed on Figure 5), the proposed MHE was implemented for the four-tank process. Figures 6 and 7 show the behavior of the system when the random delay is considered and the proposed MHE is implemented. Figure 6 shows that the estimated values achieved the real values without oscillations despite of the random delay, and the changes on the set-points of the controllable variables. Figure 7 presents the entire system behavior. In comparison with the performance of the system without delay, on the initial set-point it is observed an expected transient on the controlled variables, due to the lack of available data for the state estimation. On the second set-point change their behavior is quite similar, the proposed variable structure MHE compensated the random delay effects. However, the control actions computed by the MPC under random delay conditions and with the proposed variable structure MHE, were larger in amplitude than the control actions without delay and fixed structure MHE. In order to compare the controller performance in each of three cases presented before, it was considered as a comparison index the RMS value of the cost function ϕ_{MPC}. Table II summarizes the results obtained for each case.

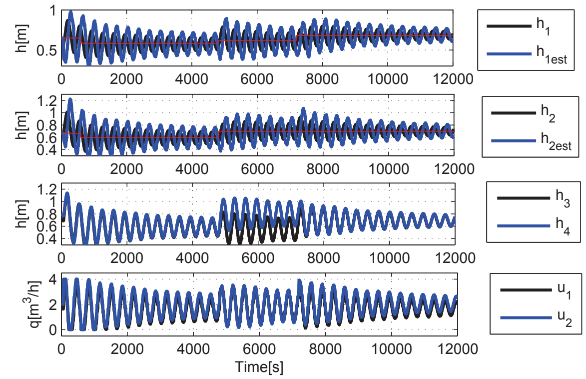

Fig. 5: Evolution of the real and estimated h_1 and h_2 levels compared to their reference value (first two panels), of the levels h_3 and h_4 (third panel), and of the control inputs q_{a} and q_{b} (forth panel), when the delay is included. The states show an oscillatory behavior produced by the delay which effect is not included on the classical MHE approach.

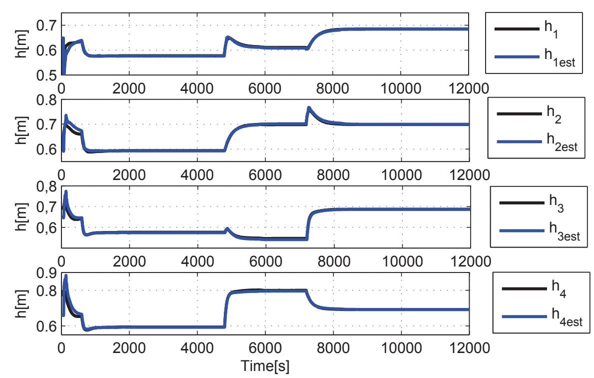

Fig. 6: Evolution of the real and estimated levels when the proposed MHE is implemented. All states converge to their set-point and an expected larger delay can be observed at the beginning of the test.

Case	RMS
Fixed MHE without delay	11.4164
Fixed MHE with delay	60.2313
Variable MHE with delay	4.6017

TABLE II: RMS value of the cost function in the three cases presented in this article

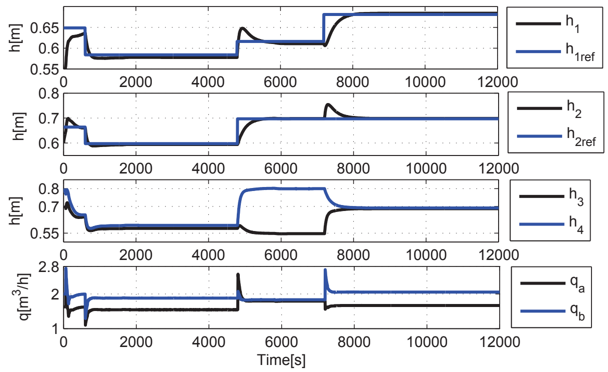

Fig. 7: Evolution of the levels h_1 and h_2, and their reference values (firs two panels), of the levels h_3 and h_4 (third panel), and of the control inputs q_a and q_b, when the proposed MHE is implemented. Larger control actions were observed but no oscillation was present.

Tests of the same experiments with kalman filters did not show large differences with the proposed variable structure MHE (not shown here). But for uniform delay conditions near the time constant of the plant ($\approx 100\ s$) as shown on Figure 8, the kalman filter-MPC pair presented oscillations when changing the set point. On the same Figure are shown the results of the proposed MHE for the same experiment with similar results than those presented in Figures 6 and 7.

