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Power Allocation Policy for Distributed Estimation in Wireless Networks

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Abstract— **We consider a sensor network monitoring a stochastic process. The sensors exchange information over wireless communication channels affected by noise. The goal of the sensors is to compute accurate estimates of the state of the stochastic process under limited energy available for communications. We introduce a distributed algorithm which computes a power allocation scheme aimed at ensuring accurate state estimates and at saving communication energy. The distributed algorithm arise from solving an approximate convex optimization problem with constraints. We present how the power allocation scheme can be used for performing distributed estimation in the case where measurements are shared between sensors.**

I. Introduction

An important problem in sensor networks is developing (distributed) algorithms for state estimation. When considering wireless sensor networks with limited energy available for communication, another important problem is designing power allocation schemes aimed at ensuring good estimation performance and network longevity. The problem increases in complexity if we impose the power allocation scheme to be obtained in a distributed manner.

We are addressing the problem of designing a power allocation scheme for a network of wireless sensors whose main functionality is to observe a process of interest and computing state estimates. Each sensor has a physical neighborhood formed by sensors with which communication is possible. Each sensor has to allocate its available energy to the communication channels used for sending information to its neighbors. The more information a sensor receives from the neighbors, the more likely the state estimate it computes is more accurate. However, for each data packet sent, a communication cost is paid and therefore communicating with the relevant sensors is of outmost importance. Therefore, the power allocation scheme must result from a tradeoff between the need for information from neighbors (for accurate estimates computations) and the need for conserving energy.

In this paper we give a distributed algorithm for computing a power allocation scheme which is obtained by solving a constraint optimization problem. The main cost reflects the relevance of the sensor measurements for the estimation process. The constraints reflect the limited energy available for communication and the need to ensure rich enough local neighborhoods for computing the state estimates. The formulation of the problem has a similar flavor with the resource allocation for wireless networks problem [5], [9].

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Notations: When referring to a directed edge (link) (j,i) of a directed graph, j denotes the destination node, while i denotes the source node. In the context of a network, a variable x related to a source (transmitter) s and a destination (receiver) d is denoted by x_{ds}.

II. Problem formulation

In this section we introduce the communication and estimation models used in the paper.

A. Communication Model

We assume that the process is monitored by a wireless network of (fixed) sensors, where each sensor is capable to simultaneously transmit to multiple other sensors, without interferences (by using for example multiple antennas and transmitting on orthogonal frequencies/channels). Let P_i^{max} denote the maximum power sensor i has available for communication and let P_{ji} be the power allocated for the channel sending information to sensor j, with the following constraint inequality

$$\sum_{j\in\mathcal{N}_i} P_{ji} \leq P_i^{max}, \tag{1}$$

where $\mathcal{N}_i$ represents the neighborhood of node i, i.e. the set of nodes within its communication range. In addition we make the assumptions that the communication topology is directed.

The power of the signal arriving at the receiver j from transmitter i is given by $G_{ji}F_{ji}P_{ji}$, where G_{ji} is a positive scalar representing the path gain from transmitter i to receiver j and F_{ji} is a random variable modeling the *fading* of the channel between i and j. The scalar G_{ji} represents for us a distance dependent power attenuation, but can also be interpreted as log-normal shadowing, cross correlation between codes in a code division multiple access system or a gain dependency on antenna direction. In particular, we assume that G_{ji} is given by

$$G_{ji} = cd_{ji}^{-\phi},$$

where c represent a constant scalar depending on the physical characteristics of the transmitter's antenna and d_{ji} is the distance between the two communicating sensors, with ϕ usually taking values between 1 and 4, depending on the environment. We assume *independent Rayleigh channels* and therefore each F_{ji} has an exponential distribution with unit mean. The channel towards the sensor j is affected by a white

