ISS-based vehicle yaw stability controller design in backstepping framework

Hongliang Zhou, Zhiyuan Liu and Hong Chen

Abstract—A vehicle yaw stability controller is proposed to make vehicle yaw stable in critical cornering manoeuvres in the paper. Considering vehicle body dynamics, wheel dynamics and tire nonlinear characteristics, a discrete nonlinear timevarying model is firstly constructed, with disturbances standing for tire/road forces estimation error and unmodelled dynamics. Then, the controller is proposed based on sliding mode control in backstepping control framework, in which the control input is calculated step-by-step. The stability of the system is analyzed through input-to-state stability (ISS) theory. Finally, a group of simulations is carried out with a multi-body vehicle dynamics software to evaluate the controller and the results indicate that it can maintain vehicle stable in critical cornering situation.

I. INTRODUCTION

Yaw stability controller of an automotive vehicle has been established as an essential safety/performance component, which generally prevents the vehicle from understeer and oversteer situations in critical cornering manoeuvres[1]. Among all the proposed control strategies, yaw stability controller based on differential braking with Electromechanical Brake module (EMB) as actuator has been widely investigated, in which brake torques on each wheel are considered as control input. As the overall vehicle dynamics is described with hard nonlinear properties (e.g. tire nonlinear characteristics, vehicle body and wheel nonlinear dynamics), simplifying the nonlinearities and deducing control oriented model are the first essential work in developing the controller. These models can be categorized into two main kinds according to their structures. The first kind is integration model including vehicle body and wheel dynamics either by excluding tire nonlinearity[2] or excluding wheel transient dynamics[3], [4], based on which Linear Quadratic Regulator theory, \mathcal{H}_{∞} theory, and sliding mode control are implemented to design yaw stability controller. However, although the closed loop systems based on integration model are simple to analyze, this kind of models fails to reveal vehicle nonlinearities when vehicle lateral acceleration is more than 0.4g, which commonly occurs in critical cornering situations. Under the assumption that the tire/road forces are estimated, the second kind of vehicle models consists of two main parts, which are vehicle body and wheel dynamics models. The significance

is introducing yaw moment as virtual control input of vehicle body model to integrate these two parts[5], [6]. This kind of models inspires hierarchical yaw stability controller, which controls yaw rate and wheel slip separately in different control loops. In order to obtain the calculated virtual control input of the upper layer controller, control allocation is implemented to allocate yaw moment to wheel slip either through fuzzy logic or through nonlinear optimizing allocation algorithm, considering nonlinear relationship from wheel slip to yaw moment. Although this kind of control strategies has advance in reflecting vehicle nonlinear characteristics, the analysis of closed loop system stability is remarkable difficult as there are two control loops and nonlinear control allocation are included.

In this paper, a vehicle yaw stability controller is proposed based on a new control oriented vehicle yaw dynamics model. Different from above two modelling strategies, the new model is discrete nonlinear time-varying model, which is strict-feedback with brake torque as input and yaw rate as output and includes vehicle yaw dynamics, wheel slip dynamics and tire nonlinear characteristics. In the model, the tire/road forces are assumed to be estimated through estimator and are considered as time-varying parameters. In order to embody estimation error of these forces and unmodelled dynamics, two bounded disturbances are introduced to the model. Then a cascade yaw stability controller is designed to keep vehicle stable through sliding mode control (SMC) in backstepping framework, in which the control input is derived step-by-step and the disturbances are estimated in approximative way. Furthermore, to analyze the closed loop stability property, input-to-state stability (ISS) is implemented as it's suitable in analyzing systems with disturbances. The paper is organized as follows. In Section II, the vehicle dynamics is briefly reviewed and the established model is presented. In Section III, controller design and closed loop system stability analysis are proposed. In Section IV, a group of simulations based on a high-precise vehicle dynamics software is carried out to evaluate the control system.

