2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Fault-tolerant Control of Dynamic Systems with Unknown Control Direction-Input Nonlinearities-Actuator Failures

X. Y. Liu[1], Y. D. Song[1,2], and Q. Song[1]

Abstract—**This paper aims at investigating fault-tolerant tracking control designs for uncertain multivariable systems involving unknown floating dynamics, fading actuators, input nonlinearities with non-symmetric saturation and dead-zones, and unknown control direction. A robust adaptive control approach with simple structure is proposed to cope with these factors concurrently, where the lack of knowledge of control sign is handled by incorporating in the control law a Nussbaum-type function. Furthermore, of practical interest, the adaptive compensation of the effects of the nonlinear inputs requires neither the knowledge of their parameters nor the construction of their inverse.**

I. Introduction

Most practical engineering systems are multivariable, and uncertain in nature, varying in both structures and parameters, due to external disturbance, harsh operational environment or plant aging, etc. Furthermore, the inputs of a practical plant are normally restricted by its physical structure and energy consumption, thus imposing nonlinear characters such as saturation and dead-zone etc. [4], [9], [10]. Moreover, component/actuator failures might occur during system operations, which are often undetectable in the sense that it is uncertain when and by how much the actuator would fail to work normally; and also the so called unknown control direction represents another challenging issue [11]-[15]. The coexistence of the abovementioned factors, among others, is not only the major source of challenge for control design, but also the key source of degradation or even instability in the performance of the system.

It is therefore of theoretical and practical importance to address the control problem of nonlinear systems taking into account these inevitable and restrictive factors jointly. It is noted that while isolated factor(s) can be dealt with by some existing work (Table 1), the coexistence of those factors cannot be handled by simply combining or collecting the existing ones. In this work, we present three control schemes to achieve tracking control of a class of MIMO dynamic systems with explicit consideration of floating dynamics, disturbances, actuator failures, unknown control direction, nonlinear inputs with non-symmetric saturation and dead-zone, and limited onboard computing/memory resources *simultaneously*. Core-information about the system is utilized, which allows for simple yet effective control schemes to be developed for the system in the face of the aforementioned restrictive and abnormal conditions without expensive or complex on-line computations or tedious design procedure. Furthermore, this approach is able to address actuator failures without the need for fault detection and diagnosis. The work extends [20] for nonlinear SISO to MIMO systems with unknown control direction.

TABLE 1.
Various Issues Addressed by the Related Works Separately

Various issues addressed		Our method	Some existing works
Actuator failures		yes	[2]-[8]
Input nonlinearity	Uncertain dead-zone	yes	[12], [15]
	Non-symmetric saturation	yes	[4], [10]
Unknown control direction		yes	[11]-[15]
Floating gain matrix		yes	[20]

II. Problem Statement

The MIMO system with floating dynamics considered in this work is of the following form

$$y_i^{(n_i)} = f_i(\mathbf{x}) + \sum_{j=1}^{m} g_{ij}^{\beta(t)}(\mathbf{x}) u_{aj}(t) + d_i(\mathbf{x},t),$$
$$i = 1, \cdots, m, \; \beta(t) \in \hbar = \{1, 2, \cdots, \mathrm{N}\} \tag{1}$$

$$u_{ai}(t) = \rho_i(t)\Phi(u_i(t)) + E_i(t) \tag{2}$$

where $\mathbf{x} = [y_1, \dot{y}_1, \cdots, y_1^{(n_1-1)}, \cdots, y_m, \dot{y}_m, \cdots, y_m^{(n_m-1)}]^{\mathrm{T}}$ is the state vector, $n_1 + \cdots + n_m = n$, y_i is the output of the i-th subsystem, $f_i(\mathbf{x})$ and $g_{ij}^{\beta(t)}(\mathbf{x})$, $i, j = 1, \cdots, m, \beta(t) \in \hbar$ represent unknown system dynamics on $\Re$, corresponding to the floating index signal of the gain function, $\beta(t)$ that takes values in the finite set $\hbar$, $d_i(\mathbf{x},t) \in \Re$ models states dependent external disturbance, $\Phi(u_i(t))$ represents the nonlinear input function.

The floating index signal is used to portray variations of the gain function of each subsystem at every time instance due to unexpected parametric or structural drifting. Here we consider that at every time instance t the index bears one unique number from $\hbar$. As the floating index signal could take any value in $\hbar$ during system operation, the temporal evolution of the system is governed by a floating/drifting set of differential equations. Plants in the form of (1) are therefore of multiple abnormal operation conditions.

Manuscript received on March 10, 2011. This work was supported in part by National Natural Science Foundation of China (60974052, 61134001).
[1]The authors are with Beijing Jiaotong University, Beijing, P. R. China 100044 (phone: 010-51684432; e-mail: ydsong@bjtu.edu.cn).
[2]The author is with University of Electronic Science and Technology of China, Chengdu, China.

Also considered here is the possible actuation failure that causes the loss of effectiveness of the actuator. In such case, the actual control signal $u_{ai}(t)$ and the designed control input $u_i(t)$ are not identical anymore, instead, they are related to each other through (2), where $0<\rho_i(t)\le 1$ is a time-varying and unknown signal called actuator efficiency factor, or "health indicator" [2], $E_i(t)$ denotes a time-varying function characterizing the portion of the control action produced by the actuator that is completely out of control (unwanted faulty control effort fed into the control loop). This model is able to describe the jump (abrupt) actuator faults [20] and the recipient actuator faults [3] and [5].

Denote $\mathbf{y}^{(n)}=[y_1^{(n_1)},\cdots,y_m^{(n_m)}]^{\mathrm{T}}$ and the composite system can be described as

$$\mathbf{y}^{(n)}=\mathbf{f}(\mathbf{x})+\mathbf{g}_{\beta(t)}(\mathbf{x})\mathbf{u}_a(t)+\mathbf{d}(\mathbf{x},t) \tag{3}$$

$$\mathbf{u}_a(t)=\boldsymbol{\rho}(t)\Phi(\mathbf{u})+\mathbf{E}(t) \tag{4}$$

where $\mathbf{d}=[d_1,\cdots,d_m]^{\mathrm{T}}$, $\mathbf{E}=[E_1,\cdots,E_m]^{\mathrm{T}}$, $\mathbf{u}_a=[u_{a1},\cdots,u_{am}]^{\mathrm{T}}$, $\mathbf{u}=[u_1,\cdots,u_m]^{\mathrm{T}}$, $\boldsymbol{\rho}=diag[\rho_1,\cdots,\rho_m]$, $\Phi(\mathbf{u})=[\Phi(u_1),\cdots,\Phi(u_m)]^{\mathrm{T}}$, $\mathbf{f}=[f_1,\cdots,f_m]^{\mathrm{T}}$, and $\mathbf{g}_{\beta(t)}=\begin{bmatrix} g_{11}^{\beta(t)} & \cdots & g_{1m}^{\beta(t)} \\ \vdots & \ddots & \vdots \\ g_{m1}^{\beta(t)} & \cdots & g_{mm}^{\beta(t)} \end{bmatrix}$.

