2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Robust Iterative Learning Control Synthesized with Sliding-mode Control for Output Tracking

Wen Chen*, Chih-Ping Yeh and Yang-Quan Chen

***Abstract*— This paper is to propose a new design of Iterative learning Control (ILC) for the purpose of output tracking. The novelty lies in the synthesis of ILC with sliding-mode control such that the tracking performance and accuracy can be improved. The considered system is first transformed into two subsystems such that the new design can be applied to broad systems. Based on the transformed systems, an ILC is designed for the first-order derivative of the control signal, instead of the control signal itself. The variable structure functions are therefore integrated such that the chattering can be eliminated accordingly. The convergence of the output-tracking error is also proved. The effectiveness of the new ILC for output tracking is verified in a simulation study.**

I. INTRODUCTION

Iterative Learning Control (ILC) is to use the repetitive actions to improve the system performance, particularly output tracking performance without seeking accurate system model knowledge. To authors' best knowledge, robust ILC is first presented in [1] where the effect of state disturbances, initial errors and output noise on a class of learning algorithms are investigated. The presented learning algorithm exhibits the bounds on asymptotic trajectory errors for the learned input and the corresponding state and output trajectories.

In recent years, various robust ILC schemes have been addressed. A nonlinear learning control scheme was developed in [2] by integrating iterative learning and adaptive robust control schemes for nonlinear systems. The main purpose of the paper [3] is to provide ILC designers with guidelines to select the learning gains to achieve arbitrarily high precision of output tracking regardless of measurement errors. In [4], a robust ILC problem for a class of nonlinear systems with structured periodic and unstructured aperiodic uncertainties is addressed. The backstepping idea is proposed to design the robust ILC systems. More research papers related to this topic can be found in [5]- [15], just to name a few.

This paper is to suggest a new robust ILC synthesized with sliding mode control. The actual control input is the integral of the designed ILC with the synthesized sliding mode control such that the control signals are continuous; thus, the continuous robust ILC can be applied broadly without damaging actuation devices.

This paper is organized as follows: in Section II, the considered system is illustrated and the objective and tasks of this paper is also addressed. System transform is first presented in Section III. The switching surface and the controller design are then followed. The convergence of the output-tracking error is also proved using Lyapunov direct method in the same section. An illustrative example is employed to demonstrate the effectiveness of the proposed robust ILC. At last, concluding remarks are made in Section V.

*W. Chen and C.-P. Yeh are with Division of Engineering Technology, Wayne State University, Detroit, MI, USA, 48202. Corresponding author: wchenc@wayne.edu.

Y.-Q. Chen is with the Department of Electrical Engineering, Utah State University, 4120 Old Main Hill, Logan, UT 84322-4120, USA.

II. PRELIMINARIES

Consider a disturbance-driven linear time-invariant system:

$$\begin{array}{rcl}\dot{x}(t) & = & Ax(t) + Bu(t) + Dd(t) \\ v(t) & = & Cx(t)\end{array} \quad (1)$$

where $x(t)$ is the n-dimensional state vector, control input $u(t)$ is an m-dimensional vector, measurable system output $v(t)$ is another m-dimensional vector, signal $d(t)$ represents bounded disturbances with dimension of q, and A, B, D, and C are $n \times n$, $n \times m$, $n \times q$, and $m \times n$ constant matrices, respectively. Without loss of generality, it is assumed that $C = [0 \;\; C_2]$ where C_2 is an $m \times m$ full-rank matrix. Even if $C = [C_1 \;\; C_2]$ for the original system, we can easily transform it to $C = [0 \;\; C_2]$. Please refer to [16] for details.

For the above system, the following assumptions are made.

Assumption 1: Matrices B and C are of full rank.

Assumption 2: The desired output trajectory $v_d(t)$ is differentiable with respect to time t up to the second-order on a finite time interval $[0, \;\; T]$, and all of its derivatives are available.

Assumption 3: The first-order derivative of the unknown disturbance, $d(t)$, is bounded such that

$$|\dot{d}(t)| \leq b_d, \; \forall t \in [0, \; T],$$

where b_d is a known constant.

Assumption 4: The initial condition $e(0) = \dot{e}(0) = \ddot{e}(0) = 0$ at any iteration $\forall t \in [0, \; T]$, such that the switching surface $S(0) = 0$, where $e(t)$ is the output tracking error that is defined as $e(t) = v(t) - v_d(t)$.

Remark 1: Assumption 3 does not include the random noise situations.

The control objective is to design an ILC synthesized with a sliding-mode control for system (1) such that system output can follow a desired one with a prescribed accuracy ϵ as follows:

$$\forall t \in [0, \; T], \; |v(t) - v_d(t)| \leq \epsilon.$$

III. MAIN RESULTS

To achieve the pre-designated objective, coordinate transformation technique will be employed. By transformation, the distribution matrix of the control input will be converted to be a non-singular matrix; then its inverse can be used for the purpose of controller design.

