2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Stable Model Predictive Control Based on TS Fuzzy Model with Application to Boiler-turbine Coordinated System

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***Abstract*—In this paper, we propose a stable fuzzy model predictive controller based on extended-fuzzy Lyapunov function. The main idea of the proposed approach is to design a free control variable and a non-parallel distributed compensation control law in such a way that an extended-fuzzy Lyapunov function is constructed with minimizing the upper bound of the infinite horizon objective function in the fuzzy model predictive control. Therefore, the predictive controller can guarantee both the stability of the closed-loop fuzzy model predictive control system and input constraints while obtaining the optimal transient performance. It is shown that the controller is obtained by solving a set of linear matrix inequalities. The extended-fuzzy Lyapunov function reduces the conservatism of common Lyapunov function and fuzzy Lyapunov function, and it also enlarges the feasible area of the predictive controller. Moreover, appropriate slack and collection matrices are used in all linear matrix inequalities, which can further reduce the conservatism. The simulations on a numerical example and a nonlinear boiler-turbine coordinated system demonstrate the advantage and effectiveness of the proposed approach.**

***Index Terms*—TS fuzzy model, model predictive control, extended-fuzzy Lyapunov function, slack and collection matrices.**

I. INTRODUCTION

MODEL predictive control (MPC) uses an explicit model to predict the future behavior of the plant and solves a constrained optimization problem on-line to obtain the optimal control sequence [1],[2]. Since MPC can guarantee the stability of the closed-loop system and deal with the input constraint in an optimal way, MPC has become a popular strategy in process control [3],[4]. Generally, there are two problems with conventional MPC; the first is that it is difficult to analyze the stability of the closed-loop system while considering the constraint, the second is that the exact model of the nonlinear plant is difficult to obtain and the computational burden of nonlinear optimization is heavy.

One way to deal with the above problems is using the combination of several linear models to approximate the behavior of nonlinear plant and adopting the infinite horizon objective function in MPC. Based on linear parameter-varying (LPV) systems, Kothare et al. [5] have used a feedback control law to minimize an upper bound of the “worst-case” infinite horizon objective function. The stability and robustness of the closed-loop system can be guaranteed through solving a convex optimization problem in the form of linear matrix inequalities (LMIs). The approaches proposed in [6], [7] have used a quasi-infinite horizon objective function to improve the method in [5]. By separating the first or several control moves from the rest of the control moves governed by the feedback law and setting them as free variables, the designed controller can be less conservative. However, these model predictive controllers are developed based on common Lyapunov function (CLF) which need to find a common positive definite matrix for all submodels. This may lead to conservatism since it is difficult to find such a matrix, especially for complex systems. Zhang et al. [8] have proposed a Takagi-Sugeno (TS) fuzzy model predictive controller based on piecewise Lyapunov function (PLF) which can reduce the conservatism in CLF. However, the stability result of this method depends on the partitions of state space. An alternative way to reduce the conservatism in CLF is to adopt fuzzy Lyapunov function (FLF) which only needs to find an independent positive definite matrix for each submodel. The FLF has been used successfully in fuzzy model predictive controllers [9]. More recently an extended-fuzzy Lyapunov function has been proposed to improve the previous results [10], and it can also be used to obtain better results of MPC.

Besides improving Lyapunov functions, recently, the technique of slack and collection matrices has been continuously developed in fuzzy control to reduce the conservatism in stability analysis or stabilization results [11]-[13] . Yu et al. [14] have used slack and collection matrices in MPC for the LPV systems; however, they do not use this technique to constrain the upper bound of the infinite horizon objective function. Moreover, with the improved Lyapunov functions more slack matrices and collection matrices can be used to further reduce the conservatism [13].

Based on discrete TS fuzzy model, a stable model predictive controller using extended-fuzzy Lyapunov function is proposed in this paper. The main idea of the

This work was supported by the National Natural Science Foundation of China (NSFC) under Grant 51076027 and Grant 51036002, the Specialized Research Fund for the Doctoral Program of Higher Education under Grant 20090092110051, the Key Project of Chinese Ministry of Education under Grant 108060, and the U.S. National Science Foundation under grant ECCS 0801440.

