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Adaptive Extended Kalman Filter for Robust Sensorless Control of PMSM Drives

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Abstract— **In this paper a robust sensorless cascade control scheme for a Permanent Magnet Synchronous Motor (PMSM) drive is proposed. A Discrete Time Variable Structure Control (DTVSC) is considered and the rotor position and speed are obtained through an Adaptive Extended Kalman Filter (AEKF). The proposed solution is experimentally tested on a commercial PMSM drive equipped with a control system based on a floating point Digital Signal Processor (DSP).**

I. INTRODUCTION

High performance control of PMSM drives requires the knowledge of the rotor shaft position and speed in order to synchronize the phase excitation pulses to the rotor position [1]. This implies the need for speed or position sensors such as an encoder or a resolver attached to the shaft of the motor. The demand of inexpensive and reliable drives now pushes applied research toward the elimination of mechanical sensors, in particular for mass-produced motors in the kW range [2]. In fact, in most applications, these sensors present several disadvantages, such as reduced reliability, susceptibility to noise, additional cost and weight and increased complexity of the drive system. The position and velocity sensorless control of PMSM drive overcome these difficulties. Therefore, sensorless control of motors based on algorithms simple enough to be executed using low-cost industrial DSP in real-time appears susceptible of industrial interest due to its cost-effective nature and wide applicability to a large class of motors [3]. A comprehensive overview of methods developed to obtain rotor position and angular speed from measurements of electric quantities is reported in [4]–[6].

In this paper, an Adaptive Extended Kalman Filter (AEKF), which is a simple and efficient state estimator for nonlinear systems with inherent robustness against parameter variations, is proposed for the estimation of rotor position and speed of PMSM drives from measurements of electric quantities. Recent advances in digital technology allow nowadays adequate data processing on cost-effective DSP-based platforms, and the EKF can be now considered a viable and computationally efficient tool for position and speed estimation [4]. Theoretical issues and digital implementation of the EKF have been deeply investigated in the past [4], [7], [8] and a novel procedure for the offline tuning of covariance matrices in EKF-based PMSM drives has been presented in [6]. Application examples reported in [5] seem to prove that some well-known pitfalls (such as the starting from unknown rotor position and the filter matrices tuning) have been successfully fixed.

Nonetheless, at least one major drawback of the EKF application to sensorless drives is still unsolved. Indeed, the use of Kalman filtering techniques requires to derive a stochastic state-space representation of the system model and of the measure process, and the design and the online tuning of the covariance matrices appearing in the EKF equations are still an open problem. Most of the EKF techniques proposed in the literature [4]–[8] for state estimation are based on some fixed values of the input and measurement noise covariance matrices. In many practical applications an *a priori* information of this kind is often unavailable and it is necessary to allow the filter to properly weight online the incoming observations. On the other hand it is well known how poor estimates of noise statistics may seriously degrade the Kalman filter performance. The main feature of the Adaptive Extended Kalman Filter (AEKF) here adopted is its capability of online adaptively estimating such unknown statistical parameters. This adaptive solution should reduce customization required by each application that makes most of the EKF-based drives incompatible with an off-the-shelf market strategy. It is worth noticing that particular attention has been paid, in developing the algorithm, to prevention of filter divergence and to the simplicity of implementation, in view of its implementation on commercial DSP.

Considering control issues requiring specific attention in electric drive systems, it is well known that electromechanical parameters are subject to significant variations. A nonlinear control strategy widely recognized and successfully applied in recent years is the Variable Structure Control (VSC) [9]–[11]. Indeed, VSC methods provide robustness to matched uncertainties [10] [12], and are computational simpler with respect to other robust control approaches, thus well suited for low-cost DSP implementation. VSC schemes are typically affected by chattering of the control signal but, as discussed in [10], this well-known implementation drawback of VSC does not cause difficulties for electric drives since the on-off operation mode is the only admissible one for power converters. For PMSM, the cascade control structure of the Field Oriented Control (FOC) is often usefully applied to achieve fast four quadrant operation, smooth starting and acceleration [13]. FOC is implemented with two current controllers in inner control loops and a speed controller in an outer control loop. The speed controller provides the reference current for one of the two inner current control

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loops; this reference current corresponds to the required motor torque. As argued in [10], VSC techniques cannot be applied for the outer speed control loop, since the reference input of the inner control loop should have bounded time derivatives.