II. VEHICLE YAW STABILITY CONTROL ORIENTED MODEL

In general, a vehicle is mainly divided into two parts: vehicle body and wheels, and the tire/road forces determine their states. The vehicle body yaw dynamics and wheel longitudinal dynamics are shown in Fig. 1 and Fig. 2 respectively, in which the symbols in the paper are illustrated[7].

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H. Zhou and Z. Liu are with Department of Control Science and Engineering, Harbin Institute of Technology, 150001, Harbin, China. E-mail: zhouhongliang, liuzy_hit@hit.edu.cn

H. Chen is with Department of Control Science and Engineering, Jilin University (Campus NanLing), and with the National Automobile Dynamic Simulation Laboratory. 130025, Changchun, China. E-mail: chenh@jlu.edu.cn.



Fig. 2. Wheel dynamics(i refers to each wheel and is in the range of [1,4])

As the function of yaw stability controller is to maintain vehicle follow driver command in critical cornering situations and yaw rate r is cornering speed, it is natural to design the controller to make r follow its reference, which is determined by steering angle δ and vehicle speed v_x and is constrained by road condition[1], [7]. The yaw rate reference is described in Eq.(1).

$$r_r = \begin{cases} r_{max} & r_d > r_{max} \\ r_d & -r_{max} < r_d < r_{max} \\ -r_{max} & r_d < -r_{max} \end{cases}$$
(1)

where

$$r_{d} = \frac{v_{x}\delta}{(l_{f}+l_{r}) + \frac{m(l_{r}C_{r}-l_{f}C_{f})v_{x}^{2}}{2(l_{f}+l_{r})C_{f}C_{r}}}$$
$$r_{max} = 0.85\frac{\mu g}{v_{x}}$$

 μ is road adhesion coefficient and C_f , C_r is tire lateral stiffness. In contrast to above design goal, vehicle slip angle β is controlled in small range in [2] and [3]. It is omitted here because β is related to r and β is kept in small range automatically when r follows r_r . These declaration will be further testified in Section IV.

It has been well addressed that the convenient way to make r follow its reference r_r is braking the most effective wheel according to vehicle situations[1]. In vehicle understeer state, the rear inner wheel is braked and the front outer wheel is braked in oversteer state. As the four wheels are braked alternatively and the characteristics of these four situations are similar, the model in the situation, which is $\delta > 0$, $r > r_r$ and front right (FR) wheel to be braked, is chosen to be discussed and it is described as

$$\dot{r} = \frac{1}{J_z} [l_f (F_{x1} + F_{x2}) \sin \delta + \frac{d}{2} (F_{x2} - F_{x1}) \cos \delta + l_f (F_{y1} + F_{y2}) \cos \delta + \frac{d}{2} (F_{y1} - F_{y2}) \sin \delta - l_r (F_{y3} + F_{y4}) + \frac{d}{2} (F_{x4} - F_{x3})]$$
(2a)

$$\dot{\lambda}_2 = -\frac{R_2}{v_x} \frac{T_{b2} + R_2 F_{x2}}{J_{\omega 2}}$$
(2b)

Eq.(2b) is derived considering that v_x varies slowly in yaw stability control process. For other three situations, the models are similar and the states are $[r, \lambda_1], [r, \lambda_3], [r, \lambda_4]$ respectively. These models are time-varying and nonlinear.

In Eq. (2a), the forces F_{xi} and F_{yi} could be categorized into two kinds: the forces acting on braked wheel and the forces acting on un-braked wheels. For the first category, the forces are related to wheel states, especially related to wheel slip λ_i , so it is natural to implement tire model to describe this relationship. The tire model is strongly nonlinear and researchers have developed some models to approximate it, such as Magic Formula[8], Dugoff Model[9], LuGre Model[10]. In the paper, the rational tire model is employed, which not only describes tire longitudinal and lateral characteristics, but also describes wheel combined characteristics[11], and it is shown in Eq. (3).

