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Robustness preserving anti-windup for SISO systems

R. M. Morales, W. P. Heath and G. Li

Abstract— **A control feedback system with saturation non-linearities is said to have robustness preserving characteristics if the constrained system is as robust as its linear counterpart. Such characteristics can be desirable. It has been proved for some special cases that the anti-windup version of the Internal Model Control architecture can offer such characteristics for first-order plants with delays against norm-bounded (not only LTI) uncertainty. This paper provides general conditions expressed in the frequency domain which allow to test for the preservation of robustness for plants of any order. In addition, a class of robustness preserving controllers is characterised in terms of the Zames-Falb conditions. The test is shown to be easily implementable and exploited for anti-windup tuning.**

I. INTRODUCTION

Saturation non-linearities are very common in feedback control systems and they can cause significant degradation in performance and stability. Usually, anti-windup loops are constructed after a linear controller to offer performance compensation against saturation degradations [16]. Early successful works on anti-windup date back to [2] and [7]. A plethora of design approaches have been proposed since then, and most of them are constructed upon nominal Linear-Time-Invariant (LTI) representations of the plant [5], [3], [21]. A popular synthesis approach exploits the sector-bound conditions on the saturation elements [10] for the construction of Linear Matrix Inequalities [18], [22], [6], [20]. Recent research explores however anti-windup design whereby explicit uncertainty representations are also considered. In this area, a major design approach is proposed by [19] and the theory of Integral Quadratic Constraints (IQCs) [12] offers a natural framework for analysis on robustness [14].

The authors in [19] introduce the concept of robustness preservation. A control system is said to preserve the robustness of the unconstrained loop if the constrained system retains the robustness properties of the corresponding linear uncertain loop. Such a characteristic could be ideal in an anti-windup scheme. In this work, the conventional Internal Model Control (IMC) structure (see Fig. 1) is shown to have such characteristics against any additive uncertainty, and hence in the SISO case against any multiplicative LTI uncertainty also. On the other hand, [14] argues that if the uncertainty is norm-bounded but not necessarily LTI, then IMC need not be a robustness preserving anti-windup for output multiplicative uncertainty. In addition, [8] illustrates with examples other anti-windup schemes where robustness is preserved for a specific class of SISO first order plants with delay.

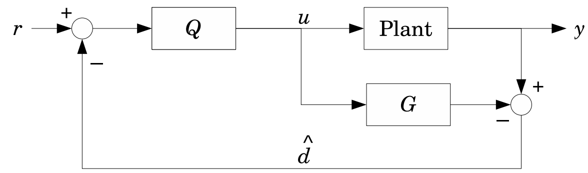

Fig. 1. Conventional IMC structure.

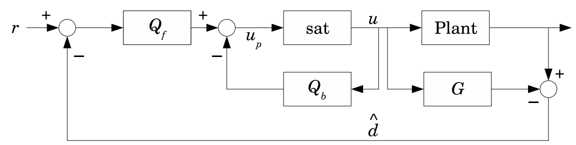

Fig. 2. IMC anti-windup structure.

The motivation behind this paper is to extend the results in [8] and find conditions that guarantee robustness preserving anti-windup which can be applied for SISO systems of any order. The treatment is constructed on the anti-windup version of the IMC structure, see Fig. 2. Two conditions are obtained using the IQC framework of [12] – we consider robustness preservation against LTI uncertainty and norm-bounded uncertainty and how they are exploited for the sake of anti-windup tuning.

The paper is structured as follows: Section II introduces briefly the IMC anti-windup structure and some preliminaries on its robustness. Sections III and IV present the main results of this work – it discusses the sufficient conditions for robustness preservation against LTI uncertainty and norm-bounded uncertainty, respectively. Section V shows how to combine the obtained conditions for the purpose of anti-windup tuning. The main outcomes are illustrated through some examples in section VI and finally, some concluding remarks are provided in section VII.

A. Notation

This work is developed on the space of square integrable signals L_2 with support on $[0,\infty)$ and its associated extended space [10] is denoted by L_{2e}. The inner product of $x, y \in L_2$ is defined in the frequency domain as shown below

$$\langle x,y\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)^* y(j\omega)\, d\omega$$

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where $x(j\omega)$ and $y(j\omega)$ denote the Fourier transforms of x and y, respectively.