To overcome this problem, different approaches have been followed, such as, for instance, the ‘direct speed control’ [10] and the ‘second-order sliding-mode technique’ [9]. Both techniques, however, share a formulation in the continuous time framework, while the practical implementation on a low-cost DSP of a real motor drive claims for a more appropriate formulation of the problem in a sampled-data systems context.

A possible solution was presented in [14] and here proposed with reference to a sensorless PMSM drive. In particular in the present study, a control policy based on Discrete-Time VSC (DTVSC) [15]–[18], endowed with a AEKF for the estimation of the rotor position and speed, is developed and experimentally tested. The introduction of a DTVSC is motivated by the need of taking directly into account the issue of control law digitalization. Reported experimental evidences seem to show that it is actually able to cope with electromechanical disturbances.

Summing up, the features of the DTVSC technique combined with the AEKF-based rotor position and speed estimator are exploited in this work to design the cascade control architecture shown in Fig. 1. In this scheme, the external velocity DTVS control loop, two internal current DTVS control loops and the AEKF-based rotor position and speed estimator can be identified. The task is to make the speed error $\omega_r^* - \hat{\omega}_r$ to tend to zero as close as possible. As well known, the discrete-time sliding mode condition can be imposed exactly only outside a given sector. This issue has been addressed using the approach known as Time Delay Control [19] [20].

The paper is organized as follows. The nonlinear state space model of the PMSM dynamics is presented in Section II. The AEKF algorithm is reported in Section III. In Section IV details on the considered DTVS controller are discussed. Results on experimental tests are reported in Section V.

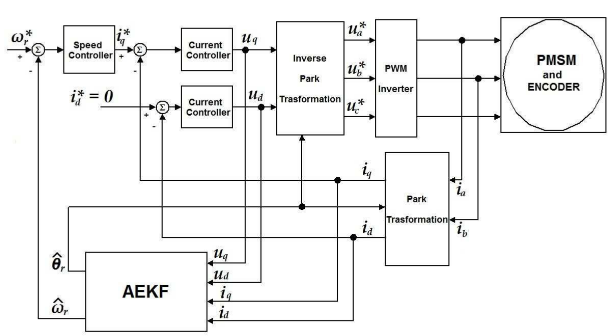

Fig. 1. Block scheme of the proposed cascade controller (FOC)

II. MOTOR DYNAMICS

In the $(d,\, q)$ reference frame, synchronously rotating with the motor rotor, the electrical equations of motion of a permanent-magnet synchronous motor can be written as [10]:

$$\frac{di_d}{dt} = -\frac{R}{L} i_d + \omega_e i_q + \frac{1}{L} u_d \tag{1}$$

$$\frac{di_q}{dt} = -\frac{R}{L} i_q - \omega_e i_d - \frac{1}{L}\lambda_0 \omega_e + \frac{1}{L} u_q \tag{2}$$

where i_d and i_q are the $d-axis$ and $q-axis$ stator currents, respectively; u_d and u_q are the $d-axis$ and $q-axis$ stator voltages, respectively; R is the winding resistance and $L = L_d = L_q$ is the winding inductance on axis d and q; λ_0 is the flux linkage of the permanent magnet and ω_e is the electrical angular speed of the motor rotor.

The electrical torque τ_e and the mechanical power P of the motor are given by $\tau_e = K_t i_q$ and $P = \tau_e \omega_r$ in which $K_t = \frac{3}{2}\lambda_0 N_r$ is the torque constant with N_r the number of pole pairs and ω_r is the mechanical angular speed of the motor rotor. The developed torque of the motor is proportional to the i_q current because of the assumption that there is no reluctance torque in the considered PMSM.

The mechanical motion equation of the motor is described by:

$$J\frac{d\omega_r}{dt} + B\omega_r = \tau_e - \tau_\ell;\ \frac{d\theta_r}{dt} = \omega_r \tag{3}$$

where J is the mechanical inertia of the motor and load, B is the coefficient of viscous friction, τ_ℓ is the load torque and θ_r denotes the mechanical angular position of the motor rotor.

For the electrical angular position/speed and the mechanical angular position/speed, these relations hold: $\omega_e = N_r \omega_r$ and $\theta_e = N_r \theta_r$.