The control objective is to design a control law $\mathbf{u}$ to force the plant output vector $\mathbf{y}=[y_1,\cdots,y_m]^{\mathrm{T}}$ to follow a desired trajectory vector $\mathbf{y}_d=[y_{d1},\cdots,y_{dm}]^{\mathrm{T}}$. Variables of functions are omitted hereafter for concise if no confusion is likely to occur.

Remark 1. The inclusion of drifting/floating in the model accounts for the situation of system structural change due to switching purposely or component(s) altered unconsciously. The control gain matrix drifts from symmetric to asymmetric with large parametric variations due to $\beta(t)$ – this is a new type of faults, i.e. component failures, loss connection, on and off, etc. To our best knowledge, very few works have dealt with such multiple abnormal operation situations.

In this work, we consider similar description of the nonlinear input function as depicted in [10], i.e. the control input u_i is constrained by the nonlinear and non-symmetric saturation with uncertain dead-zone functions instead of constants. The following assumptions are made regarding the above system and the reference signals.

Assumption 1.

i) The output of nonlinear input function $\Phi(u_i),\ \forall i=1,\cdots,m$ is not available.

ii) The dead-zone functions $u_{ri0}(t)$ and $u_{li0}(t)$ are uncertain, but with known signs, i.e.

$$0\le u_{ri0}(t)\le \overline{u}_{ri0}<u_{ri1},\ u_{li1}<\underline{u}_{li0}\le u_{li0}(t)\le 0$$

where $\overline{u}_{ri0}$ and $\underline{u}_{li0}$ are obtainable constants.

iii) Functions $\varsigma_{ri}(u_i)$ and $\varsigma_{li}(u_i)$ are smooth, and there exist unknown positive constants k_{ri0}, k_{ri1}, k_{li0}, and k_{li1} such that

$$0<k_{ri0}\le \dot{\varsigma}_{ri}(u_i)\le k_{ri1},\ u_i\in[u_{ri0}(t),u_{ri1}]$$
$$0<k_{li0}\le \dot{\varsigma}_{li}(u_i)\le k_{li1},\ u_i\in[u_{li1},u_{li0}(t)]$$

where $\dot{\varsigma}_{ri}(u_i)=d\varsigma_{ri}(z)/dz|_{z=u_i}$ and $\dot{\varsigma}_{li}(u_i)=d\varsigma_{li}(z)/dz|_{z=u_i}$.

Assumption 2. The gain matrix $\mathbf{g}_{\beta(t)},\forall\beta(t)\in\hbar$ has non-zero leading principal minors, its sign can be unknown and it must be positive or negative.

Assumption 3. Nonlinear term $f_i,\ \forall i=1,\cdots,m$ is smooth and bounded. There exists unknown constant a_f and known scalar function $\varphi_f(\mathbf{x})$ such that $\|\mathbf{f}\|\le a_f\varphi_f(\mathbf{x})$.

Assumption 4. The un-parameterized part of the actuation fault and the external disturbance are bounded in that there exist some unknown nonnegative constants a_E and a_d, known scalar function $\varphi_d(\mathbf{x})$ such that $\|\mathbf{E}\|\le a_E$ and $\|\mathbf{d}\|\le a_d\varphi_d(\mathbf{x})$.

Assumption 5. The desired trajectory $y_{di},\forall i=1,\cdots,m$ is smooth and its derivatives up to n_i -times are bounded.

Remark 2. Our motivation to consider the nonlinear relationship between the input and output of $\Phi(u_i)$ stems from the desire to capture the most realistic situation. Note that the description of the input nonlinearity is a modified version of the one addressed by the recent works [10] because the dead-zone parameters considered here are uncertain functions rather than known constants. Assumption 1 is not restrictive because the precise values of the dead-zone are not needed in control design, similar to [12] and [15], while in contrast to [9] and [10]. Assumptions 2-5 are typical for robust control design and in line with standard ones. Assumptions 2 and 3 relate to the nature and property of the system dynamics. Assumption 4 confines the external disturbance to be bounded. Assumption 5 imposes a restriction on the desired signals may be tracked.

Define the tracking error vector of the i-th subsystem as $\mathbf{e}_i(t)=[e_i,\dot{e}_i,\cdots,e_i^{n_i-1}]^{\mathrm{T}}=[y_i-y_{di},\dot{y}_i-\dot{y}_{di},\cdots,y_i^{n_i-1}-y_{di}^{n_i-1}]^{\mathrm{T}}$ and a filtered tracking error $S_i(t)\in\Re$ as

$$S_i(t)=\boldsymbol{\lambda}_i^{\mathrm{T}}\mathbf{e}_i(t) \tag{5}$$

where $\boldsymbol{\lambda}_i=[\lambda_{i,1},\cdots,\lambda_{i,n_i-1},1]^{\mathrm{T}}$ is appropriately chosen constant vector such that $\mathbf{e}_i\to 0$ exponentially as $S_i\to 0$. The control objective reduces to design u_i to ensure $S_i\to 0$ as $t\to\infty$.

Upon (1) and (2), the time derivative of S_i is written as

$$\dot{S}_i=y_i^{(n_i)}+v_i(t) \tag{6}$$

with $v_i(t)=-y_{di}^{(n_i)}+\lambda_{i,n_i-1}e_i^{(n_i-1)}+\cdots+\lambda_{i,1}\dot{e}_i$. Denote $\mathbf{v}=[v_1,\cdots,v_m]^{\mathrm{T}}$ and $\mathbf{S}(t)=[S_1,\cdots,S_m]^{\mathrm{T}}$, then from (3) and (4), we have

$$\dot{\mathbf{S}}=\mathbf{y}^{(n)}+\mathbf{v}=\boldsymbol{\eta}(\mathbf{x})+\mathbf{g}_{\beta}\boldsymbol{\rho}\Phi(\mathbf{u}) \tag{7}$$

where $\boldsymbol{\eta}(\mathbf{x})=\mathbf{f}+\mathbf{v}+\mathbf{d}+\mathbf{g}_{\beta}\mathbf{E}$, lumping nearly all uncertainties in it, represents the major source of challenge for tracking control design. Thereafter, (7) will be intensively used in the development of the controllers and the stability analysis.