An operator[1] $\Delta : L_{2e} \to L_{2e}$ is said to have finite gain or called bounded if

$$\|\Delta\| := \sup_{p \in L_2, p \neq 0} \frac{\|\Delta(p)\|}{\|p\|} < \infty$$

$\|\Delta\|$ is referred to as the gain or norm of the operator, see [9].

Two sets of uncertainty operators are distinguished in this work. They are:

- *Norm-bounded uncertainty*: This refers to the uncertainty elements with norm less than γ_Δ^{-1} which are possibly non-linear and time-varying.

$$\mathbf{U}_{NB} = \{\Delta : \ \|\Delta\| \leq \gamma_\Delta^{-1}\}$$

Such an uncertainty set can also be described by means of the following IQC

$$\left\langle \begin{bmatrix} p \\ \Delta(p) \end{bmatrix}, \begin{bmatrix} \alpha & 0 \\ 0 & -\alpha\gamma_\Delta^2 \end{bmatrix} \begin{bmatrix} p \\ \Delta(p) \end{bmatrix} \right\rangle \geq 0, \ \forall \omega \in \mathbb{R} \quad (1)$$

where $\alpha \geq 0$ is a constant.

- *LTI uncertainty*: This corresponds to the subset of $\mathbf{U}_{NB}$ with LTI stable elements with norm less than γ_Δ^{-1}

$$\mathbf{U}_{LTI} = \{\Delta : \ \|\Delta\|_\infty \leq \gamma_\Delta^{-1}\}$$

where

$$\|\Delta\|_\infty := \max_{\omega \in \mathbb{R}} |\Delta(j\omega)|$$

Within the IQC framework, such a set is described by

$$\left\langle \begin{bmatrix} p \\ \Delta(p) \end{bmatrix}, \begin{bmatrix} \alpha(j\omega) & 0 \\ 0 & -\gamma_\Delta^2 \alpha(j\omega) \end{bmatrix} \begin{bmatrix} p \\ \Delta(p) \end{bmatrix} \right\rangle \geq 0, \ \forall \omega \in \mathbb{R} \quad (2)$$

where $\alpha(j\omega) \geq 0$ is a bounded measurable function. For more details refer to [12].

II. IMC ANTI-WINDUP

The conventional IMC structure illustrated in Fig. 1 is a useful control technique for stable plants and is often taught in control textbooks (e.g. [17]). The controller is indicated by Q and G denotes the plant model. Its structure corresponds to the Youla parametrisation for stable plants and a robust treatment is found in [15].

IMC anti-windup has also proved a fruitful structure for the design of anti-windup control. Zheng et al. [24] introduce the structure illustrated in Fig. 2 with Q_f and Q_b as the anti-windup elements. Input constraints are indicated by the saturation function sat(.). To retain the behaviour of the IMC linear controller, the following relation should hold

$$(I + Q_b)^{-1} Q_f = Q \quad (3)$$

Two anti-windup choices stand out:

Case 1:

Set $Q_f = Q$ and $Q_b = 0$. This is the conventional IMC structure which is known to provide nice stability properties, but the system performance can be sluggish – see [24].

Case 2:

Set $Q_f = Q(\infty)$ and $Q_b = Q(\infty)Q^{-1} - I$ as proposed in [5]. This choice was shown to improve performance but nothing was said on its robustness properties [24].

[1]The development applies to operators acting upon the extended function space L_{2e} since boundedness of an operator acting on L_2 implies boundedness on the extended space L_{2e}, see [9].

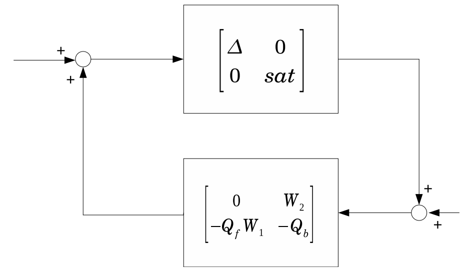

Fig. 3. IMC anti-windup expressed in the standard feedback connection.

