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Asymptotic Tracking Control of Piezoelectric Actuators with Hysteresis

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***Abstract*— A piezoelectric actuator (PEA) with hysteretic characteristics is usually difficult to precisely control because the unmeasurable hysteretic force is typically generated by a complicated nonlinear model. In this paper, the global robust tracking problem of a PEA with hysteresis is first studied within an internal model architecture. With the proposed controller, the PEA is able to asymptotically track a desired reference trajectory in the presence of plant uncertainties.**

***Index Terms*— Hysteresis, piezoelectric actuator, nonlinear systems, output regulation**

I. INTRODUCTION

Piezoelectric actuators (PEAs) have been recognized as the most appropriate actuation devices for accomplishing high-precision motion tasks in the field of micro/nano manipulation, such as scanning probe microscopy (SPM), scanning tunnelling microscopy (STM), atomic force microscope (AFM), optical alignments, active vibration control of rotor bearing systems, and tracking control of hard disk drivers. In PEAs, the dynamic characteristics between the displacement and the electric field strength involve a class of hysteretic nonlinearities, which sometimes increase the complexity in precise positioning control. In particular, the errors caused by hysteretic effects may lead to undesirable inaccuracies and even instability. Over the past decade, many efforts have been devoted to establish different models to describe the hysteretic dynamics. The relevant works include the Preisach model [1], the Duhem model [2], the Maxwell slip model [3], the constant phase lag approximation [4], and the Bouc-Wen model [5], [6], among which the last one has gained a special attention due to its capability of capturing the analytical principle for a range of shapes of hysteretic loops. Typically, a second order mechanical system coupled with a Bouc-Wen model has been widely used to describe the dynamics of physical devices with hysteresis, e.g., PEAs (see., e.g., [6], [7]).

The difficulty in the precise positioning control problem for a simple linear model coupled with a hysteresis model lies in two facts that the hysteretic model is highly nonlinear and the hysteretic force is unmeasurable. The problem has been studied for long time due to its academic and practical interests. In the early work, Tao and Kokotovic presented adaptive controllers with adaptive hysteresis inverse [8] and saturation-based controllers [9] to deal with hysteretic nonlinearities. Ge and Jouaneh [1] developed a controller consisting of a feed-forward tracking component and a PID feedback controller. Chang and Sun [10] proposed a feed-forward model in a feedback control with an input shaper. Shieh and Hsu [11] designed an adaptive back-stepping approach. Tan and Baras [12] addressed a Preisach operator-based recursive identification algorithm together with an adaptive inverse controller. Zaccarian and Teel [13] designed a saturation-based nonlinear scheduled anti-windup method for linear systems, with which hysteresis switching among a family of linear gains was employed for performance improvement. The method was further developed for nonlinear systems in [14]. Moreover, a large volume of literature from MEMS community has also reported the progresses and challenges of hysteresis control in an industrial perspective, e.g., [15], [16].

Recently, the precise tracking problem has attracted more attentions. A feed-forward hysteresis compensation algorithm was proposed by Song *et al.* [17] to reduce tracking errors. Later on, variable structure control (VSC) laws were proposed by Hwang *et al.* [18] and Liaw *et al.* [19], to achieve exact tracking. But an obvious disadvantage of a variable structure controller is that it is discontinuous and it gives rise to control chattering unless a boundary saturation function is introduced at the cost of loss of accuracy. Rather than the discontinuous VSC laws, a continuous linear PID controller was proposed by Ikhouane and Rodellar [6] which is also able to achieve exact tracking provided the reference trajectory and its derivatives exponentially approach zero. From the above, the asymptotic tracking control problem has yet to be further investigated, which motivates the present research.

In this paper, we aim to design a global robust tracking controller for a PEA coupled with a Bouc-Wen model when the reference trajectories are in a more general form (not necessarily approaching zero). The problem can be put in the framework of output regulation problem (see, e.g. [20]) and an internal model based controller is thus proposed. The major idea is to construct a nonlinear observer (called an internal model) for the unmeasurable hysteretic force and hence use the estimated state in the controller design. Moreover, a sufficiently high gain is dynamically generated to dominate the unknown parameters in the system. With the proposed controller, the PEA displacement is able to asymptotically track a desired reference trajectory for any initial conditions of the closed-loop system.