III. ADAPTIVE ESTIMATION OF ROTOR POSITION AND SPEED

The proposed AEKF providing online estimates of rotor position and speed is derived in this section. Denote with $X(t) := \begin{bmatrix} \omega_r(t) & i_d(t) & i_q(t) & \vartheta_r(t) \end{bmatrix}$ the motor state and with $U(t) := \begin{bmatrix} u_d(t) & u_q(t) \end{bmatrix}$ the motor control input. The motor nonlinear dynamic state space model can be written in the compact form of the following stochastic differential equation:

$$dX(t) = F(X(t), U(t))dt + d\eta(t), \tag{4}$$

where $F(X(t), U(t))$ is obtained by (1), (2), (3) and $\eta(t)$ is a Wiener process such that $E(d\eta(t)d\eta(t)^T) = Q(t)dt$. Its weak mean square derivative $d\eta(t)/dt$ is a white noise process $\sim N(0, Q(t))$ representing the model inaccuracies. Assuming a constant sampling period $\Delta t_k = T_c$ and denoting t_{k+1} by $(k+1)T_c$, the following sampled nonlinear measure equation can be associated to equation (4):

$$Z((k+1)T_c) = G(X((k+1)T_c)) + v(kT_c), \tag{5}$$

where $Z(kT_c)$ is the vector containing measures of motor phase currents and $v(kT_c)$ is a white sequence $\sim$

$N(0, R(kT_c))$. The measure vector $Z(kT_c)$ is composed of two elements, i.e. $Z(kT_c) = [z_1(kT_c)\ \ z_2(kT_c)]^T$, where $z_1(kT_c) = i_d(kT_c) + v_1(kT_c)$, $z_2(kT_c) = i_q(kT_c) + v_2(kT_c)$.

By definition of the measurement vector one has that the output function $G(X((k+1)T_c))$ has the following form:

$$G(X(kT_c)) = \begin{bmatrix} i_d(kT_c) & i_q(kT_c) \end{bmatrix}^T = C(kT_c)X(kT_c) \quad (6)$$

where

$$C(kT_c) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (7)$$

and $v(kT_c) = [v_1(kT_c)\ v_2(kT_c)]^T$. Assume $U(t) = U(kT_c)$ for $t \in [kT_c, (k+1)T_c)$. To obtain an extended Kalman filter with an effective state prediction equation in a simple form, model (1) and (2) has been linearized about the current state estimate $\hat{X}(kT_c, kT_c)$ and the control input $U((k-1)T_c)$ applied until the linearization instant. Subsequent discretization with period T_c of the linearized model results in the EKF reported in [21] (where explicit dependence on T_c has been dropped for simplicity of notation). In particular in [21], a simplification assumption has been introduced to obtain an input noise covariance matrix $Q_d(k)$ which is completely known up to the unknown multiplicative scaling factor $\sigma_\eta^2(k)$. Moreover, the covariance matrix $R(k)$ is assumed to have the following diagonal form:

$$R(k) = diag[\sigma_{v,1}^2(k), \sigma_{v,2}^2(k)]; \quad (8)$$

this means that no correlation is assumed between the measurement errors introduced by the sensors [21].

The EKF can be implemented once estimates of $Q_d(k)$ and $R(k)$ are available. In general, a complete and reliable information about these matrices is not available; on the other hand it is well known how poor knowledge of noise statistics may seriously degrade the Kalman filter performance. This problem is here dealt with introducing an adaptive adjustment mechanism of $Q_d(k)$ and $R(k)$ values in the EKF equations.

A. Adaptive Estimation of $Q_d(k)$ and $R(k)$

A considerable amount of research has been carried out in the adaptive Kalman filtering area [22]–[24], but in practice it is often necessary to redesign the adaptive filtering scheme according to the particular characteristics of the problem faced. Following [22], in view of real time applications, a particular attention has been here devoted to simplicity of implementation and to prevention of filter divergence, moreover, the particular structure of the input noise covariance matrix $Q_d(k)$, which is completely known save that for a multiplicative scalar, has been suitably taken into account.

The following nearly stationarity assumption is made: the parameters $\sigma_{v,i}^2(k)$, $i = 1, 2$, and $\sigma_\eta^2(k)$ are nearly constant over $n_v \geq 2$ and $n_\eta \geq 2$ samples respectively [22].

Define $\gamma_i(k+1) = z_i(k+1) - G_i(\hat{X}(k+1,k))$, where $z_i(k+1)$ and $G_i(\hat{X}(k+1,k))$ are the i-th component of $Z(k+1)$ and $G(\hat{X}(k+1,k))$, respectively. For analogy with the linear case, residuals $\gamma_i(k+1), i = 1, 2$, are called the innovation process samples and are assumed to be well described by a white sequence $\sim N(0, s_i(k+1))$, where $s_i(k+1), i = 1, 2$ can be expressed as

$$\begin{aligned} s_i(k+1) &= C_i(k+1)P(k+1,k)C_i^T(k+1) \\ &+ \sigma_{v,i}^2(k+1) \\ &= C_i(k+1)[A_d(k)P(k,k)A_d^T(k) \\ &+ \sigma_\eta^2(k)\bar{Q}(k)]C_i^T(k+1) + \sigma_{v,i}^2(k+1) \end{aligned} \quad (9)$$

This simplifying assumption is as more valid as discretization and linearization of (4) is more accurate and makes it possible to apply the methods of the adaptive filtering theory developed for the linear case.

