2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Formation Control of Weak Autonomous Robots

Huan Zhang   Pubudu N. Pathirana

Abstract— **Formation of autonomous mobile robots to an arbitrary geometric pattern in a distributed fashion is a fundamental problem in formation control. This paper presents a new fully distributed, memoryless (oblivious) algorithm to the formation control problem via distributed optimization techniques. The optimization minimizes an appropriately defined difference function between the current robot distribution and target geometric pattern. The optimization processes are performed independently by individual robots in their local coordinate system. A movement strategy derived from the results of the distributed optimizations guarantees that every movement makes the current robot configuration approaches the target geometric pattern until the final pattern is reached.**

I. Introduction

This paper studies the problem of coordinating a group of autonomous mobile robots to form a prescribed geometric pattern in a two-dimensional plane. Study of cooperative behavior in a group of autonomous agents is a topic that attracts increasing attention in biology, robotics and control community [8],[10],[11],[12],[13],[18]. Swarm behaviors in certain living beings have been observed for a long time. Examples are birds flocks, fish schools, animal herds. The cooperative behavior is very important to the survival of these species since it has certain advantages such as avoiding predators and increasing the chance of finding food. Similarly, cooperative behaviors are also important for many engineering systems such as multi-robot teams, autonomous air vehicles and automated highway systems [2],[5],[9].

The formation problem we study in this paper can be described by an example. Suppose a school teacher asks his (her) students to form a particular shape, say a fixed size ellipse, in the ground to play a game. However, the teacher does not designate the position and orientation of the ellipse. Hence the students don't have a common idea of where and what orientation the final ellipse should be. Furthermore the students are asked not to talk to each other and the only information they can use to form the ellipse is the observed relative positions of other students.

This class of autonomous agents formation problem was formulated and studied by Suzuki and Ramashita [16], Flocchini et. al. [3] and [7]. The main feature of this form of formation problem can be described by the so called "hard task" for "weak robots "[7]. For this, we mean that the robots are anonymous, memoryless with only local views, limited movement capabilities and without communication capabilities, albeit they are required to achieve a given Target Geometric Pattern (TGP) from any given initial robots configuration. This formation problem have been investigated mainly by simulation and experimental approaches [2],[14],[15], where heuristic algorithms were proposed and the correctness of the algorithms were tested by either simulations or experiments. Recently, concerns on computability and complexity of coordination problems have motivated researchers to approach the problem from a computational point of view [1],[3],[16]. For example, Cieliebak et.al. proposed a computationally tractable approach that solved the gathering problem.

In this paper, we propose and formalize the following formation idea. Let us consider the example of ellipse formation problem by the students again. In determining what the position a student should move to at any time, he or she first relocate and rotate the target ellipse (since they are not prefixed by the teacher) to compare with the form of current positions he/she observed and find the best matching location and orientation of the target ellipse. And the students then move to the corresponding destination points found in the best matching target form.

The process of finding the best matching position and orientation involves solving an optimization problem of minimizing an appropriately defined cost function measuring the difference between the target and current forms. Although the comparing processes are performed in a distributed fashion by the students independently in their local coordinate systems, we show that the results are global, that is, different students will come up with the same "best matching"ellipse location and orientation. Consequently, every movement according to the same best matching target form results in a new distribution for the students more similar to the final ellipse form.

Compared with the existing heuristic solutions for the similar problem, the optimization based formation ideas provide a general, yet mathematically rigorous algorithm for solving the distributed formation problem. The mathematical framework also allows a rigorous analysis of the performance of the algorithm including the speed of convergence.

II. Formation Problem Statement

We consider the problem of coordinating a group of autonomous, mobile robots to form a specific geometric pattern in a plane. A *geometric pattern* is a distribution of the robots in the plane such as a uniform distribution on a circle. However, the location and orientation of the geometric pattern are not fixed in advance. The formation

H. Zhang is with the Department of Electrical and Electronic Engineering, University of Melbourne, Melbourne, VIC 3010, Australia. hzhang5@unimelb.edu.au.

P.N. Pathirana is with School of Engineering and Information Technology, Deakin University, Pigdons Road, Geelong, VIC 3217, Australia. Pubudu.Pathirana@deakin.edu.au.

This is work is supported by Australian Research Council.

is performed by autonomous robots which means there is no central control scheme available and each robot moves based on local computations.