The rest of this paper is organized as follows. In Section II, the main problem is formulated in a so-called tracking

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model. In Section III, the global robust controller is proposed together with the stability analysis for the closed-loop system. Afterwards, numerical simulation results are given in Section IV to demonstrate the feasibility and superiority of the proposed controller. Finally, conclusions are drawn in Section V.

II. Problem Formulation

An ideal PEA has the following second order mechanical model

$$m\ddot{x} + c\dot{x} + kx = u$$

with x the displacement and $v := \dot{x}$ the velocity. When the mass $m > 0$, the damping $c > 0$, and the stiffness $k > 0$ are all precisely known, the tracking problem is straightforward. Specifically, for any sufficiently smooth reference trajectory $y(t)$, the asymptotic performance $\lim_{t\to\infty} x(t) - y(t) = 0$ is achieved by a feed-forward compensator

$$u = m\ddot{y} + c\dot{y} + ky.$$

However, when the parameters $\mu := [k, c, m]^\intercal$ are unknown, this feed-forward compensator is not implementable. Nevertheless, the tracking problem is still solvable in the framework of robust output regulation (see, e.g., [20]) as simply re-visited below.

Throughout the paper, we assume that the smooth reference trajectory $y(t)$ is generated by the following linear autonomous system, called an exosystem:

$$y = C\varsigma,\ \dot{\varsigma} = A\varsigma. \tag{1}$$

for two constant matrices A and C. This assumption is widely used in the literature of output regulation problem. It accommodates many typical signals in practice including constant, ramp, polynomial, sinusoid, exponential, etc. With the exosystem (1), we define a function

$$\mathbf{u}(\mu, \varsigma) = mCA^2\varsigma + cCA\varsigma + kC\varsigma,$$

and obviously, $\mathbf{u}(\mu, \varsigma) = m\ddot{y} + c\dot{y} + ky$. Since the function $\mathbf{u}(\mu, \varsigma)$ depends on the unknown parameter μ, it can not be directly compensated. However, since ς is generated by the exoystem (1), there exists a so-called steady state generator:

$$\begin{aligned} \dot{\theta}(\mu, \varsigma) &= \Phi\theta(\mu, \varsigma) \\ \mathbf{u}(\mu, \varsigma) &= \Psi\theta(\mu, \varsigma) \end{aligned} \tag{2}$$

for an observable pair (Ψ, Φ). Next, pick a pair of matrices (M, N) such that M is Hurwitz and has a disjoint spectrum with Φ, and (M, N) is controllable. Then, there exists a nonsingular matrix T satisfying the Sylvester equation

$$T\Phi = MT + N\Psi.$$

Define a vector $\vartheta(\mu, \varsigma) := T\theta(\mu, \varsigma)$. Then, the steady state generator can be rewritten as

$$\begin{aligned} \dot{\vartheta}(\mu, \varsigma) &= T\Phi T^{-1}\vartheta(\mu, \varsigma) \\ \mathbf{u}(\mu, \varsigma) &= \Psi T^{-1}\vartheta(\mu, \varsigma). \end{aligned} \tag{3}$$

Now, it is ready to construct an observer (called an internal model) for ϑ and hence a feedback controller as follows:

$$\begin{aligned} u &= \Psi T^{-1}\eta + h \\ \dot{\eta} &= M\eta + Nu \end{aligned} \tag{4}$$

where h is an additional control to be designed later. Under a new coordinate

$$\chi = x - y,\ \nu = v - \dot{y},\ z = \eta - \vartheta - Nm\nu \tag{5}$$

the closed-loop system becomes

$$\begin{aligned} \dot{z} &= Mz + Nk\chi + (MNm + Nc)\nu \\ \dot{\chi} &= \nu \\ m\dot{\nu} &= \Psi T^{-1}z - k\chi + (\Psi T^{-1}Nm - c)\nu + h. \end{aligned} \tag{6}$$

Define the following matrices

$$\mathcal{A} = \begin{bmatrix} M & Nk & (MNm + Nc) \\ 0 & 0 & 1 \\ \frac{\Psi T^{-1}}{m} & \frac{-k}{m} & \frac{\Psi T^{-1}Nm - c}{m} \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0 & 0 & 1/m \end{bmatrix}^\intercal,\ \mathcal{C} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

If a row vector $L \in \mathbb{R}^{1\times 2}$ is such that $\mathcal{A} + \mathcal{B}L\mathcal{C}$ is Hurwitz, the stabilization problem of the system (6), and hence the original tracking problem is solved by a simple linear controller $h = L[\chi, \nu]^\intercal$.