The two above assumptions will allow us to define a simple and efficient estimation algorithm based on the condition of consistency, at each step, between the observed innovation process samples $\gamma_i(k+1), i = 1, 2$ and their predicted statistics $E\{\gamma_i^2(k+1)\} = s_i(k+1)$. Imposing such a condition, one stage estimates $\hat{\sigma}_\eta^2(k)$ and $\hat{\sigma}_{v,i}^2(k+1), i = 1, 2$, of $\sigma_\eta^2(k)$ and $\sigma_{v,i}^2(k+1), i = 1, 2$, respectively are obtained at each step. To increase their statistical significance, the one stage estimates $\hat{\sigma}_\eta^2(k)$ and $\hat{\sigma}_{v,i}^2(k+1), i = 1, 2$, are averaged obtaining the relative smoothed versions $\bar{\hat{\sigma}}_\eta^2(k)$ and $\bar{\hat{\sigma}}_{v,i}^2(k+1), i = 1, 2$.

From (9) it is apparent that the statistical information carried by each $\gamma_i(k+1), i = 1, 2$, depends, at the same time, on the two unknown parameters $\sigma_\eta^2(k)$ and $\sigma_{v,i}^2(k+1)$.

This indeterminateness is here dealt with using a number (say n'_η) of innovation process samples $\gamma_i(k+1), i = 1, 2$, to estimate $\sigma_\eta^2(k)$ and the others (say n'_v) to estimate $\sigma_{v,i}^2(k+1)$. In the light of the nearly stationarity assumption, the two integers n'_η and n'_v are chosen such that $n'_\eta / n'_v = n_v / n_\eta$.

Assume $n_v \geq n_\eta$, let α and β two coprime integers such that $\alpha/\beta = n_v/n_\eta$ and let q and r two integers such that $\alpha = \beta q + r$; then, the innovation process sequence is subdivided into intervals $I_{\alpha+\beta}$ composed of $\alpha + \beta$ samples. Each interval contains β sequences of q samples used to estimate $\sigma_\eta^2(k)$ (the faster varying parameter), the ensembles of q samples are separated by β sequences of one sample used to estimate $\sigma_{v,i}^2(k+1), i = 1, 2$ (the more slowly varying parameter), the last r samples of each $I_{\alpha+\beta}$ interval are used to estimate $\sigma_\eta^2(k)$. This scheme minimizes the interval of time over which either one step estimate is not updated. A symmetric situation holds if $n_\eta \geq n_v$.

When the one step estimate $\hat{\sigma}_\eta^2(k)(\hat{\sigma}_{v,i}^2(k+1), i = 1, 2)$ is updated, the other single stage estimate $\hat{\sigma}_{v,i}^2(k+1), i = 1, 2, (\hat{\sigma}_\eta^2(k))$ is kept constant, so that the symbol $\hat{\sigma}_\eta^2(k)(\hat{\sigma}_{v,i}^2(k+1))$ does not necessarily imply that this estimate has been computed using the last observed innovation process sample $\gamma_i(k+1), i = 1, 2$.

Because of the particular form of $Q_d(k)$ and of the sequential scalar processing of measures, 2 one stage estimates $\hat{\sigma}_{\eta,i}^2(k)$ of the unknown $\sigma_\eta^2(k), i = 1, 2$ can be determined maximizing the probability of observing the corresponding i-th component of the predicted residual $\gamma_i(k+1), i = 1, 2$ [22]. Namely, each $\hat{\sigma}_{\eta,i}^2(k)$ is determined by the operation

$$\max \text{prob}_{\sigma_{\eta,i}^2(k+1) \geq 0} \gamma_i(k+1).$$

The maximizing $\hat{\sigma}^2_{\eta,i}(k)$ is obtained by imposing the condition of consistency between residuals and their predicted statistics, namely $\gamma_i^2(k+1) = E\{\gamma_i^2(k+1)\} = s_i(k+1)$. Using (9) and replacing $\sigma^2_{v,i}(k+1)$ with $\bar{\hat{\sigma}}^2_{v,i}(k+1)$ one has

$$\begin{aligned}\hat{\sigma}^2_{\eta,i}(k) &= \max\left\{(C_i(k+1)\bar{Q}(k)C_i^T(k+1))^{-1}[\gamma_i(k+1)^2\right.\\ &\quad -C_i(k+1)A_d(k)P(k,k)A_d^T(k)C_i^T(k+1)\\ &\quad \left.-\bar{\hat{\sigma}}^2_{v,i}(k+1)],0\right\}. \qquad (10)\end{aligned}$$

