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A novel active fault tolerant control design with respect to actuators reliability

Ahmed Khelassi, Didier Theilliol, Philippe Weber and Dominique Sauter

Abstract— **In this paper, actuators reliability analysis is considered in the design of active fault tolerant control system. The main objective consists on the synthesis of a fault tolerant controller gain which guarantees a highest overall system reliability. Benefit of incorporating reliability indicators on the controller gain design is to manage effectively the control inputs in order to increase the remaining life time of a system in the presence of faults. A reliability analysis of system modeled by a state space representation is proposed. A controller gain design is reformulated as an Linear Matrix Inequality (LMI) problem and synthesized through an effective admissible model matching approach.**

I. INTRODUCTION

the control design of safety-critical systems, reliability, maintainability, and safety are the basic design requirements. Safety critical systems should be able to maintain a satisfactory closed-loop performance in nominal case and to adapt with faulty situations. These types of adaptive systems are known as Fault Tolerant Control Systems (FTCS). The aim of FTC is to keep plant available by the ability to achieve the objectives assigned in the faulty behavior and to accept reduced performance when critical faults occur [3]. Currently, various approaches for FTCS design have been developed and and proposed in the literature. Overviews on the development of FTCS have been provided in survey papers and books such as [3] or [15]. Among these approaches, some of them require an on-line Fault Detection and Isolation (FDI) mechanism. These approaches are called Active Fault Tolerant Control Systems (AFTCS).

Therefore, it is important to enhance the system safety not only by improving reliability of individual components or by designing control systems to compensate the effect of fault on the dynamic behavior but also by taking into consideration the degradation of components health. One motivation to integrate information about actuator health in the controller design is to improve the safety and the life time of the reconfigurable systems. Indeed, system safety and overall system reliability can be improved by Fault Tolerant Control Design based on an effective control inputs management. Consequently, the system remains operational for a longer duration with respect to the acceptable control performance requirements.

The authors are with the Centre de Recherche en Automatique de Nancy, CRAN-CNRS UMR 7039, Nancy University, BP 70239, 54500 Vandeuvre Cedex, France. (E-mail: ahmed.khelassi@cran.uhp-nancy.fr, didier.theilliol@cran.uhp-nancy.fr, philippe.weber@cran.uhp-nancy.fr, dominique.sauter@cran.uhp-nancy.fr). This work was supported by the SAFFE project 2010-2011 (Ageing Management in fault tolerant control system design project) from GIS 3SGS - France (https://www.gis-3sgs.fr/).

Some research works have introduced reliability analysis for FTCS. [14] has used Markov chain to model the system reliability where subsystems are supposed to reach two states: intact (available) or failed (unavailable) states. In [13] and [11], the authors proposed a sensor and actuator reconfiguration strategy based on physical redundancy. The reliability analysis provides some indicators to select the optimal set for reconfiguration strategy. In a similar way, [9] has considered the reliability of sensor faults in the filtering design issue. A reconfiguration mechanism of FTC strategy incorporating reliability analysis under a dynamic behavior constraints has been proposed in [8].

In this paper, actuator criticality evaluation is integrated in the design of active fault tolerant controller gain. The novelty of the proposed approach is to design a fault tolerant controller gain which achieves the control objective with a highest overall system reliability level by respecting the criticality of each actuator. Indeed, this objective improves the safety of the reconfigurable system and keeps the set of actuators available as long as possible by minimizing the use of the critical actuators. To select the critical actuators, sensitivity analysis of the overall system reliability is proposed *a priori*.

The problem of fault tolerant control design with respect to actuator criticality is reformulated as an Linear Matrix Inequalities (LMIs) problem . Among various methods, the active fault tolerant controller gain synthesis which guarantees the overall system reliability and safety is achieved through an admissible model matching (AMM) method.