The solution to this problem depends on a number of problem specifications. In particular, the degree of common knowledge, the capability of a single robot and the geometric pattern the robots are required to form have prominent impact on the structure and complexity of the solution. The amount of available common knowledge is vital to the solvability of the problem [7]. For example, if there is a common (global) coordinate system available to the group of robots, then the problem is almost trivial since every robot can be given the explicit target position in the geometric pattern in terms of this coordinate system. It becomes more difficult if all the robots can only view the world in its own local coordinate system. The capabilities of a single robot such as the computational power, memory size and movement ability etc. also affect the performance of the solution. For example, it has been proven in [16] that if the robots have limited memory size and can only make a decision of movement based on the current view of the world (*oblivious*), then it is impossible for two robots to gather at one point in finite steps, while they can do it if the robots can remember and utilize the past observations (*nonoblivious*). The target geometric pattern is also important to the solution. While solutions exist for forming simple symmetric patterns like a circle, some geometric patterns may not even be achievable [16].

In this section we carefully formulate our autonomous robots formation problem. That is, we clearly specify factors such as common knowledge, the robot capabilities and properties of target geometric patterns. The aim is to formulate a formation problem in which a group of "*weak robots*"accomplish a "*hard task*".

A. Weak Robots

A robot is defined as a mobile computational unit equipped with sensors. It is viewed as a point in the plane, and hence the problem of collision does not arise. We assume the robots are identical and *anonymous*. This implies that a robot does not have an identity to use in the computation. We also assume that there is no direct communication between the robots. The only direct information a robot can get from other robots is their locations which are obtained by passive observations via sensors. The locations of other robots are expressed in terms of the observer's *local* (Cartesian) coordinate system $\mathcal{Z}_i = (\mathbf{o}_i, \mathbf{x}_i, \mathbf{u}_i)$. Here $\mathbf{o}_i$ is the location of the origin, $\mathbf{x}_i$ is the direction of the positive $\mathbf{x}$ axis and $\mathbf{u}_i$ is the size of the unit distance. For simplicity of presentation, we assume that all robots have a common sense of distance, i.e. $\mathbf{u}_i = u, \forall i$. It is also assumed that all robots have a common sense of orientation so that the positive $\mathbf{y}_i$-direction is 90 degrees counterclockwise from the positive $\mathbf{x}_i$-direction.

We assume that the robots have only limited memory capacity such that they can only remember the current location information of other robots. The robots always move in strait lines and the maximum distance a robot can move in one step is $\epsilon > 0$.

B. Hard Task

The task for robots is to form a particular target geometric pattern (TGP) from an arbitrary initial distribution. We assume the robots initially occupy different points in the plane. The TGP is passed to all robots at the beginning without assigning its location and orientation. Different robots view the TGP in their local coordinate systems and may have different understanding of the location and orientation of the TGP as shown in Fig. 1. However, we do fix the size of the TGP which means it is not scalable as those in [7],[16]. The only assumption we impose on the TGP is that the target robot positions are all distinct. The formation process must be performed in a fully *distributed* fashion by which we mean each robot executes the same algorithm independently to determine its movement. There is no central scheme coordinating the collective behavior of the group of robots. This is the reason that the robots are termed as *autonomous*.

We assume that the formation process takes place in a discrete time sequence $T = \{t_0, t_1, t_2, \cdots\}$. At any time instance t_k, a robot goes through a circle of three actions: *LOOK*, *THINK* and *MOVE*. The *LOOK* action returns the robot with the locations of all other robots at that moment via its sensors. This establishes the current robot configuration (CRC). In the *THINK* action, the robot executes the algorithm and makes a decision of next movement based on the established CRC. In the *MOVE* action, the robot simply moves to the destination point along the strait line connecting the current and destination points. If the destination point is less than ϵ away, then it reaches it directly. Otherwise it travels ϵ towards the destination in the line. In that case, the robot is not able to remember the current computational result. At he next time instance, it has to go through the circle again and recalculate the new destination position.

Since the robots have only limited memory size, they don't remember any past observations and computation results. When a robot comes to calculate the next destination position, the only information it can use is the current locations of all robots. An algorithm makes use of only the current robots configuration to determine its movement is called *oblivious*. Otherwise it is termed as *nonoblivious*. In formation problem, we also require the robots achieve the target geometric pattern in finite time steps.

Now we summarize the formation design problem as follows: **For a group of robots with limited memory size and limited mobility, we seek an oblivious, distributed algorithm such that each robot uses its observations to determine its movement to form a given target geometric pattern from any arbitrary initial configuration in the plane without the presence of a global coordination scheme and direct communication**.