To obtain a unique estimate of $\sigma^2_\eta(k)$ and to increase the statistical significance of estimators (10), which are based on only one component $\gamma_i(k+1)$, the following smoothed estimate is computed

$$\bar{\hat{\sigma}}^2_\eta(k) = \frac{1}{2(l_\eta+1)}\sum_{j=0}^{l\eta}\sum_{i=1}^{2}\hat{\sigma}^2_{\eta,i}(k-j), \qquad (11)$$

where l_η denotes the number of one-stage estimates $\hat{\sigma}^2_{\eta,i}(\cdot)$ yielding the smoothed estimate.

In a recursive form the proposed estimate of $\sigma^2_\eta(k)$ is

$$\begin{aligned}&\bar{\hat{\sigma}}^2_\eta(k) = \bar{\hat{\sigma}}^2_\eta(k-1)\\ &+\frac{1}{2(l_\eta+1)}\left[\sum_{i=1}^{2}\left(\hat{\sigma}^2_{\eta,i}(k)-\hat{\sigma}^2_{\eta,i}(k-(l_\eta+1))\right)\right]. \qquad (12)\end{aligned}$$

Analogously, the operation

$$\max \mathrm{prob}_{\sigma^2_{v,i}(k+1)\geq 0}\gamma_i(k+1)$$

and (9) give the following one stage estimate of $\sigma^2_{v,i}(k+1), i=1,2,$

$$\begin{aligned}\hat{\sigma}^2_{v,i}(k+1) &= \max\{\gamma_i^2(k+1)-[C_i(k+1)A_d(k)P(k,k)\\ &\quad A_d^T(k)C_i^T(k+1)+C_i(k+1)\bar{\hat{\sigma}}^2_{\eta,i}(k)\bar{Q}(k)\\ &\quad C_i^T(k+1)],0\}, \qquad (13)\end{aligned}$$

the smoothed version $\bar{\hat{\sigma}}^2_{v,i}(k+1)$ is

$$\bar{\hat{\sigma}}^2_{v,i}(k+1) = \frac{1}{l_v+1}\sum_{j=0}^{l_v}\hat{\sigma}^2_{v,i}(k+1-j), \qquad (14)$$

where l_v denotes the number of one-stage estimates $\hat{\sigma}^2_{v,i}(\cdot)$ yielding the smoothed estimate.

In a recursive form the proposed estimates of $\sigma^2_{v,i}(k+1)$ becomes

$$\bar{\hat{\sigma}}^2_{v,i}(k+1) = \bar{\hat{\sigma}}^2_{v,i}(k)+\frac{1}{l_v+1}(\hat{\sigma}^2(k+1)-\hat{\sigma}^2(k-l_v)). \quad (15)$$

The proposed adaptive estimation algorithm is given by equations (12), (15) and is able to prevent filter divergence. In fact, as long as the innovation samples $\gamma_i(k+1), i=1,2$ are sufficiently small and consistent with their statistics, the filter operates satisfactorily and the noise model is kept small (or null) by (10). If a sudden increase of the absolute value of the innovation process samples is observed, equation (10) provides an increased $\hat{Q}_d(k) = \bar{\hat{\sigma}}^2_\eta(k)\bar{Q}(k)$, and hence an augmented filter gain, thus preventing filter divergence.

Parameters l_η and l_v of estimators (12) and (15) are chosen on the basis of two antagonist considerations: low values would produce noise estimators which are not statistically significant, large values would produce estimators which are scarcely sensitive to possible rapid fluctuations of the true $\sigma^2_\eta(k)$ and $\sigma^2_{v,i}(k), i=1,2$. During filter initialization, the starting values $\hat{\sigma}^2_\eta(0)$ and $\hat{\sigma}^2_{v,i}(0), i=1,2,$ of $\hat{\sigma}^2_\eta(k)$ and $\hat{\sigma}^2_{v,i}(k)$ respectively, must be chosen on the basis of the *a priori* available information. In the case of a lack of such information, a large value of $P(0,0)$ is useful to prevent divergence.