As indicated in [12], the AMM method is able to obtain admissible solutions in cases where the classical Pseudo-Inverse Method (PIM) leads to unstable behaviors [6]. In the proposed method, effective admissible solution is proposed in nominal case based on reliability analysis and actuators criticality. After fault occurrence, the LMI-based regional pole placement is used to reach the admissible behavior by considering the actuators aging. The actuators criticality is re-actived by taking into account the actuators aging caused before faults occurrence.

This paper is organized as follows. Section II presents the general framework of fault tolerant control design for a reliability objective requirement. Preliminaries on AMM FTC and the considered faulty system are introduced. In Section III, reliability analysis procedure of closed-loop systems is presented and an actuators criticality indicator is given. In Section IV, active fault tolerant control design maximizing the overall system reliability with respect to actuators criticality is proposed. In Section V, the application of the proposed approach on a linearized flight control example are given. Finally, Section VI concludes the contribution of this paper.

II. Preliminaries and problem statement

Let us consider the linearized dynamic of the system given by:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_u \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are respectively, the state, the control and the output matrices. $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the system output and (A, B) is assumed to be stabilizable.

In nominal operation, the control law $u(t)$ is designed with the classical state feedback as:

$$u(t) = -Kx(t) \tag{2}$$

Indeed, several methods have been proposed to design a feedback controller K which satisfies $A-BK \in \mathcal{M}_a$ where, $\mathcal{M}_a$ presents a family of acceptable closed-loop behaviors called *admissible behaviors* [13].

***Definition** 1 (Admissibility):* The triple $(A, B, K) \in \mathcal{M}_a$ is called admissible if and only if,

$$\mathcal{M}_a = \{(A, B, K) : \Phi_{\mathcal{M}}(A, B, K) \leq 0\} \tag{3}$$

where $\Phi_{\mathcal{M}}(A, B, K)$ are the set of constraints that guarantee $(A - BK) \in \mathcal{M}_a$ is achieved with the control law $u(t) = -Kx(t)$.

Then, when actuator faults occurrence, the system (2) can be modeled in degraded functional mode as follows:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_f u(t)\\ y(t) &= Cx(t)\end{aligned} \tag{4}$$

where the matrix B_f is written according to the nominal control input matrix B and the control effectiveness factors $\gamma_i \in [0,\ 1]$, $i = 1, \ldots, m$, as follows:

$$B_f = B\Gamma, \quad \Gamma = diag([\gamma_1, \gamma_2, \ldots, \gamma_m]) \tag{5}$$

Indeed, if $\gamma_i = 1$, then the i^{th} actuator is considered to be fault-free. Nevertheless, when $0 < \gamma_i \leq 1$, the considered faults is a partial loss of control effectiveness. Moreover, when $\gamma_i = 0$ critical failure is considered and the actuator is out of order.

In this paper, admissible model matching FTC is considered. The main idea of admissible model matching is that instead of looking for a controller gain that provides an exact (or best) matching to a nominal behavior after the fault appearance.

***Definition** 2:* The faulty system (A, B_f) is fault-tolerant using active FTC if and only if the set:

$$\mathcal{K}_f(A, B_f) = \{K_f : \Phi_{\mathcal{M}}(A, B_f, K_f) \leq 0\} \tag{6}$$

is not empty. Indeed, there exists a $K_f \in \mathcal{K}_f : A - B_f K_f \in \mathcal{M}_a$.

The main contribution of this work is to consider the overall system reliability as a principle objective that guide the design of the controller gain. Thus, to design an effective fault tolerant controller gain with an overview on the overall system reliability, the generated control inputs should be applied to the system taken into account the actuators aging. This objective leads to design an optimal controller $K_f^* \in \mathcal{K}_f$ with an optimal value of the overall system reliability $R_g(t_M)$ where, the acceptable performance is guaranteed until the end of the mission with a hight probability. The problem of effective fault tolerant control design is defined as follows:

***Definition** 3:* The system (A, B_f) is fault-tolerant and effective using active fault tolerant control for $K_f^* \in \mathcal{K}_f$ defined by:

$$K_f^* = \{K_f : \Phi_{\mathcal{M}}(A, B_f, K_f) \leq 0,\ R_g(t_M) \longrightarrow R_g^{max}\} \tag{7}$$

where R_g^{max} is the maximum value of reliability that can be obtained using $\mathcal{K}(A, B_f)$. $R_g(t_M)$ is the overall system reliability estimated for $t = t_M$. t_M is the predefined time of the end of mission.