The relation between IMC anti-windup and other anti-windup structures is well-understood (e.g. [11]). For multi-variable plants it is common to replace the saturation function with a constrained optimiser: see [1] for an overview and [13] for a particular application.

A. Preliminaries on Robustness

We model our plant with uncertainty as

$$y = (G + W_1 \Delta W_2)u + d \quad (4)$$

with G the nominal model used in the controller, W_1 and W_2 known frequency weighting functions, Δ representing uncertain dynamics and d some exogenous disturbance. We will consider the following special cases:

Additive uncertainty:

$W_1 = I$, $W_2 = I$.

Input multiplicative uncertainty:

$W_1 = G$, $W_2 = I$.

Output multiplicative uncertainty:

$W_1 = I$, $W_2 = G$.

Standard application of the small gain theorem gives robustness of the linear loop provided

$$\gamma_\Delta^{-1} < \frac{1}{\|W_2 Q W_1\|_\infty} \quad (5)$$

where γ_Δ^{-1} is an indication of the "size" of the uncertainty which the linear loop is known to tolerate.

For robustness analysis purposes, the IMC anti-windup structure is expressed in terms of the standard feedback connection – see Fig. 3. Let our saturation operator sat(.) be defined in terms of IQCs as

$$\left\langle \begin{bmatrix} u \\ \text{sat}(u_p) \end{bmatrix}, \begin{bmatrix} 0 & Z^* \\ Z & -Z - Z^* \end{bmatrix} \begin{bmatrix} u \\ \text{sat}(u_p) \end{bmatrix} \right\rangle \geq 0 \quad (6)$$

where $Z(j\omega)$ satisfies conditions for Zames-Falb multipliers [23].

It follows from the IQC theorem [12] that the loop in Fig 3 is robust to LTI uncertainty provided there is some $\alpha(j\omega) \geq 0$ such that

$$\begin{bmatrix} 0 & W_2 \\ -Q_f W_1 & -Q_b \\ I & 0 \\ 0 & I \end{bmatrix}^* \Pi \begin{bmatrix} 0 & W_2 \\ -Q_f W_1 & -Q_b \\ I & 0 \\ 0 & I \end{bmatrix} < 0 \quad (7)$$

evaluated at all frequencies ω, with

$$\Pi = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & Z^* \\ 0 & 0 & -\alpha\gamma_\Delta^2 & 0 \\ 0 & Z & 0 & -Z-Z^* \end{bmatrix} \quad (8)$$

This reduces to the condition that there exists some $\alpha(j\omega) \geq 0$ such that

$$\alpha^2 - 2\frac{\text{Re}\{ZQ_fQ^{-1}\}}{|W_2|^2}\alpha + \frac{|ZQ_fW_1|^2}{\gamma_\Delta^2|W_2|^2} < 0 \quad (9)$$

is evaluated at all frequencies ω with α, Z, Q, Q_f, W_1 and W_2 being frequency dependent.

III. Robustness preservation against LTI uncertainty

A sufficient condition for the preservation of robustness is provided below. We consider first robustness preservation against LTI uncertainty.

Result 1. IMC anti-windup is guaranteed to preserve the robustness of the linear loop against LTI uncertainty if

$$\frac{|X|}{\|X\|_\infty} \leq \frac{\text{Re}\{Q_fQ^{-1}Z\}}{|Q_fQ^{-1}Z|}, \ \forall\omega \in \mathbb{R} \quad (10)$$

with

$$X(j\omega) := W_1(j\omega)Q(j\omega)W_2(j\omega) \quad (11)$$

Proof. The robustness of the constrained loop with respect to $\mathbf{U}_{LTI}$ is guaranteed if there exists an $\alpha(j\omega) \geq 0$ such that (9) holds. We first demonstrate that (9) is equivalent to

$$\gamma_\Delta^{-1} \leq \frac{|W_2|\text{Re}\{Q_fQ^{-1}Z\}}{|ZQ_fW_1||W_2|^2} = \frac{\cos(\angle Q_fQ^{-1}Z)}{|W_2QW_1|}, \ \forall\omega \in \mathbb{R} \quad (12)$$

First, choose

$$\alpha(j\omega) = \frac{\text{Re}\{ZQ_fQ^{-1}\}}{|W_2|^2}$$

then (12) is sufficient for (9) . To show (12) is necessary, assume it does not hold. In this case (9) is only satisfied for $\alpha(j\omega) < 0$.