Remark 3.1: As stated in [4], to reduce the computational effort for a real time implementation of the EKF an acceptable approximation is to use a diagonal covariance matrix $Q_d(k)$.

IV. CONTROL DESIGN

The discretization of the model equations with a sampling time T_c according to standard techniques gives [21]:

$$\begin{aligned}\omega_e(k+1) &= A_\omega\omega_e(k)+B_\omega(K_t i_q(k)-\tau_\ell) &(16)\\ i_d(k+1) &= A_i i_d(k)+B_i u_d(k)+f_1(\omega_e,i_q,k) &(17)\\ i_q(k+1) &= A_i i_q(k)+B_i u_q(k)-f_2(\omega_e,i_d,k). &(18)\end{aligned}$$

To account for possible model uncertainties, it is assumed that model parameters may differ from their nominal values for some unknown but bounded quantities:

$$\begin{aligned}A_\omega = \bar{A}_\omega+\Delta A_\omega;\quad & B_\omega = \bar{B}_\omega+\Delta B_\omega;\\ |\Delta A_\omega|\leq\rho_{A\omega};\quad & |\Delta B_\omega|\leq\rho_{B\omega}\\ A_i = \bar{A}_i+\Delta A_i;\quad & B_i = \bar{B}_i+\Delta B_i;\\ |\Delta A_i|\leq\rho_{Ai};\quad & |\Delta B_i|\leq\rho_{Bi}. \qquad (19)\end{aligned}$$

Define the following discrete-time sliding surfaces:

$$\begin{aligned}s_\omega(k) &= (\hat{\omega}_e(k)-\omega_e^*(k))\\ &+ \lambda_\omega(\hat{\omega}_e(k-1)-\omega_e^*(k-1)) = 0 &(20)\\ s_{iq}(k) &= (i_q(k)-i_q^*(k))\\ &+ \lambda_q(i_q(k-1)-i_q^*(k-1)) = 0 &(21)\\ s_{id} &= i_d(k)+\lambda_d i_d(k-1) = 0 &(22)\end{aligned}$$

where $\lambda_\omega,\lambda_q,\lambda_d\in(-1,1)$, $\hat{\omega}_e(k)$ is the estimate of $\omega_e(k)$ provided by the AEKF, $\omega_e^*(k)$ is the given reference value for the angular velocity, and $i_q^*(k)$ will be defined in the following.

As well known, a quasi sliding motion on the surface $s_\omega(k)=0$ can be achieved imposing the following discrete time sliding mode existence condition [16], [17]:

$$s_\omega(k)\Delta s_\omega(k+1) < -\frac{1}{2}\left[\Delta s_\omega(k+1)\right]^2 \qquad (23)$$

being $\Delta s_\omega(k+1) = s_\omega(k+1)-s_\omega(k)$. It can be easily verified that condition (23) is ensured by the control law $i_q^*(k) = i_q^{eq}(k)+i_q^n(k)$, where the equivalent control is given by:

$$i_q^{eq}(k) = \frac{1}{\bar{B}_\omega K_t}(\omega_e^*(k+1)-\bar{A}_\omega\hat{\omega}_e(k)-\lambda_\omega(\hat{\omega}_e(k)-\omega_e^*(k))) \qquad (24)$$

As usual, the discontinuous control i_q^n is such that the sliding condition can be imposed exactly only outside a given sector.

Inside such sector the sliding condition can be imposed only approximately. To this purpose we made resort to the approach known as Time Delay Control [19], obtaining

$$i_q^n(k) = \begin{cases} \theta_\omega \dfrac{|s_\omega(k)| - \rho_\omega}{\bar{B}_\omega K_t} & if\ |s_\omega(k)| > \rho_\omega \\ -\dfrac{s_\omega(k) - \bar{B}_\omega i_q^n(k-1)}{\bar{B}_\omega K_t} & if\ |s_\omega(k)| \leq \rho_\omega \end{cases} \tag{25}$$

with $|\theta_\omega| \leq 1$, and with

$$\rho_\omega = (|\bar{B}_\omega| + \rho_{B\omega})\rho_\tau + \rho_{A\omega}\omega_e^{max} + K_t \rho_{B\omega} i_q^{max}$$

ρ_τ being the constant bound of the unknown load which can affect the motor, i.e. $|\tau_\ell| \leq \rho_\tau$. Note that ω_e^{max} and i_q^{max} are the largest speed achievable by the motor and the largest current which can be supplied, respectively, according to its constructive limits.