In the sequel, we called the system (6) a *tracking model* because the asymptotic stability of the equilibrium point $\mathrm{col}(z, \chi, \nu) = 0$ of (6) implies the tracking performance

$$\begin{aligned} \lim_{t\to\infty} x(t) - y(t) &= \lim_{t\to\infty} \chi(t) = 0 \\ \lim_{t\to\infty} v(t) - \dot{y}(t) &= \lim_{t\to\infty} \nu(t) = 0. \end{aligned}$$

Now, it is ready to formulate the main problem based on the tracking model. In particular, we consider the influence of a hysteretic force bw appearing in the control channel of the tracking model. Thus, the system (6) with the additional term bw can be put in a more compact form

$$\begin{bmatrix} \dot{z} \\ \dot{\chi} \\ \dot{\nu} \end{bmatrix} = \mathcal{A}\begin{bmatrix} z \\ \chi \\ \nu \end{bmatrix} + \frac{1}{m}\begin{bmatrix} 0 \\ 0 \\ -bw + h \end{bmatrix}. \tag{7}$$

Here, w is the hysteretic force described by a Bouc-Wen model:

$$\begin{aligned} \dot{w} &= \rho(v + g(v, w)),\ v = \nu + \dot{y} \\ & \quad g(v, w) := -\sigma|v||w|^{n-1}w + (\sigma - 1)v|w|^n \end{aligned} \tag{8}$$

where the parameters are $b > 0$, $\rho > 0$, $\sigma \geq 1/2$, and $n \geq 1$. The main objective is to design the control h to compensate the influence of bw. The problem becomes more complicated because the state w is not measurable and it can not be directly cancelled. In particular, we note that $w = 0$ is not an asymptotically stable equilibrium point of the system (8) for any v. To illustrate the influence of the hysteretic force, we have a motivation example below.

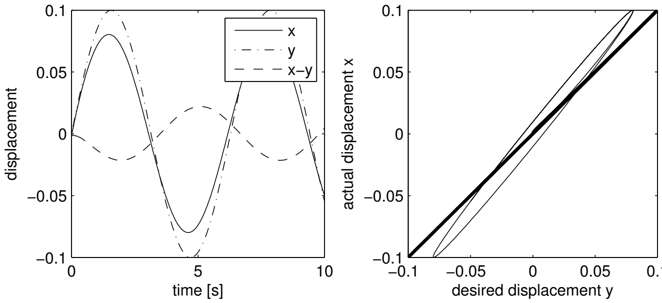

Fig. 1. Profile of trajectories with hysteresis for a sinusoidal reference $y(t) = 0.1\sin(t)$. When the hysteresis disappears, the hysteretic loop in the right graph reduces to the bold straight line.

Example 1: A motivation example is given in Fig. 1 to illustrate the influence of the hysteresis term bw. In particular, when $b = 0$, the tracking performance under the aforementioned controller $h = L[\chi, \nu]^\intercal$ is represented by the bold straight line in the right graph for $y(t) = 0.1\sin(t)$. With the same controller but a non-zero b, the tracking performance is lost and the influence of the term bw is represented by the hysteresis loop in the right graph. In this case, the trajectories of x, y and $x - y$ in time course are depicted in the left graph. ∎

In this paper, we aim to propose a novel controller to deal with the influence caused by the hysteresis term. In the system composed of (7) and (8), only the states χ and ν are available for control feedback. The precise formulation is given below.

Global robust asymptotic tracking problem: Consider the system composed of (7) and (8) with states col(z, χ, ν, w) and an input h. Assume the external reference signal $\dot{y}$ is bounded. Find a continuous controller of the following form

$$\begin{aligned} h &= \phi(\chi, \nu, \zeta) \\ \dot{\zeta} &= \psi(\chi, \nu, \zeta, \dot{y}) \end{aligned} \tag{9}$$

such that the trajectories of the closed-loop system composed of (7), (8) and (9), from any initial condition, are bounded and satisfy $\lim_{t\to\infty} \text{col}(\chi(t), \nu(t)) = 0$.