III. Reliability analysis of closed-loop systems

A. Reliability computation

The life time of the system can be quantified by the overall system reliability estimation.

***Definition** 4:* Reliability $R(t)$ is defined as the probability that units, components, equipments and systems will accomplish their intended function for a specified period of time under some operating conditions and specific environments [7].

In the useful period of life, the component can be characterized at a given time t by a baseline reliability measure $R^0(t)$. In the following, $R^0(t)$, associated to the reliability of the actuator, is obtained under nominal conditions in the useful period of life defined such as:

$$R_i^0(t) = exp(-\lambda_i^0 t), \quad i = 1 \ldots m \tag{8}$$

where λ_i^0 is a baseline failure rate of the i^{th} actuator obtained under a nominal operating condition defined by a specific applied load level.

However, a realistic health measurement should also include the trend of actuator aging according to the variation of the operating conditions. Indeed, in many situations and especially in the considered study, failure rates are obtained from actuators under different levels of loads depending on the applied control input. Several mathematical models have been developed to introduced the impact of load level in the reliability estimation. Proportional hazard model firstly proposed by [5] is used.

***Definition** 5:* In the case of time variation of operating condition, the reliability component can be estimated according to the nominal failure rate as follows:

$$\begin{aligned}R_i(t) &= \exp(-\lambda_i t)\\ \lambda_i &= \lambda_i^0 \times g_i(\ell, \vartheta)\end{aligned} \tag{9}$$

where λ_i^0 represents the baseline failure rate (nominal failure rate) for the i^{th} subsystem or component and $g_i(\ell, \vartheta)$ is a function (independent of time) taking into account the effects of applied loads called *stress* with ℓ presenting an image of the load and ϑ defining component parameters.

***Assumption** 1:* In this study, the exponential distribution for reliability estimation is considered. The function load introduced in (9) will be defined according to the applied load until the end of the mission $t = t_M$, assumed to be known. In this framework, the component reliability measure will be evaluated for $t = t_M$ noted as $R(t_M)$.

Different definitions of the load function $g_i(\ell, \vartheta)$ exist in the literature where, the exponential form is commonly used. In this study, the load function is defined according to the root-mean-square of the applied control input as follows:

$$g_i(\ell, \vartheta) = \exp(\beta_i \int_0^{t_M} u_i^2(t)dt), \quad i = 1, \ldots, m \tag{10}$$

where β_i is an actuator parameter defined as follows:

$$\beta_i = (t_M(\overline{u}_i - \underline{u}_i))^{-1}, \quad i = 1, \ldots, m \tag{11}$$

where $\overline{u}_i$ and $\underline{u}_i$ are the physical saturations of $u_i(t)$.

In fact, based on (9) and (10), the actuators health degradation at a given time t_M is modeled as a function of the applied load.

B. Reliability analysis: parallel and series case

Based on the previous definitions, the actuator reliability $R_i(t_M)$ can be estimated depending on the baseline reliability $R_i^0(t_M)$ as follows:

$$\begin{aligned} R_i(t_M) &= \alpha_i(u) R_i^0(t_M), \quad i = 1, \ldots m \\ \alpha_i(u) &= \exp(-\lambda_i^0 \beta_i \|u_i\|^2) \end{aligned} \tag{12}$$

where, $\|.\|$ is the Euclidien norm. $\alpha_i(u_i) \leq 1$ represents the rate of reliability degradation due to the applied load during the mission. In fact, the following relation is satisfied: $if \quad \|u_i\|^2 \to 0 : R(t_M) \to R^0(t_M)$ where $\|u_i\|^2$ is the variation of the applied load compared to the nominal level of load. Thus, the component reliability decreases for a large variation of the applied load from the nominal level.