The control law $i_q^*(k)$ is fed as reference value, which is the required motor torque, to one of the two inner current control loops. The tracking of such reference is ensured by the imposition of a quasi sliding motion of the surface $s_{iq}(k) = 0$. Following the same lines as before, it can be easily verified that the sliding condition on $s_{iq}(k) = 0$ is ensured by the control law $u_q(k) = u_q^{eq}(k) + u_q^n(k)$, where:

$$u_q^{eq}(k) = \frac{1}{\bar{B}_i}\left[i_q^*(k) - \bar{A}_i i_q(k) - \lambda_q(i_q(k) - i_q^*(k))\right] \tag{26}$$

$$u_q^n(k) = \begin{cases} \theta_q \dfrac{|s_{iq}(k)| - \rho_q}{\bar{B}_i} & if\ \ |s_{iq}(k)| > \rho_q \\ -\dfrac{s_{iq}(k) - \bar{B}_i u_q^n(k-1)}{\bar{B}_i} & if\ \ |s_{iq}(k)| \leq \rho_q \end{cases} \tag{27}$$

where $|\theta_q| \leq 1$, $\rho_q = \rho_{Ai} i_q^{max} + \rho_{Bi} u_q^{max} + \rho + \omega_e^{max}(i_d^{max} + \frac{\lambda_0}{L})T_c$, ρ being the bound of $\Delta i_q^*(k) = |i_q^*(k+1) - i_q^*(k)|$.

Finally, the achievement of a quasi sliding motion on $s_{id}(k) = 0$ guarantees the vanishing of the variable $i_d(k)$, and is ensured by the control law:

$$u_d^{eq}(k) = -\frac{(\bar{A}_i + \lambda_d) i_d(k)}{\bar{B}_i} \tag{28}$$

$$u_d^n(k) = \begin{cases} \theta_d \dfrac{|s_{id}(k)| - \rho_d}{\bar{B}_i} & if\ \ |s_{id}(k)| > \rho_d \\ -\dfrac{s_{id}(k) - \bar{B}_i u_d^n(k-1)}{\bar{B}_i} & if\ \ |s_{id}(k)| \leq \rho_d \end{cases} \tag{29}$$

where $|\theta_d| \leq 1$ and $\rho_d = \rho_{Ai} i_d^{max} + \rho_{Bi} u_d^{max} + \omega_e^{max} i_q^{max} T_c$.

V. EXPERIMENTAL RESULTS

The proposed DTVS controller and AEKF-based rotor position and speed estimator have been implemented on the Technosoft MCK28335-Pro DSP motion control kit [25]. In the proposed solution the reference of the direct current component (i_d^*) is set to zero (see Fig. 1). This case corresponds to the motion of the motor in the normal speed range, without considering possible field weakening operations [10]. The sampling frequency is selected as 1 kHz for the velocity control loop and 10 kHz for the current control loops.

A sample of the performed speed-tracking experiments considering the proposed DTVSC equipped with the AEKF-based rotor position and speed estimator is shown in Fig. 2. A comparison with the performance of a PI-based FOC equipped with a conventional backward-difference method for speed estimation, using sampled position measurements provided by a digital incremental encoder, has been also made.

In particular, the Fig. 2 shows one of the tests performed with a time-varying disturbance acting on the i_q current for the rectangular reference velocity profile. It is apparent from the inspection of Fig. 2 that, with the PI-based FOC with the encoder and the backward-difference based speed estimator, in response to disturbances acting in the electrical subsystem the actual velocity deviates significantly from the reference (Fig. 2(b)), while the DTVSC-based FOC with the AEKF-based rotor position and speed estimator performs a more accurate tracking (Fig. 2(a)).

In Fig. 2(c), the AEKF-based estimated rotor position (blue continuous line) is compared with the encoder-based measured one (red dashed line); the estimated position shows good correspondence to the measured rotor position. The criterium IAE, i.e. the integral of the absolute value of the speed-tracking error and of the error between the estimated and the encoder-based measured rotor position, is used to summarize the above experimental result (see Table I). In Table I are also reported results for the motor following speed trajectories chosen with trapezoidal and sinusoidal shapes and without the time-varying disturbance. Fig. 2(d) shows

TABLE I
PERFORMANCE COMPARISON.