Gaussian noise of power σ_{ji}^2. Given that agent j is a receiver and agent i is a transmitter, the signal to noise ratio SNR_{ji} at agent j is given by

$$SNR_{ji} = \frac{F_{ji}G_{ji}P_{ji}}{\sigma_{ji}^2}. \quad (2)$$

We refer to the *outage* of a link $i \to j$ as the *packet loss event*. The outage event can be translated as the event when the SNR_{ji} is less than a threshold S_{ji}. The threshold S_{ji} depends on the physical layer parameters such as rate transmissions, modulation and coding. The probability a packet is successfully transmitted from node i to j is given by $p_{ji} = Pr(SNR_{ji} \geq S_{ji})$ and can be explicitly written as

$$p_{ji} = Pr(SNR_{ji} \geq S_{ji}) = e^{-\frac{S_{ji}\sigma_{ji}^2}{G_{ji}P_{ji}}} = e^{-\frac{\gamma_{ji}}{P_{ji}}}, \quad (3)$$

where $\gamma_{ji} = \frac{S_{ji}\sigma_{ji}^2}{G_{ji}}$. The above probability model was inspired from [6], except that we omitted the terms corresponding to interferences.

B. Estimation model

We assume that a network of N sensors observes a random process modeled by a discrete-time linear dynamic equation

$$x(k+1) = Ax(k) + w(k), \; x(0) = x_0, \quad (4)$$

where $x(k) \in \mathbb{R}^n$ is the state vector and $w(k) \in \mathbb{R}^n$ is a driving noise, assumed Gaussian with zero mean and covariance matrix Σ_w. The initial condition x_0 is assumed to be Gaussian with mean μ_0 and covariance matrix Σ_0. The sensing model of the sensor i is given by

$$y_i(k) = C_i x(k) + v_i(k), \; i = 1 \ldots N, \quad (5)$$

where $y_i(k) \in \mathbb{R}^{r_i}$ is the observation made by sensor i and $v_i(k) \in \mathbb{R}^{r_i}$ is the measurement noise, assumed Gaussian with zero mean and covariance matrix Σ_{v_i}. We assume that the matrices $\{\Sigma_{v_i}\}_{i=1}^N$ and Σ_w are positive definite and that the initial state x_0, the noises $v_i(k)$ and $w(k)$ are independent for all $k \geq 0$. In addition, we assume that all sensor have knowledge of the parameter of the process, i.e. matrices A, Σ_w, μ_0 and Σ_0.

We denote by $\hat{x}_i(k)$ the state estimate computed by sensor i and by $e_i(k) = x(k) - \hat{x}_i(k)$ the estimation error. If a (directed) communication link exists between two agents, they can exchange information such as local measurements $y_i(k)$, the local sensing model parameters given by matrices C_i and Σ_{v_i} and the local estimates $\hat{x}_i(k)$. This information is used to update the local estimates $\hat{x}_i(k)$.

III. Power Allocation Scheme Design

The communication model described in the previous section induces a random topology with probability distribution depending on the powers allocated for communication and some parameters of the physical layer. Our goal is to design a power allocation scheme for state estimation purposes which considers the communication costs inherent to a wireless network. In essence, this entails the design of a (random) communication network.

For each sensor i we define the cost

$$\mathcal{J}_i(P_{ij}, j \in \mathcal{N}_i) = E\left[\sum_{j \in \mathcal{N}_i} \alpha_{ij} l_{ij}\right] = \sum_{j \in \mathcal{N}_i} \alpha_{ij} p_{ij} = \sum_{j \in \mathcal{N}_i} \alpha_{ij} e^{-\frac{\gamma_{ij}}{P_{ij}}}, \quad (6)$$

where E is the expectation operator, $\alpha_{ij} = \frac{\alpha_j}{\alpha_i} = \frac{tr(C_j'\Sigma_{v_j}^{-1}C_j)}{tr(C_i'\Sigma_{v_i}^{-1}C_i)}$, $\gamma_{ij} = \frac{S_{ij}\sigma_{ij}^2}{G_{ij}}$, l_{ij} is a random variable taking values zero or one describing the presence of a link from sensor j to i with probability distribution $Pr(l_{ij} = 1) = p_{ij}$. Note that in the following, for notational simplicity, we use $\mathcal{J}_i(P_{ij})$ to denote $\mathcal{J}_i(P_{ij}, j \in \mathcal{N}_i)$.