III. CORRESPONDENCE AND ORIENTATION BASED OPTIMIZATION

A. Overview

In this section, we present a solution to the above formation problem. The general ideas are sketched in Fig. 1-Fig. 3. In step 1 (Fig. 1), the robots are assigned a task to form an ellipse in the plane. The location and orientation of the ellipse are not prefixed. Different robots view the current robot position distribution differently in their local coordinate systems and also have different understanding of the target ellipse.

In step 2 (Fig. 2), any active robot (here robot R_1) performs optimization to find the best location of the ellipse. We will prove in Theorem 3.1 for the cost function we defined below, the best location for the ellipse is at such a position that the center of gravities of current robot configuration and the ellipse are at the same point. To help performing a rotation optimization in step 3, move the origin of the local coordinate system $\mathcal{Z}_1$ to the center of gravity.

In step 3 (Fig. 3), do a rotation of the TGP to find the best orientation of the ellipse with respect to the current configuration. The best matching TGP will provide robot R_1 the destination of itself as well as all other robots.

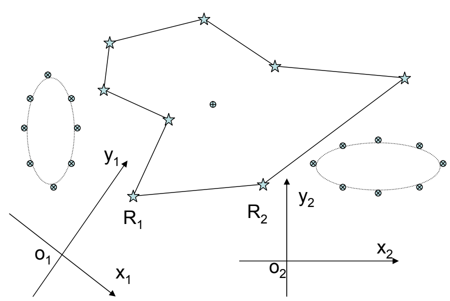

Fig. 1. step 1: the assignment of a target form (an ellipse) to the robots

Another robot R_2 performs the same as R_1, we shall prove in Theorem 3.3 that R_2 will get the same best matching

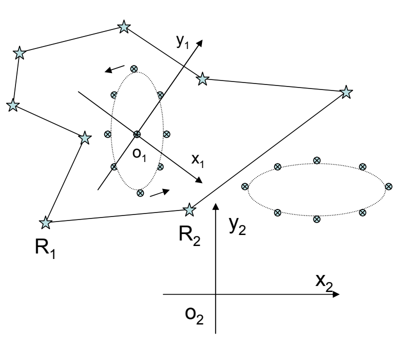

Fig. 2. step 2: robot $\mathbf{R}_1$ finds the optimal location for the ellipse with respect to the current configuration

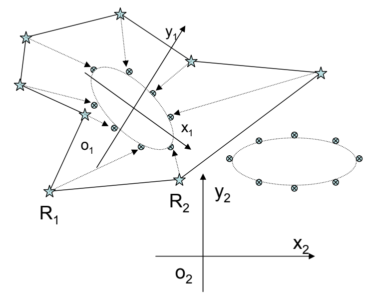

Fig. 3. step 3: robot $\mathbf{R}_1$ performs a rotation to find the best orientation of the TGP, and at the best location and orientation, it returns with the destinations for every robot.

location and orientation for the ellipse. So R_2 will come up with the same destination points for all robots as R_1 did. That is how the distributed computation leads to a common action strategy.

Now all the robots move to the destinations obtained from the best matching TGP location and orientation. Note that the location and orientation obtained are not necessary the final ellipse location and orientation, it may not be possible to reach all the destinations in one step due to the mobility limitations of the robots. But we will show that any movement of the robots will lead to a new robot configuration which is more similar to the target ellipse. Also because the oblivious character of the formation, the robots need to go through step 1 to step 3 again to obtain a new best matching location and orientation of the ellipse with respect to the updated robot distribution. We will prove in Theorem 4.1 that the process will achieve the formation in finite steps.

B. Measure of Difference Between CRC and TGP

The key to the above sketched algorithm is to guarantee any movement will make the CRC more similar to the TGP. Mathematically, it can be expressed as reducing some quantity measuring the difference between CRC and TGP. We now define such a function in an arbitrary local coordinate system.