III. Control Design

Control designs for the following two cases are worth investigating:

- Case i): MIMO systems with known sign of the structural floating gain matrix;
- Case ii): MIMO systems with unknown sign of the structural floating gain matrix.

Case i) corresponds to known control direction while Case ii) represents unknown control direction. The control design for the system governed by (3) under the above two cases is based on the following useful lemma.

Lemma 1. *For any real matrix $\mathbf{g}(\mathbf{x}) \in \Re^{m\times m}$ with non-zero leading principal minors and any real diagonal positive-definite matrix $B(\cdot) = diag[b_{11}(\cdot), b_{22}(\cdot), \cdots, b_{mm}(\cdot)]$, the matrix $\mathbf{g}(\mathbf{x})B(\cdot)$ can be decomposed as follows:*

$$\mathbf{g}(\mathbf{x})B(\cdot) = \mathbf{g}_{sb}(\cdot)D_b T_b(\cdot) \tag{8}$$

with $\mathbf{g}_{sb}(\cdot) \in \Re^{m\times m}$ symmetric positive-definite matrix, $D \in \Re^{m\times m}$ diagonal matrix whose elements are +1 or -1, and $T_b(\cdot) \in \Re^{m\times m}$ unity upper triangular matrix. Moreover, the diagonal elements d_{bii} of D_b are nothing than the ratios of the signs of the leading principal minors of $\mathbf{g}(\mathbf{x})$, i.e. $D_b = D$.

Proof. The result in this lemma is an extension of the work [1] and [14], and the proof is outlined below.

We start by showing the existence of this decomposition. Under Assumption 2, (8) exists by [1], if the matrix $\mathbf{g}(\mathbf{x})B(\cdot)$ has non-zero leading principal minors. The calculation of the minors of $\mathbf{g}(\mathbf{x})B(\cdot)$ gives $\Delta_{bi} = b_{11}(\cdot)\cdots b_{ii}(\cdot)\Delta_i$, $i = 1,\cdots,m$, where Δ_i are the minors of $\mathbf{g}(\mathbf{x})$. Since $B(\cdot)$ is positive-definite, i.e. $b_{ii}(\cdot) > 0$, thus $\Delta_{bi} \neq 0, \forall i = 1,\cdots,m$, and hence (8) exists.

Apparently, $D_{bii} = sign(\Delta_{bi}/\Delta_{bi-1}) = sign(b_{ii}(\cdot)\Delta_i/\Delta_{i-1}) = D_{ii}$ holds, which means that the diagonal elements of D_b are the ratios of the signs of the minors of $\mathbf{g}(\mathbf{x})$. This completes the proof.

Remark 3. Note that the result in this lemma includes that of [1] and [14] as a special case in that only constant matrix $B(\cdot)$ is considered therein. The decomposition (8) is crucial to deal with the impact of actuator failures as seen later.

A. Known Control Direction

In this subsection, the sign of the floating gain matrix is known. Without loss of generality, we shall assume it positive. According to Assumption 2 and Lemma 1, we have the decomposition $\mathbf{g}_\beta = \mathbf{g}_{\beta s} T_\beta$ ($D = I_m$).

1) Robust Adaptive Fault-tolerant Control

A robust adaptive tracking control law is proposed corresponding to Case 1), and the assumption below is made.

Assumption 6. The symmetric and positive-definite matrix $\mathbf{g}_{\beta s}$ has the following properties:

$$\left\|\mathbf{g}_{\beta s}^{-1}\right\|_F \le a_s\varphi_s(\mathbf{x}),\ \frac{1}{2}\|\dot{\mathbf{g}}_{\beta s}^{-1}\|_F \le a_{ds}\varphi_{ds}(\mathbf{x}),\ \forall\beta\in\hbar \tag{9}$$

where $\mathbf{g}_{\beta s}^{-1}$ is the inverse of $\mathbf{g}_{\beta s}$, a_s and a_{ds} are unknown nonnegative constants, $\varphi_s(\cdot)$ and $\varphi_{ds}(\cdot)$ are known positive nonlinear functions.

Remark 4. Compared with the assumptions in [13] and [14], where $\partial g_{ij}(\mathbf{x})/\partial y_i^{(n_i-1)} = 0, \forall i, j = 1,\cdots,m$ is assumed, the assumption imposed herein is not so restrictive because we only require the boundedness of $\left\|\mathbf{g}_{\beta s}^{-1}\right\|$ and $\left\|\dot{\mathbf{g}}_{\beta s}^{-1}\right\|$.

In light of Lemma 1, (7) can be rearranged as

$$\mathbf{g}_{\beta s}^{-1}\dot{\mathbf{S}} = \mathbf{g}_{\beta s}^{-1}\boldsymbol{\eta} + T_\beta\boldsymbol{\rho}\Phi(\mathbf{u}) = \boldsymbol{\alpha}(\mathbf{x}) + \boldsymbol{\rho}\Phi(\mathbf{u}) - \frac{1}{2}\dot{\mathbf{g}}_{\beta s}^{-1}\mathbf{S} \tag{10}$$

with $\boldsymbol{\alpha}(\mathbf{x}) = \mathbf{g}_{\beta s}^{-1}\boldsymbol{\eta} + [T_\beta - I_m]\boldsymbol{\rho}\Phi(\mathbf{u}) + \dot{\mathbf{g}}_{\beta s}^{-1}\mathbf{S}/2$. Note that $\boldsymbol{\alpha}$ is essentially unavailable because of its inclusion of the uncertain terms, to derive a feasible and practical control scheme such $\boldsymbol{\alpha}$ should not be used directly. Now we will show that $\boldsymbol{\alpha}$ has bounded norm. By inserting the expression of $\boldsymbol{\eta}$, we see that $\|\boldsymbol{\alpha}\|$ is less than or equal to the sum of five terms: $\left\|\dot{\mathbf{g}}_{\beta s}^{-1}\mathbf{S}\right\|/2$, $\left\|\mathbf{g}_{\beta s}^{-1}(\mathbf{f}+\mathbf{d}+\mathbf{v})\right\|$, $\left\|T_\beta\mathbf{E}\right\|$, $\left\|T_\beta\boldsymbol{\rho}\Phi(\mathbf{u})\right\|$, and $\left\|\boldsymbol{\rho}\Phi(\mathbf{u})\right\|$.