Remark 3.1: The term $\alpha_j = tr(C_j'\Sigma_{v_j}^{-1}C_j)$ is a *Fisher* like information metric, emphasizing the *quality* of the measurements taken by sensor j. A similar metric was used in [8], in the context of distributed tracking with information driven mobility. In our problem, "information" will not drive mobility but rather formation of links. The scalar $\alpha_{ij} = \frac{\alpha_j}{\alpha_i}$ tells us how much sensor i benefits from receiving measurements from sensor j. Note that a ratio α_{ij} smaller than one means that agent i' measurements are of better quality than agent j's. The larger the value of α_{ij} is, the larger the corresponding probability p_{ij} should be, such that the value of the cost function $\mathcal{J}_i(P_{ij})$ is made large.

If the pair (A, C_i) is not detectable (in the sense of the standard definition for the linear time-invariant systems), the stability of the estimation error produced by the Kalman filter, when *only local measurements are used*, can not be guaranteed. Therefore an additional constraint which ensures that enough power is allocated to a sufficient number of neighbors of sensor i such that the local detectability is achieved by using additional measurements, may be needed. Given that $|\mathcal{N}_i|$ is the cardinality of set $\mathcal{N}_i$, there are a number of $L_i = 2^{|\mathcal{N}_i|}$ combination of sensors belonging to $\mathcal{N}_i$ which can send information to sensor i. Let l denote such an instance of all possible combinations and let $\mathcal{S}_i^l$ be the set of sensors that are successfully sending information to agent i and corresponding to the instance l. Inspired by the work of [4] (Proposition 4.1), a necessary and sufficient condition for detectability at the node i (i.e. $\lim_{k\to\infty} E[\|e_i(k)\|^2] < \infty$), when the measurements from neighbors are also considered, is given by

$$\prod_{j \in \mathcal{S}_l^i} (1 - p_{ij})|\rho_l^i|^2 < 1, \; l = 1 \ldots, L_i, \quad (7)$$

where ρ_l^i is the spectral radius of the unobservable part of matrix A when the pair (A, C_l^i) is put in the observer canonical form, with $C_l^{i'} = [C_i', C_j', j \in \mathcal{S}_l^{ic}]$, with $\mathcal{S}_l^{ic}$ being the set of sensors not sending information under the instance l. To avoid the existence of an infinity of solutions for an optimization problem formulated in what follows, we introduce a positive scalar ϵ, much smaller than one and we replace the strict inequality (7) with the inequality

$$\prod_{j \in \mathcal{S}_l^i} (1 - p_{ij})|\rho_l^i|^2 \leq 1 - \epsilon, \; l = 1, \ldots, L_i. \quad (8)$$

In addition of the previous constraint, we recall the power constraint expressed in (1).

By summing up the local cost functions over all sensors, we obtain a global optimization cost

$$\mathcal{J}(P_{ij}) = \sum_{i=1}^{N} \mathcal{J}_i(P_{ij}), \quad (9)$$

which together with the constraints (1) and (7) make up the constraint optimization problem whose solution generates a random communication topology (through the power allocation scheme) aimed at obtaining good estimates and at keeping the communication costs limited.

Our goal is to *distributively* solve the following optimization problem

$$\min_{P_{ij}\geq 0} \quad -\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\alpha_{ij}e^{-\frac{\gamma_{ji}}{P_{ji}}} \quad (10)$$

$$\text{s.t.:} \quad \prod_{j\in\mathcal{S}_l^i}\left(1-e^{-\frac{\gamma_{ji}}{P_{ji}}}\right)|\rho_l^i|^2 \leq 1-\epsilon,\ l=1\dots L_i,\ i=1\dots N,$$

$$\sum_{j\in\mathcal{N}_i} P_{ji} \leq P_i^{max},\ i=1\dots N,$$

where the first inequality constraints are imposed to ensure detectability of the process at sensor i and the second inequality limits the communication power of sensor i.

Note that although the constraints (1) are convex, both the cost function $\mathcal{J}(P_{ij})$ and the constraints (8) needed for maintaining local detectability at each sensor are not.