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proposed approach is to design a free control variable and a non-parallel distributed compensation (non-PDC) control law in such a way that an extended-fuzzy Lyapunov function is constructed with minimizing the upper bound of the infinite horizon objective function in the fuzzy model predictive control. Therefore, the predictive controller can guarantee both the stability of the closed-loop fuzzy model predictive control system and input constraints while obtaining the optimal transient performance. Combined with some new results in fuzzy control, we use appropriate slack and collection matrices with extended-fuzzy Lyapunov function to further reduce the conservatism and enlarge the feasible area of the predictive controller. The simulation on a numerical example demonstrates the advantage and effectiveness of the proposed approach. Then, the proposed stable model predictive controller will be applied to a nonlinear boiler-turbine coordinated system of a power unit to change the load in a wide range.

II. Preliminaries

A. TS Fuzzy Model

Suppose a nonlinear discrete system can be represented as the following TS fuzzy model:

$$\begin{aligned} &R^i : IF\ z_1(k)\ \ is\ \ \mathrm{M}_1^i, z_2(k)\ \ is\ \ \mathrm{M}_2^i, ..., z_n(k)\ \ is\ \ \mathrm{M}_n^i \\ &\quad THEN\quad x(k+1) = A_i x(k) + B_i u(k) \\ &\quad i = 1,2,...r \end{aligned} \tag{1}$$

where R^i denotes the i -th fuzzy inference rule, r is the number of inference rules, $\mathrm{M}_j^i\,(j=1,2,...,n)$ are fuzzy sets, $x(k) \in \mathrm{R}^n$, $u(k) \in \mathrm{R}^m$ are respectively the system state and input variables, $z_k = (z_1(k), z_2(k), ..., z_n(k))$ are premise variables such as known state variables, and (A_i, B_i) is the i -th local model of the fuzzy system.

By using fuzzy blending, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{r} w_i(z_k)(A_i x(k) + B_i u(k)) \\ &= A_z x(k) + B_z u(k) \end{aligned} \tag{2}$$

where $A_z = \sum_{i=1}^{r} w_i(z_k) A_i$, $B_z = \sum_{i=1}^{r} w_i(z_k) B_i$

$$w_i(z_k) = \frac{\mathrm{M}^i(z_k)}{\sum_{i=1}^{r} \mathrm{M}^i(z_k)},\quad \mathrm{M}^i(z_k) = \prod_{j=1}^{n} \mathrm{M}_j^i(z_j(k)).$$

B. Extended-Fuzzy Lyapunov Function

As shown in [10], an extended-fuzzy Lyapunov function is defined by:

$$V(x(k), z_k) = x(k)^T \Big(\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z_k) w_j(z_k) S_{ij}\Big)^{-1} x(k) \tag{3}$$

For notational convenience, we denote the following notations for a matrix X :

$$\mathrm{X}_z = \sum_{i=1}^{r} w_i(z)\mathrm{X}_i = \sum_{i=1}^{r} w_i(z_k)\mathrm{X}_i$$

$$\mathrm{X}_{z+} = \sum_{i=1}^{r} w_i(z_+)\mathrm{X}_i = \sum_{i=1}^{r} w_i(z_{k+1})\mathrm{X}_i$$

$$\mathrm{X}_{zz} = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z) w_j(z)\mathrm{X}_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z_k) w_j(z_k)\mathrm{X}_{ij}$$

$$\mathrm{X}_{zz+} = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z_+) w_j(z_+)\mathrm{X}_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z_{k+1}) w_j(z_{k+1})\mathrm{X}_{ij}$$

thus the extended-fuzzy Lyapunov function can be simply described as:

$$V(x(k), z_k) = x(k)^T S_{zz}^{-1} x(k) \tag{4}$$

Then we have:

$$\Delta V(x(k), z_k) = x(k+1)^T S_{zz+}^{-1} x(k+1) - x(k)^T S_{zz}^{-1} x(k) \tag{5}$$

If the Lyapunov function (4) satisfies $\Delta V < 0$ for all time, the system can be guaranteed to be stable in the sense of Lyapunov.

III. Stable MPC Based on TS Fuzzy Model

The closed-loop system stability as well as a satisfactory control performance is more desirable for industrial processes such as fossil fired power plant, and MPC is an excellent strategy to achieve these goals.