Remark 1: The controller (9) to be constructed in this paper is robust with respect to the mechanical model parameters k, c, m. In other words, no knowledge on these parameters is required a prior. In particular, it is not necessary to assume them lie in known intervals as in [6]. ∎

Remark 2: In this problem formulation, a tracking performance $\lim_{t\to\infty} \nu(t) = 0$, i.e., $\lim_{t\to\infty} v(t) - \dot{y}(t) = 0$ is a main requirement. Therefore, at the steady space $v - \dot{y} = 0$, the Bouc-Wen model becomes $\dot{w} = \rho(\dot{y} + g(\dot{y}, w))$. Generally, we do not have the property that $\lim_{t\to\infty} \dot{w}(t) = 0$ unless $\lim_{t\to\infty} \dot{y}(t) = 0$. In fact, the condition $\lim_{t\to\infty} \dot{y}(t) = 0$ (more precisely, $y(t)$ and its derivatives exponentially approach zero) is critical for the tracking problem using a PID controller [6]. Obviously, this condition is very restrictive in practice, which motives us to consider a more general tracking problem to remove this condition. ∎

III. Main Results

The main difficulty in the global robust asymptotic tracking problem is caused by unknown term bw in the system (7). Otherwise, the problem is directly solved by a controller $h = bw + L[\chi, \nu]^\intercal$. When the unmeasurable state w in (7) is regarded as an exogenerous signal, the problem can be formulated as an output regulation problem with (8) an exosystem [20]. However, the existing framework for output regulation problem is not ready for the current situation, because the framework accommodates autonomous exosystem of the form $\dot{w} = a(w)$ while the Bouc-Wen model is non-autonomous. Moreover, the exosystem in the existing framework is usually linear, especially for global cases, while the Bouc-Wen model is highly nonlinear. Nevertheless, the internal model principle for output regulation problem can be further developed in this section to accommodate the present scenario. First of all, we list some technical lemmas below.

Lemma 1: The function g defined in (8) satisfies the property: $\xi[g(v, \xi + d) - g(v, d)] \leq 0$, $\forall \xi, v, d \in \mathbb{R}$. ∎

Proof: It suffices to show for any given v, the function $g(v, w)$ is monotonically decreasing (not necessarily strictly) with respect to w. If $v = 0$, $g(0, w) = 0$, the statement is trivially true. Next, we consider the case with $v \neq 0$.

Define a new function

$$g_1(s, w) := |w|^{n-1}[\sigma w - (\sigma - 1)s|w|]$$

where $s = \text{sign}(v)$ is 1 or -1. It suffices to show that the function $g_1(s, w)$ is monotonically increasing for $s = 1$ and $s = -1$. In particular, we note that

$$g_1(s, w) \begin{cases} > 0, & w > 0 \\ = 0, & w = 0 \\ < 0, & w < 0 \end{cases}$$

For $w > 0$, $g_1(s, w) = w^n[\sigma - (\sigma - 1)s]$ with $[\sigma - (\sigma - 1)s] > 0$ is obviously monotonically increasing. For $w < 0$, $g_1(s, w) = -|w|^n[\sigma + (\sigma - 1)s]$ with $[\sigma + (\sigma - 1)s] > 0$ is also monotonically increasing. ■

Lemma 2: For the function g defined in (8),

$$|g(\nu + \delta_1, \delta_2\nu + w) - g(\nu + \delta_1, w)| \leq |\nu|\hbar(\delta_1, \delta_2, w)\ell(|\nu|),$$

$\forall \nu, \delta_1, \delta_2, w \in \mathbb{R}$ for some smooth functions $\hbar(\delta_1, \delta_2, w) > 0$ and $\ell(|\nu|) \geq 1$. ∎

Proof: First, we define a function

$$f_1(\delta_2, w, \nu) = \begin{cases} |\delta_2\nu|^{n-1}\delta_2, & w = 0 \\ \frac{|\delta_2\nu + w|^{n-1}(\delta_2\nu + w) - |w|^{n-1}w}{\nu} & w \neq 0, \nu \neq 0 \\ n|w|^{n-1}\delta_2 & w \neq 0, \nu = 0 \end{cases}$$

where we note

$$\lim_{\nu\to 0} \frac{|\delta_2\nu + w|^{n-1}(\delta_2\nu + w) - |w|^{n-1}w}{\nu} = n|w|^{n-1}\delta_2.$$

Clearly, we have

$$|\delta_2\nu + w|^{n-1}(\delta_2\nu + w) - |w|^{n-1}(w) = f_1(\delta_2, w, \nu)\nu.$$