In the following, by using a change of variable, we *approximate* the original optimization problem (10), with a convex optimization problem. Consider the change of variable $z_{ij} = \log(1-p_{ij}) = \log\left(1-e^{-\frac{\gamma_{ij}}{P_{ij}}}\right)$. Let us also consider the simplifying notation $\beta_l^i = \log(1-\epsilon) - 2\log|\rho_l^i|$. Under the new variables and notation, the optimization problem (10) becomes

$$\min_{z_{ij}\leq 0} \quad \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\alpha_{ij}e^{z_{ij}} \quad (11)$$

$$\text{s. t.:} \quad \sum_{j\in\mathcal{S}_l^i} z_{ij} \leq \beta_l^i,\ \forall l,i, \quad (12)$$

$$\sum_{j\in\mathcal{N}_i}\gamma_{ji}g(z_{ji}) \leq P_i^{max}, \forall i, \quad (13)$$

where

$$g(z) = -\frac{1}{\log(1-e^z)}.$$

Under the new variables, the cost function and the constraints (12) are convex. Although, the function $g(z)$ has the appearance of a convex function, in fact looses convexity at small values of z. Therefore, the constraints (13) are non-convex. We define the function

$$f(z) = \begin{cases} g(z) & z \leq \tilde{z} \\ g'(\tilde{z})z + b & z > \tilde{z} \end{cases}$$

where $\tilde{z}$ is the solution of $g''(z) = 0$ (i.e. the inflexion point) given by $\tilde{z} \approx -0.2271$, $g'(\tilde{z})$ is the slope of $g(z)$ at $\tilde{z}$ given by $g'(\tilde{z}) \approx -1.54413$ and $b \approx 0.27677$. We note that $f(z)$ approximates $g(z)$ by a line on the interval $[\tilde{z},0]$ and more importantly is a convex function and $g(z) \leq f(z)$, $\forall z \leq 0$.

By substituting $g(z)$ with $f(z)$, the set defined by the constraints (13) is approximated by a convex set and we obtain a constraint convex optimization problem of the form

$$\min_{z_{ij}\leq 0} \quad \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\alpha_{ij}e^{z_{ij}} \quad (14)$$

$$\text{s. t.:} \quad \sum_{j\in\mathcal{S}_l^i} z_{ij} \leq \beta_l^i,\ \forall l,i, \quad (15)$$

$$\sum_{j\in\mathcal{N}_i}\gamma_{ji}f(z_{ji}) \leq P_i^{max}, \forall i, \quad (16)$$

for which we can use the primal-dual decomposition theory to solve it.

Since the minimum of $f(z)$ is $b \approx 0.27677$ and is attained at zero, in order for the constraints (16) to be feasible, we make the following additional assumption.

Assumption 3.1: The following inequality holds

$$\sum_{j\in\mathcal{N}_i}\gamma_{ji}b < P_i^{max}, \forall i.$$

A. Distributed iterative algorithm for solving the power allocation problem

The optimization problem (14) is convex with convex constraints and from Proposition 6.4.3 of [1] (where we use also the Assumption 3.1), it follows that there is no duality gap. Let

$$\mathcal{L}(z_{ij},\lambda_i,\mu_i^l) = \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\alpha_{ij}e^{z_{ij}} + \sum_{i=1}^{N}\lambda_i\left(\sum_{j\in\mathcal{N}_i}\gamma_{ji}f(z_{ji}) - P_i^{max}\right) +$$

$$+\sum_{i=1}^{N}\sum_{l=1}^{L_i}\mu_i^l\left(\sum_{j\in\mathcal{S}_l^i} z_{ij} - \beta_l^i\right) \quad (17)$$

denote the Lagrangian corresponding to the convex optimization problem (14). Algorithm 1 computes iteratively the solution of the dual of the optimization problem (14).

In Algorithm 1, $[\cdot]^+$ represents the projection operator onto the set of positive real numbers and ζ represents a small positive scalar which controls the precision of the algorithm. The parameter $\delta(t)$ represents the stepsize of the algorithms. Conditions the stepsize must satisfy such that the langrange multipliers $\lambda_i(t)$ and $\mu_i^l(t)$ converge, can be found in Chapter 8 of [1], for example. We now turn to the step 2 of Algorithm 1, i.e. solving the optimization problem

$$z_{ji}(t) = \arg\min_{z_{ji}\leq 0}\mathcal{L}(z_{ji},\lambda_i(t),\mu_i^l(t)). \quad (18)$$