***Lemma** 1:* For a system composed by m redundant actuators, the overall system reliability $Rg(t_M, u) \longrightarrow Rg^{max}$ for an effective controller $K^* \in \mathcal{K}$ where the control inputs $u^*(t) = -K^* x(t)$ stress more the less reliable actuators and satisfy the following condition as close as possible:

$$K^* \mapsto \{u^*(t) \quad | \quad \min \frac{\alpha_i(u_i^*)}{\alpha_j(u_j^*)} \quad \text{if} \quad \frac{\lambda_i^0}{\lambda_j^0} < 1\},$$

where $i \in [1, \ldots, m]$; $j \in [1, \ldots, m]$ and $i \neq j$. Rg^{max} is the optimal value of overall system reliability that can be obtained at $t = t_M$.

Proof: Systems composed by m redundant actuators can be presented by a parallel scheme of Reliability Block Diagram (RBD). The associated overall system reliability under nominal conditions $Rg^0(t_M)$ can be obtained as follows:

$$Rg^0(t_M) = 1 - \prod_{k=1}^{m} (1 - R_k^0(t_M)) \tag{13}$$

The sensitivity of the overall system reliability s_i versus the i^{th} component can be evaluated as:

$$\begin{aligned} s_i &= \frac{\partial Rg^0(t_M)}{\partial R_i^0(t_M)} = \prod_{k=1;\ k \neq i}^{m} (1 - R_k^0(t_M)) \\ &= \prod_{k=1}^{m} (1 - R_k^0(t_M))(1 - R_i^0(t_M))^{-1} \end{aligned} \tag{14}$$

It can be shown that, if $R_i^0 > R_j^0$ (or $\lambda_i^0 < \lambda_j^0$) then $s_i < s_j$.

This result means that a small degradation $\alpha_i(u_i)$ of the i^{th} more critical actuator (for that the system reliability is more sensitive) causes a large degradation of the overall system reliability $R_g(t_M)$ and vice-versa for the j^{th} actuator. ■

***Corollary** 1:* For a system composed by m series actuators, the overall system reliability under nominal conditions $Rg^0(t_M)$ can be obtained as follows:

$$Rg^0(t_M) = \prod_{k=1}^{m} R_k^0(t_M) \tag{15}$$

It can be seen clearly that the overall system reliability $Rg(t_M, u) \longrightarrow Rg^{max}$ for an effective controller $K^* \in \mathcal{K}$ where the control inputs $u^*(t) = -K^* x(t)$ stress more the more reliable actuators and satisfy the following condition:

$$K^* \mapsto \{u^*(t) \quad | \quad \min \frac{\alpha_i(u_i)}{\alpha_j(u_j)} \quad \text{if} \quad \frac{\lambda_i^0}{\lambda_j^0} > 1\},$$

where $i \in [1, \ldots, m]$; $j \in [1, \ldots, m]$. Rg^{max} is the optimal value of overall system reliability that can be obtained at $t = t_M$.

C. Actuators criticality analysis: General case

In the more general case, a global reliability $R_g(t_M)$ is computed based on the reliabilities of elementary components or subsystems. In this context, $R_g(t_M)$ depends on the actuators's connection which can generally be decomposed on elementary combinations of series and parallel components.

In the context of fault tolerant control, it is crucial to know a priori how the actuators are connected in order to analyze the actuators criticality which guide the design of an effective controller. In fact, the variation of the overall system reliability when the applied load is considered depends on structure of the system and the applied load corresponding to each actuator. By considering the matrix $B = [b_1 b_2 \ldots b_m], \quad b_i \in, \mathbb{R}^{n \times 1}$, reliability bloc diagram of the actuators can be obtained by testing the controllability condition for the different combinations of b_i. For a control problem, the intended function for which the reliability bloc diagram will be established is to guarantee the system controllability. This conditions make the design of the control law possible.

