2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

State Feedback Control Against Sensor Faults for Lipschitz Nonlinear Systems via New Sliding Mode Observer Techniques

Ming Liu, Lixian Zhang, Peng Shi, and Hamid Reza Karimi

Abstract— **This paper investigates the problem of simultaneous state and fault estimation and observer-based fault tolerant controller design for Lipschitz nonlinear systems with sensor failure. A new estimation technique is presented in this paper to deal with this design problem. In the proposed approaches, the original system is first augmented by a descriptor model transformation, then a new *Proportional and Derivative sliding mode* observer technique is developed to obtain accurate estimations of both system states and sensor faults. The designing observer is generalized from the PD observer in [3], but is not a trivial extension. Based on the state estimates, a observer-based control strategy is developed to stabilize the resulting closed-loop system. Finally, a numerical example is presented to illustrate the effectiveness and applicability of the proposed technique.**

I. INTRODUCTION

In practical industrial process, sensor and actuator faults may possibly result in unsatisfactory performance or even instability, especially for complex safety critical systems, e.g. space craft and nuclear power plant, aircraft space, etc. To pursue an ideal performance of the control system, it is desirable for faults and failures to be detected and estimated, or to design control scheme in the presence of faults such that the stability and performance of the closed-loop system can be maintained. Hence, fault tolerance can be classified into the following two categories: 1) Fault Detection Isolation (FDI), that is, detect, estimate or reconstruct faults by developing effective filters or observers techniques [2]; 2) Fault Tolerant Control (FTC), that is, design reliable control schemes independent of fault/failure, main approaches include passive redundancy controller, H_∞ reliable control [4], [5]. In particular, sliding mode control scheme has been applied to fault estimation and tolerant control [6], [7] since such type of control strategy is robust to system uncertainties [8], [9], [10].

This work was supported by the Key Laboratory of Integrated Automation for the Process Industry (Northeastern University), Ministry of Education, China, by the Fundamental Research Funds for the Central Universities (Grant No. HIT.KLOF.2010019 and HIT. NSRIF. 201161), by the National Natural Science Foundation of China (61104101).

Ming Liu is with the School of Astronautics, Harbin Institute of Technology, Harbin 150001, China. Email: robustcontrol23@gmail.com.

Lixian Zhang is with the Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin, Heilongjiang 150001, China. Email: lixianzhang@hit.edu.cn.

Peng Shi is with the Department of Computing and Mathematical Sciences, University of Glamorgan, Pontypridd, CF37 1DL, U.K., with the School of Engineering and Science, Victoria University, Melbourne, 8001 Vic., Australia. Email: peng.shi@vu.edu.au.

H. R. Karimi is with the Department of Engineering, Faculty of Technology and Science, University of Agder, 4898 Grimstad, Norway. Email: hamid.r.karimi@uia.no.

It is worth pointing out that, recently an significant and effective fault estimation scheme has been designed in [3], where a proportional and derivative observer technique is proposed to deal with the sensor/actuator faults, such that the fault estimation can be obtained. However, in the approach of [3] the *derivative observer gain* is required to be a high gain, which results in a higher cost in practical application. Motivated by this observation and based on the work of [3], in this paper, we will develop a new fault estimation scheme incorporated with the characterizations of both PD observer and sliding mode observer for a class of Lipschitz nonlinear systems. Compared with the approach of [3], the main advantage of our method is that a discontinuous control term is injected into the observer to eliminate the effects of sensor faults instead of designing the derivative observer gain as a high gain. Hence, compared with the approach of [3], our method is more feasible effective to be performed in practice.

In this paper, the objective is to design an effective observer technique and an observer-based fault tolerant control scheme for a class of Lipschitz nonlinear systems in the presence of sensor failures. The design procedure is composed by the following several steps: (i) the original plant is first augmented into a descriptor system, in which the sensor fault is assembled into the state vector of the augmented singular system to facilitate the analysis and synthesis. (ii) a new *PD sliding mode observer* is proposed which is generalized from the PD observer in [3] by injecting a discontinuous input into the observer to reject the sensor faults and Lipschitz nonlinearity. In light of the proposed estimation technique, accurate estimation of the system state and the sensor fault can be obtained simultaneously. (iii) based on the state estimation, an observer-based fault tolerant control strategy is developed to stabilize the plant.

It is worth pointing out that our work is not a simple extension of [3]. The main difficulties come from the sliding surface function design based on the state estimation error vector and the stability and reachability analysis for the resulted sliding-mode dynamics. Thus, how to design an appropriate sliding surface function and perform stability and reachability analysis for the resulted sliding-mode dynamics are the main problems to be solved in this paper.

This paper is organized as follows. Section 2 provides preliminaries for subsequent developments. It is followed by the PD sliding mode observer design in section 3. In section 4, the observer-based fault tolerant control strategy is investigated. Finally, in Section 5, a numerical example is provided to demonstrate the effectiveness of the proposed methods.