Note that as a result of our approximation, problem (18) is convex and therefore we can use efficient algorithms to find a solution. For simplicity, in what follows we will omit the time index t associated with the lagrange multipliers. The lagrangian function can also be represented as

$$\mathcal{L}(z_{ji},\lambda_i,\mu_i^l) = \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}F_{ji}(z_{ji},\lambda_i,\mu_i^l),$$

where

$$F_{ji}(z_{ji},\lambda_i,\mu_i^l) = \alpha_{ji}e^{z_{ji}} + \mu_{ij}z_{ji} + \lambda_i\gamma_{ji}f(z_{ji}),$$

Algorithm 1: Subgradient algorithm for solving the dual optimization problem

Input: $\lambda_i(0)$, $\mu_i^l(0)$, $i=1\dots N$, ζ

Initialization: $t=0$

do Solve the primal optimization problem:

$$(z_{ji}(t)) = \arg\min_{z_{ji}\leq 0} \mathcal{L}(z_{ji},\lambda_i(t),\mu_i^l(t)),\ i=1\dots N, j\in\mathcal{N}_i$$

Update the lagrange multipliers:

$$\lambda_i(t+1) = \left[\lambda_i(t)+\delta(t)\left(\sum_{j\in\mathcal{N}_i}\gamma_{ji}f(z_{ji}(t)) - P_i^{max}\right)\right]^+,\ i=1\dots N$$

$$\mu_i^l(t+1) = \left[\mu_i^l(t)+\delta(t)\left(\sum_{j\in S_i^l} z_{ij}(t)-\beta_i^l\right)\right]^+,\ i=1\dots N, l=1\dots L_i$$

Update the time step:

$$t=t+1$$

while $|\lambda_i(t)-\lambda_i(t-1)|\leq\zeta$ and $|\mu_i^l(t)-\mu_i^l(t)|\leq\zeta$

Compute

$$P_{ji} = -\frac{\gamma_{ji}}{\log(1-e^{z_{ji}(t)})},\ i=1\dots N, j\in\mathcal{N}_i$$

with

$$\mu_{ij} = \sum_{l=1\dots L_j}\mu_j^l \mathbb{1}_{\{i\in S_j^l\}},$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function. We can see that $\mathcal{L}(z_{ij},\lambda_i,\mu_i^l)$ is minimized if each of the functions $F_{ji}(\cdot)$ is minimized. Basically, minimizing (18) is reduced to solving

$$z^* = \arg\min_{z\leq 0} F(z), \tag{19}$$

where $F(z)=\alpha e^z+\mu z+\lambda\gamma f(z)$ for some $\alpha,\mu,\lambda,\gamma\geq 0$, with

$$f(z)=\begin{cases} -1/\log(1-e^z) & z\leq -0.2271\\ -1.54413z+0.27677 & z>-0.2271.\end{cases}$$

It turns out that $F(z)$ is a strictly convex function on $\mathbb{R}$ and therefore admits a unique minimizer. The solution of (19) is given by

$$z^*=\min\{\tilde{z},0\},$$

where

$$\tilde{z}=\arg\min_{z\in\mathbb{R}} F(z),$$

and where the above optimization problem can be solve efficiently by using a *line search method* [10], for example.

Note that the above algorithm can implemented in a *distributed manner* since each agent uses only local information. From this perspective, each agent i controls the variables $z_{ji}(t)$, $j\in\mathcal{N}_i$ (which is an estimate of the optimizer z_{ji}^* at time instant t), $\lambda_i(t)$ (which can be interpreted as the *price* paid in terms of energy for sending data to its neighbors) and $\mu_i^l(t)$ (which can be interpreted as the *reward* node i gets, in terms of the detectability property, under some combination of neighbors sending data to it). Indeed, for updating $z_{ji}(t)$, node i needs from its neighbors the quantities μ_{ij}, $j\in\mathcal{N}_i$ and for updating $\mu_i^l(t)$ node i needs only $z_{ij}(t)$ for $j\in\mathcal{N}_i$. The quantity $\mu_{ij}(t)$ can be interpreted as the total reward nodes j gets when node i shares information with j. This terms mitigates the variable $z_{ji}(t)$ in the function $F_{ji}(\cdot)$, which is a representation of how much power nodes i spends for transmitting to node j (i.e. the larger the power the smaller z_{ji} becomes).