Now compare (12) and (5). Ensuring the constrained system to be as robust as the unconstrained counterpart demands that

$$\frac{1}{\|W_2QW_1\|_\infty} \leq \frac{\cos(\angle Q_fQ^{-1}Z)}{|W_2QW_1|}, \ \forall\omega \in \mathbb{R}$$

Hence Result 1. □

Corollary 1. The condition in Result 1 can also be expressed as

$$\frac{|X|}{\|X\|_\infty} \leq \frac{\text{Re}\{(I+Q_b)Z\}}{|(I+Q_b)Z|}, \ \forall\omega \in \mathbb{R} \quad (13)$$

or

$$\frac{|X|}{\|X\|_\infty} \leq \cos(\angle Q_f + \angle Z - \angle Q), \ \forall\omega \in \mathbb{R} \quad (14)$$

□

Remarks:

- The task of searching for a valid IQC multiplier $\alpha(j\omega)$ is not required in the condition for robustness preservation making the computation of the test greatly simpler.
- Choose $Z = I$. If $Q_f = Q$ (conventional IMC) then robustness is preserved for any valid choice of W_1, W_2 and Q. This result agrees with the discussion on robustness preservation of [19] and [14].
- For robustness preservation against LTI multiplicative uncertainty choose $W_2 = G$ without loss of generalisation.
- Result 1 states that the gain of the product W_2QW_1 is penalised when the product Q_fZ deviates in direction from Q.
- Notice that the robustness condition of the constrained system requires

  $$\text{Re}\{Q_fQ^{-1}Z\} = \text{Re}\{(Q_b+I)Z\} \geq 0, \quad \forall\omega \in \mathbb{R} \quad (15)$$

  This is equivalent to the stability result for the nominal constrained loop by means of valid multipliers $Z(j\omega)$ to reduce conservatism, see [23].
- The phase difference between $(Q_f + Z)$ and Q cannot be greater than 90°.

Finally, note that if the product Q_fZ is in the same direction of Q at all frequencies then robustness is preserved. Hence, a class of robustness-preserving anti-windup controllers against LTI uncertainty is expressed in terms of the Zames-Falb multipliers [23]

$$Z = I + H \quad (16)$$

where the impulse response of H, denoted by $h(t)$, satisfies

$$\int_{-\infty}^{\infty} |h(t)|dt < 1 \quad (17)$$

Corollary 2. If

$$\begin{aligned} Q_f &= \frac{Q}{I+H} \\ Q_b &= \frac{Q_f}{Q} - I \end{aligned} \quad (18)$$

with $h(t)$ satisfying (17), then preservation of robustness is guaranteed.
Proof. Since $Q_f = Q/Z$, the condition in Result 1 is automatically satisfied because

$$|X| \leq \|X\|_\infty, \forall\omega \in \mathbb{R}$$

□

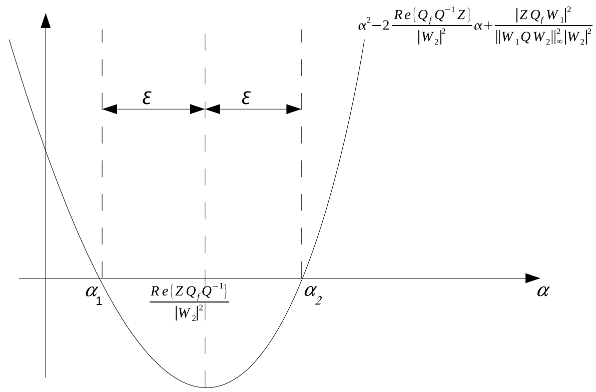

Fig. 4. Parabola involved in the robustness preservation condition against $\mathbf{U}_{NB}$.

IV. Robustness preservation against norm-bounded uncertainty

If Δ is a norm-bounded uncertainty (possibly non-linear and/or time-varying) with size γ_Δ^{-1}, then a stronger condition must be satisfied to guarantee robustness preservation.