The dynamic fuzzy model (1) can be rewritten in a predictive form as:

$$x(k+s+1\,|\,k) = A_z x(k+s\,|\,k) + B_z u(k+s\,|\,k) \tag{6}$$

Considering the infinite horizon objective function:

$$\begin{aligned} J_0^{\infty}(k) = \sum_{s=0}^{\infty} [&x(k+s\,|\,k)^T Q_0 x(k+s\,|\,k) + \\ &u(k+s\,|\,k)^T R_0 u(k+s\,|\,k)] \end{aligned} \tag{7}$$

where $Q_0 = Q_0^T > 0,\ R_0 = R_0^T > 0$ are respectively symmetric weighting matrices of states and control moves, we present the following main result.

Theorem 1: For the discrete TS fuzzy system (1) under input constraint: $|u_p(k+s\,|\,k)| \le u_{p,\max}$, $s \ge 0, p=1,2,...,m$, if there exist control variable $u(k\,|\,k)$, matrices $Y_i, G_i, Q_{ij}^{kk} = (Q_{ji}^{kk})^T\ \ (j>i)$, $Q_{ij}^{kl} + Q_{ij}^{lk} = (Q_{ji}^{kl})^T + (Q_{ji}^{lk})^T\ \ (j>i, l>k)$, $\Theta_{kl} = (\Theta_{lk})^T\ \ (l>k)$, $m_{ij} = (m_{ji})^T$

$(j>i)$, $c_{ij}=(c_{ji})^T$ $(j>i)$ and symmetric matrices $\tilde{S}_{ij}=\tilde{S}_{ji}$ $(j>i)$, $Q_{ii}^{kl}=Q_{ii}^{lk}$ $(l\geq k)$, Θ_{kk}, m_{ii}, c_{ii} such that the following convex optimization problem is feasible:

$$\min_{\gamma,u(k|k),Y_i,G_i,Q_{ij}^{kl},\Theta_{kl},m_{ij},c_{ij},\tilde{S}_{ij}} \gamma \qquad s.t.(9)-(22) \tag{8}$$

then, the control action $u(k|k)$ and non-PDC law $u(k+s|k)=Y_zG_z^{-1}x(k+s|k)$,$s>0$ minimize the upper bound of the objective function (7) while guaranteeing the stability of the closed-loop system.

$$r_{ii}^{kk}\geq Q_{ii}^{kk} \qquad i,k=1,2,...,r \tag{9}$$

$$r_{ij}^{kk}+r_{ji}^{kk}\geq Q_{ij}^{kk}+Q_{ji}^{kk} \quad j>i;\ i,j,k=1,2,...,r \tag{10}$$

$$r_{ii}^{kl}+r_{ii}^{lk}\geq Q_{ii}^{kl}+Q_{ii}^{lk} \quad l>k;\ i,k,l=1,2,...,r \tag{11}$$

$$r_{ij}^{kl}+r_{ji}^{kl}+r_{ij}^{lk}+r_{ji}^{lk}\geq Q_{ij}^{kl}+Q_{ji}^{kl}+Q_{ij}^{lk}+Q_{ji}^{lk} \quad j>i,\ l>k;\ i,j,k,l=1,2,...,r \tag{12}$$

$$\Psi=\begin{bmatrix}\Theta_{11} & \Theta_{12} & \cdots & \Theta_{1r}\\ \Theta_{21} & \Theta_{22} & \vdots & \vdots\\ \vdots & \vdots & \ddots & \Theta_{(r-1)r}\\ \Theta_{r1} & \cdots & \Theta_{r(r-1)} & \Theta_{rr}\end{bmatrix}>0 \tag{13}$$

$$\Phi^{kk}\geq 0 \qquad k=1,2,...,r \tag{14}$$

$$\Phi^{kl}+\Phi^{lk}\geq 0 \qquad l>k;\ k,l=1,2,...,r \tag{15}$$

$$n_{ii}\geq m_{ii} \qquad i=1,2...r \tag{16}$$

$$n_{ij}+n_{ji}\geq m_{ij}+m_{ji} \qquad i,j=1,2,...,r \tag{17}$$

$$\mathrm{M}=\begin{bmatrix}m_{11} & m_{12} & \cdots & m_{1r}\\ m_{21} & m_{22} & \vdots & \vdots\\ \vdots & \vdots & \ddots & m_{(r-1)r}\\ m_{r1} & \cdots & m_{r(r-1)} & m_{rr}\end{bmatrix}>0 \tag{18}$$