B. *Sensors satisfying the local detectability property*

In this subsection we study how Algorithm 1 simplifies when the sensors are locally detectable, i.e. the pairs (A,C_i) are detectable. The first consequence is that the set of constraints (12) are no longer necessary. Therefore the optimization problem (11) becomes

$$\min_{z_{ij}\leq 0} \quad \textstyle\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\alpha_{ij}e^{z_{ij}} \tag{20}$$

$$\text{subject to:}\quad \textstyle\sum_{j\in\mathcal{N}_i}\gamma_{ji}g(z_{ji})\leq P_i^{max},\forall i, \tag{21}$$

where

$$g(z)=-\frac{1}{\log(1-e^z)}.$$

Note that as before, the above problem becomes convex if we approximate $g(z)$ with $f(z)$, defined above. Using the same ideas as before, for solving the primal problem during the iterative algorithm for solving the dual problem, each sensor will have to locally minimize a set of problems of the form

$$F(z)=\alpha e^z+\lambda\gamma f(z),$$

over the interval $(-\infty\ 0]$, where

$$f(z)=\begin{cases} g(z) & z\leq\tilde{z}\\ g'(\tilde{z})z+b & z>\tilde{z}.\end{cases}$$

Differentiating $F(z)$ and solving $F'(z)=0$ for z, we obtain that the minimal point of $F(z)$ on the interval $(-\infty\ 0]$ is given by

$$z^*(\eta)=\begin{cases} 0 & \eta>\eta_1\\ \log(1.54413\eta) & \eta_2<\eta\leq\eta_1\\ \log\left(1-\frac{1}{4}\frac{\eta}{W\left(-\frac{1}{2}\sqrt{\eta}\right)^2}\right) & 0<\eta\leq\eta_1\end{cases}, \tag{22}$$

where $\eta=\frac{\lambda\gamma}{\alpha}$, $W(\eta)$ is the *Lambert W function*, $\eta_1\approx 0.64761$ and $\eta_2\approx 0.51602$. Interestingly, we can interpret the parameter η as the weighted transmission price, where the weight is given by the factor $\frac{\gamma}{\alpha}$, with γ increasing with the distance between the source and destination, and α reflecting the *informational richness* of the measurements.

IV. Distributed estimation under lossy communication links

In this section we combine the power allocation scheme presented in the previous sections with the state estimation process. We study here only the case where the sensors use only the measurements of their neighbors, and not their estimates. We make the following fundamental assumptions. We assume that the estimation process is split in two steps. In the *first step*, the sensors *negotiate* a topology *suitable* for the state estimation, by deriving a power allocation scheme. During this step the sensors implement distributively Algorithm 1. In the *second step*, the sensors perform the state estimation process.

Assumption 4.1: We assume that during the first step, the agents invest sufficient energy for communication such that the agents communicate without errors with the neighbors.

This assumption makes sense if the time horizon of the estimation process is very large compared with the time necessary for the topology negotiation and therefore the energy consumed during the first step is much smaller compared to the energy spent for exchanging information during the estimation process.

In our numerical example we consider a grid network formed of sixteen sensors (Figure 2 monitoring a linear stochastic process, with parameters

$$A = \begin{pmatrix} 0.9996 & -0.03 \\ 0.03 & 0.9996 \end{pmatrix},\ \Sigma_w = 0.1I,\ \mu_0 = \begin{pmatrix} 10 \\ 10 \end{pmatrix}, \Sigma_0 = I.$$

The parameters of the sensing models are as follows: $C_i =$

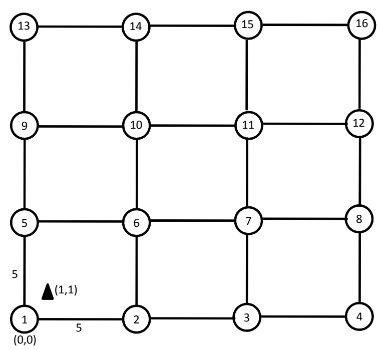

Fig. 1. Grid sensor network

$[1\ 0]$ and the measurement noise $\sigma_{v_i}^2$ depends on the distance, i.e. $\sigma_{v_i}^2 = d_i^2$, where d_i is the distance between the sensor i and the object. In our example we assume that the first sensor is at the origin of the 2D coordinate frame, that the vertical and horizontal distances between sensors is five distance units and that the coordinates of the observed object are $(1,1)$ distance units.