Result 2. Robustness is preserved against $\mathbf{U}_{NB}$ if

$$\max_\omega \ \alpha_1(j\omega) \leq \min_\omega \ \alpha_2(j\omega) \tag{19}$$

with

$$\alpha_1(j\omega) := \frac{1}{|W_2|^2}\left(\text{Re}\{ZQ_fQ^{-1}\} - \varepsilon\right) \tag{20}$$

$$\alpha_2(j\omega) := \frac{1}{|W_2|^2}\left(\text{Re}\{ZQ_fQ^{-1}\} + \varepsilon\right) \tag{21}$$

and

$$\varepsilon(j\omega) := \sqrt{\left(\text{Re}\{ZQ_fQ^{-1}\}\right)^2 - \frac{|ZW_2Q_fW_1|^2}{\|W_2QW_1\|_\infty^2}}$$

□

Proof. Robustness of the constrained loop for this type of uncertainty is guaranteed provided there exists a constant $\alpha > 0$ such that (9) is satisfied for all frequencies. In addition, robustness is preserved if the constrained loop is as robust as the linear system. This is enforced by setting

$$\gamma_\Delta^{-1} = \|W_2QW_1\|_\infty^{-1}$$

in the robustness condition of the constrained loop. It follows then that robustness is preserved against norm-bounded uncertainty if (19) holds where $\alpha_1(j\omega)$ and $\alpha_2(j\omega)$ are respectively the smaller and greater non-negative roots of the polynomial

$$\alpha^2 - 2\frac{\text{Re}\{ZQ_fQ^{-1}\}}{|W_2|^2}\alpha + \frac{|ZQ_fW_1|^2}{\|W_1QW_2\|_\infty^2|W_2|^2}$$

See Fig. 4. □

Remarks:

- Note that Result 1 is necessary for Result 2. Should Result 1 not be satisfied at a certain frequency then $\varepsilon(j\omega) \notin \mathbb{R}$; hence Result 2 is not satisfied either.
- The ordering of W_1 and W_2 with respect to the uncertainty Δ is significant as they need not commute when Δ is nonlinear.
- For the conventional IMC structure and choosing $Z = I$, Result 2 becomes

$$\alpha_1(j\omega) := \frac{1}{|W_2|^2} - \sqrt{1 - \frac{|W_2QW_1|^2}{\|W_2QW_1\|_\infty^2}}$$

$$\alpha_2(j\omega) := \frac{1}{|W_2|^2} + \sqrt{1 - \frac{|W_2QW_1|^2}{\|W_2QW_1\|_\infty^2}}$$

If W_2 is constant, then robustness of the unconstrained loop is preserved against norm-bounded uncertainty provided it is preserved against LTI uncertainty. This is not necessarily true for the case where W_2 is a function of the frequency – there can exist a W_2 for which preservation of robustness is not guaranteed against $\mathbf{U}_{NB}$ but it does for $\mathbf{U}_{LTI}$. This agrees with the results in [14] in the sense that conventional IMC is known to preserve the robustness for input multiplicative uncertainty ($W_2 = 1$) but not otherwise.

V. Tuning

The linear controller Q is usually designed by the standard methodology proposed in [4]. Assume the plant G is decomposed as

$$G = G_- G_+$$

where G_+ is the transfer function that collects all stable poles and zeros. Then the linear controller is chosen as

$$Q = G_+^{-1} F$$

with F selected so the controller Q is realisable and to set an specific closed-loop performance, see the complementary sensitivity function

$$T = GQ = G_- F$$

A suitable form for tuning the anti-windup elements is

$$Q_f = Q\lambda + (1-\lambda)Q(\infty) \tag{22}$$

with $0 \leq \lambda \leq 1$, see [11]. The choices $\lambda = 1$ and $\lambda = 0$ corresponds to Case 1 and Case 2, respectively, as discussed in section II.