II. Problem Formulation

Consider the following continuous-time Lipschitz nonlinear system

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + f_d(t,x,u), \\ y_s(t) &= Cx(t) + f_s(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, $y_s(t) \in \mathbb{R}^p$ is the fault measurement output, $f_d(t) \in \mathbb{R}^n$ is the real nonlinear vector function, $f_s(t) \in \mathbb{R}^p$ is the unknown sensor fault vector. Throughout this paper, the following assumptions are made:

(A1) The pair $(A,\ C)$ is observable;

(A2) The nonlinear function $f_d(t)$ satisfies the Lipschitz constraint

$$\begin{aligned} &\|f_d(t,\hat{x},u) - f_d(t,x,u)\| \\ \le\ & \|GC(\hat{x}(t) - x(t))\| \\ \le\ & \eta_0 \|C(\hat{x}(t) - x(t))\| \\ \le\ & \eta \|(\hat{x}(t) - x(t))\|, \end{aligned} \quad (2)$$

and $\eta_0 \triangleq \|G\|$, $\eta \triangleq \|GC\|$ are positive scalars;

(A3) The sensor fault vector $f_s(t)$ satisfies the following norm bounded constraint:

$$\|f_s(t)\| \le \alpha, \quad (3)$$

and $\alpha > 0$ is a known constant.

In this paper, the main objectives to be addressed are formulated as follows: (i) An effective estimation technique is proposed for system (1) to obtain the accurate estimations of $x(t)$ and $f_s(t)$ simultaneously. (ii) Based on the state estimation, an observer-based control scheme is developed such that the closed-loop system is asymptotically stable.

III. Design of the Observer Dynamics

To begin the presentation of our design approach, we define the following augmented variables and matrices

$$\begin{aligned} \bar{x}(t) &\triangleq \begin{bmatrix} x(t) \\ f_s(t) \end{bmatrix},\ \bar{A} \triangleq \begin{bmatrix} A, & 0 \\ 0, & -I_p \end{bmatrix}, \\ \bar{B} &\triangleq \begin{bmatrix} B \\ 0 \end{bmatrix},\ \bar{C} \triangleq \begin{bmatrix} C, & I_p \end{bmatrix}, \\ \bar{E} &\triangleq \begin{bmatrix} I_n, & 0 \\ 0, & 0_{p\times p} \end{bmatrix},\ \bar{N} \triangleq \begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}, \\ \bar{F} &\triangleq \begin{bmatrix} I_n & 0^T_{p\times n} \end{bmatrix}^T, \end{aligned} \quad (4)$$

and we construct the following augmented plant

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}u(t) + \bar{F}f_d(t,x,u) \\ & \quad +\bar{N}f_s(t), \\ y_s(t) &= \bar{C}\bar{x}(t). \end{cases} \quad (5)$$

System (5) is a *descriptor nonlinear* model where the state vector $x(t)$ and the sensor fault variable $f_s(t)$ are both the components of the descriptor state vector. If an ideal state observer can be constructed for the augmented plant (5), the accurate estimation of system state $x(t)$ and sensor fault $f_s(t)$ can be obtained simultaneously. It is observed that the system matrices $\bar{E}$ and $\bar{C}$ has the following property

$$\text{rank}\begin{bmatrix} \bar{E} \\ \bar{C} \end{bmatrix} = \text{rank}\left[\begin{array}{c|c} I_n & 0 \\ 0 & 0_{p\times p} \\ \hline C & I_p \end{array}\right] = n + p,$$

and thus an appropriate gain $\bar{L}_D \in \mathbb{R}^{(n+p)\times p}$ can be selected such that the new defined matrix $\bar{S} \triangleq (\bar{E} + \bar{L}_D\bar{C})$ is non-singular. We decompose $\bar{L}_D$ into the following form $\bar{L}_D = \begin{bmatrix} \bar{L}^T_{D1} & \bar{L}^T_{D2} \end{bmatrix}^T$, and it is derived that

$$\begin{aligned} \bar{S}^{-1} &= \begin{bmatrix} I_n, & -\bar{L}_{D1}(\bar{L}_{D2})^{-1} \\ -C, & (I_p + C\bar{L}_{D1})(\bar{L}_{D2})^{-1} \end{bmatrix} \\ \bar{C}\bar{S}^{-1}\bar{L}_D &= I_p. \end{aligned} \quad (6)$$

Motivated by this observation, the following *PD sliding mode observer* is presented for system (5)

$$\begin{cases} \bar{S}\dot{\bar{z}}(t) &= (\bar{A} - \bar{L}_p\bar{C})\bar{z}(t) + \bar{A}\bar{S}^{-1}\bar{L}_D y_s(t) \\ & \quad +\bar{B}u(t) + \bar{L}_s u_s(t) + \bar{F}f_d(t,\hat{x},u) \\ \hat{\bar{x}}(t) &= \bar{z}(t) + \bar{S}^{-1}\bar{L}_D y_s(t), \end{cases} \quad (7)$$