$$\begin{aligned} F_{xi}(\alpha_i, \lambda_i, \mu, F_{zi}) &\approx \frac{\mu F_{zi}}{\mu_0 F_{zi0}} \frac{\chi_z}{\chi_\lambda \lambda_i^{2+1}} \frac{K_\lambda}{\chi_\alpha \alpha_i^{2+1}} \lambda_i \\ F_{yi}(\alpha_i, \lambda_i, \mu, F_{zi}) &\approx \frac{\mu F_{zi}}{\mu_0 F_{zi0}} \frac{\eta_z}{\eta_\lambda \lambda_i^{2+1}} \frac{C_2}{\eta_\alpha \alpha_i^{2+1}} \alpha_i \end{aligned} \tag{3}$$

where F_{zi} is wheel load, and χ_z , χ_λ , χ_α , η_z , η_λ and η_α are all tire parameters. Wheel load F_{zi} is mainly affected by vehicle longitudinal acceleration a_x , lateral acceleration a_y , and they can be calculated as Eq. (4):

$$F_{z1} = \frac{mgl_r}{2(l_f+l_r)} - \frac{ma_xh}{2(l_f+l_r)} - \frac{ma_yl_rh}{2d(l_f+l_r)} F_{z2} = \frac{mgl_r}{2(l_f+l_r)} - \frac{ma_xh}{2(l_f+l_r)} + \frac{ma_yl_rh}{2d(l_f+l_r)} F_{z3} = \frac{mgl_f}{2(l_f+l_r)} + \frac{ma_xh}{2(l_f+l_r)} - \frac{ma_yl_fh}{2d(l_f+l_r)} F_{z4} = \frac{mgl_f}{2(l_f+l_r)} + \frac{ma_xh}{2(l_f+l_r)} + \frac{ma_yl_fh}{2d(l_f+l_r)}$$
(4)

As in [12], the time-varying parameters, wheel slip angle α_i , are calculated as below:

$$\begin{aligned}
\alpha_1 &= \delta - \arctan\left(\frac{v_y + l_f r}{v_x - \frac{b}{2}r}\right) \\
\alpha_2 &= \delta - \arctan\left(\frac{v_y + l_f r}{v_x + \frac{b}{2}r}\right) \\
\alpha_2 &= -\arctan\left(\frac{v_y - l_f r}{v_x - \frac{b}{2}r}\right) \\
\alpha_4 &= -\arctan\left(\frac{v_y - l_f r}{v_x + \frac{b}{2}r}\right)
\end{aligned}$$
(5)

As for the tire/road forces acting on un-braked wheels in Eq. (2a), it is not needed to include tire nonlinear model, as these wheel slip λ_i are not essential to be considered as model states. Although these forces are hard to measure, there are many estimation methods proposed, such as extended Kalman filter[13], sliding mode observer[14] and the estimation performance are evaluated in normal and critical situations. Thus, F_{xi} and F_{yi} can be replaced by their estimation \hat{F}_{xi} and \hat{F}_{yi} , and they are considered as time-varying parameters of Eq. (2). Another problem of Eq. (2) is its parameter uncertainties and unmodelled dynamics, as using much simple model to describe vehicle main dynamics through excluding other dynamics (e.g. vehicle roll, pitch dynamics). To comprise the forces estimation error and unmodelled dynamics, disturbances d_{12} and d_{22} are added to Eq. (2). Furthermore, as the vehicle yaw stability controller has to be discrete to be implemented in a vehicle, the model is converted to discrete form through Euler approximation. As Eq.(6), the model of braking FR wheel is brought forth to represent one of the four vehicle yaw stability control conditions.

$$\begin{aligned} r(k+1) &= f_{11}(r(k)) + f_{12}(r(k))g(\lambda_2(k)) + d_{11}(k) + d_{12}(k) \\ & (6a) \\ \lambda_2(k+1) &= f_{21}(\lambda_2(k)) + f_{22}(k)T_{b2}(k) + d_{21}(k) + d_{22}(k) \end{aligned}$$