Similarly, we can define a function $f_2(\delta_2, w, \nu)$ such that

$$|\delta_2\nu + w|^n - |w|^n = f_2(\delta_2, w, \nu)\nu.$$

As a result,

$$|g(\nu + \delta_1, \delta_2\nu + w) - g(\nu + \delta_1, w)| \le |\nu| f_3(\delta, \nu)$$

for

$$\begin{aligned} \delta &:= \text{col}(\delta_1, \delta_2, w) \\ f_3(\delta, \nu) &:= |\sigma(\nu + \delta_1) f_1(\delta_2, \nu, w)| + \\ &\quad |(\sigma - 1)(\nu + \delta_1) f_2(\delta_2, \nu, w)|. \end{aligned}$$

Next, we define

$$\hbar_o(s) = \sup_{|\nu| \le \|\delta\| \le s} f_3(\delta, \nu),\ \ell_o(s) = \sup_{\|\delta\| \le |\nu| \le s} f_3(\delta, \nu).$$

As a result, we have $f_3(\delta, \nu) \le \hbar_o(\|\delta\|) + \ell_o(|\nu|)$. It suffices to pick $\hbar$ and ℓ such that

$$\hbar_o(\|\delta\|) + \ell_o(|\nu|) < (\hbar_o(\|\delta\|) + 1)(\ell_o(|\nu|) + 1) \le \hbar(\delta)\ell(|\nu|).$$

■

The next lemma can be found in [6]. For the convenience of reader, we list the lemma below with a self-contained proof.

Lemma 3: Let w be governed by the dynamics $\dot{w} = \rho(v + g(v, w))$ with g defined in (8) where v is an arbitrary external signal and continuous in time. Then $|w(t)| \le \max\{1, |w(0)|\}$, $\forall t \ge 0$. ■

Proof: Denote $r = \max\{1, |w(0)|\}$. First, we will show $w(t) \le r$. Pick $\tau \ge 0$ be the first instant when $w(\tau) = r$. We will show that $\dot{w}(\tau) \le 0$ and hence the trajectory does not go through the boundary point $w = r$ increasingly. That is, $w(t) \le r$. To show $\dot{w}(\tau) \le 0$, we have

$$\dot{w}(\tau) = \rho(v - \sigma|v|r^n + (\sigma - 1)vr^n) \le 0$$

by noting $r \ge 1$ and $\sigma \ge 1/2$.

The proof for $w(t) \ge -r$ is in a dual procedure. Pick $\tau \ge 0$ be the first instant when $w(\tau) = -r$. We will show that $\dot{w}(\tau) \ge 0$ and hence the trajectory does not go through the boundary point $w = -r$ decreasingly. That is, $w(t) \ge -r$. To show $\dot{w}(\tau) \ge 0$, we note

$$\dot{w}(\tau) = \rho(v + \sigma|v|r^n + (\sigma - 1)vr^n) \ge 0.$$

From the above, in both cases, the trajectory $w(t)$ does not leave the boundary $|w| = r$. The proof is thus complete. ■

Now, we move to the construction of the controller h. The controller is first explicitly listed as follows

$$\begin{aligned} h &= b\varpi - \kappa\beta(\chi, \nu) \\ \dot{\varpi} &= \rho(\nu + \dot{y} + g(\nu + \dot{y}, \varpi)) - \alpha\kappa\beta(\chi, \nu) \\ \dot{\kappa} &= \gamma(\chi, \nu) \end{aligned} \tag{10}$$

where $\alpha > 0$ is an arbitrary constant, and the functions β and γ will be determined later. In the controller (10), ϖ is the estimation of w, and κ is a dynamically generated gain to handle the unknown parameter μ, the unknown boundary of $w(t)$ (depending on $w(0)$), and the unknown boundary of the reference signal $\dot{y}(t)$. It is noted that the controller (10) is in the form of (9) with $\zeta = \text{col}(\varpi, \kappa)$.

What left is to explicitly construct the functions β and γ such that the closed-loop system has the desired stability property. The main result is summarized in the following theorem.