No dist.	AEKF-speed	backward-difference	AEKF-position
Rectangular	1.98	2.90	0.16
Trapezoidal	0.35	1.01	0.14
Sinusoidal	0.35	2.19	0.30
Dist.	AEKF-speed	backward-difference	AEKF-position
Rectangular	2.12	3.04	0.18
Trapezoidal	0.39	1.23	0.15
Sinusoidal	0.43	2.29	0.31

the behavior of the estimated $\bar{\hat{\sigma}}_\eta^2(k)$ assuming $l_\eta = 5$ and the initial value $\bar{\hat{\sigma}}_\eta^2(0) = 0.3$ for the rectangular velocity profile. These figures evidence increases of $\bar{\hat{\sigma}}_\eta^2(k)$ in correspondence of the initial time instant, time instants when step changes of the reference trajectory occur and also when electrical disturbances act on the motor drive.

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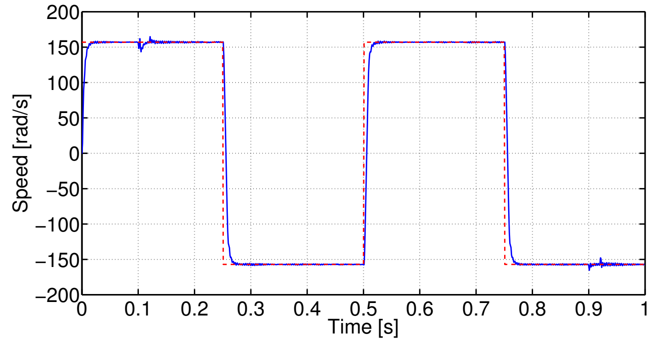
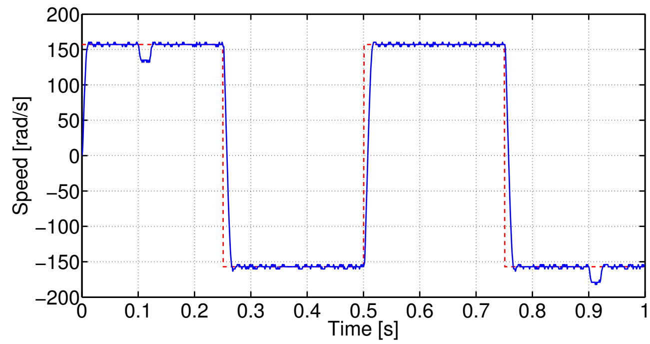


(a)



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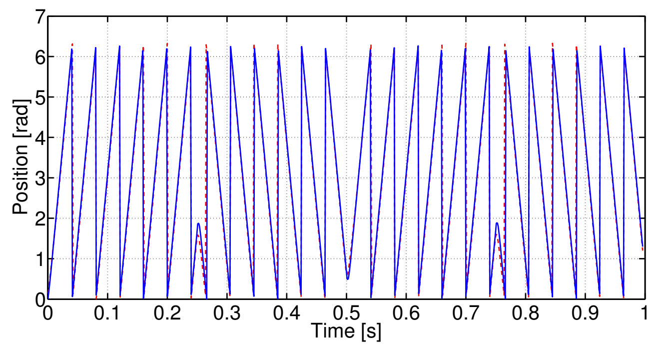

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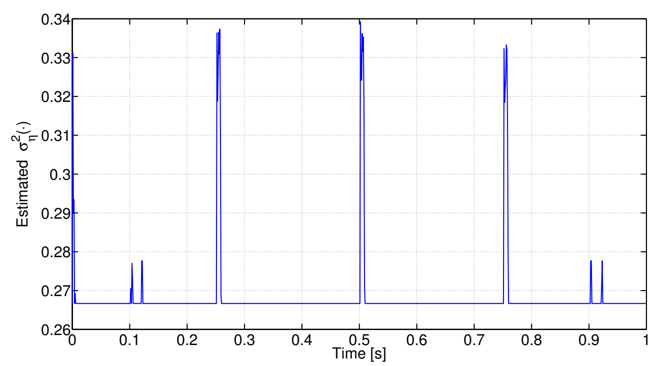

(d)

Fig. 2. Trapezoidal velocity profile; a time-varying disturbance acts on the i_q current. Actual (blue continuous line) and reference (red dashed line) velocities: (a) DTVSC-based FOC with the AEKF-based rotor position and speed estimator; (b) PI-based FOC with the encoder and the backward-difference based speed estimator; (c) AEKF-based estimated rotor position (blue continuous line) and encoder-based measured rotor position (red dashed line); (d) Behavior of the estimated $\bar{\hat{\sigma}}^2_\eta(\cdot)$ assuming $l_\eta = 5$ and $\bar{\hat{\sigma}}^2_\eta(0) = 0.3$.