where

$$\begin{split} f_{11}(r(k)) &= r(k) \\ f_{12}(r(k)) &= \frac{T_{s}\mu F_{z2}(k)}{J_{z\mu_0} \bar{f}_{z20}} \frac{\chi_z K_\lambda}{\chi_\alpha \alpha_2(k)^2 + 1} (l_f \sin \delta(k) + \frac{d}{2} \cos \delta(k)) \\ g(\lambda_2(k)) &= \frac{\lambda_2(k)}{\chi_\lambda \lambda_2(k)^2 + 1} \\ d_{11}(k) &= \frac{T_s}{J_s} [-l_r(\bar{F}_{y3}(k) + \hat{F}_{y4}(k)) + \frac{d}{2} (\hat{F}_{x4}(k) - \hat{F}_{x3}(k)) \\ &\quad l_f \bar{F}_{x1}(k) \sin \delta(k) + l_f \cos \delta(k) (\hat{F}_{y1}(k) + \hat{F}_{y2}(k)) \\ &\quad -\frac{d}{2} \hat{F}_{x1}(k) \cos \delta(k) + \frac{d}{2} (\hat{F}_{y1}(k) - \hat{F}_{y2}(k)) \sin \delta(k)] \\ f_{21}(\lambda_2(k)) &= \lambda_2(k) \\ f_{22}(k) &= -\frac{T_s R_2}{J_{\omega 2} v_x(k)} \\ d_{21}(k) &= -\frac{T_s R_2}{J_{\omega 2}} \frac{R_2 F_{x2}(k)}{v_x(k)} \\ T_s \text{ is sampling period} \end{split}$$

It is important to note that $d_{12}(k)$ and $d_{22}(k)$ are assumed to be slowly time-varying comparing with sampling rate of the controller, and $(F_{xi}(k) - \hat{F}_{xi}(k))$, $(F_{yi}(k) - \hat{F}_{yi}(k))$, $d_{12}(k), d_{22}(k) \in \mathcal{L}_{\infty}$. Eq. (6) can reflect vehicle body, wheel dynamics and tire nonlinear characteristics as it is obtained from integrating vehicle yaw dynamics, wheel dynamics and rational tire model instead of using linear tire model[3] or using direct yaw moment to leave tire nonlinearity behind[5].

III. VEHICLE YAW STABILITY CONTROLLER DESIGN

The output of Eq. (6) is r and its input is $T_{b2}(k)$, so it is in single-input-single-output strict-feedback form and it is convenient to implement backstepping control theory, which is a systematic method for designing a controller to track reference signal step-by-step through calculating control (virtual control) at each step. Furthermore, to improve robustness of control systems, backstepping and sliding mode control (SMC) have been combined and implemented[15]. In the section, the proposed controller is designed according to the integration of backstepping and sliding mode control theory and the closed loop system stability is testified through input-to-state stability theory, which is well developed for analyzing systems with uncertainties.

A. Yaw stability controller design

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The yaw stability controller calculating $T_{b2}(k)$ to control r(k) follow $r_r(k)$ is designed in the following two steps.

Step 1. Yaw rate control This step is based on Eq. (6a) and $\lambda_2(k)$ is taken as virtual control input. The yaw rate tracking error $S_r(k) = r(k) - r_r(k)$ is selected as sliding surface and the tracking velocity is designed as

$$S_r(k+1) = \beta_r S_r(k)$$
 (-1 < β_r < 1) (7)

where β_r is used to balance tracking velocity and control intensity.

Substituting Eq.(6a) to Eq.(7) and rearranging, the following equation is obtained

$$g(\lambda_2(k)) = [f_{12}(r(k))]^{-1} [\beta_r S_r(k) - f_{11}(k) - d_{11}(k) - d_{12}(k) + r_r(k+1)]$$
(8)

To calculate $g(\lambda_2(k))$, $r_r(k+1)$ and $d_{12}(k)$ must be obtained in every sampling period. Considering vehicle velocity $v_x(k)$ and steering angle $\delta(k)$ are continuous and the sampling period T_s is small, the reference yaw rate of next sampling period $r_r(k+1)$ is approximated by current reference yaw rate $r_r(k)$. $d_{12}(k)$ is assumed to be slowly time-varying, so it can be estimated in each step as follows[16]:

$$\hat{d}_{12}(k) \approx d_{12}(k-1) = r(k) - f_{11}(r(k-1)) - f_{12}(r(k-1))g(\lambda_2(k-1)) - d_{11}(k-1)$$