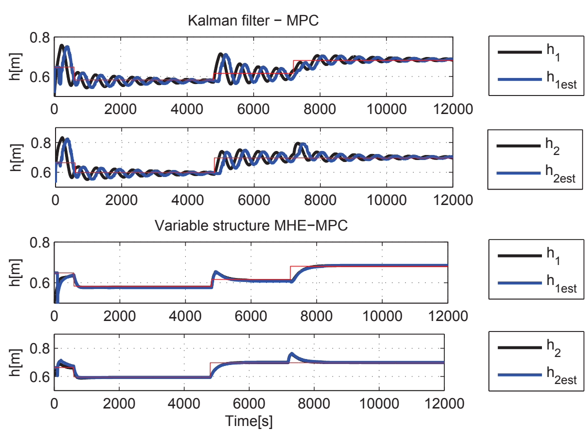

Fig. 8: Large variable delay test on the four tank process for the MPC with a Kalman filter based estimation (first two panels), and a variable structure MHE (last two panels). Even the Kalman filter-MPC can control the h_1 and h_2 levels, there are oscillations not present on the variable structure MHE-MPC

IV. Conclusions and Future Work

In this work the problem of the random delay in the measurements of the states was considered. Here, the delay was assumed random, known, and n times the sample time ($n \in \mathbb{N}$). In order to handle this problem a variable structure MHE was proposed, where the delayed measurements of the states were arranged in a vector and placed on the equivalent position of their true arriving time.

The four tank system was used for validation. A pair MHE-MPC was implemented in order to control it, with two MHE structures: fixed and variable. Variations on the reference value of the controlled variables were made with the purpose of testing the performance of the pair MHE-MPC. When a random delay was included into the measurements of the states, the pair MHE-MPC with fixed MHE structure fell into an oscillatory behavior. Under the same conditions, the proposed MHE improved the performance of the pair MHE-MPC exhibiting a performance similar than the pair MHE-MPC without delay. Then, it is concluded that the MHE with variable structure proposed on this work compensates the effect of the random delay on the state.

References

[1] L. Schenato, “Optimal estimation in networked control systems subject to random delay and packet drop,” *IEEE TRANSACTIONS ON AUTOMATIC CONTROL*, vol. 53, pp. 1311–1317, 2008.

[2] E. Besada-Portas, J. A. Lopez-Orozco, J. A. Besada, and J. M. de la Cruz, “Multisensor out of sequence data fusion for estimating the state of discrete control systems,” *IEEE TRANSACTIONS ON AUTOMATIC CONTROL*, vol. 54, pp. 1728–1732, 2009.

[3] G. C. Goodwin, H. Haimovich, D. E. Quevedo, and J. S. Welsh, “A moving horizon approach to networked control system design,” *Automatic Control, IEEE Transactions on*, vol. 49, no. 9, pp. 1427–1445, sep. 2004.

[4] H. Michalska and D. Q. Mayne, “Moving horizon observers and observer-based control,” *Automatic Control, IEEE Transactions on*, vol. 40, no. 6, pp. 995–1006, jun. 1995.

[5] C. V. Rao, J. B. Rawlings, and D. Q. Mayne, “Constrained state estimation for nonlinear discrete-time systems: Stability and moving horizon approximations,” *IEEE Transactions on Automatic Control*, vol. 48, pp. 246–258, 2003.

[6] A. Bemporad, D. Mignone, and M. Morari, “Moving horizon estimation for hybrid systems and fault detection,” in *Proceedings of the American Control Conference*, 1999, pp. 2471–2475.

[7] N. Haverbeke, T. Van Herpe, M. Diehl, G. Van den Berghe, and B. De Moor, “Nonlinear model predictive control with moving horizon state and disturbance estimation - application to the normalization of blood glucose in the critically ill,” in *Proceedings of the 17th IFAC World Congress*, 2008, pp. 9069–9074.

[8] F. Allgöwer, T. A. Badgwell, S. J. Qin, J. B. Rawlings, and S. J. Wright, “Nonlinear predictive control and moving horizon estimation - an introductory overview,” in *Proceedings of the Advances in Control - Highlights of ECC ’09*, 1999, pp. 391–449.

[9] K. Findeisen, “Moving horizon state estimation of discrete-time systems,” Master’s thesis, University of Wisconsin-Madison, 1997.

[10] C. Rao, “Moving horizon strategies for the constrained monitoring and control of nonlinear discrete-time systems,” Ph.D. dissertation, University of Wisconsin-Madison, 2000.

[11] J. Garcia and J. Espinosa, “Moving horizon estimators for large-scale systems,” *Journal of Control Engineering and Applied Informatics - CEAI-*, vol. 11, no. 3, pp. 49–56, 2009.

[12] M. Farina, G. Ferrari-Trecate, and R. Scattolini, “Distributed moving horizon estimation for sensor networks.” *IFAC Workshop on Estimation and Control of Networked Systems (NecSys’09)*, 2009.

[13] K. H. Johansson, A. Horch, O. Wijk, and A. Hansson, “Teaching multivariable control using the quadruple-tank process,” in *Proceedings of the 1999 IEEE Conference on Decision and Control*, Phoenix, AZ, 1999.