Theorem 1: Consider the system composed of (7) and (8) with a bounded reference signal $\dot{y}(t)$. Then, there exist continuous functions β and γ such that the trajectories of the closed-loop system composed of (7), (8) and (10), from any initial condition, are bounded and satisfy $\lim_{t\to\infty} col(\chi(t), \nu(t)) = 0$. That is, the global robust asymptotic tracking problem is solved. ■

Proof: First of all, we introduce a coordinate transformation

$$\xi = \varpi - w - \alpha m\nu.$$

Under this coordinate, the closed-loop system is composed of (8) and the following one:

$$\begin{aligned} \dot{\xi} &= -\alpha b\xi + \rho g(v, \xi + \alpha m\nu + w) - \rho g(v, w) \\ &\quad -\alpha[\Psi T^{-1}z - k\chi + (\Psi T^{-1}Nm - c + \alpha bm)\nu] \\ \dot{z} &= Mz + Nk\chi + (MNm + Nc)\nu \\ \dot{\chi} &= \nu \\ m\dot{\nu} &= \Psi T^{-1}z - k\chi + (\Psi T^{-1}Nm - c + b\alpha m)\nu \\ &\quad +b\xi - \kappa\beta(\chi, \nu) \\ \dot{\kappa} &= \gamma(\chi, \nu). \end{aligned} \tag{11}$$

In the following analysis, we will study the stability of the system (11) taking w as an external signal, which is bounded by Lemma 3. To this end, we define Laypunov function candidates for all subsystems and verify their properties along the trajectories of (11).

Let $V_1(\xi) = \xi^2/(\alpha b)$ and $\bar{\nu} = \nu + \chi$. Before examining the property V_1, we note that

$$\begin{aligned} &\xi[g(v, \xi + \alpha m\nu + w) - g(v, w)] \\ &= \xi[g(v, \xi + \alpha m\nu + w) - g(v, \alpha m\nu + w)] \\ &\quad +\xi[g(v, \alpha m\nu + w) - g(v, w)]. \end{aligned}$$

By Lemma 1, we have

$$\xi[g(v, \xi + \alpha m\nu + w) - g(v, \alpha m\nu + w)] \le 0$$

and by Lemma 2,

$$|g(v, \alpha m\nu + w) - g(v, w)| \le |\nu|\hbar_1(m, w, \dot{y})\ell_1(|\nu|)$$

for some smooth functions $\hbar_1$ and $\ell_1 \ge 1$. It is easy to check

$$|\nu|\ell_1(|\nu|) \le |\bar{\nu}|\ell_2(|\bar{\nu}|) + |\chi|\ell_3(|\chi|)$$

for some smooth functions $\ell_2, \ell_3 \geq 1$. As a result, one has

$$\frac{dV_1(\xi)}{dt} \leq -2\xi^2 + 2\rho\xi|\nu|\hbar_1(m,w,\dot{y})\ell_1(\nu)/(\alpha b)$$
$$+2\xi[\Psi T^{-1}z - k\chi + (\Psi T^{-1}Nm - c + \alpha bm)\nu]/b$$
$$\leq -\xi^2 + \hbar_2(\mu,w,\dot{y})[\bar{\nu}^2\ell_2^2(|\bar{\nu}|) + \chi^2\ell_3^2(|\chi|)] + 2\|\Psi T^{-1}/b\|^2\|z\|^2$$

for some smooth function $\hbar_2$.

Let $V_2(z) = z^\intercal P z$ where P is a symmetric positive definite matrix to the Lyapunov equation $PM + M^\intercal P = -2I$. Then,

$$\frac{dV_2(z)}{dt} \leq -2\|z\|^2 + 2z^\intercal P[Nk\chi + (MNm + Nc)\nu]$$
$$\leq -\|z\|^2 + \hbar_3(\mu)(\chi^2 + \bar{\nu}^2)$$

for some smooth function $\hbar_3$.

Next, pick a non-decreasing function

$$\varrho(s) \geq \ell_3^2(\sqrt{s}),\ s \geq 0$$

and let $V_3(\chi) = \int_0^{\chi^2} \varrho(s)ds$. Then,

$$\frac{dV_3(\chi)}{dt} = 2\varrho(\chi^2)\chi(-\chi + \bar{\nu}) \leq -\varrho(\chi^2)\chi^2 + 4\varrho(4\bar{\nu}^2)\bar{\nu}^2$$
$$\leq -\ell_3^2(|\chi|)\chi^2 + \ell_4^2(|\bar{\nu}|)\bar{\nu}^2.$$

for some smooth function $\ell_4 \geq 1$.

Then, letting $V_4(\bar{\nu}) = \bar{\nu}^2$ gives

$$\frac{dV_4(\bar{\nu})}{dt} = \hbar_4(\mu)(\bar{\nu}^2 + \chi^2 + \|z\|^2 + \xi^2) - 2\bar{\nu}\kappa\beta(\chi,\nu)/m$$

for some smooth function $\hbar_4$.