In order to obtain the relation $R_g(t) = f(R_1(t), R_2(t), \ldots R_m(t))$ for the complex systems, the following subsets $\mathcal{L}_k, \quad k = 1, \ldots, n_L$ are considered as follows:

$$\mathcal{L}_k = \{R_i, rank[Ctr(A, [\rho_1 b_1 \ldots \rho_m b_m])] = n\}, i = 1, \ldots, m \tag{16}$$

where, $Ctr(A,B)$ is the controllability Grammien of the system defined by the matrices (A,B), and

$$\begin{cases} \rho_i = 0, & \text{the } i^{th}\text{actuator is not considered in } \mathcal{L}_k \\ \rho_i = 1, & \text{the } i^{th}\text{actuator is considered in } \mathcal{L}_k \end{cases} \quad (17)$$

In fact, $\mathcal{L}_k, k = 1, \ldots, n_L$ are all the subsets schemes of the actuators for which the controllability condition is satisfied and so a fault tolerant controller can be calculated. $\mathcal{L}_k$ represents the success solution of controllability similar to the success path defined in the standard reliability analysis.

Then, based on the *Poincare Theorem* [10], the overall system reliability $R_g(t)$ can be expressed as a function of $R_i(t)$ as follows,

$$R_g(t) = \sum_{j=1}^{n_L} \prod_{i \in \mathcal{L}_j} R_i(t) - \sum_{j=2}^{n_L} \sum_{k=1}^{j-1} \mathcal{P}(\mathcal{L}_k \cap \mathcal{L}_j) + \ldots + (-1)^m \mathcal{P}(\mathcal{L}_1 \cap \ldots \cap \mathcal{L}_m) \quad (18)$$

where

$$\mathcal{P}(\mathcal{L}_k \cap \mathcal{L}_j) = \prod_{i \in \{\mathcal{L}_k, \mathcal{L}_j\}} R_i(t) \quad (19)$$

The *Poincare Theorem* is generally used to calculate the overall system reliability of complex systems. Hence, this theorem is adapted in this work to obtain the relation of the overall system reliability for the controlled system defined by the matrices (A,B). The proposed evaluation is applied off-line where the reliability bloc diagram and the connection between the actuators is fixed in the system design stage and does not change.

Consequently, the actuators criticality can be evaluated by the following indicator obtained based on the sensitivity study such as:

$$s_i = \frac{\partial R_g^0(t_M)}{\partial R_i^0(t_M)} \quad (20)$$

In order to maximize the overall system reliability, the applied loads depending on the effective controller $K^* \in \mathcal{K}$ should guarantee the stability of the faulty closed-loop system with respect to the actuators criticality.

IV. Active Fault Tolerant Controller Gain Synthesis Versus Reliability

Before presenting the main contribution on the effective fault tolerant controller design, let us consider the following lemma:

***Lemma** 2 ([4]):* The applied load of the control input $u(t) = -Kx(t)$ evaluated as the norm $\|u(t)\|$ can be enforced to respect an upper bound $\|u(t)\| < \mu$ at all times $t \geq 0$ if the LMI

$$\begin{bmatrix} P & S^T \\ S & \mu^2 \end{bmatrix} \geq 0 \quad (21)$$

holds for a given $\mu > 0$, where $P > 0$, $x(0)^T P^{-1} x(0) \leq 1$ and $K = SP^{-1}$ such that S satisfies the stabilizing condition,

$$AP + PA^T + BS + S^T B^T < 0 \quad (22)$$

Admissible model matching method proposed in [12] is considered to design a fault tolerant controller gain such that the poles of the closed-loop system are inside a pre-established region even in faulty case. This algorithm will be combined with an additional LMI constraint that enforce the applied load to respect as possible to predefined level with a priority to the actuators based on its criticalities. The set of admissible behaviors $\mathcal{M}_a$ can be proposed as:

$$\mathcal{M}_a = \{(A, B_f, \mathcal{K}_f) : \Lambda(A - B_f \mathcal{K}_f) \in \mathcal{D}_\alpha\} \quad (23)$$