Via transformation, the original system has the following form:

$$\begin{aligned} \dot{z}_1(t) &= \bar{A}_{11}z_1(t) + \bar{A}_{12}z_2(t) + \bar{D}_1 d(t) \\ \dot{z}_2(t) &= \bar{A}_{21}z_1(t) + \bar{A}_{22}z_2(t) + D_2 d(t) + B_2 u(t) \\ v(t) &= \bar{C}z_2(t). \end{aligned} \tag{2}$$

where $z_1(t) \in R^{n-m}$, $z_2(t) \in R^m$, $\bar{A}_{11} \in R^{(n-m)\times(n-m)}$, $\bar{A}_{12} \in R^{(n-m)\times m}$, $\bar{A}_{21} \in R^{m\times(n-m)}$, $\bar{A}_{22} \in R^{m\times m}$, $CT^{-1} = [0 \;\; \bar{C}]$ and $\bar{C} \in R^{m\times m}$.

Based on the transformed system (2), we shall design a robust ILC for the purpose of output tracking.

A. Design of Sliding Surface Dynamics

The first-order derivative of the output-tracking error can be obtained from the above transformed system equation as follows:

$$\begin{aligned} \dot{e} &= \dot{v}(t) - \dot{v}_d(t) \\ &= \bar{C}\bar{A}_{21}z_1 + \bar{C}\bar{A}_{22}z_2 + \bar{C}D_2 d + \bar{C}B_2 u(t) - \dot{v}_d \end{aligned} \tag{3}$$

If a sliding surface dynamics, at kth iteration, is designed as

$$S_k = \alpha e_k(t) + \dot{e}_k(t), \tag{4}$$

where α is a positive constant.

Differentiating the sliding surface dynamics leads to

$$\begin{aligned} \dot{S}_k = \; & \bar{C}\bar{A}_{21}\dot{z}_{1k} + \bar{C}\bar{A}_{22}\dot{z}_{2k} + \bar{C}D_2\dot{d}_k + \bar{C}B_2\dot{u}_k(t) \\ & + \alpha\bar{C}\dot{z}_{2k} - \alpha\dot{v}_d - \ddot{v}_d \end{aligned} \tag{5}$$

B. Design of Iterative Learning Control

The first-order derivative of the control signal is designed as:

$$\begin{aligned} \dot{u}(t) = \; & \left[\bar{C}B_2\right]^{-1} \Big[-\beta S_k(t) + \alpha\dot{v}_d - \bar{C}\bar{A}_{21}\hat{\dot{z}}_{1k} - E_2\hat{\dot{z}}_{2k} \\ & -\rho_d \mathrm{sgn}(S_k) - \rho_1 \mathrm{sgn}(S_k) - \rho_2 \mathrm{sgn}(S_k) + \ddot{v}_d\Big] \end{aligned} \tag{6}$$

The estimates of two derivatives, $\hat{\dot{z}}_{1k}$ and $\hat{\dot{z}}_{2k}$, can be realized by the following iterative learning laws

$$\hat{\dot{z}}_{1k}(t) = \hat{\dot{z}}_{1(k-1)}(t) + q_1(\bar{C}\bar{A}_{21})^\top S(t) \tag{7}$$

$$\hat{\dot{z}}_{2k}(t) = \hat{\dot{z}}_{2(k-1)}(t) + q_2(E_2)^\top S(t) \tag{8}$$

where q_1 and q_2 are two positive constants.

Three signum functions are used to counteract the effects of the disturbances. If their effects on sliding surface dynamics can be eliminated completely, then, the sliding surface will converge to zero such that output-tracking error will converge to zero accordingly.

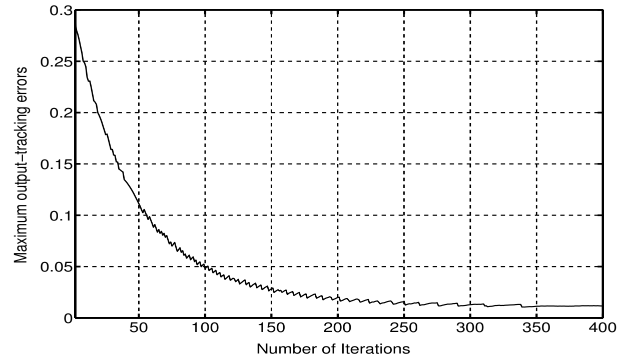

Fig. 1. Maximum output-tracking errors.

C. Convergence of the Output-Tracking Errors

A Lyapunov function, at k-th iteration, is chosen as

$$V_k(t) = \frac{1}{2}S_k^\top S_k + \frac{1}{2q_1}\int_0^t \tilde{\dot{z}}_{1k}^\top \tilde{\dot{z}}_{1k} + \frac{1}{2q_2}\int_0^t \tilde{\dot{z}}_{2k}^\top \tilde{\dot{z}}_{2k} \tag{9}$$

It is used for proof of the convergence of the output-tracking error. The detailed proof is omitted.

Theorem 1: Considering system (1) satisfying Assumptions (1) through (4). If it can be transformed to (2) and $\rho_d = b_d\|\bar{C}D_2\|$, $\rho_1 = b_1\|\bar{C}\bar{A}_{21}\|$, and $\rho_2 = b_2\|E_2\|$, then, control laws (6), (7), and (8) make the sliding surface dynamics and the output-tracking errors converge to zero.