Assume a robot $\mathcal{R}_i$ observes and records all the current robot positions $\{z_j^{i,t} = [x_j, y_j]_i'^t, j = 1, 2, \cdots, n\}$ at a time instant t, which establishes the CRC at time t. Here j represents the robot $\mathcal{R}_j$ and the subscript i means that the locations are expressed in the local coordinate system $\mathcal{Z}_i = (\mathbf{o}_i, \mathbf{x}_i, \mathbf{u}_i)$ of robot $\mathcal{R}_i$. When the local coordinate system and time instant are clear from context, we save notation and denote the positions as $\{z_j = [x_j, y_j]', j = 1, 2, \cdots, n\}$. Without loss of generality, we can assume the origin $\mathbf{o}_i$ of $\mathcal{Z}_i$ is at the Center of Gravity (CoG) $\bar{z} = [\bar{x}, \bar{y}]'$ of the CRC according to

$$\bar{z} = \frac{1}{n} \sum_{j=1}^{n} z_j. \tag{1}$$

Under such a coordinate system, the robot understands the TGP by marking the coordinates of the target robot positions

$\{\gamma_j = [\alpha_j, \beta_j]', j = 1, 2, \cdots, n\}$. With no loss of generality, we assume the CoG of the TGP $\bar{\gamma} = \frac{1}{n}\sum_{j=1}^{n}\gamma_j$ is at the origin $\mathbf{o}_i$, i.e. $\bar{\gamma} = [\bar{\alpha}, \bar{\beta}]' = [0, 0]'$.

Now a robot can give the order $[\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_n]$ and $[P_1, P_2, \cdots, P_n]$ to the current and target robot positions according to the increasing order of the angles between the positive $\mathbf{x}$-axis and the half line that starts from the origin and passes through the current (target) robot positions. If the angle is the same for two robots, then it is ordered according to the length of the segment between the origin and the robot position. With this ordering at hand, the coordinates of the current and target robot positions can be expressed in a more compact form as row vectors

$$Z = [z_1, z_2, \cdots, z_n], \Gamma = [\gamma_1, \gamma_2, \cdots, \gamma_n]. \tag{2}$$

Under the ordering convention, we can also define a notion of *correspondence* between the robot positions of CRC and TGP. Denote $\delta = [\delta_1, \delta_2, \cdots, \delta_n], \delta_j \in \{1, 2, \cdots, n\}$ a permutation of $[1, 2, \cdots, n]$, and denote the set of all permutations $\Delta = \{\delta \text{ is a permutation}\}$. We define that a permutation $\delta = [\delta_1, \delta_2, \cdots, \delta_n]$ designates a *correspondence* between the current and target robot positions as

$$\mathcal{R}_j \leftrightarrow P_{\delta_j}, j = 1, 2, \cdots, n.$$

Designate $\Gamma_\delta = [\gamma_{\delta_1}, \gamma_{\delta_2}, \cdots, \gamma_{\delta_n}]$ the vector of target robot positions of ordering $\delta = [\delta_1, \delta_2, \cdots, \delta_n]$. All permutations Δ designates all possible one to one correspondences between the current and target robot positions under the CRC and TGP in the current local coordinate system.

Now we define a function of difference between CRC and TGP given that the coordinates of current and target robot positions are expressed in the local coordinate system $\mathcal{Z}_i$. Given a correspondence $\delta = [\delta_1, \delta_2, \cdots, \delta_n]$ and the coordinates of current and target robot positions in the form of vectors Z and Γ respectively, we define a difference function for the CRC and TGP viewed by robot $\mathcal{R}_i$ in its coordinate system $\mathcal{Z}_i$ as

$$\begin{aligned} \mathbf{J}^i(\delta) &= \|Z - \Gamma_\delta\|^2 \\ &\doteq \sum_{j=1}^{n}\|z_j - \gamma_{\delta_j}\|^2 \\ &= \sum_{j=1}^{n}[(x_j - \alpha_{\delta_j})^2 + (y_j - \beta_{\delta_j})^2]. \end{aligned} \tag{3}$$

$\mathbf{J}^i(\delta)$ is the total Euclidean distances of all pairs $(\mathcal{R}_j \leftrightarrow P_{\delta_j}), j = 1, 2, \cdots, n$. It is obvious that $\mathbf{J}^i(\delta) \geq 0$ and $\mathbf{J}^i(\delta) = 0$ iff $x_j = \alpha_{\delta_j}, y_j = \beta_{\delta_j}, j = 1, 2, \cdots, n$, i.e. iff the robots achieve the TGP.