So, Eq.(8) is obtained as

$$g(\lambda_2(k)) = [f_{12}(r(k))]^{-1} [\beta_r S_r(k) - f_{11}(k) - d_{11}(k) - \hat{d}_{12}(k) + r_r(k)]$$
(9)

Then the virtual control input $\lambda_{r2}(k)$ to make r(k) track $r_r(k)$ is obtained in the following progress:

- (1) Substitute above $g(\lambda_2(k))$ to its definition in Eq.(6) and solve the equation to get two results;
- (2) Choose the larger one of above results as λ_{r2}(k) to reduce braking intensity. If it is out of [-1,0], set it 0 when overshooting on the upside and set it -1 when overshooting on the downside.

Step 2. wheel slip control This step is to control $\lambda_2(k)$ to track $\lambda_{r2}(k)$, the tracking error, $S_{\lambda}(k) = \lambda_2(k) - \lambda_{r2}(k)$, is chosen as sliding surface as Step 1, and the desired tracking velocity is defined as

$$S_{\lambda}(k+1) = \beta_{\lambda}S_{\lambda}(k) \qquad (-1 < \beta_{\lambda} < 1) \tag{10}$$

Substitute Eq.(6b) to Eq. (10) and it is obtained

$$T_{b2}(k) = \frac{v_x(k)}{T_s R_2} [-\beta_\lambda S_\lambda(k) + f_{21}(\lambda_2(k)) + d_{21}(k) + d_{22}(k) - \lambda_{r2}(k+1)]^{-1}$$
(11)

In Eq.(11), the unknown items are $\lambda_{r2}(k+1)$ and $d_{22}(k)$. Similarly to **Step 1**, $\lambda_{r2}(k+1)$ is approximated by its current value $\lambda_{r2}(k)$ and $d_{22}(k)$ is approximated as below considering its slowly time-varying characteristics.

$$\hat{d}_{22}(k) \approx d_{22}(k-1) = \lambda_2(k) - f_{21}(\lambda_2(k-1)) \\ + \frac{T_s R_2}{v_r(k-1)} T_{b2}(k-1) - d_{21}(k-1)$$

So, the control input is rewritten as

$$T_{b2}(k) = \frac{v_x(k)J_{\omega_2}}{T_s R_2} [-\beta_\lambda S_\lambda(k) + f_{21}(\lambda_2(k)) + d_{21}(k) + \hat{d}_{22}(k) - \lambda_{r2}(k)]^{-1}$$
(12)

Similarly, the calculation of control input $T_{b1}(k)$, $T_{b3}(k)$ and $T_{b4}(k)$ for the other three vehicle situations follows **Step 1** and **Step 2** in the same way.

Remark 1. Usually, there are two other ways to handle the disturbances $d_{12}(k)$ and $d_{22}(k)$, one is to neglect them and to design controller based on nominal system, and the other is to use the bounds of them and to design robust controller[17]. In the paper, they are approximated in each sampling period in order to compensate them considering their characteristics to obtain better performance. This design principle will be testified in Section IV.

B. Stability analysis

Although backstepping theory is one of control-Lyapunov method and the stability can be guaranteed in normal case, there are disturbances estimation and reference approximation in above design. So the closed-loop system stability is evaluated in this section and the following theorem is obtained.