If $Z = I$ and the choice of anti-windup elements is dictated by the rule (22), robustness against LTI uncertainty is preserved provided

$$\frac{|X|}{\|X\|_\infty} \leq \cos(\angle Q_f - \angle Q), \ \forall \omega \in \mathbb{R} \tag{23}$$

with

$$\angle Q_f - \angle Q = \arctan\left(\frac{-\text{Im}\{Q\}(1-\lambda)Q(\infty)}{\lambda|Q|^2 + (1-\lambda)Q(\infty)\text{Re}\{Q\}}\right)$$

and $X(j\omega)$ defined in (11).

Results 1 and 2 can be exploited for tuning the anti-windup scheme in such a way that the system aims at performance while guaranteeing robustness preservation. A line search can be carried out by means of (22) for tuning the anti-antiwindup architecture according to the uncertainty set for which robustness is desired to be preserved. We propose:

- Robust anti-windup against $\mathbf{U}_{NB}$:

$$\min_{\lambda\in[0,1]} \lambda$$
$$\text{s.t.}$$
$$\max_{\omega} \alpha_1(j\omega) \le \min_{\omega} \alpha_2(j\omega), \ \forall\omega\in\mathbb{R}$$

  with $\alpha_1(j\omega)$ and $\alpha_2(j\omega)$ given by (20) and (21), respectively.

- Robust anti-windup against $\mathbf{U}_{LTI}$:

$$\min_{\lambda\in[0,1]} \lambda$$
$$\text{s.t.}$$
$$\frac{|X|}{\|X\|_\infty} \le \frac{\mathrm{Re}\{Q_f Q^{-1} Z\}}{|Q_f Q^{-1} Z|}, \ \forall\omega\in\mathbb{R}$$

  with $X(j\omega)$ expressed in (11).

Obviously, if the conditions for robustness preservation are not satisfied even with $\lambda = 1$, then robustness preserving anti-windup can not be guaranteed. The multiplier $Z(j\omega)$ is chosen to reduce conservatism.

VI. EXAMPLES

The following examples consider a second order plant

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

A. Example 1

This example illustrates that robustness can be preserved against norm-bounded uncertainty for Case 2. With this aim, we consider $w_n = 10, \zeta = 0.5$ and a linear controller in the form

$$Q(s) = \frac{s^2 + 2\zeta w_n s + w_n^2}{w_n^2} \ \frac{c^2}{(s+c)^2}$$

with $c = 5$. In addition, assume $W_1 = W_2 = Z = I$ for simplicity. We observe from the simulations that by choosing $\lambda = 0$

$$Q_f = Q(\infty) = 0.25$$
$$Q_b = \frac{75}{s^2 + 10s + 100}$$

Robustness preservation is guaranteed against norm-bounded uncertainty since the condition in Result 2 is satisfied - see Fig. 5. Observe that in this case

$$\max_{\omega} \alpha_1(j\omega) = \min_{\omega} \alpha_2(j\omega) = \mathrm{Re}\{Q_f Q^{-1}(j0)\} = 0.25$$

Note also from Fig. 6 that robustness preservation is ensured against $\mathbf{U}_{LTI}$. As expected, $|\angle Q_f - \angle Q| \le 90^\circ$ for all frequencies and $(Q_b + 1)$ is positive real.

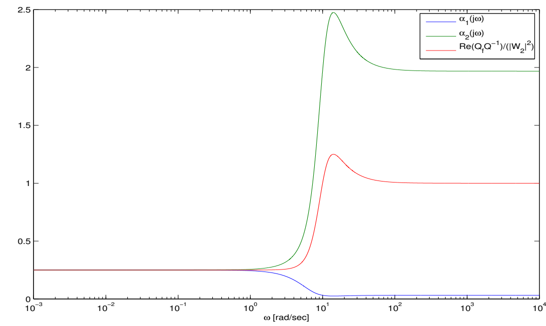

Fig. 5. Example 1 – Robustness preservation test against $\mathbf{U}_{NB}$.

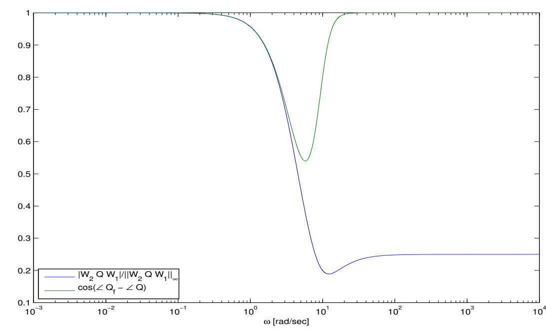

Fig. 6. Example 1 – Robustness preservation test against $\mathbf{U}_{LTI}$.