where $\Lambda(.)$ is the set of the eigenvalues of the matrix $(.)$. $\mathcal{D}_\alpha$ is a desired region included in the unite circle with an affix $(-q, 0)$ and a radius r such that $(q+r) < 1$ is fixed. These two scalars q and r are used to determine a specific region included in the unite circle. According to [1], (23) can be rewritten as follows:

$$\begin{bmatrix} -rP & qP + PM^T \\ qM + MP & -rM \end{bmatrix} < 0 \quad (24)$$

where $M = A - B_f K_f$ and $P > 0$. $\mathcal{D}_a$ is the region encompassing the desired system poles Λ^d and the admissible ones.

In the following, we propose to design an effective nominal controller gain K^* in order to maximize the overall system reliability and to improve the system dependability. The proposed controller gain K^* places the system eigenvalues $\Lambda(M)$ in the admissible region as close as possible to the desired eigenvalues Λ^d and manages effectively the actuators. For that, an additional LMI constraint is considered in the controller design and the effective controller gain K^* can be obtained in the nominal functional operation as follows:

***Theorem** 1:* Given a scalar μ and the criticality indicators s_i defined by (20), an effective nominal controller gain $K^* = SP^{-1}$ can realize $\Lambda(A - BK^*) \in \mathcal{D}_\alpha$ and $R_g(t_M) \to R_g^{max}$ by solving the following LMIs problem:

$$\begin{bmatrix} -rP & qP + PA^T - S^T B^T \\ qP + AP - BS & -rP \end{bmatrix} < 0 \quad (25)$$

$$\begin{bmatrix} P & S^T \\ S & (\Omega^{-1}\mu)^2 \end{bmatrix} \geq 0, \quad i = 1 \ldots m \quad (26)$$

where $P > 0 \in \mathbb{R}^{m \times m}$, $S \in \mathbb{R}^{n \times m}$ and $\Omega = \text{diag}\ \{\Omega_1, \Omega_2, \ldots, \Omega_m\}$ with

$$\Omega_i = \frac{s_i}{s_{min}}, \quad , s_{min} = \min(s_i) \quad (27)$$

Proof: Based on lemma (2), $\|u\| < \mu$ can be satisfied by considering the weighing norm $\|\Omega u\|$ where

$$\|u\| \leq \|\Omega u\| < \mu \quad (28)$$

for $\Omega = diag\{\Omega_1, \Omega_2, \ldots, \Omega_m\} \geq 0$, and $\Omega_i \geq 1, \quad i \in [1, \ldots, m]$.

Based on the weighted norm $\|\Omega u\|$, the applied load constraint $\|u\| < \mu$ is satisfied by considering a specified priority level to each actuator. In fact the applied load of each control input $u_i(t)$ can be enforced to respect a fixed upper bound for each actuator. The values of Ω_i are chosen as (27) in order to manage the control inputs based on the criticality indicators where the required applied loads of the actuators are bounded by considering their criticality. This additional property can be expressed as follows:

$$\begin{bmatrix} P & S^T \\ S & (\Omega^{-1}\mu)^2 \end{bmatrix} > 0 \tag{29}$$

The stability condition and the admissibility solution are both guaranteed by the LMI region constraint of poles placement (25). ■

After the fault occurrence at $t = t_f$, the criticality indicator s_i can be re-estimated on-line by considering the failure rates λ_i defined in (9). The applied load in this case is calculated until $t_M = t_f$ and evaluated as:

$$g_i(\ell, \vartheta) = \exp(\beta_i \int_0^{t_f} u_i^2(t)dt), \quad i = 1, \ldots, m \tag{30}$$

Thus, the stability and the admissibility of the closed-loop system in the faulty case can be guaranteed by considering $K_f^* = SP^{-1}$ in the inequality (25) and the new criticality indicators s_i in the inequality (27) computed based on the re-estimated actuator failure rates λ_i .