The robot $\mathcal{R}_i$ can rotate the TGP such that the TGP matches the CRC better, i.e. results in a smaller value of $\mathbf{J}^i$. A rotation operation of $\theta \in [-\pi, \pi)$ counterclockwise (a negative angle $\theta \in (-\pi, 0)$ means a rotation of $-\theta$ clockwise) transforms the coordinates of the robot target positions Γ to

$$\Gamma(\theta) = \Pi(\theta)\Gamma$$

where rotation matrix $\Pi(\theta)$ is given by

$$\Pi(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ +\sin(\theta) & \cos(\theta) \end{bmatrix}.$$

Now the difference function after rotation is

$$\begin{aligned} \mathbf{J}^i(\delta, \theta) &= \|Z - \Gamma_\delta(\theta)\|^2 \\ &= \|Z - \Pi(\theta)\Gamma_\delta\|^2. \end{aligned} \tag{4}$$

Furthermore, the robot can translate the TGP to find a better matching between the TGP and CRC. If the CoG of the TGP moves to $\psi = [\alpha, \beta]'$, the translation can be defined by the transform of the post-rotation target robot positions $\Gamma(\theta)$

$$\Gamma(\theta, \psi) = \Gamma(\theta) + \Psi$$

where $\Psi = [\psi, \psi, \cdots, \psi]$. After the rotation and translation, the difference function (3) becomes

$$\begin{aligned} \mathbf{J}^i(\delta, \theta, \psi) &= \|Z - \Gamma_\delta(\theta, \psi)\|^2 \\ &= \|Z - \Pi(\theta)\Gamma_\delta - \Psi\|^2. \end{aligned} \tag{5}$$

C. Finding the Best Matching TGP

In our algorithm, the most important calculation for a robot to determine the next destination is to find the best matching TGP position and orientation with respect to the CRC. This involves solving an optimization problem over the difference function defined above.

Given the coordinates of the current and target robot positions in terms of the coordinate system with the origin located at the CoG of the CRC. We define the best matching TGP with respect to the CRC is the pair $(\delta^*, \theta^*, \psi^*)$ minimizing the difference function $\mathbf{J}^i(\delta, \theta, \psi)$ in (5)

$$\mathbf{J}^i(\delta^*, \theta^*, \psi^*) = \min_{\delta \in \Delta, \theta \in [-\pi, \pi), \psi \in \mathbf{R}^2} \mathbf{J}^i(\delta, \theta, \psi). \tag{6}$$

The optimization problem (6) can be broken in two steps:

Step 1: First fix a particular $\delta \in \Delta$ and minimize for the orientation and location to find the best (θ^*, ψ^*)

$$\begin{aligned} \mathbf{V}^i(\delta) &= \|Z - \Pi(\theta^*)\Gamma_\delta - \Psi^*\|^2 \\ &= \min_{\theta \in [-\pi, \pi), \psi \in \mathbf{R}^2} \mathbf{J}^{i,\delta}(\theta, \psi) \\ &= \min_{\theta \in [-\pi, \pi), \psi \in \mathbf{R}^2} \|Z - \Pi(\theta)\Gamma_\delta - \Psi\|^2. \end{aligned} \tag{7}$$

Step 1 optimization can be solved explicitly as shown in the following theorem.

Theorem 3.1: Given the coordinates Z and Γ of the CRC and TGP in the local coordinate system of robot $\mathcal{R}_i$ and a given correspondence $\delta \in \Delta$, the minimum of the difference function $\mathbf{J}^{i,\delta}$ is

$$(\theta^*(\delta), \psi^*(\delta)) = (-\arctan(\frac{B^\delta}{A^\delta}), [0, 0]'), \tag{8}$$

where

$$A^\delta = \sum_{j=1}^{n}(x_j\alpha_{\delta_j} + y_j\beta_{\delta_j}),$$
$$B^\delta = \sum_{j=1}^{n}(x_j\beta_{\delta_j} - y_j\alpha_{\delta_j}).$$

Step 2: Find the optimal correspondence δ^* by minimizing over $\delta \in \Delta$

$$\begin{aligned} \mathbf{V}^i &= \min_{\delta \in \Delta} \mathbf{V}^i(\delta) \\ &= \min_{\delta \in \Delta} \|Z - \Pi(\theta^*(\delta))\Gamma_\delta\|. \end{aligned} \tag{9}$$

Since Δ is a finite set, the optimal δ^* can be found by simple compare and search method. The minimum is δ^*. And the minimum point for the difference function $\mathbf{J}^i$ in (5) is $(\delta^*, \theta^*(\delta^*), \psi^*(\delta^*))$ where $\psi^*(\delta^*) = [0, 0]'$.