Theorem 1. Define

$$\Lambda_1(k) = f_{12}(r(k))\widetilde{g}(\lambda_2(k)) + d_{12}(k) - d_{12}(k) + r_r(k) - r_r(k+1) \Lambda_2(k) = d_{22}(k) - \widehat{d}_{22}(k) + \lambda_{r2}(k) - \lambda_{r2}(k+1)$$

where $\tilde{g}(\lambda_2(k))$ is the error between Eq. (9) and its real value in each sampling period. Considering that the tire/road forces observer is stable (i.e. $(F_{xi}(k) - \hat{F}_{xi}(k)), (F_{yi}(k) - \hat{F}_{yi}(k)) \in \mathcal{L}_{\infty})$ and disturbances $d_{12}(k), d_{22}(k)$ are slowly time-varying and bounded (i.e. $d_{12}(k), d_{22}(k) \in \mathcal{L}_{\infty}$), the closed-loop system of plant Eq.(6) and controller Eq.(12) is ISS with the consideration that $d(k) = [\Lambda_1(k), \Lambda_2(k)]^T$ is lumped disturbance.

Proof. For the system, define $S(k) = [S_r(k), S_\lambda(k)]^T$ and choose Eq.(13) as candidate Lyapunov function

$$V(S(k)) = V_1(S_r(k)) + V_2(S_\lambda(k))$$
(13)

where

$$V_1(S_r(k)) = \frac{1}{2}S_r(k)^2, \quad V_2(S_\lambda(k)) = \frac{1}{2}S_\lambda(k)^2$$

Firstly, the difference of $V_1(S_r(k))$ is computed as

$$\Delta V_1(S_r(k)) = V_1(S_r(k+1)) - V_1(S_r(k))$$

= $\frac{1}{2}(f_{11}(k) + f_{12}(r(k))g(\lambda_2(k)) + d_{11}(k) + d_{12}(k) - r_r(k+1))^2 - \frac{1}{2}S_r(k)^2$ (14)

Substitute Eq.(9) to above equation,

$$\Delta V_1(S_r(k)) = \frac{\beta_r^2 - 1}{2} S_r(k)^2 + \beta_r S_r(k) \Lambda_1(k) + \frac{1}{2} \Lambda_1(k)^2$$
(15)

According to Young's inequality,

$$|\beta_r S_r(k)\Lambda_1(k)| \le \kappa_1 \beta_r^2 S_r(k)^2 + \frac{1}{4\kappa_1}\Lambda_1(k)^2 \qquad (16)$$

where $\kappa_1 > 0$. Substitute Eq.(16) to Eq.(15),

$$\Delta V_1(S_r(k)) \le (\frac{1}{2}\beta_r^2 - 0.5 + \kappa_1\beta_r^2)S_r^2(k) + (\frac{1}{2} + \frac{1}{4\kappa_1})\Lambda_1(k)^2$$
(17)

In the same way,

$$\Delta V_2(S_{\lambda}(k)) \leq (\frac{1}{2}\beta_{\lambda}^2 - 0.5 + \kappa_2 \beta_{\lambda}^2) S_{\lambda}^2(k) + (\frac{1}{2} + \frac{1}{4\kappa_2}) \Lambda_2(k)^2$$

where $\kappa_2 > 0$.

The difference of the above candidate Lyapunov function shown in Eq.(13) is

$$\Delta V(S(k)) \le -\alpha(S(k)) + \gamma(d(k)) \tag{18}$$

where

$$\begin{aligned} \alpha(S(k)) &= (0.5 - \frac{1}{2}\beta_r^2 - \kappa_1\beta_r^2)S_r^2(k) \\ &+ (0.5 - \frac{1}{2}\beta_\lambda^2 - \kappa_2\beta_\lambda^2)S_\lambda^2(k) \\ \gamma(d(k)) &= (\frac{1}{2} + \frac{1}{4\kappa_1})\Lambda_1(k)^2 + (\frac{1}{2} + \frac{1}{4\kappa_2})\Lambda_2(k)^2 \end{aligned}$$

Through choosing $|\beta_r| < \sqrt{\frac{0.5}{0.5+\kappa_1}}$ and $|\beta_\lambda| < \sqrt{\frac{0.5}{0.5+\kappa_2}}$, $\alpha(S(k)) > 0$ is obtained. And it is easy to find $\widetilde{\alpha}(|S(k)|) \in \mathcal{K}_{\infty}$ and $\widetilde{\gamma}(|d(k)|) \in \mathcal{K}$ to meet $\alpha(S(k)) > \widetilde{\alpha}(|S(k)|)$ and $\gamma(d(k)) < \widetilde{\gamma}(|d(k)|)$. So Eq.(19) is rewritten as

$$\Delta V(S(k)) \le -\widetilde{\alpha}(|S(k|)) + \widetilde{\gamma}(|d(k)|) \tag{19}$$

Based on the analysis, V(S(k)) is Lyapunov-ISS function and the closed-loop system of vehicle yaw stability is ISS according to [18].