By Assumptions 3 and 6, we have $\left\|\dot{\mathbf{g}}_{\beta s}^{-1}\mathbf{S}\right\|/2 \le a_{ds}\varphi_{ds}\|\mathbf{S}\|$ and

$$\left\|\mathbf{g}_{\beta s}^{-1}(\mathbf{f}+\mathbf{d}+\mathbf{v})\right\| \le \left\|\mathbf{g}_{\beta s}^{-1}\right\|_F\left[\|\mathbf{f}\| + \|\mathbf{d}\| + \|\mathbf{v}\|\right] \le a_1\varphi_1(\mathbf{x})$$

with $a_1 \ge a_s\max\{a_f, a_d, 1\}$ and $\varphi_1(\mathbf{x}) = \varphi_s(\varphi_f + \varphi_d + \|\mathbf{v}\|)$. Define positive constants $\lambda_{T\rho} \ge \lambda_{\max}[(T_\beta\boldsymbol{\rho})^{\mathrm{T}}(T_\beta\boldsymbol{\rho})]$, $\lambda_T \ge \lambda_{\max}[T_\beta^{\mathrm{T}}T_\beta], \forall\beta\in\hbar$, $\kappa \ge \max_{1\le i\le m}\{|\bar{u}_{ri\max}|, |\underline{u}_{li\max}|\}$, and $\lambda_\rho \ge \lambda_{\max}[\boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{\rho}]$, which can be unknown but always exist, then we can get

$$\left\|T_\beta\boldsymbol{\rho}\Phi(\mathbf{u})\right\| = \sqrt{\Phi^{\mathrm{T}}(\mathbf{u})(T_\beta\boldsymbol{\rho})^{\mathrm{T}}(T_\beta\boldsymbol{\rho})\Phi(\mathbf{u})} \le \sqrt{\lambda_{T\rho}\sum_{i=1}^{m}\Phi^2(u_i)} \le \kappa\sqrt{m\lambda_{T\rho}}$$

$$\left\|\boldsymbol{\rho}\Phi(\mathbf{u})\right\| \le \kappa\sqrt{m\lambda_\rho},\ \left\|T_\beta\mathbf{E}\right\| \le a_E\sqrt{\lambda_T}$$

Upon the above analysis, it can be established that

$$\|\boldsymbol{\alpha}\| \le a\varphi(\mathbf{x}) \tag{11}$$

with $a \ge \max\{a_1, a_{ds}/2, \kappa\sqrt{m}(\sqrt{\lambda_{T\rho}} + \sqrt{\lambda_\rho})\}$ unknown constant and $\varphi(\mathbf{x}) = \varphi_1 + \varphi_{ds}\|S\| + 1$ known function, respectively.

The first robust adaptive fault-tolerant control scheme with the following form is developed

$$u_i = \begin{cases} -(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)S_i + \bar{u}_{ri0} & \text{if} \quad S_i < 0 \\ 0 & \text{if} \quad S_i = 0 \\ -(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)S_i + \underline{u}_{li0} & \text{if} \quad S_i > 0 \end{cases} \tag{12}$$

where $k_1 > 0$ is a free design parameter, other variables are defined as before, and $\hat{a}$, the estimate of a, is updated by

$$\dot{\hat{a}} = \|\mathbf{S}\|\varphi,\ \hat{a}(0) \ge 0 \tag{13}$$

Before proving the stability of the control scheme, we carry out some analysis on the nonlinear input function $\Phi(u_i)$. Define $\bar{u}_i = [u_i, u_{li1}, u_{ri1}, u_{li0}, u_{ri0}]^{\mathrm{T}}$ and according to [10], there exist $\upsilon_{ri}(u_i) \in [u_{ri0}, u_{ri1}]$ and $\upsilon_{li}(u_i) \in [u_{li1}, u_{li0}]$ such that

$$\Phi(u_i) = \begin{cases} \chi(\bar{u}_i)(u_i - u_{ri0}) & \text{if} \quad u_i \geq u_{ri0} \\ \chi(\bar{u}_i)(u_i - u_{li0}) & \text{if} \quad u_i \leq u_{li0} \\ 0 & \text{else} \end{cases} \tag{14}$$

with $\chi(\bar{u}_i) = \begin{cases} u_{ri\max}/(u_i - u_{ri0}) & \text{for} \quad u_i > u_{r1} \\ \dot{\varsigma}_{ri}(\upsilon_{ri}(u_i)) & \text{for} \quad u_{ri0} \leq u_i \leq u_{ri1} \\ \dot{\varsigma}_{li}(\upsilon_{li}(u_i)) & \text{for} \quad u_{li1} \leq u_i \leq u_{li0} \\ u_{li\max}/(u_i - u_{li0}) & \text{for} \quad u_i < u_{li1} \end{cases}$.

Obviously $\chi(\bar{u}_i) > 0,\ \forall i = 1, \cdots, m$ hold by Assumption 1.

To show that the above control scheme (12) is able to deal with Case 1), we need the following Lemma.

Lemma 2. *For nonlinear input function satisfying Assumption 1, the control law (12) ensures that*

$$\mathbf{S}^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) \leq -\lambda_m(k_1\|\mathbf{S}\|^2 + \hat{a}\varphi\|\mathbf{S}\|) \tag{15}$$

with $0 < \lambda_m \leq \lambda_{\min}[\boldsymbol{\rho}\boldsymbol{\chi}(\bar{\mathbf{u}})]$ and $\boldsymbol{\chi}(\bar{\mathbf{u}}) = diag[\chi(\bar{u}_1), \cdots, \chi(\bar{u}_m)]$.