The sensors first execute Algorithm 1, in order to determine the power allocation scheme. In our numerical simulations we assume that $P_i^{max} = 3$ power units and that the parameters $\gamma_{ji} = 1$ power units for all i and $j \in \mathcal{N}_i$. The numerical simulation results of Algorithm 1 are presented in Figure 2, where the numerical values on the links refer to the percentage of the available power of each sensor allocated for the respective links. In this numerical simulation we do not consider the detectability constrains (8) since the pairs (A, C_i) are detectable for all i. Note that as expected, the sensors tend to allocate more power to the neighbors further away from the tracked object, since they have less accurate measurements. In addition, each sensor allocates at least 9% of its power for each link in its neighborhood, as per constraints (16).

For the estimation process, each sensor uses the measurements and the sensing models from the neighbors and employs a linear filter under intermittent measurements; intermittence induced by the random nature of the communication channels. This approach is a particular case of the

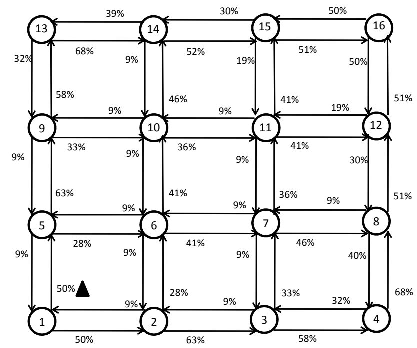

Fig. 2. Random network architecture as a result of Algorithm 1

state estimation for Markovian jump linear systems, detailed in [3]. We would like to point out that the analysis can be extended to the case where the estimation is performed using a consensus-based linear filter, where the linear filter under intermittent measurements is combined with a consensus step and where the estimates from neighbors are also used. This approach was studied extensively in [2], [7], [11], [12] under a deterministic communication topology. Due to space limitation however, such analysis is not done in this paper.

A. Distributed Estimation - Measurements Sharing

From the point of view of the information received, sensor i can be in $2^{|\mathcal{N}_i|}$ modes of operation. Let l be a mode of operation and let S_i^l be the set of neighbors of agent i which successfully transmit information to sensor i, in mode l. Denoting by $\theta_i(k)$ the mode of sensor i at time k, we get that

$$Pr(\theta_i(k) = l) = \prod_{j \in S_i^l} p_{ij} \prod_{j \in S_i^{lc}} (1 - p_{ij}) = q_{i,l},$$

where S_i^{lc} is the complement of S_i^l (i.e. the set of neighbors of agent i with unsuccessful transmissions) and p_{ij} are probabilities of successful transmissions determined in the first stage.

At each time instant k, the measurements available at sensor i are given by the vector $\mathbf{y}_{\theta_i(k)}(k)' = [y_i(k)', y_j(k)', j \in S_i^{\theta_i(k)}]'$ with sensing model

$$\mathbf{y}_{\theta_i(k)}(k) = \mathbf{C}_{\theta_i(k)} x(t) + \mathbf{v}_{\theta_i(k)}(k), \tag{23}$$

where $\mathbf{C}'_{\theta_i(k)} = [C_i', C_j', j \in S_i^{\theta_i(k)}]'$ and $\mathbf{v}_{\theta_i(k)}(k)' = [v_i(k)', v_j(k)', j \in S_i^{\theta_i(k)}]'$ a vectored valued zero mean Gaussian noise with covariance matrix $\Sigma_{\theta_i(k)} = diag(\Sigma_{v_i}, \Sigma_{v_j}, j \in S_i^{\theta_i(k)})$. Note that for all possible modes of operations, matrix $\Sigma_{i,\theta_i(k)}$ is invertible. In addition, the dimensions of the vectors $\mathbf{y}_{\theta_i(k)}(k)$, $\mathbf{v}_{\theta_i(k)}(k)$ and matrices $\mathbf{C}_{\theta_i(k)}$, $\Sigma_{\theta_i(k)}$ varies for each mode of operation.