where $\bar{z}(t) \triangleq [z^T_x(t),\ z^T_f(t)]^T$ is the middle variable; $\hat{\bar{x}}(t) \triangleq [\hat{x}^T(t),\ \hat{f}^T_s(t)]^T$ is the estimation of $\bar{x}(t)$; $\bar{L}_p$ and $\bar{L}_D \in \mathbb{R}^{n\times p}$ are the proportional gain and derivative gain matrices respectively to be designed; $\bar{S} \triangleq \bar{E} + \bar{L}_D\bar{C}$ is a non-singular parameter matrix by selecting appropriate $\bar{L}_D$ (such $\bar{L}_D$ can be always found as discussed below); $u_s(t)$ with the gain $L_s \in \mathbb{R}^{(n+p)\times p}$ is the discontinuous input term which is robust to system uncertainties and nonlinearities. We now derive the error system. From observer (7), the following can be derived

$$\begin{aligned} \bar{S}\dot{\hat{\bar{x}}}(t) &= (\bar{A} - \bar{L}_p\bar{C})\hat{\bar{x}}(t) + \bar{L}_p\bar{C}\bar{S}^{-1}\bar{L}_D y_s(t) \\ &\quad +\bar{L}_D\dot{y}_s(t) + \bar{B}u(t) \\ &\quad +\bar{L}_s u_s(t) + \bar{F}f_d(t,\hat{x},u). \end{aligned} \quad (8)$$

Recall the second equation in (6), equation (8) now becomes

$$\begin{aligned} \bar{S}\dot{\hat{\bar{x}}}(t) &= (\bar{A} - \bar{L}_p\bar{C})\hat{\bar{x}}(t) + \bar{L}_p y_s(t) + \bar{L}_D\dot{y}_s(t) \\ &\quad +\bar{B}u(t) + \bar{L}_s u_s(t) + \bar{F}f_d(t,\hat{x},u). \end{aligned} \quad (9)$$

On the other hand, if we add $\bar{L}_D\dot{y}_s(t)$ to both sides of the plant (5), one can obtain

$$\begin{aligned} \bar{S}\dot{\bar{x}}(t) &= (\bar{A} - \bar{L}_p\bar{C})\bar{x}(t) + \bar{L}_p y_s(t) + \bar{L}_D\dot{y}_s(t) \\ &\quad +\bar{B}u(t) + \bar{F}f_d(t,x,u) + \bar{N}f_s(t). \end{aligned} \quad (10)$$

Define the following error variables

$$\begin{aligned} \bar{e}(t) &\triangleq \hat{\bar{x}}(t) - \bar{x}(t), \\ f_e(t,x,u) &\triangleq f_d(t,\hat{x},u) - f_d(t,x,u), \end{aligned} \quad (11)$$

and subtracting (9) from (10), one can obtain the following error system

$$\begin{aligned} \dot{\bar{e}}(t) &= \bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C})\bar{e}(t) + \bar{S}^{-1}\bar{L}_s u_s(t) \\ &\quad -\bar{S}^{-1}\bar{N}f_s(t) + \bar{S}^{-1}\bar{F}f_e(t,x,u). \end{aligned} \quad (12)$$

In error dynamics (12), the derivative gain $\bar{L}_D$ has been designed. The subsequent analysis should be focused on the design of proportional gain $\bar{L}_p$ and discontinuous input $u_s(t)$, such that the error system (12) achieves asymptotically stable. The remaining part of this section is divided into two parts: ***III-A.*** Design of the observer gain $\bar{L}_p$, ***III-B.*** Stability analysis of the error dynamics.

A. Design of observer gain $\bar{L}_p$

In this Subsection, our objective is to design the proportional gain $\bar{L}_p$, which plays a crucial role in our whole design work. We first introduce the following Lemma, which will be used in further analysis below.

Lemma 1: [1] Given a pair of matrix $(\tilde{A}, \tilde{C})$ with $\tilde{A} \in \mathbb{R}^{n\times n}$, $\tilde{C} \in \mathbb{R}^{p\times n}$, the following two conditions are equivalent:

(i) The matrix $\tilde{A}$ is stable;
(ii) If the pair $(\tilde{A},\ \tilde{C})$ is observable, then the following Lyapunov equation
$$\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} = -\tilde{C}^T\tilde{C} \tag{13}$$
has unique solution.

We now consider the matrix $\bar{S}^{-1}\bar{A}$, it is noticed that a positive number β can always be chosen such that $\text{Re}\left[\lambda_i(\bar{S}^{-1}\bar{A})\right] > -\beta$, or equivalently $\text{Re}\left[\lambda_i(-(\beta I + \bar{S}^{-1}\bar{A}))\right] < 0$, $\forall\ i \in \{1, 2, \cdots, n+p\}$. It is easy to see that $\forall s \in \mathcal{C}_+$, the following holds rank $\begin{bmatrix} sI_{n+p} - \bar{S}^{-1}\bar{A} \\ \bar{C} \end{bmatrix} =$ rank $\begin{bmatrix} sI_n - A \\ C \end{bmatrix} + p$. This condition implies that $[\bar{S}^{-1}\bar{A}, \bar{C}]$ is observable, and thus $[-\bar{S}^{-1}\bar{A},\ -\bar{C}]$ is observable. Hence, there exits a matrix L^* such that $-\bar{S}^{-1}\bar{A} - L^*\bar{C}$ is stable. This implies that $-\beta I - \bar{S}^{-1}\bar{A} - L^*\bar{C}$ is stable (note that $\beta > 0$), which further implies that $(-\beta I - \bar{S}^{-1}\bar{A},\ \bar{C})$ is observable. Hence, there exists a positive definite matrix $\bar{X}$ such that
$$-(\beta I + \bar{S}^{-1}\bar{A})^T\bar{X} - \bar{X}(\beta I + \bar{S}^{-1}\bar{A}) = -C^TC. \tag{14}$$
We choose the observer gain $\bar{L}_p$ as
$$\bar{L}_p = \bar{S}\bar{X}^{-1}\bar{C}^T, \tag{15}$$
Hence, one can obtain $[\beta I + \bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C})]^T\bar{X} + \bar{X}\left[\beta I + \bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C})\right] = -C^TC$, which implies that $\text{Re}\left[\lambda_i(\bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C}))\right] < -\beta, \forall\ i \in \{1, 2, \cdots, n+p\}$. As a result, by selecting $\bar{L}_p$ as (15), the matrix $\bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C})$ is designed to be stable. Hence, the design of proportional gain $\bar{L}_p$ has been well proceeded.