***Corollary** 2:* The proposed method can be generalized to tracking problem:

$$u^*(t) = -K_f^* x(t) + K_r^* r(t) \tag{31}$$

where $r(t) \in \mathbb{R}^{n_r}$ is the desired reference to be tracked by the system input. The gain K_r^* can be designed according to the state feedback gain K_f^* as follows:

$$K_r^* = (C(B_f K_f^* - A)^{-1} B_f)^+ \tag{32}$$

V. An application to flight control

In order to illustrate the novel fault tolerant controller gain synthesis, a linearized flight control problem considered in [2] is adopted where $x(t) = [\tilde{p}, \tilde{q}, \tilde{r}, \Delta_\alpha, \beta, \phi, \theta]^T$ is the state. $u(t) = [\delta_a, \delta_r, \delta_e]^T$ is the control input, where δ_a is the aileron deflection angle, δ_r is the rudder deflection angle and δ_e is the elevator deflection angle. The controlled outputs $y(t) = [\beta, \phi, \theta]$ are respectively, the angle of sideslip, the angle of bank and pitch angle.

To illustrate the proposed approach in a short time window, the values of the baseline actuators failure rates λ_i^0 are considered with a very huge values and given as follows:

$$\lambda^0 = \{0.005,\ 0.09,\ 0.01\} \text{ min}^{-1}$$

The degraded closed-loop behavior is considered admissible if the eigenvalues of M lie in a disk around $q = 3.5$ with a radius of 1. The corresponding admissible behavior is defined as

$$\mathcal{M}_a = \{M : \Lambda_i(A - B_f K_f^*) \in \mathcal{D}_\alpha(3.5, 1)\} \quad \forall i \in [1, 2, 3]$$

where $\Lambda(A - BK) \in \mathcal{D}_\alpha(3.5, 1)\}$ is verified in the nominal case.

In order to obtain the criticality indicator (20), the reliability block diagram of the system (A, B) can be obtained by testing the controllability condition as in (16).

For simplicity, complete failure of actuator 1 is not considered and the overall system reliability $R_g^0(t)$ can be expressed as:

$$R_g^0(t) = R_3^0(t) + R_1^0(t)R_2^0(t)(1 - R_3^0(t))$$

where the criticality indicator is obtained for $t_M = 20sec$ as, $s = [0.03,\ 0.164,\ 0.8504]$. Based on (26), the corresponding weighting matrix Ω is equal to,

$$\Omega = diag\{28.3821,\ 5.185,\ 1\}$$

It can be seen that actuator 1 is the most critical actuator in term of reliability.

Figure (1) shows the evaluation of the required outputs with a constant degraded mode characterized by $\Gamma = [1,\ 0.5,\ 1]$. The fault is assumed occurred at $t_f = 7sec$ where a time delay of few samples is considered for fault detection, isolation and magnitude estimation and consequently to modify the controller gain. Three controller gains have been synthesized in order to illustrate our method:
- a controller gain K in nominal case;
- an admissible model matching controller gain K_f in faulty case solution of (21) and (25) without respect to the reliability analysis;
- a reliable admissible controller gain K_f^* in faulty case obtained by solving the LMI problems (25) and (26).

The controller gains have been obtained by using YALMIP to solve the LMIs problems where $\mu = 50$ is considered. As presented in Figure (1), the dynamic behavior has been affected by the actuator fault. Without fault tolerant control approach, the controller K is not able to guarantee the performance of the closed-loop. However, with the fault tolerant controller gain methods (K_f or K_f^*), the outputs reach their reference with a time delay due to the FDI module (see Figure (2)). Indeed, the reliable controller K_f^* is calculated in order to find a novel distribution of the desired efforts that tracks the reference with an admissible behavior and stabilizes the system taking into account the actuators criticality. The aim is to increase the overall system reliability by minimizing the use of the critical actuators as much as possible.