Proof. Since $k_1 > 0$, $\hat{a} \geq 0$, and $\varphi > 0$, it is readily verified that $-(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)$ is strictly non-positive for $S_i \neq 0$. Using this fact, it is obtained from (12) that when $S_i < 0$, we easily get $u_i \geq \bar{u}_{ri0} \geq u_{ri0}$. It follows from (14) that

$$(\mathbf{u} - \bar{\mathbf{u}}_{r0})^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) = -(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)\mathbf{S}^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u})$$
$$\geq (\mathbf{u} - \bar{\mathbf{u}}_{r0})^{\mathrm{T}}\boldsymbol{\rho}\boldsymbol{\chi}(\bar{\mathbf{u}})(\mathbf{u} - \bar{\mathbf{u}}_{r0}) \geq \lambda_m(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)^2\|\mathbf{S}\|^2$$

Similarly, when $S_i > 0$, it holds that

$$(\mathbf{u} - \bar{\mathbf{u}}_{r0})^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) = -(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)\mathbf{S}^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) \geq \lambda_m(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)^2\|\mathbf{S}\|^2$$

where $\bar{\mathbf{u}}_{r0} = [\bar{u}_{r10}, \cdots, \bar{u}_{rm0}]^{\mathrm{T}}$ and $\underline{\mathbf{u}}_{l0} = [\underline{u}_{l10}, \cdots, \underline{u}_{lm0}]^{\mathrm{T}}$ are defined.

Dividing the above two inequalities by $-(k_1 + \hat{a}\varphi/\|\mathbf{S}\|)$ leads to (15). Based upon this lemma, the following tracking result is established.

Theorem 1. *Consider system (3) corresponding to Case i) with Assumptions 1-6 satisfied, if the proposed RAFTC (12) is applied, asymptotically stable tracking is ensured in that* $\lim_{t\to\infty} e_i^{(j)}(t) \to 0$ *for* $i = 1, \cdots, m$ and $j = 0, \cdots, n_i - 1$.

Proof. Choose the following Lyapunov function candidate

$$V(t) = \frac{\mathbf{S}^{\mathrm{T}}g_{\beta s}^{-1}\mathbf{S}}{2} + \frac{\tilde{a}^2}{2\lambda_m} \tag{16}$$

where λ_m is defined as before, $\tilde{a}$ is a generalized estimation error defined as $\tilde{a} = a - \lambda_m\hat{a}$, which is introduced to facilitate stability analysis as seen shortly. Utilizing (10), it follows that

$$\dot{V} = \mathbf{S}^{\mathrm{T}}\mathbf{g}_{\beta s}^{-1}\dot{\mathbf{S}} + \frac{1}{2}\mathbf{S}^{\mathrm{T}}\dot{\mathbf{g}}_{\beta s}^{-1}\mathbf{S} + \tilde{a}(-\dot{\hat{a}}) = \mathbf{S}^{\mathrm{T}}\boldsymbol{\alpha} + \mathbf{S}^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) + \tilde{a}(-\dot{\hat{a}})$$

Utilizing (11) and Lemma 2, we easily obtain

$$\dot{V} \leq -k_1\lambda_m\|\mathbf{S}\|^2 + \tilde{a}(\|\mathbf{S}\|\varphi - \dot{\hat{a}}) \tag{17}$$

Upon inserting the parameter adaptation law (13), one gets

$$\dot{V} \leq -k_1\lambda_m\|\mathbf{S}\|^2 \leq 0 \tag{18}$$

Therefore, it holds that $V \in \ell_\infty$, $\hat{a} \in \ell_\infty$, $S_i \in \ell_\infty \cap \ell_2, i = 1, \cdots, m$, which ensures $u_i \in \ell_\infty$, from which we get $\dot{S}_i \in \ell_\infty$. Since S_i is uniformly continuous, and by Barbalat Lemma [16], the asymptotically stable tracking is guaranteed, this completes the proof.

Remark 5. i) The proposed RAFTC (12) is structurally simple and computationally inexpensive. The stability analysis is relatively simple and different from that pursued in [12] and [19]. Furthermore, there is no need for precise information on the fault magnitude and fault occurrence time instance. ii) It is interesting to note that while $\mathbf{g}_{\beta s}^{-1}$ and λ_m are used for stability analysis, they are not involved in the control scheme, and thus there is no need for estimating or computing such matrix and parameter. This significantly simplifies the design and implementation procedures.

2) Neural Network Based Adaptive Fault-tolerant Control

In this section, a model-independent NN-based robust adaptive fault-tolerant control scheme is proposed to relax $\varphi(\mathbf{x})$ in (12). The main idea is to use RBF neural network to approximate $\mu(\mathbf{x})$, the upper bound of $\|\boldsymbol{\alpha}\|$, through

$$\mu(\mathbf{x}) = \omega^{*\mathrm{T}}\phi(\mathbf{x}) + \varepsilon,\ \forall \mathbf{x} \in \Omega \tag{19}$$

where $\phi(\cdot) = [\phi_1(\cdot), \cdots, \phi_p(\cdot)]^{\mathrm{T}} \in \Re^p$ (p is the total number of the neurons in the hidden layer) is the basis function vector, ε is the network reconstruction error, and $\omega^* \in \Re^p$ is the ideal (optimal) weight vector, $\Omega \subseteq \Re^p$ is a compact set.

Remark 6. It is noted that by using RBFNN to deal with the unknown scalar function $\mu(\mathbf{x})$, rather than the vector function $\boldsymbol{\alpha}$, the number of neurons and the on-line weight updating computations needed are largely reduced.

We make the following assumption.

Assumption 7. The approximation error ε is upper bounded by some unknown constant ε_0, i.e. $|\varepsilon| \leq \varepsilon_0 < \infty$.

The following NN-base robust adaptive fault-tolerant tracking control algorithm is proposed,

$$u_i = \begin{cases} -[k_2 + (\hat{\omega}^{\mathrm{T}}\phi + \hat{\varepsilon}_0)/\|\mathbf{S}\|]S_i + \bar{u}_{\mathrm{r}i0} & \text{if} \quad S_i < 0 \\ 0 & \text{if} \quad S_i = 0 \\ --[k_2 + (\hat{\omega}^{\mathrm{T}}\phi + \hat{\varepsilon}_0)/\|\mathbf{S}\|]S_i + \underline{u}_{li0} & \text{if} \quad S_i > 0 \end{cases} \tag{20}$$

where $k_2 > 0$ is a free design parameter, other variables and parameters are defined as before, $\hat{\omega}$ and $\hat{\varepsilon}_0$ are the estimates of ω^* and ε_0, respectively, and are updated by

$$\dot{\hat{\omega}} = \|\mathbf{S}\|\phi\ (\hat{\omega}(0) \geq 0),\ \dot{\hat{\varepsilon}}_0 = \|\mathbf{S}\|\ (\hat{\varepsilon}_0(0) \geq 0) \tag{21}$$

For nonlinear input function $\Phi(u_i)$ satisfying Assumption 1, the control law (20) ensures that

$$\mathbf{S}^{\mathrm{T}}\boldsymbol{\rho}\Phi(\mathbf{u}) \le -\lambda_m (k_2 \|\mathbf{S}\|^2 + \hat{\omega}^{\mathrm{T}}\phi\|\mathbf{S}\| + \hat{\varepsilon}_0\|\mathbf{S}\|) \tag{22}$$

The proof is similar to Lemma 2, thus omitted for brevity.