B. Example 2

This example is taken from [19] and the second-order plant is considered to have $w_n = \sqrt{10}$ and $\zeta = 1$. The damping in the perturbed plant is reduced significantly

$$\hat{G}(s) = \frac{10}{s^2 + 0.01s + 10}$$

In [19], it is shown that a static anti-windup design leads to instability. A dynamic anti-windup scheme is synthesised with the method proposed therein and the obtained controller is essentially the conventional IMC structure. In this Example, we exploit Corollary 2 and find an anti-windup controller with the same robustness preservation guarantees as that of the conventional IMC structure but with slightly improved performance.

To account for this uncertainty in the LTI case, the following uncertainty weights are chosen

$$W_1(s) = \frac{-7.176\times 10^{-15}s^2 + 100.9s}{s^4 + 10.01s^3 + 20.1s^2 + 100.1s + 100}$$

and $W_2(s) = 1$. The corresponding linear IMC controller is obtained from the feedback controller in [19]

$$Q(s) = \frac{135(s+8.873)(s+5)^2(s+1.127)}{(s+55.56)(s+25.8)(s^2+8.639s+23.54)}$$

The anti-windup elements are chosen as indicated in Corollary 2 (see (18)) with a corresponding dynamic multiplier

$$Z(s) = \frac{s+5}{s+10} \tag{24}$$

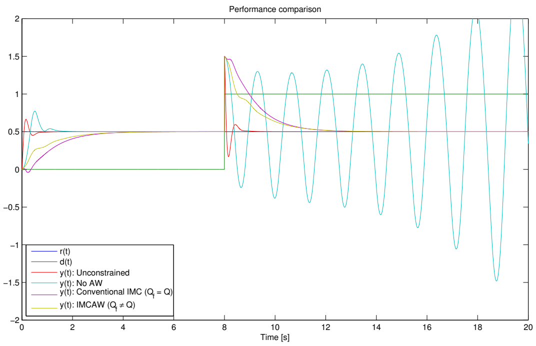

Fig. 7. Example 2 – Performance comparison

Hence the above anti-windup controller is guaranteed to preserve robustness against LTI uncertainty. Furthermore, since W_2 is constant, then robustness is also guaranteed against norm-bounded uncertainty. Notice that (24) is a valid multiplier since

$$H(s) = Z(s) - I = \frac{-5}{s+10}$$

and

$$\int_0^\infty |-5e^{-10t}| dt = \frac{1}{2} < 1$$

Figure 7 shows the performance among the different scenarios. We observe that when the constraints become active, the closed-loop becomes unstable if no anti-windup is included. In addition, we behold that the IMC anti-windup controller offer a slight improvement with respect to the conventional IMC in the sense that no undershoot is present for reference tracking and the settling and rising times have been decreased.

VII. Conclusions

This paper has presented two conditions for robustness preserving anti-windup against LTI and norm-bounded uncertainty. Such characteristics can be ideal for anti-windup structures. The conditions have been developed within the framework of IQCs and it has been expressed in terms of multipliers for descriptions of the saturation operator to improve conservatism. This work also characterises a set of robustness-preserving anti-windup controllers expressed in terms of the Zames-Falb conditions. The conditions have been exploited for the sake of anti-windup design.

Although the treatment has been limited for the SISO scenario, the work herein is believed to provide a neat insight sometimes difficult to achieve for MIMO systems. Conditions for the preservation of robustness are expressed in [14] for the MIMO case. However, the selection of the IQC multiplier for the uncertainty is required in the conditions hence increasing the complexity in the implementation.

The reliability of the different inequality tests depends on the resolution and range of the frequency variable ω. For high-order cases, the resolution must be increased adequately at the cost of an increased computational time to minimise the risk of missing crucial frequencies.

References

[1] A. A. Adegbege and W. P. Heath. Internal model control design for input-constrained multivariable processes. *To appear in AIChE Journal*, 2011.