B. Stability analysis of the error dynamics

In the previous analysis, the observer gains $\bar{L}_p$ and $\bar{L}_D$ have been designed. In this subsection, we shall focus on the design of the discontinuous input $u_s(t)$ and establish sufficient condition for the stability of the error dynamics (12).

We define the following sliding surfaces $s(t)$
$$s(t) = \bar{N}^T\bar{S}^{-T}\bar{P}\bar{e}(t), \tag{16}$$
where the Lyapunov matrix $\bar{P} > 0$ satisfies the following constraint
$$\bar{N}^T\bar{S}^{-T}\bar{P} \;=\; H\bar{C}, \tag{17}$$
and $H \in \mathbb{R}^p$ is the matrix be determined.

In (17), it is noted that rank$(\bar{N}^T\bar{S}^{-T}\bar{P}) =$ rank$(\bar{C}) = p$, which implies that rank$(H) = p$. Since $H \in \mathbb{R}^p$, this means that H is non-singular. Hence, we can design the discontinuous input $u_s(t)$ as the following sliding mode control form
$$u_s(t) \;=\; -\left(\alpha + \gamma_1 + \eta_0\|\bar{P}\bar{S}^{-1}\|\|H^{-1}\|\right) \times \text{sgn}(s(t)), \tag{18}$$
where $\gamma_1 > 0$ is parameter to be designed, $\alpha > 0$ are defined as in Assumption 3.

Theorem 1: Apply the sliding mode control input $u_s(t)$ (18) to the error dynamics (12) , if there exist nonsingular matrices $\bar{P}$, $\bar{L}_P$, $H \in \mathbb{R}^{p\times p}$ with appropriate dimensions, such that the following matrix constraint holds,
$$\bar{P}\bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C}) + (\bar{A} - \bar{L}_p\bar{C})^T\bar{S}^{-T}\bar{P} \;<\; 0, \tag{19}$$
$$\bar{N}^T\bar{S}^{-T}\bar{P} \;=\; H\bar{C} \tag{20}$$
then the error dynamics (12) is asymptotically stable.

Furthermore, the observer gain $\bar{L}_s$ is given by
$$\bar{L}_s \;=\; \bar{S}\bar{P}^{-1}\bar{C}^TH^T = \bar{N}. \tag{21}$$

Proof: We choose the following Lyapunov function $V_e(t) = \bar{e}^T(t)\bar{P}\bar{e}(t)$, and take the time derivative along the state trajectories of (12), it follows that $\dot{V}_e(t) = \bar{e}^T(t)[\bar{P}\bar{S}^{-1}(\bar{A}-\bar{L}_p\bar{C}) + (\bar{A}-\bar{L}_p\bar{C})^T\bar{S}^{-T}\bar{P}]\bar{e}^T(t) + 2\bar{e}^T(t)\bar{P}\bar{S}^{-1}(\bar{L}_su_s(t) - \bar{N}f_s(t)) - 2\bar{P}\bar{S}^{-1}\bar{F}f_e(t,x,u)$. Hence, it is derived that $2\bar{e}^T(t)\bar{P}\bar{S}^{-1}(\bar{L}_su_s(t) - \bar{N}f_s(t)) \leq -2\eta_0\|\bar{P}\bar{S}^{-1}\|\|H^{-1}\|\|s(t)\| - 2\gamma_1\|s(t)\|$. On the other hand, by Assumption 2, it is derived that$-2\bar{e}^T\bar{P}\bar{S}^{-1}\bar{F}f_e(t,x,u) \leq 2\|\bar{e}^T\|\ \|\bar{P}\bar{S}^{-1}\|\|\bar{F}\|\|f_d(t,\hat{x},u) - f_d(t,x,u)\| = 2\eta_0\|\bar{P}\bar{S}^{-1}\|\|H^{-1}\|\|s(t)\|$. As a result, we have$\dot{V}_e(t) = \bar{e}^T(t)[\bar{P}\bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C}) + (\bar{A} - \bar{L}_p\bar{C})^T\bar{S}^{-T}\bar{P}]\bar{e}^T(t) - 2\gamma_1\|s(t)\| \leq -\lambda_{\min}(-\Gamma)\|e(t)\|^2 - 2\gamma_1\|s(t)\| < 0$.This completes the proof. ■