To establish the stability of the control scheme (20), two generalized parameter estimation errors $\tilde{\omega} = \omega^* - \lambda_m\hat{\omega}$ and $\tilde{\varepsilon}_0 = \varepsilon_0 - \lambda_m\hat{\varepsilon}_0$ are introduced, which leads to the following modified Lyapunov function candidate

$$V(t) = \frac{\mathbf{S}^{\mathrm{T}}\mathbf{g}_{\beta s}^{-1}\mathbf{S}}{2} + \frac{\tilde{\omega}^{\mathrm{T}}\tilde{\omega}}{2\lambda_m} + \frac{\tilde{\varepsilon}_0^{\,2}}{2\lambda_m} \tag{23}$$

Following the same lines as in the proof of Theorem 1, the following result can be established.

Theorem 2. *For* system *(3) corresponding to Case i) under Assumptions 1-7, the control scheme (20) can guarantee asymptotically stable tracking, i.e.* $\lim_{t\to\infty} e_i^{(j)}(t) \to 0$ for $i = 1,\cdots,m$ and $j = 0,\cdots,n_i - 1$.

B. Unknown Control Direction

The two control schemes presented previously are applicable only for MIMO dynamic systems with known sign of the floating gain matrix. In this section, an adaptive control algorithm is proposed to deal with the case that the control direction is unknown as assumed in Case ii), which does not require a priori knowledge of the sign of the gain matrix. A Nussbaum function is incorporated in the control input. A function $N(\zeta)$ is called a Nussbaum-type function if it has the following properties:

$$\text{(i)}\limsup_{t\to\infty}\frac{1}{t}\int_0^t N(\zeta)d\zeta = +\infty\,,\ \text{(ii)}\liminf_{t\to\infty}\frac{1}{t}\int_0^t N(\zeta)d\zeta = -\infty\,.$$

Commonly used Nussbaum functions include: $\zeta^2\cos(\zeta)$, $\zeta^2\sin(\zeta)$, and $\exp(\zeta^2)\cos(\pi\zeta/2)$ [15], [18], [17]. For clarity, the even Nussbaum function $N(\zeta) = e^{\zeta^2}\cos(\pi\zeta/2)$ is used here.

To develop the control law, we rewrite $\Phi(u_i) = \dot{\varsigma}_i^{\mathrm{T}}\gamma_i u_i + l_i(u_i)$, where $\dot{\varsigma}_i = [\dot{\varsigma}_{ri}(\upsilon_{ri}(u_i)),\ \dot{\varsigma}_{li}(\upsilon_{li}(u_i))]^{\mathrm{T}}$ and $\dot{\varsigma}_i = [\dot{\varsigma}_{ri}(\upsilon_{ri}(u_i)),\ \dot{\varsigma}_{li}(\upsilon_{li}(u_i))]^{\mathrm{T}}$, $\gamma_i = [\gamma_{ri},\ \gamma_{li}]^{\mathrm{T}}$, $\gamma_{ri} = \begin{cases}1 & \text{if} \quad u_i \ge u_{li0} \\ 0 & \text{else}\end{cases}$, $\gamma_{li} = \begin{cases}1 & \text{if} \quad u_i \le u_{ri0} \\ 0 & \text{else}\end{cases}$, and

$$l_i(u_i) = \begin{cases} -\dot{\varsigma}_{ri}u_i + u_{ri\max} & \text{for} \quad u_i > u_{ri1} \\ -\dot{\varsigma}_{ri}u_{ri0} & \text{for} \quad u_{ri0} \le u_i \le u_{ri1} \\ -(\dot{\varsigma}_{ri} + \dot{\varsigma}_{li})u_i & \text{for} \quad u_{li0} \le u_i \le u_{ri0} \\ -\dot{\varsigma}_{li}u_{li0} & \text{for} \quad u_{li1} \le u_i \le u_{li0} \\ -\dot{\varsigma}_{li}u_i + u_{li\max} & \text{for} \quad u_i < u_{li1} \end{cases}.$$

Then $\Phi(\mathbf{u})$ can be expressed as

$$\Phi(\mathbf{u}) = R\mathbf{u} + L(\mathbf{u}) \tag{24}$$

with $R = diag[\dot{\varsigma}_1^T\gamma_1,\cdots,\dot{\varsigma}_m^T\gamma_m]$ and $L(\mathbf{u}) = [l_1(u_1),\cdots,l_m(u_m)]^{\mathrm{T}}$.

It is easily verified that R is symmetric and positive-definite by the fact that $R_{ii} = \dot{\varsigma}_i^{\mathrm{T}}\gamma_i \in [\min\{k_{r0},k_{l0}\}, k_{r1} + k_{l1}] > 0$, from which we get $\boldsymbol{\rho}R$ is also positive-definite, then there exists some symmetric and positive-definite matrix $\mathbf{g}_{\beta sr}$ and unity upper triangular matrix $T_{\beta r}$ such that $\mathbf{g}_\beta\boldsymbol{\rho}R = \mathbf{g}_{\beta sr}(\cdot)DT_{\beta r}(\cdot)$ by Lemma 1. Using (24) and this decomposition, (7) can be rearranged as

$$\mathbf{g}_{\beta sr}^{-1}\dot{\mathbf{S}} = \boldsymbol{\alpha}_r + D\mathbf{u} - \frac{1}{2}\dot{\mathbf{g}}_{\beta sr}^{-1}\mathbf{S} \tag{25}$$

with $\boldsymbol{\alpha}_r = \mathbf{g}_{\beta sr}^{-1}(\boldsymbol{\eta} + \mathbf{g}_\beta\boldsymbol{\rho}L) + D(T_{\beta r} - I_m)\mathbf{u} + \dot{\mathbf{g}}_{\beta sr}^{-1}\mathbf{S}/2$. Similar to the treatment to $\boldsymbol{\alpha}$ in the previous section, here we approximate $\mu_r(\mathbf{x})$, the upper bound of $\|\boldsymbol{\alpha}_r\|$, through a RBF neural network $\mu_r(\mathbf{x}) = \omega_r^{*\mathrm{T}}\phi(\mathbf{x}) + \varepsilon_r, \forall\mathbf{x}\in\Omega\subseteq\Re^p$, similar to Assumption 7, here we assume the reconstruction error ε_r is bounded by some unknown constant ε_{r0}, i.e. $|\varepsilon_r| \le \varepsilon_{r0} < \infty$.