Remark 3.2: Theorem 3.1 implies that the best location for the TGP is that the CoG of TGP should be located at the CoG of the CRC. Hence from now on, we assume, without loss of generality, the CoG of TGP is always at the CoG of the CRC. The only computation for the robot is to rotate the TGP at this convenient location according to $\theta^*(\delta)$ to find the best TGP orientation. This equivalents to optimizing the difference function $\mathbf{J}^i(\delta, \theta)$ in (4). We denote the minimum to be $(\delta^*, \theta^*(\delta^*))$.

D. Determining The Move Strategy

Solution to the optimization problem provides the optimizing robot with the *best matching* TGP given in terms of the post rotation target robot positions and orientation $\hat{\Gamma} = \Pi(\theta^*(\delta^*))\Gamma$ and the best correspondence δ^* between the current and target robot positions. $\hat{\Gamma}$ and δ^* determine a *move strategy*

$$Z_j \to \hat{\gamma}_{\delta_j^*}, j = 1, 2, \cdots, n, \tag{10}$$

that is, the current robot $\mathcal{R}_j$ should move towards the point of target robot position $\hat{\gamma}_{\delta_j^*}$. This move strategy is based on the local computation of robot $\mathcal{R}_i$ and independent of the computation of all other active robots. The key thing that makes the optimization useful to our formation problem is that the simultaneous computations of all other robots will result in the same move strategy. The idea of distributed computation resulting in a global consequence is the key to our autonomous robot formation solution as stated in the following theorem.

Theorem 3.3: Assume two robots $\mathcal{R}_i, i = a, b$ perform the above optimization processes independently with unique minimum in their own local coordinate systems $\mathcal{Z}_i, i = a, b$. Assume under an ordering convention $[\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_n]$ and $[P_1, P_2, \cdots, P_n]$, the optimal correspondences are $\delta_i^*, i = a, b$ and under a third global coordinate system $\mathcal{Z}$, the post-rotation target robot positions are $\hat{\Gamma}^i = [\hat{\gamma}_1^i, \hat{\gamma}_2^i, \cdots, \hat{\gamma}_n^i], i = a, b$. then it holds

$$\delta_a^* = \delta_b^* \tag{11}$$

and

$$\hat{\Gamma}^a = \hat{\Gamma}^b. \tag{12}$$

Theorem 3.3 shows that the move destinations obtained from local computations are global to all robots. We have the following result.

Corollary 3.4: Assume the minimums for the optimizations performed in local coordinate systems $\mathcal{Z}_i, i = a, b$ for robots $\mathcal{R}_i, i = a, b$ at time t are $\mathbf{V}^i, i = a, b$, then

$$\mathbf{V}^a = \mathbf{V}^b. \tag{13}$$

Proof: From Theorem 3.3, we know that the optimal correspondence $\delta_i^*, i = a, b$ and the post rotation TGP positions $\hat{\Gamma}_i, i = a, b$ are the same under a common coordinate system. It implies that the relative positions between the CRC and post rotation TGP are the same for robots $\mathcal{R}_i, i = a, b$. According to the definition of the difference function $\mathbf{J}^i$, we know that $\mathbf{V}^a = \mathbf{V}^b$.

■

From now on, we can omit the superscripts of $\mathbf{V}^i$ for different $\mathcal{R}_i$ and write them as common $\mathbf{V}$. We now show that any movement according to the move strategy leads to a robot configuration more similar to the TGP.

Theorem 3.5: Assume the active robots move according to the move strategy (10) obtained from the distributed optimizations. If $\mathbf{V}_k, k = 1, 2, \cdots$ are the minimums of the difference function (5) calculated in the local coordinate system of any robot at time k. Then

$$\mathbf{V}_k \geq \mathbf{V}_{k+1}, \forall k \geq 0. \tag{14}$$

IV. Formation Algorithm

We suggest an algorithm for the TGP formation problem based on the above move strategy.