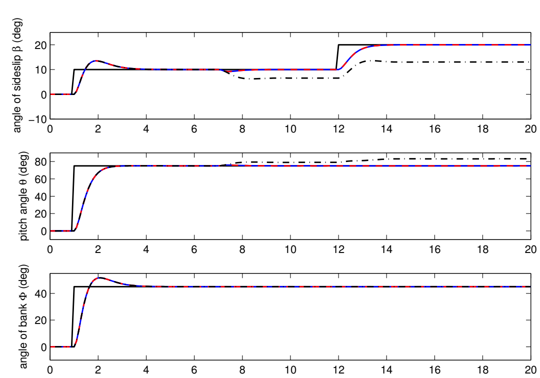

Fig. 1. Outputs responses in the faulty case without FTC used K (dot-dashed line), with K_f (solide line) and with K_f^* (dashed line).

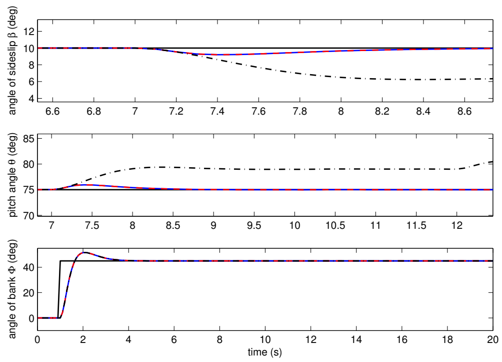

Fig. 2. Zoomed on outputs responses in the faulty case without FTC used K (dot-dashed line), with K_f (solide line) and K_f^* (dashed line).

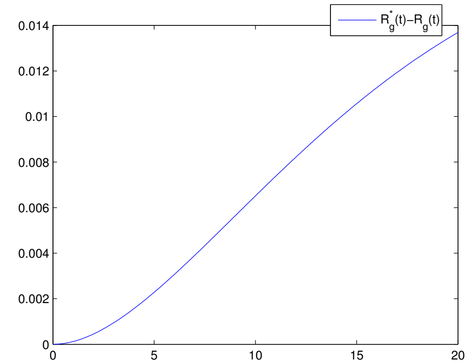

Fig. 4. Evaluation of the difference between the reliabilities

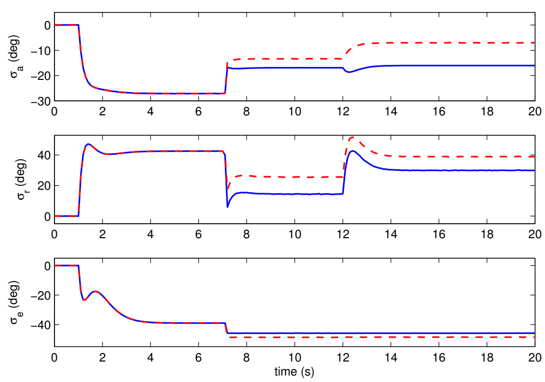

Fig. 3. Inputs responses in the faulty case with K_f (solide line) and K_f^* (dashed line).

As illustrated in Figure (3), the applied load of the most critical actuator (*Actuator 1*) is minimized. This implies the increasing of the applied load corresponding to the less critical actuators. It can be seen also that after fault occurrence, the faulty actuator 2 for K_f^* is stressed taken into account the fault magnitude in addition to their criticality. The developed method preserved the critical actuators which cause a large degradation of the residual system life time for a large applied loads. The increasing of the overall system reliability improves the safety of the reconfigurable system where it can be operate until the end of the mission with a hight reliability. As presented in Figure (4), the probability that the system remains operational with K_f^* is larger compared to K_f.

VI. CONCLUSION

In this paper, a novel active fault tolerant controller gain design based on actuators criticality is proposed. Under the presence of faults, the aim is to design an effective controller gain where the desired efforts are applied to the system with an overview on the overall system reliability. The solution is performed by considering the sensitivity of the overall system reliability in the controller gain synthesis through linear matrix inequality technique. Effective admissible model matching fault tolerant control is proposed and illustrated by a flight control application. The proposed approach improves the overall system reliability and the system dependability.

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