IV. AN ILLUSTRATIVE EXAMPLE

A circuit system is used to illustrate the effectiveness of the designed ILC.

It has the following forms:

$$\dot{x}(t) = \begin{bmatrix} -\frac{R_1L_2}{L_1L_2-M^2} & \frac{R_2M}{L_1L_2-M^2} \\ \frac{R_1M}{L_1L_2-M^2} & -\frac{R_2L_1}{L_1L_2-M^2} \end{bmatrix} x(t) + \begin{bmatrix} \frac{L_2-M}{L_1L_2-M^2} \\ \frac{L_1-M}{L_1L_2-M^2} \end{bmatrix} u(t) \tag{10}$$

where resistors $R_1 = 1\Omega$, and $R_2 = 1\Omega$. Inductors $L_1 = 0.36H$ and $L_2 = 0.5H$, and the mutual inductor $M = 0.15H$. System states are the loop currents. Control signal $u(t)$ is an input voltage.

We now designate a desired output, $v_d(t) = \sin(t)$, for the system output to follow. We display the maximum out-tracking errors for iterations up to 400 in Fig. 1 which has revealed the effectiveness of the proposed ILC.

V. CONCLUSIONS

This paper has presented a new design of robust iterative learning controller synthesized with sliding mode control. The first-order derivative of the control input is designed with ILC and sliding mode control such that the actual control signal is the integral of the designed one. Thus, the control input is a continuous signal. Moreover, the new synthesized ILC can be applied to broader systems without any constraints on the configuration of the system dynamics. The simulation results clearly verify the effectiveness of the synthesized ILC and the sliding mode control for output tracking.

REFERENCES

[1] G. Heinzinger, D. Fenwick, B. Paden, and F. Miyazaki, "Stability of learning control with disturbances and uncertain initial conditions," *IEEE Trans Auto. Contr.*, vol. 31, no. 1, 1992, pp. 110-114.

[2] J. X. Xu and B. Viswanathan, "Adaptive robust itarative learning control with dead zone scheme," *Automatica*, vol. 36, 2000, pp. 91-99.

[3] S. S. Saab, "Selection of the Learning Gain Matrix of an Iterative Learning Control Algorithm in Presence of Measurement Noise," *IEEE Trans Auto. Contr.*, vol. 50, no. 11, 2005, pp. 1761-1774.

[4] Y.-P. Tian, X. H. Yu, "Robust learning control for a class of nonlinear systems with periodic and aperiodic uncertainties," *Automatica*, vol. 39, 2003, pp. 1957 1966.

[5] D. Wang, "Convergence and robustness of discrete time nonlinear systems with iterative learning control," *Automatica*, vol. 34, no. 11, 1998, pp. 14451448.

[6] J. X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches," *Automatica*, vol. 38, 2002, pp. 1867 1880.

[7] D.-Y. Pi and K. Panaliappan, Robustness of discrete nonlinear systems with open-closed-loop iterative learning control, in *Proc. 2002 Int. Conf. Mach. Learning Cybern.*, Nov. 2002, pp. 12631266.

[8] A. Tayebi and J. X. Xu, Observer-based iterative learning control for a class of time-varying nonlinear systems, *IEEE Trans. Circuits Syst. IFundam. Theory Appl.*, vol. 50, no. 3, pp. 452455, 2003.

[9] A. Tayebi and M. B. Zaremba, Robust ILC design is straightforward for uncertain LTI systems satisfying the robust performance condition, *IFAC 15th World Congr.*, Barcelona, Spain, Jul. 2002.

[10] A. Tayebi and M. B. Zaremba, "Internal model-based robust iterative learning control for uncertain LTI systems," *Proceedings of the 39th IEEE Conference on Decision and Control,"* Sydney, Australia, December, 2000, pp.3439 - 3444.

[11] H.-S. Ahn, Y. Q. Chen and K. L. Moore, "Intermittent Iterative Learning Control," *Proceedings of the 2006 IEEE International Symposium on Intelligent Control*, Munich, Germany, October 4-6, 2006, pp. 832-837.

[12] H.-S. Ahn, Y. Q. Chen and K. L. Moore, " Stability analysis of discrete-time iterative learning control systems with interval uncertainty," *Automatica*, vol. 43, no. 5, 2007, pp. 892-902.

[13] J.-H. Moon, T.-Y. Doh and M. J. Chung, "A robust approach for iterative learning control design for uncertain systems," *Automatica*, vol. 34, no. 8, pp. 1001-1004.

[14] J. X. Xu and Z. Qu, "Robust Iterative Learning Control for a Class of Nonlinear Systems," *Automatica*, vol. 34, no. 8, 1998, pp. 983-988.

[15] J. X. Xu and W. J. Cao, "Learning variable structure control approaches for repeatable tracking control tasks," *Automatica*, vol. 37, no. 7, 2001, pp. 997-1006.

[16] C. Edwards and S. Spurgeon, "On the development of discontinuous observers," *International Journal of Control*, vol. 59, 1994, pp. 1211-1229.