Algorithm: FORM-TGP–BY-SYNCHRONOUS-ROBOTS

At every step $t = t_0, t_1, t_2, \cdots$, each robot $\mathcal{R}_i$ **Do**

1) **LOOK & RECORD**: Establish the CRC (current robot configuration) by recording the current robot positions $Z = [Z_1, Z_2, \cdots, Z_n]$ from the sensor readings in the local coordinate system $\mathcal{Z}_i$. At the same time, establish an understanding of the TGP (target geometric pattern) by denoting the coordinates for the target robot positions $\Gamma = [\gamma_1, \gamma_2, \cdots, \gamma_n]$.
2) **THINK to Obtain the Best move strategy**:
   - Do translations such that the origin of the local coordinate system is at the CoG of the CRC and TGP.
   - Optimize the cost function (4) and obtain the minimum $(\delta^*, \theta^*(\delta^*))$ and the optimum cost $\mathbf{J}^i(\delta^*, \theta^*(\delta^*))$. Rotate the TGP to the optimal orientation with coordinates $\Gamma^* = \Pi(\theta^*(\delta^*))\Gamma$ according to $\theta^*(\delta^*)$. The best action for robot $\mathcal{R}_i$ is to move from the current position Z_i to $\gamma_{\delta_i^*}^* = \Pi(\delta^*)\gamma_{\delta_i^*}$.
3) **If** $\mathbf{J}^i(\delta^*, \theta^*(\delta^*)) = 0$, then **STOP**, return with TGP achieved and output the coordinates of the robot positions of CRC and TGP. **Else** make a **MOVE** to $\gamma_{\delta_i^*}^*$ along the line connecting Z_i and $\gamma_{\delta_i^*}^*$.

Theorem 4.1: For given initial robot configuration described by a position vector Z and TGP described by target robot position vector Γ in any local coordinate systems, the Algorithm **FORM-TGP–BY-SYNCHRONOUS-ROBOTS** achieves the TGP in finite steps N, furthermore

$$N \leq \frac{\mathbf{V}_0}{\epsilon} + 1. \tag{15}$$

Proof: From Theorem 3.3, we know the minimum value $\mathbf{V}_0$ of the difference function (4) for the initial configuration and TGP is independent of the local coordinate system. Let's denote

$$d^k(Z_i, \gamma^*_{\delta^*_i}), i = 1, 2, \cdots, n$$

the distances between the current robot positions Z^k_i and their destination positions $\gamma^*_{\delta^*_i}$ at time k. Now if $\max_{i\in\{1,2,\cdots,n\}} d^k(Z_i, \gamma^*_{\delta^*_i}) \leq \epsilon$, then the robots achieves the TGP at time $k+1$ by directly moving to the destination positions. Hence only $\max_{i\in\{1,2,\cdots,n\}} d^{N-1}(Z_i, \gamma^*_{\delta^*_i}) \leq \epsilon$, while for all $0 \leq k \leq N-1$, it holds $\max_{i\in\{1,2,\cdots,n\}} d^k(Z_i, \gamma^*_{\delta^*_i}) > \epsilon$, i.e. there is at least one robot who moves a distance at least ϵ. From Theorem 3.5 we know

$$\mathbf{V}_k \geq \mathbf{V}_{k+1} + \epsilon, 0 \leq k \leq N-1.$$

We obtain

$$\mathbf{V}_0 - \epsilon * (N-1) \geq \mathbf{V}_{N-1} \geq 0$$

which is

$$N \leq \frac{\mathbf{V}_0}{\epsilon} + 1.$$

■

V. Simulations

We develop simulations based on the suggested formation control algorithms. The simulation results are shown in Fig. 4. The aim is to ask a group of 8 autonomous robots to form an ellipse somewhere in the plane as shown in Fig. 4. It is assumed that the maximum distance a robot can travel at one step is 0.2. Fig. 4 shows the formation by the synchronous algorithm. As it should be, the optimal cost sequence $\{\mathbf{V}_k\}$ decreases. In our simulation, the optimal cost decreases to 0 in 17 steps.

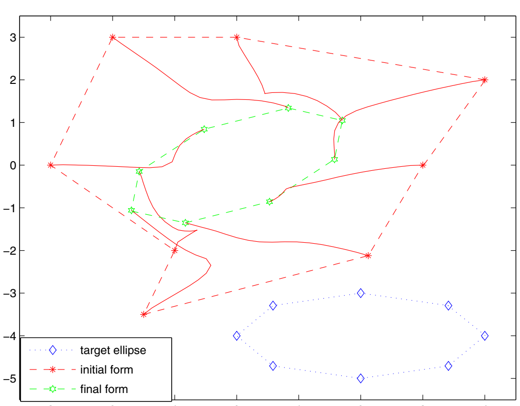

Fig. 4. Synchronous formation of an ellipse

VI. Concluding Remarks

We have developed a new distributed solution to the formation problem for autonomous robots. The key to the solution is to solve the optimization problem at every time step to find the best matching orientation and correspondence between the CRC and TGP such that every movement of the active robots will make the CRC more similar with the TGP until it is completely achieved.