C. Calculation of the constrained linear matrix equality

In the previous Section, we have presented the sufficient condition for the asymptotic stability of the dynamics (12). It is noted that, the condition proposed in Theorem 3 includes a linear matrix equality which is difficult to directly solve by Matlab toolbox. We consider the linear equality condition (20). In fact, it can be rewritten as the following equivalent form: Trace$[(\bar{N}^T\bar{S}^{-T}\bar{P} - H\bar{C})^T(\bar{N}^T\bar{S}^{-T}\bar{P} - H\bar{C})] = 0$. We introduce the condition
$$(\bar{N}^T\bar{S}^{-T}\bar{P} - H\bar{C})^T(\bar{N}^T\bar{S}^{-T}\bar{P} - H\bar{C}) < \theta I, \tag{22}$$
by Schur complement, (22) is equivalent to
$$\begin{bmatrix} -\theta I, & \Pi^T \\ \Pi, & -I \end{bmatrix} < 0 \tag{23}$$
with $\Pi = \bar{N}^T\bar{S}^{-T}\bar{P} - H\bar{C}$. Hence, the design problem of PD sliding mode observer is now converted into a problem of finding a global solution of the following minimization

problem:

$$\min \theta, \text{ subject to } (19) \text{ and } (23) \tag{24}$$

This problem is a minimization problem which can be solved by using the Solvers $mincx$ in the LMI toolbox of Matlab.

D. Reachability condition of $s(t)$

In this subsection, we investigate the reachability of the following sliding surface (16)

$$s(t) = \bar{N}^T \bar{S}^{-T} \bar{P} \bar{e}(t) \tag{25}$$

in the estimation error space. The following theorem shows that the SMC $u_s(t)$ in (18) will guarantees the state-estimation error trajectories uniformly convergent to the sliding surface $s(t) = 0$.

Theorem 2: If there exists a nonsingular matrix $\bar{P}$ and parameter matrix H with appropriate dimension such that the matrix constraint (19) and (20) holds, and the observer gains $\bar{L}_D$ and $\bar{L}_p$ are designed as in Section III-A, then the sliding mode control law (18) guarantees that the sliding motion is attained on the sliding surfaces $s(t) = 0$.

Proof: Choose the Lyapunov function candidate as follows: $V_s(t) = 0.5 s^T(t)(\bar{N}^T \bar{S}^{-T} \bar{P} \bar{S}^{-1} \bar{N})^{-1} s(t)$. We denote $N_s := \bar{N}^T \bar{S}^{-T}$ for simplicity, and it is derived that $\dot{V}_s(t) = s^T(t)(N_s \bar{P} N_s^T)^{-1} N_s \ \bar{P} \bar{S}^{-1}[(\bar{A} - \bar{L}_p \bar{C})\bar{e}(t) + \bar{L}_s u_s(t) - \bar{N} f_s(t) + \bar{F} f_e(t, x, u)]$. On the other hand, it is clear that $s^T(t)(N_s \bar{P} N_s^T)^{-1} \ N_s \bar{P} \bar{S}^{-1}[\bar{L}_s u_s(t) - \bar{N} f_s(t)] = s^T(t)(u_s(t) - f_s(t)) < -2\eta_0 \|\bar{P}\bar{S}^{-1}\| \|H^{-1}\| \|s(t)\| - 2\gamma_1 \|s(t)\|$. As a result, one can obtain $\dot{V}_s(t) < -2\gamma_1 \|s(t)\| + \|s(t)\| \|(N_s \bar{P} N_s^T)^{-1} N_s \bar{P} \bar{S}^{-1}\| \|\bar{C}\| \|\bar{e}(t)\| + \|s(t)\| \|(N_s \bar{P} N_s^T)^{-1} N_s \bar{P} \bar{S}^{-1}(\bar{A} - \bar{L}_p \bar{C})\| \|\bar{e}(t)\|$. We define $\delta_1 = \|(N_s \bar{P} N_s^T)^{-1} N_s \ \bar{P} \bar{S}^{-1} \ (\bar{A} - \bar{L}_p \bar{C})\| + \|(N_s \bar{P} N_s^T)^{-1} N_s \bar{P} \bar{S}^{-1}\| \|\bar{C}\|$, it is shown that $\dot{V}_s(t) < -\|s(t)\|(2\gamma_1 - \delta_1 \|\bar{e}(t)\|)$. We define following domain: $\Omega(\delta_1) = \{2\gamma_1 - \delta_1 \|\bar{e}(t)\| > 0\}$. Recall that in Theorem 1 it has been proved that the error system (12) is stochastically stable. Hence, the trajectories of $\bar{e}(t)$ will enter Ω in finite time and remains there. As a result, it is shown that the trajectories of $\bar{e}(t)$ will attain the sliding surface $s(t, i) = 0$ in finite time. This completes the proof. ■