Finally, we define a Laypunov function candidate

$$W(\xi,z,\chi,\nu,\kappa) = \epsilon_1 V_1(\xi) + \epsilon_2 V_2(z) + \epsilon_3 V_3(\chi) + V_4(\bar{\nu}) + (\kappa - \kappa^*)^2/(\gamma_o m)$$

for some positive numbers ϵ_i, γ_o, and κ^*. In particular, we pick the following parameters

$$\begin{aligned}
\epsilon_1 &\geq 1 + \hbar_4(\mu) \\
\epsilon_2 &\geq 1 + \epsilon_1 2\|\Psi T^{-1}/b\|^2 + \hbar_4(\mu) \\
\epsilon_3 &\geq 1 + \epsilon_1\hbar_2(\mu,w,\dot{y}) + \epsilon_2\hbar_3(\mu) + \hbar_4(\mu)
\end{aligned}$$

such that

$$\frac{dW(\xi,z,\chi,\nu,\kappa)}{dt} \leq -\xi^2 - \|z\|^2 - \ell_3^2(|\chi|)\chi^2$$
$$+\bar{\nu}^2\big[\epsilon_1\hbar_2(\mu,w,\dot{y})\ell_2^2(|\bar{\nu}|) + \epsilon_2\hbar_3(\mu) + \epsilon_3\ell_4^2(|\bar{\nu}|) + \hbar_4(\mu)\big] - 2\bar{\nu}\kappa^*\beta(\chi,\nu)/m$$
$$-2\bar{\nu}(\kappa - \kappa^*)\beta(\chi,\nu)/m + 2(\kappa - \kappa^*)\gamma(\chi,\nu)/(\gamma_o m)$$
$$\leq -\xi^2 - \|z\|^2 - \chi^2 - \bar{\nu}^2.$$

The last inequality holds if the functions β and γ are chosen as follows

$$\begin{aligned}
\gamma(\chi,\nu) &= \gamma_o(\nu + \chi)\beta(\chi,\nu),\ \gamma_o > 0 \\
\beta(\chi,\nu) &= (\nu + \chi)\beta_o(\nu + \chi)
\end{aligned}$$

and β_o and κ^* satisfy

$$\kappa^*\beta_o(\bar{\nu}) \geq (m/2)\big[\epsilon_1\hbar_2(\mu,w,\dot{y})\ell_2^2(|\bar{\nu}|) + \epsilon_2\hbar_3(\mu) + \epsilon_3\ell_4^2(|\bar{\nu}|) + \hbar_4(\mu) + 1\big].$$

It suffices to pick β_o and κ^* such that:

$$\beta_o(\bar{\nu}) = \ell_2^2(|\bar{\nu}|) + \ell_4^2(|\bar{\nu}|)$$

and

$$\kappa^* \geq (m/2)\max\{\epsilon_1\hbar_2(\mu,w,\dot{y}), \epsilon_3, \epsilon_2\hbar_3(\mu) + \hbar_4(\mu) + 1\}.$$

The existence of k^* is guaranteed by the fact that $\mu, w, \dot{y}$ are all bounded.

From the above, we have

$$\frac{dW(\xi,z,\chi,\nu,\kappa)}{dt} \leq -\xi^2 - \|z\|^2 - \chi^2 - \bar{\nu}^2.$$

The proof is thus complete by applying LaSalle-Yoshizawa Theorem. ■

IV. Numerical Examples

We consider a reference trajectory $y(t) = 0.1\sin(t)$. Obviously, the trajectory is generated by the exosystem (1) with $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $\varsigma(0) = [0\ \ 1]^\intercal$. A simple calculation shows $\mathbf{u}(\mu,\varsigma) = k\varsigma_1 + c\omega\varsigma_2 - m\omega^2\varsigma_1$ and $\Phi = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \Psi = \begin{bmatrix} 1 & 0 \end{bmatrix}$. For this case, the influence of hysteresis on the tracking performance has been demonstrated in Example 1. Following the procedure in Theorem 1, the controller (10) can be explicitly constructed. The simulation results under the controller are given in Figs. 2 and 3. In particular, the tracking performance in time course is given in the left top graph where the x and y trajectories match well due to the good tracking performance and the tracking error is detailed in the bottom graph. The relationship between the actual displacement $x(t)$ and the desired displacement $y(t)$ is also captured by the straight line in the right top graph.