We have divided the optimization process into two steps. The first step is to find the best orientation of the TGP relative to the CRC which can be solved explicitly. But for the second step which is to find the best correspondence, we have to search a finite set of $n!$ elements. Practically, it is only possible for small n. Reducing the search space for computational efficiency is an important and interesting topic of future research.

References

[1] H. Ando, Y. Oasa, I. Suzuki and M. Yamashita. A Distributed Memoryless Point Convergence Algorithm for Mobile Robots with Limited Visibility. IEEE Trans. on Robotics and Automation. Vol. 15, No. 5, pp. 818-828, 1999.

[2] T. Balch and R.C. Arkin. Behavior-Based Formation Control for Multirobot Teams. IEEE Trans. on Robotics and Automation. Vol. 14, No. 6. 1998.

[3] M. Cieliebak, P. Flocchini, G. Prencipe and N. Santoro, Solving the Robots Gathering Problem. In LNCS 2719, pp. 1181-1196, 2003.

[4] J. Cortes, S. Martinez and F. Bullo. Robust Rendezvous for Mobile Autonomous Agents via Proximity Graphs in Arbitrary Dimensions. IEEE Trans. on Automatic Control. Vol. 51, No. 8, pp. 1289-1298, 2006.

[5] S.Darbha and K.R.Rajagopal. Intelligent Cruise Control Systems and Traffic Flow Stability. Transportation Research Part C, Vol. 7, pp. 329-352, 1999.

[6] J.A. Fax and R.M.Murray. Information Flow and Cooperative Control of Vehicle Formations. IEEE Trans. on Automatic Control. Vol. 49, No. 9, pp. 1465-1476, 2004.

[7] P. Flocchini, G. Prencipe, N. Santoro and P. Widmayer, Hard Task for Weak Robots: The Role of Common Knowledge in Pattern Formation by Autonomous Mobile Robots. In ISAAC'99, vol. 1741 of LNCS, pp. 93-102, 1999.

[8] V. Gazi and K.M. Passino. Stability Analysis of Swarms. IEEE Trans. Automatic Control. Vol.48. No. 4, pp.692-697, 2003

[9] F. Giulietti, L. Pollini and M. Innocenti. Autonomous Formation Flight. IEEE Control Systems Magazine. Vol. 20, No. 6, pp. 34-44, 2000.

[10] A. Jadbabaie, J. Lin and A.S. Morse. Coordination of Groups of Mobile Autonomous Agents Using Nearest Neighbor Rules. IEEE Trans. on Automatic Control. Vol. 48, No. 6, pp. 988-1001, 2003.

[11] Z. Lin, B. Francis and M. Maggiore. Necessary and Sufficient Graphical Conditions fo Formation Control of Unicycles. IEEE Trans. on Automatic Control. Vol. 50, No. 1, 2005.

[12] Y. Liu and K.M. Passino. Stable Social Foraging Swarms in a noisy environment. IEEE Trans. on Automatic Control. Vol. 49, No. 1, pp. 30-44, 2004.

[13] S. Martinez, J. Cortes and F. Bullo. Motion Coordination With Distributed Information. IEEE Control Systems Magazine, pp. 75-88, August 2007.

[14] K. Sugihara and I. Suzuki. Distributed Motion Coordiantion of Multiple Mobile Robots. In Proceedings of the 5th IEEE International Sympsium on Intellligent Control. Philadelphia, PA, pp. 138-143, 1990.

[15] K. Sugihara and I. Suzuki. Distributed Algorithms for Formation of Geometric Patterns With Many Mobile Robots. Journal of Robotic Systems, Vol. 13, pp. 127-139, 1996.

[16] I. Suzuki and M. Yamashita. Distributed Anonymous Mobile Robots: Formation of Geometric Pattern. SIAM J. of Computing, 28(4):1347-1363, 1999.

[17] C.B. Yu, J.M. Hendrickx, B. Fidan, B.D.O. Anderson and V.D. Bloner. Three and Higer Dimensional Autonomous Formations: Rigidity, Persistence and Structural Persistence. Automatica. Vol. 43, pp. 387-402, 2006.

[18] K. Warburton and J. Lazarus. Tendency-distance Models of Social Cohesion in Animal Groups. J. of Theoretical Biology. Vol. 150, pp. 473-488, 1991.