[2] K. J. Aström and L. Rundqwist. Integrator windup and how to avoid it. *Proceedings of the American Control Conference*, pages 1693–1698, 1989.

[3] P. J. Campo and M. Morari. Robust control of processes subject to saturation nonlinearities. *Computers chem. Engng.*, 14(4/5):343–358, 1990.

[4] C. E. Garcia and M. Morari. Internal Model Control 1. A unifying review and some new results. *Ind. Eng. Chem. Process Des. Dev.*, 21(2):308–323, 1982.

[5] G. C. Goodwin, S. F. Graebe, and W. S. Levine. Internal model control of linear systems with saturating actuators. pages 1072–1077, Groningen, The Netherlands, 1993. Proceedings of the European Control Conference.

[6] G. Grimm, J. Hatfield, I. Postlethwaite, A. R. Teel, M. C. Turner, and L. Zaccarian. Anti-windup for stable linear systems with input saturations: An LMI based approach. *IEEE Transactions on Automatic Control*, 48(9):1509–1525, 2003.

[7] R. Hanus, M. Kinnaert, and J.-L. Henrotte. Conditioning technique, a general anti-windup and bumpless transfer method. *Automatica*, 23(6):729 – 739, 1987.

[8] W. Heath, R. Morales, and G. Li. Robustness preserving antiwindup: examples and counterexamples. Denver, USA, 2011. To appear in the Proceedings of the IEEE Multiconference on Systems and Control.

[9] U. Jönsson. Lecture notes on integral quadratic constraints. Department of Mathematics, KTH, Stockholm. ISBN 1401-2294, available at http://www.math.kth.se/˜uj/, 2000.

[10] H. K. Khalil. *Nonlinear Systems (third edition)*. Prentice Hall, Upper Saddle River, 2002.

[11] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett. A unified framework for the study of anti-windup designs. *Automatica*, 30(12):1869–1883, 1994.

[12] A. Megretski and A. Rantzer. System analysis via integral quadratic constraints. *IEEE Transactions on Automatic Control*, 42(6):819–830, 1997.

[13] R. M. Morales and W. P. Heath. The robustness and design of constrained cross-directional control via integral quadratic constraints. *To appear in IEEE Transactions on Control Systems Technology*, 2011.

[14] R. M. Morales, G. Li, and W. P. Heath. Anti-windup and the preservation of robusnetss against structured norm-bounded uncertainty. Seoul, Korea, 2008. IFAC World congress.

[15] M. Morari and E. Zafirou. *Robust Process Control*. Prentice Hall, Englewood Cliffs, 1989.

[16] E. F. Mulder, M. V. Kothare, and M. Morari. Multivariable anti-windup controller synthesis using linear matrix inequalities. *Automatica*, 37:1407–1416, 2001.

[17] D. E. Seborg, T. F. Edgar, and D. A. Mellichamp. *Process Dynamics and Control*. John Wiley & Sons, 1989.

[18] A. R. Teel and N. Kapoor. The l2 anti-windup problem: its definition and solution. Brussels, Belgium, 1997. Proceedings of the fourth European Control Conference.

[19] M. C. Turner, G. Herrmann, and I. Postlethwaite. Incorporating robustness requirements into anti-windup design. *IEEE Transactions on Automatic Control*, 52(10):1842–1855, 2007.

[20] M. C. Turner and I. Postlethwaite. A new perspective on static and low order anti-windup synthesis. *International Journal of Control*, 77(1):27 – 44, 2004.

[21] P. F. Weston and I. Postlethwaite. Linear conditioning for systems containing saturating actuators. *Automatica*, 36:1347 – 1354, 2000.

[22] L. Zaccarian and A. R. Teel. A common framework for anti-windup, bumpless and realiable designs. *Automatica*, 38:1735 – 1744, 2002.

[23] G. Zames and P. L. Falb. Stability conditions for systems with monotone and slope-restricted nonlinearities. *SIAM Journal of Control*, 6(1):89 – 108, 1968.

[24] A. Zheng, M. V. Kothare, and M. Morari. Anti-windup design for internal model control. *International Journal of Control*, 60(5):1015–1024, 1994.