Next, we consider a triangular wave as the reference trajectory. Obviously, this trajectory is not always smooth and it cannot be asymptotically tracked during the entire time course. However, we note that each segment of the signal is generated by the exosystem (1) with $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ with different initial conditions. A simple calculation shows $\mathbf{u}(\mu,\varsigma) = k\varsigma_2 + c\varsigma_1$ and $\Phi = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \Psi = \begin{bmatrix} 1 & 0 \end{bmatrix}$. Similarly, the simulation results with the controller proposed in Theorem 1 are given in Figs. 4 and 5. It is noted that the peak values in the error curse correspond to the abrupt changes in the reference trajectory.

V. Conclusion

A novel internal model based controller has been proposed for a class of PEAs with hysteresis characterized by a Bouc-Wen model. The features of the controller include: an asymptotic tracking performance is achieved for a broad class of reference trajectories; the controller is in a global sense that the initial states of the closed-loop system can be arbitrary; and the controller is robust with respect to the system parameters (mass, damping, and stiffness).

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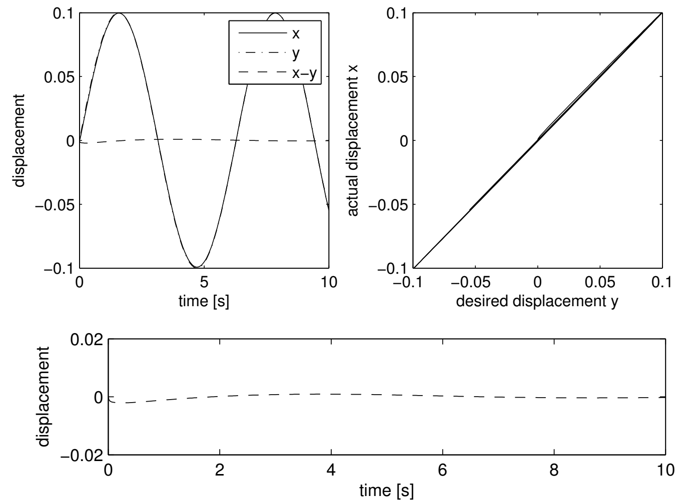

Fig. 2. Profile of trajectories for a sinusoidal reference.

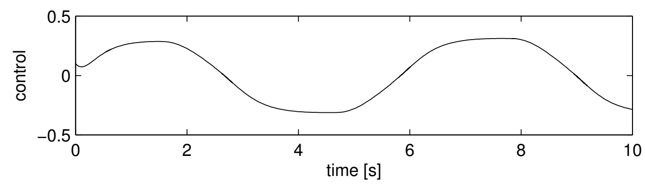

Fig. 3. Profile of control input for a sinusoidal reference.

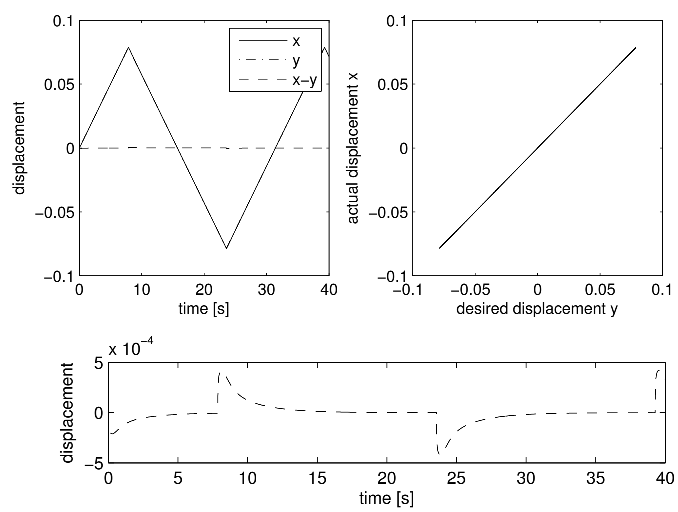

Fig. 4. Profile of trajectories for a triangular wave reference.

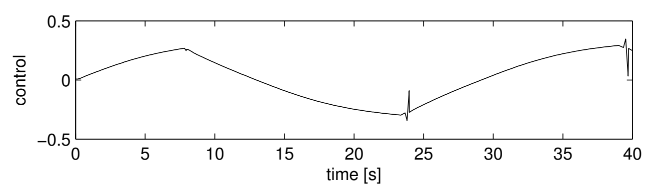

Fig. 5. Profile of control input for a triangular wave reference.