IV. Design of observer-based controller

In this section, the goal is to investigate the design problem of the observer-based controller for plant (1). We construct the following observer-based controller

$$\begin{cases} \bar{S}\dot{\bar{z}}(t) &= (\bar{A} - \bar{L}_p \bar{C})\bar{z}(t) + \bar{A}\bar{S}^{-1}\bar{L}_D y_s(t) \\ & \quad + \bar{B}u(t) + \bar{L}_s u_s(t) + \bar{F} f_d(t, \hat{x}, u) \\ \hat{\bar{x}}(t) &= \bar{z}(t) + \bar{S}^{-1}\bar{L}_D y_s(t) \\ u(t) &= K\hat{x}(t) = \bar{K}\hat{\bar{x}}(t) \\ \bar{K} &= [K, \ 0_{m\times p}] \end{cases} \tag{26}$$

in which $\bar{L}_D$, $\bar{L}_p$ and $u_s(t)$ have been designed in Section III-A, whilst state-feedback gain K will be designed in this section. Applying controller (26) to the open-loop system (1), we can obtain the following closed-loop system

$$\dot{x}(t) = (A + BK)x(t) + B\bar{K}\bar{e}(t) + f_d(t, x, u). \tag{27}$$

For the nonlinear function $f_d(t, x, u)$, a further derivation is performed. Let $\hat{x}(t) = 0$ in (2), one can obtain $f_d(t, \hat{x}, u) = 0$, which further implies that $\|f_d(t, x, u)\| \le \eta \|x(t)\|$. This inequality will be useful in the following discussion. We now provide how to design K such that the closed-loop system (27) is asymptotically stable.

Theorem 3: If there exist positive and definite matrix $Z \in \mathbb{R}^{n\times n}$ and matrix $K \in \mathbb{R}^{m\times n}$, such that the following matrix constraints hold

$$(A + BK)^T Z + Z(A + BK) + I_n + \eta^2 Z^T Z < 0, \tag{28}$$

then the closed-loop system (27) is asymptotically stable.

Proof: We define the Lyapunov function $V_x(t) = x^T(t) Z x(t)$ for system (27), it is derived that $\dot{V}_x(t) = x^T(t)\left[(A + BK)^T Z + Z(A + BK)\right] x(t) + 2x^T(t) Z(B\bar{K}\bar{e}(t) + f_d(t, x, u))$,

Note that $2x^T(t) Z f_d(t, x, u) \le x^T(t) x(t) + \eta^2 x^T(t) Z^T Z x(t)$, which implies that $\dot{V}_x(t) = x^T(t)[(A + BK)^T Z + Z(A + BK) + I_n + \eta^2 Z^T Z] x(t) - 2x^T(t) Z B\bar{K}\bar{e}(t) \le -\lambda_{\min}(-\Pi)\|x(t)\|^2 + 2\|ZB\bar{K}\| \cdot \|x(t)\| \cdot \|\bar{e}(t)\|$. We now define a new Lyapunov function $V(t) = V_x(t) + g_0 V_e(t)$, where g_0 is a positive number to be designed and $V_e(t) = \bar{e}^T(t)\bar{P}\bar{e}(t)$ with $\bar{P} > 0$. From the proof of Theorem 1, it is known that $\dot{V}_e(t) \le -\epsilon_1 \|\bar{e}(t)\|^2$, with $\epsilon_1 = \lambda_{\min}([\bar{P}\bar{S}^{-1}(\bar{A} - \bar{L}_p\bar{C}) + (\bar{A} - \bar{L}_p\bar{C})^T \bar{S}^{-T}\bar{P} + \bar{P}]) > 0$. We furthermore define $\epsilon_2 = \lambda_{\min}(-\Pi) > 0$, $\epsilon_3 = 2\|ZB\bar{K}\| > 0$, and select the scalar g_0 as $g_0 > \epsilon_3^2/\epsilon_2\epsilon_1$. Then, taking the derivative along the trajectory of (27), one can obtain $\dot{V}(t) = \dot{V}_x(t) + g_0\dot{V}_e(t) = -\epsilon_2\|x(t)\|^2 + \epsilon_3\|x(t)\|\|\bar{e}(t)\| - \epsilon_1 g_0\|\bar{e}(t)\| \le -\epsilon_2\|x(t)\|^2 + \sqrt{\epsilon_2\epsilon_1 g_0}\|x(t)\|\|\bar{e}(t)\| - \epsilon_1 g_0\|\bar{e}(t)\| \le -0.5\epsilon_2\|x(t)\|^2 - 0.5\epsilon_1 g_0\|\bar{e}(t)\| < 0$, which implies that $x(t) \to 0$ and $\bar{e}(t) \to 0$ as $t \to 0$. This completes the proof. ■

It is noticed that (28) is a nonlinear matrix inequality, we thus have to transform (28) into the LMI form.

Theorem 4: The closed-loop system (27) is asymptotically stable, if there exist a positive and definite matrix $Y \in \mathbb{R}^{n\times n}$, and a matrix $W \in \mathbb{R}^{n\times m}$ such that

$$\begin{bmatrix} \Gamma_{11}, & Y \\ Y, & -I_n \end{bmatrix} < 0, \tag{29}$$

with $\Gamma_{11} = AY + YA^T - BW^T - WB^T + \eta^2 I_n$. Furthermore, the state-feedback gain can be calculated as $K = W^T Y^{-1}$.