According to Eq.(7) and Eq.(10), the parameters β_r and β_{λ} are critical to the controller performance and are needed to be tuned through simulations both to satisfy **Theorem 1** and to balance those two issues below:

1.Driver expectation. The driver expects vehicle follow commands as fast as possible. To reach the expectation, it's desirable to set β_r and β_{λ} small values, which means vehicle states approach their references fast and requires large brake torque.

2.Wheel forces saturation. As tire/road forces are nonlinear with the increase of wheel slip and wheel slip angle, and the wheel may even locked due to large brake torque. As a result, these parameters needs to be chosen close to 1 to waken the brake intensity and to prevent wheel from being locked.

IV. SIMULATIONS

To evaluate the performance of the proposed vehicle yaw stability controller, a comparative group of simulations is carried out in this section and the simulations are designed to control a high-precision modular multi-body vehicle model ve-DYNA[19], whose parameters have been configured to simulate the dynamics of CA7180A4E produced by FAW (China First Automobile Works), and the parameters are listed in Table 1. Particularly, the tire parameters of Eq.(3) is obtained through fitting tire characteristics of ve-DYNA and the comparison is illustrated in Fig. 3, which shows that Eq.(3) fits ve-DYNA well in small wheel slip and slip angle. That is accurate enough in vehicle yaw stability control as the wheel slip is controlled in small range.

TABLE I

VEHICLE PARAMETERS

Parameter	Symbol	Value
Vehicle mass (kg)	m	1300
Yaw moment of inertia $(kg \cdot m^2)$	J_z	2167.56
Wheel rational inertia($kg \cdot m^2$)	$J_{\omega i}$	0.8
Distance from mass center to front axle (m)	l_f	1.45
Distance from mass center to rear axle (m)	l_r	1.07
Wheel base(m)	d	1.40
tire longitudinal parameter	χ_z	0.22
tire longitudinal parameter	χ_{λ}	10
tire longitudinal parameter	χ_{lpha}	50
tire longitudinal parameter	K_{λ}	72000
tire lateral parameter	η_z	1.5
tire lateral parameter	η_{lpha}	100
tire lateral parameter	η_{λ}	50
tire lateral parameter	C_{α}	35000

To testify the performance of the proposed controller, Sine Steer test method is implemented, which is a conventional



Fig. 3. Tire characteristics comparison between Eq.(3) and ve-DYNA

way to evaluate vehicle yaw dynamics. In the group of comparative simulations, the vehicle accelerates to 80 km/h and is steered sinusoidally with amplitude of 100° and frequency of 0.5Hz, and the road adhesion coefficient is 0.8. The simulations are carried out five times:

Sim1. without yaw stability controller.

Sim2. with yaw stability controller. The parameters of ve-DYNA and controller are listed in Table 1.

Sim3. with yaw stability controller. The parameters of ve-DYNA are set as m = 1400kg, $J_z = 2600kg \cdot m^2$, $K_{\lambda} = 50000$, $C_{\alpha} = 25000$, $l_f = 1.7m$, $l_r = 0.82m$ and $J_{\omega_i} = 0.6kg \cdot m^2$ to simulate vehicle load variation and tires wear. This simulation is to evaluate the robustness of the controller, as these parameters are critical in affecting vehicle yaw dynamics and wheel dynamics.

Sim4. with yaw stability controller without $\hat{d}_{12}(k)$ and $\hat{d}_{22}(k)$ in Eq.(8) and Eq.(12).