The following control law is developed to achieve the control objective,

$$u_i = [k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|]\psi(\|\mathbf{S}\|\,|\,c)N(\zeta)S_i \tag{26}$$

where $\psi(\|\mathbf{S}\|\,|\,c) = \begin{cases}1 & \text{if} \quad \|\mathbf{S}\| \ge \mathrm{c} \\ 0 & \text{else}\end{cases}$. c and k_3 are two free design positive constants, $\hat{\omega}_r$ and $\hat{\varepsilon}_{r0}$ are the estimates of ω_r^* and ε_{r0}, respectively, which are updated by

$$\dot{\hat{\omega}}_r = \|\mathbf{S}\|\phi\ (\hat{\omega}_r(0) \ge 0),\ \dot{\hat{\varepsilon}}_{r0} = \|\mathbf{S}\|\ (\hat{\varepsilon}_{r0}(0) \ge 0) \tag{27}$$

and $N(\zeta) = e^{\zeta^2}\cos(\pi\zeta/2)$ with

$$\dot{\zeta} = [k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|]\psi(\|\mathbf{S}\|\,|\,c)\|\mathbf{S}\|^2 \tag{28}$$

Theorem 3. *For* system *(3) correspond to Case ii) under Assumptions 1-5, the proposed control scheme (26) can guarantee that the closed-loop system is stable and the output tracking errors converge to zero asymptotically.*

Proof. Similar to the proof of Theorem 2, here we introduce $\tilde{\omega}_r = \omega_r^* - \hat{\omega}_r$ and $\tilde{\varepsilon}_{r0} = \varepsilon_{r0} - \hat{\varepsilon}_{r0}$, consider the following Lyapunov function candidate,

$$V(t) = \frac{\mathbf{S}^{\mathrm{T}}\mathbf{g}_{\beta sr}^{-1}\mathbf{S}}{2} + \frac{\tilde{\omega}_r^{\mathrm{T}}\tilde{\omega}_r}{2} + \frac{\tilde{\varepsilon}_{r0}^2}{2} \tag{29}$$

If $\|\mathbf{S}\| \ge c$, we have $\mathbf{u} = [k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|]N(\xi)\mathbf{S}$, and (25) can be rewritten as

$$\begin{aligned}\mathbf{g}_{\beta sr}^{-1}\dot{\mathbf{S}} = &-\frac{1}{2}\dot{\mathbf{g}}_{\beta sr}^{-1}\mathbf{S} + \boldsymbol{\alpha}_r - \left[k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|\right]\mathbf{S} \\ &+ \left[k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|\right]\left[I_{\mathrm{m}} + DN(\xi)\right]\mathbf{S}\end{aligned} \tag{30}$$

Upon using (30), it follows that

$$\begin{aligned}\dot{V} &= \mathbf{S}^{\mathrm{T}}\boldsymbol{\alpha}_r - \mathbf{S}^{\mathrm{T}}\left[k_3 + (\hat{\omega}_r^{\mathrm{T}}\phi + \hat{\varepsilon}_{r0})/\|\mathbf{S}\|\right]\mathbf{S} + [1 + gN(\zeta)]\dot{\zeta} + \tilde{\omega}_r\dot{\tilde{\omega}}_r + \tilde{\varepsilon}_{r0}\dot{\tilde{\varepsilon}}_{r0} \\ &\le -k_3\|\mathbf{S}\|^2 + [1 + gN(\zeta)]\dot{\zeta} + \tilde{\omega}_r(\|\mathbf{S}\|\phi - \dot{\hat{\omega}}_r) + \tilde{\varepsilon}_{r0}(\|\mathbf{S}\| - \dot{\hat{\varepsilon}}_{r0})\end{aligned}$$

where $g = d_{ii}, i = 1,\cdots,m$, with d_{ii} the diagonal elements of D.

Upon inserting the adaptation laws (27), one gets

$$\dot{V} \le -k_3\|\mathbf{S}\|^2 + [1 + gN(\zeta)]\dot{\zeta} \tag{31}$$

Integrating (31) over $[0,t]$, we have

$$V(t) \le V(0) + \int_0^t [1 + gN(\zeta)]\dot{\zeta} d\tau$$

Thus $V(t)$, $\int_0^t [1+gN(\zeta)]\dot{\zeta} d\tau$, and ζ are all bounded on $[0,t_f)$ by [19]. The above conclusion is also true for $t_f = +\infty$ by [18] and [19]. It is straightforward to show that $\tilde{\omega}_r \in \ell_\infty$ and $\int_0^\infty \|\mathbf{S}\|^2 d\tau$ exists, i.e. $S_i \in \ell_2$. By the boundedness of S_i, $\tilde{\omega}_r$, and ζ, we can conclude the boundedness of $\hat{\omega}_r$ and u_i.

In order to show the boundedness of $\dot{S}_i$, multiply $\mathbf{g}_{\beta sr}$ on both sides of (30), and since S_i, $\tilde{\omega}_r$, ω_r, u_i, ε_r, $L(\mathbf{u}) \in \ell_\infty$, $\mathbf{g}_{\beta sr}^{-1}$ is a positive-definite matrix (i.e. $\exists \sigma_0 > 0$, such that $\|\mathbf{g}_{\beta sr}^{-1}\| \ge \sigma_0$), and $\mathbf{g}_{\beta sr}^{-1}$, $\dot{\mathbf{g}}_{\beta sr}^{-1}$ are continuous, thus we have $\dot{S}_i \in \ell_\infty$.

Finally, since $S_i \in \ell_2 \cap \ell_\infty$ and $\dot{S}_i \in \ell_\infty$, we can conclude with Barbalat Lemma [16] that $S_i(t) \to 0$ as $t \to \infty$ which means the asymptotically tracking is achieved.

If $\|\mathbf{S}\| < c$, it means $\psi(\|\mathbf{S}\| \,|\, c) = 0$, $u_i = 0, \forall i = 1,\cdots,m$, and $\dot{\zeta} = 0$, thus $\dot{\hat{\omega}}_r$, and $\dot{\hat{\varepsilon}}_{r0}$ are bounded, i.e. ζ, $\hat{\omega}_r$, and $\hat{\varepsilon}_{r0}$ are kept bounded.