Proof: Pre- and post-multiplying Z^{-1} on the inequality (28), and letting $Z^{-1} = Y$, $W = YK^T$, using the Schur complement, the inequality (29) can be obtained. This completes the proof. ■

V. Simulation

Consider the plant (1) and augmented form (5), the system data are chosen as follows:

$$A = \begin{bmatrix} -1.5, & 0, & -0.2 \\ 0.3, & -0.7, & 0.5 \\ -0.3, & 0.5, & -1.2 \end{bmatrix}, \ B = \begin{bmatrix} 1, & 1 \\ 1, & 0 \\ 1, & 1 \end{bmatrix},$$

$$C = \begin{bmatrix} -2, & 1, & 1 \\ 1, & -2, & 1 \end{bmatrix},$$

$$f_d(t) = 0.1310 \times \begin{bmatrix} \sin(-2x_1(t) + x_2(t) + x_3(t)) \\ \sin(x_1(t) - 2x_2(t) + x_3(t)) \\ 0 \end{bmatrix}.$$

It can be checked that $(A,\ C)$ is observable pair, and the Lipschitz condition (2) is given as $\eta = 0.42$. It is assumed that $f_s(t) = \begin{bmatrix} f_{s1}^T, & f_{s2}^T \end{bmatrix}^T$ has the following form

$$f_{s1}(t) = \begin{cases} 0, & 0 \le t \le 2, \\ 0.4, & 2 < t < 5, \\ 0.1\sin(2t) + 0.2\cos(2t), & t \ge 5, \end{cases}$$

$$f_{s2}(t) = \begin{cases} 0.5, & 0 \le t \le 3, \\ 0.3\sin(2t) + 0.1, & t > 3, \end{cases}$$

and we choose $\alpha = 0.4$ to satisfy the constraint in Assumption 3.

(i) *Design of* $\bar{L}_D$ *and* $\bar{L}_p$: In the first step to design the estimator (7), we choose the derivative gain $\bar{L}_D$ as $\bar{L}_D = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T$, such that the system matrix $\bar{S} = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ -2 & 1 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 & 1 \end{bmatrix}$ is nonsingular. It can be calculated that $\lambda_{\min}(\bar{S}^{-1}\bar{A}) = -1$, and we choose $\beta = -\lambda_{\min}(\bar{S}^{-1}\bar{A}) + 0.01 = 1.01$. Solve the Lyapunov equation (14), one can obtain $\bar{X}$ as

$$\begin{bmatrix} 15.9817, & -25.0122, & 56.3332, & -2.7050, & 11.0916 \\ -25.0122, & 41.9654, & -95.4749, & 3.3708, & -19.2389 \\ 56.3332, & -95.4749, & 228.6139, & -6.3596, & 43.8952 \\ -2.7050, & 3.3708, & -6.3596, & 1.0933, & -1.0074 \\ 11.0916, & -19.2389, & 43.8952, & -1.0074, & 9.3192 \end{bmatrix}$$

According to (15), the proportional gain $\bar{L}_p$ is selected as

$$\bar{L}_p = \begin{bmatrix} 0.7210, & -0.0452 \\ -0.3090, & -0.6330 \\ 0.6068, & -1.0267 \\ 3.2420, & -2.1109 \\ -2.1109, & 4.0580 \end{bmatrix}.$$

(ii) *Design of control input* $u_s(t)$: Next, we design the sliding mode function (16). Solve LMI conditions (23) by applying LMI-Toolbox in Matlab environment, we have that $\theta \approx 4.3288 \times 10^{-12}$ (This means that the linear constraint $\bar{N}^T\bar{S}^{-T}\bar{P} = H\bar{C}$ is satisfied). According to (21), the observer gain $\bar{L}_s$ is designed as $\bar{L}_s = \bar{N} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$, and select $\gamma_1 = 0.1$, the discontinuous input $u_s(t)$ is given by $s(t) = \bar{N}^T\bar{S}^{-T}\bar{P} \times \bar{e}(t)$, $u_s(t) = -2.8674 \times \text{sgn}(s(t))$. To prevent the control signals from chattering, we replace $\text{sgn}(s(t))$ with $s(t)/(\|s(t)\| + 0.01)$ in (18). The simulation results are provided in the following Figures 1-3. The state trajectories of $x(t)$ is shown in Figure 1; the trajectories of sensor faults $f_s(t)$ and its estimation are shown in Figures 2-3. It can be seen that the asymptotic stability of the closed-loop system is guaranteed, and the tracking performance of system states and sensor faults have achieved an ideal performance.