Inspired by the Markovian jump linear system estimation theory [3], in the following we introduce a linear filtering scheme for computing the state estimates. To simplify notation, the index i corresponding the an agent is omitted. Denoting by $\hat{x}(k)$ the estimate, a linear filter for computing the estimate is given by

$$\hat{x}(k+1) = A\hat{x}(k) + M_{\theta(k)}\left(y_{\theta(k)} - \mathbf{C}_{\theta(k)}\hat{x}(k)\right),\ \hat{x}(0) = \mu_0.$$

Proposition 4.1: The finite horizon quadratic filtering cost

$$\sum_{k=0}^{K} E[\|e(k)\|^2],$$

where $e(k) = x(k) - \hat{x}(k)$ denotes the estimation error is minimized by the filtering gains

$$M_l(k) = AY(k)\mathbf{C}_l'\left(\Sigma_{v_l} + \mathbf{C}_l Y(k)\mathbf{C}_l'\right)^{-1}, \tag{24}$$

where $Y(k) = E[e(k)e(k)']$ is the covariance matrix of the estimation error whose dynamics is given by

$$Y(k+1) = \sum_l q_l(A - \mathbf{C}_l M_l)Y(k)(A - \mathbf{C}_l M_l)' + \sum_l q_l M_l \Sigma_l M_l' + \Sigma_w, \tag{25}$$

with $Y(0) = \Sigma_0$, $\mathbf{C}_l' = [C_i', C_j', j \in S_i^l]'$ and $\Sigma_l = diag(\Sigma_{v_i}, \Sigma_{v_j}, j \in S_i^l)$.

Proof: The proof of this result can be mimicked after the proof of Theorem 5.5 of [3], where $\theta(k)$ is a finite state Markov chain. Although in the aforementioned theorem the parameters of the sensing model are assumed to have the same dimension across the operating modes (e.g. $\mathbf{C}_l$ has the same dimension for all l), it can be easily shown that that optimal filter (24) remains valid even under our assumptions. ■

Remark 4.1: It turns out that the infinite horizon quadratic filtering cost given by

$$\lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{N} E[\|e(k)e(k)\|^2],$$

is minimized by the same filtering gains as in (24) (for $k \geq 0$), and in addition if the optimization problem (14) is feasible and consequently the constraint (8) is satisfied, the infinite horizon filtering cost is upper-bounded since the trace of the covariance matrix $Y(k)$ remains upper-bounded for all $k \geq 0$. Sufficient conditions under which $Y(k)$ also converges can be found in Appendix A of [3].

Figure 3 shows the average estimation error (in means square sense) taken over all sensors, given by $\epsilon_{av}(k) = \frac{1}{N}\sum_{i=1}^{N} E[\|e_i(k)\|^2]$ under two power allocation schemes. With $\epsilon_{av}^{(1)}(k)$ we denoted the average estimation error when the power allocation scheme is computed using Algorithm 1, while by $\epsilon_{av}^{(2)}(k)$ we denoted the average estimation error when sensors *equally* distribute the available power to their neighbors. Figure 3 does show an improvement of the average estimation error when the power allocation scheme given by Algorithm 1 is used.

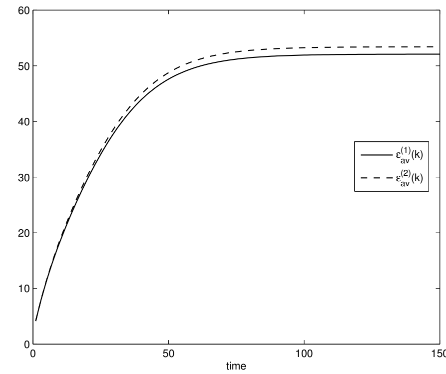

Fig. 3. Average estimation error under two power allocation schemes

V. Conclusions

In this paper we addressed the problem of designing a power allocation scheme for distributed estimation executed by a sensor network. The power allocation scheme results from solving an (approximate) constrained convex optimization problem and reflects the tradeoff between the need for accurate and stable estimates and the need for network longevity. We gave an iterative algorithm for solving the optimization problem which can be implemented in a distributed manner. Finally we show how the power allocation scheme can be used to perform distributed estimation in the case where measurements are shared between sensors.

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