Therefore, we can conclude from the above two cases that all the closed-loop signals are semi-globally uniformly ultimately bounded.

Remark 7. It is worth stressing that when the underlying system has only limited onboard computing/memory resources, computationally inexpensive control algorithms are always favorable for practical application. The proposed control schemes involve fairly simple and inexpensive computations, yet capable of coping with various anomaly factors in the system simultaneously.

Remark 8. i) The control schemes (12), (20) and (26) involve the switch term $S_i/\|\mathbf{S}\|$, $i = 1,\cdots,m$ which might cause chattering as $\|\mathbf{S}\| \to 0$. A simple and effective solution is to replace it by $S_i/(\|\mathbf{S}\| + z)$, where z is small positive constant. ii) Also, to prevent the parameter estimation drifting, the following modified adaptive laws can be used,

$$\dot{\hat{a}} = -\delta_a \hat{a} + \sigma_a \|\mathbf{S}\| \varphi, \; \dot{\hat{\omega}} = -\delta_\omega \hat{\omega} + \sigma_\omega \|\mathbf{S}\| \phi, \; \dot{\hat{\varepsilon}}_0 = -\delta_\varepsilon \hat{\varepsilon}_0 + \sigma_\varepsilon \|\mathbf{S}\|,$$

$$\dot{\hat{\omega}}_r = -\delta_{\omega r} \hat{\omega}_r + \sigma_{\omega r} \|\mathbf{S}\| \phi, \; \dot{\hat{\varepsilon}}_{r0} = -\delta_{\varepsilon r} \hat{\varepsilon}_{r0} + \sigma_{\varepsilon r} \|\mathbf{S}\|.$$

where δ_a, σ_a, δ_ω, σ_ω, δ_ε, σ_ε, $\delta_{\omega r}$, $\sigma_{\omega r}$, $\delta_{\varepsilon r}$, and $\sigma_{\varepsilon r}$ are positive constants. It should be emphasized Lemma 2 and (22) are still valid under the modified adaptive laws. As a result, the stable tracking is still ensured. In this case, the tracking errors is ensured to be uniformly ultimately bounded.

IV. Conclusion

Fault-tolerant tracking control design for a class of MIMO dynamic systems with floating structures, unknown control direction, nonlinear input with non-symmetric saturation and dead-zones as well as actuator failures is studied in this work. A robust adaptive control approach is proposed to cope with these factors simultaneously. Utilizing core-information about the system, three control schemes are developed with emphasis on their effectiveness and affordability – the resultant control algorithms bear simple structures and demand inexpensive computations, yet capable of handling the coexistence of the aforementioned anomaly factors.

References

[1] R. R. Costa, Liu Hsu, A. K. Imai, P. Kokotovic. Lyapunov-based adaptive control of MIMO systems. Automatica, 2003, 39(7): 1251−1257.

[2] W. C. Cai, X. H. Liao, Y. D. Song. Indirect robust adaptive fault-tolerant control for attitude tracking of spacecraft. Journal of Guidance, Control and Dynamics, 2008, 31(5): 1456−1463.

[3] X. D. Tang, G. Tao, S. M. Joshi. Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application. Automatica, 2007, 43, 1869-1883.

[4] Z. Q. Zuo, Dainel W. C. Ho, Y. J. Wang. Fault tolerant control for singular systems with actuator saturation and nonlinear perturbation. Automatica, 2010, 46, 569−576.

[5] G. Tao, S. M. Joshi, X. L. Ma. Adaptive state feedback and tracking control of systems with actuator failures. IEEE Trans. on Automatic Control, 2001, 46(1): 78−95.

[6] B. Jiang, M. Staroswiecki, V. Cocquempot. Fault accommodation for nonlinear dynamic systems. IEEE Trans. on Auto. Control, 2006, 51(9): 1578−1583,.

[7] Z. Q. Zhang, S. Y. Xu, Y. Guo, Y. M. Chu. Robust adaptive output-feedback control for a class of nonlinear systems with time-varying actuator faults. Int. Journal of Adaptive Control and Signal Processing, 2010.

[8] H. Yang, B. Jiang, M. Staroswiecki. Supervisory fault tolerant control for a class of uncertain nonlinear systems. Automatica, 2009, 45, 2319−2324.

[9] Q. L. Hu, G. F. Ma, L. H. Xie. Robust and adaptive variable structure output feedback control of uncertain systems with input nonlinearity. Automatica, 2008, 44, 552−559.

[10] M. Chen, S. S. Ge, Bernard Voon Ee How. Robust adaptive neural network control for a class of uncertain MIMO nonlinear systems with input nonlinearities. IEEE Trans. Neural Networks, 2010, 21(5): 1−17.

[11] X. D. Ye. Asymptotic regulation of time-varying uncertain nonlinear systems with unknown control directions. Automatica, 1999, 35, 929-935.

[12] T. P. Zhang, S. S. Ge. Adaptive neural control of MIMO nonlinear state time-varying delay systems with unknown dead-zones and gain signs. Automatica, 2007, 43, 1021−1033.

[13] A. Boulkroune, M. Tadjine, M. M'Saad, M. Farza. Fuzzy adaptive controller for MIMO nonlinear systems with known and unknown control direction. Fuzzy Sets and Systems, 2010, 161, 797−820.

[14] A. Boulkroune, M. M'Saad, H. Chekireb. Design of a fuzzy adaptive controller for MIMO nonlinear time-delay systems with unknown actuator nonlinearities and unknown control direction. Information Sciences, 2010, 180, 5041−5059.

[15] S. S. Ge, F. Hong, T. H. Lee. Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients. IEEE Trans. on SMC - Part B: Cybernetics, 2004, 34(1): 499−516.

[16] J. E. Slotine, W. Li. Applied nonlinear control [M]. Englwood Cliffs, NJ, Prentice Hall, 1991.

[17] R. D. Nussbaum. Some remarks on the conjecture in parameter adaptive control. Systems and Control Letters, 1983, 1(3): 243–246.

[18] E. P. A. Ryan. Universal adaptive stabilizer for a class of nonlinear systems. Systems and Control Letters, 1991, 16, 209–218.

[19] S. S. Ge, J. Wang. Robust adaptive neural control for a class of perturbed strict feedback nonlinear systems. IEEE Trans. on Neural Networks, 2002, 13(6): 1409–1419.

[20] X. Y. Liu, Y. D. Song. Robust adaptive fault-tolerant control of dynamic systems with floating nonlinearities and fading actuator. Systems & Control Letters (under review).