VI. Conclusion

In this work, the problem of state and fault estimation and observer-based control for Lipschitz nonlinear systems with sensor failure has been investigated . A new *Proportional*

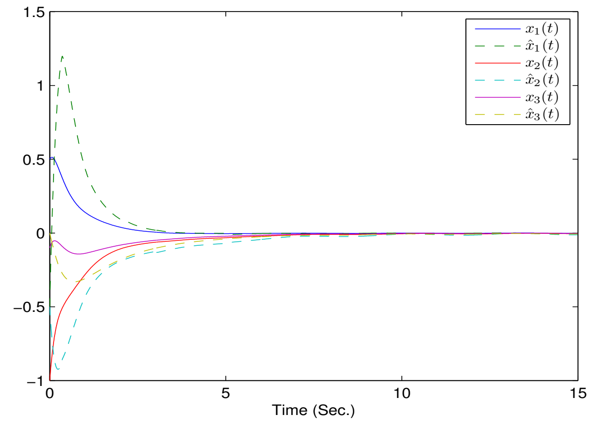

Fig. 1: $x(t)$ and the estimation

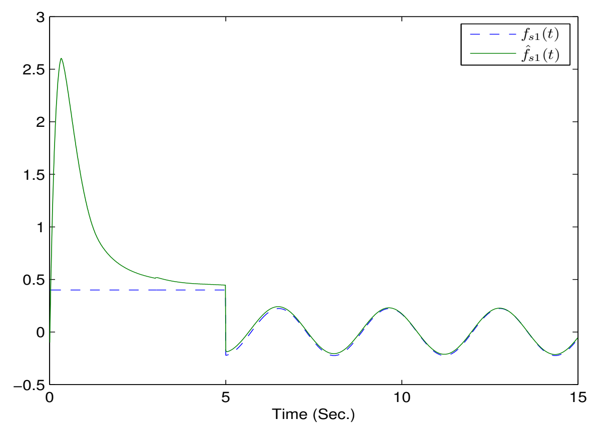

Fig. 2: Component 1 of sensor fault $f_s(t)$ and its estimation

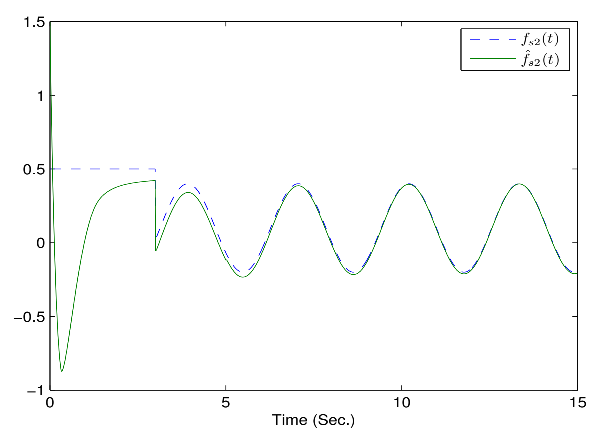

Fig. 3: Component 2 of sensor fault $f_s(t)$ and its estimation

and Derivative sliding mode observer has been proposed to construct accurate estimation of both system states and sensor faults. Based on the state estimation, an observer-based control scheme has been designed to stabilize the closed-loop system. Future work will be focused on applying the proposed estimation technique to more complicated systems such as Markovian jump systems and fuzzy systems.

ACKNOWLEDGEMENTS

The authors thank the referees and the editor for their valuable comments and suggestions.

REFERENCES

[1] C. Chen. *Linear system theory and design*. New York: Oxford, 1999.
[2] Z. Gao, B. Jiang, R. Qi, Y. Xu, and Y. Cheng. Fuzzy observer design for near space vehicle with application to sensor fault estimation. *ICIC Express Letter*, 4(1):177–182, 2010.
[3] Z. Gao and H. Wang. Descriptor observer approaches for multivariable systems with measurement noises and application in fault detection and diagnosis. *Systems & Control Letters*, 55(4):304–313, 2006.
[4] Y. Guo, B. Jiang, and P. Shi. Delay-dependent adaptive reconfiguration control in the presence of input saturation and actuator faults. *International Journal of Innovative Computing, Information and Control*, 6(4):1873–1882, 2010.
[5] B. Jiang and F. Chen. Direct self-repairing control for a small helicopter via fuzzy adaptive technique. *ICIC Express Letter*, 4(3(A)):641–646, 2010.
[6] B. Jiang and F. Chen. Piecewise sliding mode decoupling fault tolerant control system. *ICIC Express Letter*, 4(4):1215–12226, 2010.
[7] B. Jiang, P. Shi, and Z. Mao. Sliding mode observer-based fault estimation for nonlinear networked control systems. *Circuits, Systems, and Signal Processing*, 30(1):1–16, 2011.
[8] Z. Lin, Y. Xia, P. Shi, and H. Wu. Robust sliding mode control for uncertain linear discrete systems independent of time-delay. *International Journal of Innovative Computing Information and Control*, 7(2):869–881, 2011.
[9] J. Wang, B. Jiang, and P. Shi. Adaptive observer based fault diagnosis for satellite attitude control systems. *International Journal of Innovative Computing, Information and Control*, 4(8):1921–1930, 2008.
[10] L. Wu, P. Shi, and H. Gao. State estimation and sliding-mode control of Markovian jump singular systems. *IEEE Transactions on Automatic Control*, 55(5):1213–1219, 2010.