Sim5. with yaw stability controller but $\hat{d}_{12}(k)$ and $\hat{d}_{22}(k)$ are replaced by $\rho_1 \operatorname{sat}(\frac{r-r_r}{\varepsilon_1})$ and $\rho_2 \operatorname{sat}(\frac{\lambda_i - \lambda_{ri}}{\varepsilon_2})$ respectively, where $\operatorname{sat}(\cdot)$ is saturation function, ρ_1 and ρ_2 are bounds of $d_{12}(k)$ and $d_{22}(k)$, and ε_1 , ε_2 are parameters to be tuned[17].

In above simulations, the sampling period is 0.005s and the parameters β_r and β_{λ} in the last four simulations are tuned as 0.995 and 0.85 respectively through simulations to achieve satisfied performance.



Fig. 4. Vehicle states comparison of the first three simulations

Vehicle yaw rate, reference yaw rate and slip angle in the first three simulations are shown in Fig. 4. As illustrated in it, the vehicle is unstable in **Sim1** as yaw rate r(k) could not follow its reference $r_r(k)$ and vehicle slip angle $\beta(k)$ is very large, nearly -20° . The vehicle is in understeer and oversteer states alternately and the driver could not control it as desired. While the vehicle is stable both in both **Sim2** and

Sim3, as r(k) follows $r_r(k)$ and $\beta(k)$ is kept in small range. Comparing with Sim2, r(k) in Sim3 just oscillates in small extent. Furthermore, the results testifies that it's equivalent to control r(k) and to control both r(k) and $\beta(k)$.



Fig. 5. FR wheel states in Sim2







Fig. 7. Estimation of d_{12} and d_{22} in Sim2

Fig. 5 and Fig. 6 describe FR wheel states in **Sim2** and **Sim3** respectively. The wheel slip and brake torque in **Sim3** both change faster than them in **Sim2**. Fig. 7 and Fig. 8 present $d_{12}(k)$, $d_{22}(k)$ and their approximation values in the these two simulations and the disturbances are relatively



Fig. 8. Estimation of d_{12} and d_{22} in Sim3

small comparing with r(k) and $\lambda_2(k)$. $d_{12}(k)$ and $d_{22}(k)$ in **Sim3** both oscillate too. From these results, it is validated that although the control performance is degraded, the proposed controller is robust to vehicle parameter uncertainties.

To check the effect of compensating $d_{12}(k)$ and $d_{22}(k)$ through $\hat{d}_{12}(k)$ and $\hat{d}_{22}(k)$ more deeply, vehicle yaw rates in **Sim2**, **Sim4** and **Sim5** are compared. In **Sim5**, ρ_1 and ρ_2 are set to 0.01 and 0.03 respectively according to Fig. 7, and ε_1 , ε_2 are set to 0.2 and 0.4 respectively through simulation tuning. Vehicle yaw rate is described in Fig. 9, which indicates that r in **Sim2** is much smoother than it in **Sim4** and **Sim5**, as the parameter uncertainties have been already complemented as discussed in **Remark 1**.



Fig. 9. Vehicle yaw rate comparison in Sim2, Sim4 and Sim5

This group of simulations indicates that the proposed controller is able to make vehicle yaw stability in critical cornering manoeuvre and is robust to parameters uncertainties.

V. CONCLUSION

To control vehicle yaw rate follow its reference, a vehicle yaw stability controller is proposed based on discrete nonlinear time-varying model. The model is firstly established through considering vehicle yaw dynamics, braked wheel dynamics and tire nonlinear characteristics. The forces of un-braked wheels are considered as time-varying parameters and they are assumed to be estimated. In order to compensate estimation error and unmodelled dynamics, uncertainties are added to yaw dynamics and wheel dynamics. Then, yaw stability controller is designed in the framework of backstepping control, and control (virtual control) input is derived under sliding mode control method. Based on ISS theory, the closed-loop system is proved to be stable. Finally, with a high precision vehicle dynamics model, a group of critical cornering simulations validate that this vehicle yaw stability controller meets the control requirement.

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