$$u_p(k|k)\leq u_{p,\max} \quad p=1,2,...,m \tag{19}$$

$$o_{ii}\geq c_{ii} \qquad i=1,2...r \tag{20}$$

$$o_{ij}+o_{ji}\geq c_{ij}+c_{ji} \qquad i,j=1,2,...,r \tag{21}$$

$$\Gamma=\begin{bmatrix}c_{11} & c_{12} & \cdots & c_{1r}\\ c_{21} & c_{22} & \vdots & \vdots\\ \vdots & \vdots & \ddots & c_{(r-1)r}\\ c_{r1} & \cdots & c_{r(r-1)} & c_{rr}\end{bmatrix}>0 \tag{22}$$

where

$$r_{ij}^{kl}=\begin{bmatrix}G_i+G_j^T-\tilde{S}_{ij} & * & * & *\\ (A_iG_j+B_iY_j) & \tilde{S}_{kl} & 0 & 0\\ G_j & 0 & \gamma Q_0^{-1} & 0\\ Y_j & 0 & 0 & \gamma R_0^{-1}\end{bmatrix} \quad i,j,k,l=1,2,...,r \tag{23}$$

$$\Phi^{kl}=\begin{bmatrix}Q_{11}^{kl}-\Theta_{kl} & Q_{12}^{kl}-\Theta_{kl} & \cdots & Q_{1r}^{kl}-\Theta_{kl}\\ Q_{21}^{kl}-\Theta_{kl} & Q_{22}^{kl}-\Theta_{kl} & \vdots & \vdots\\ \vdots & \vdots & \ddots & Q_{(r-1)r}^{kl}-\Theta_{kl}\\ Q_{r1}^{kl}-\Theta_{kl} & \cdots & Q_{r(r-1)}^{kl}-\Theta_{kl} & Q_{rr}^{kl}-\Theta_{kl}\end{bmatrix} \quad k,l=1,2,...,r \tag{24}$$

$$n_{ij}=\begin{bmatrix}1 & * & * & *\\ A_z(k)x(k|k)+B_z(k)u(k|k) & \tilde{S}_{ij} & 0 & 0\\ Q_0^{1/2}x(k|k) & 0 & \gamma I & 0\\ R_0^{1/2}u(k|k) & 0 & 0 & \gamma I\end{bmatrix} \quad i,j=1,2...r \tag{25}$$

$$o_{ij}=\begin{bmatrix}U & Y_j\\ Y_j^T & G_i+G_j^T-\tilde{S}_{ij}\end{bmatrix} \quad i,j=1,2,...,r$$

$$U_{pp}\leq u_{p,\max}^2 \quad ,p=1,2,...,m \tag{26}$$

where U_{pp} is the diagonal elements of the matrix U.

"*" in a matrix stands for the corresponding terms of a symmetric matrix.

Proof:

Part 1(minimizing the upper bound of infinite horizon objective function): For the infinite horizon objective function (7), divide it into two parts [6]:

$$J_0^{\infty}(k)=x(k|k)^TQ_0x(k|k)+u(k|k)^TR_0u(k|k)+J_1^{\infty}(k) \tag{27}$$

Suppose the extended FLF satisfies:

$$V(x(k+s+1|k))-V(x(k+s|k))\leq-[x(k+s|k)^T\times Q_0x(k+s|k)+u(k+s|k)^TR_0u(k+s|k)] \tag{28}$$

Summing (28) from $s=1$ to $s=\infty$, and with $x(\infty|k)=0$ and $V(x(\infty|k))=0$, we get:

$$J_1^{\infty}(k)\leq V(x(k+1|k))=x(k+1|k)^TS_{zz}^{-1}x(k+1|k) \tag{29}$$

Thus we can get the upper bound of $J_0^{\infty}(k)$:

$$J_0^{\infty}(k)\leq x(k|k)^TQ_0x(k|k)+u(k|k)^TR_0u(k|k)+x(k+1|k)^TS_{zz}^{-1}x(k+1|k) \tag{30}$$

We also divide the control input into two parts: $U_0^{\infty}=[u(k|k),U_1^{\infty}]$, where the first computed move $u(k|k)$ is defined as a free input variable and implemented on the plant, while U_1^{∞} are given through the non-PDC law [15],[10]:

$$u(k+s|k)=Y_zG_z^{-1}x(k+s|k),\quad s>0 \tag{31}$$

At sampling time k, since the current premise variables z_k are assumed to be available, the current model $\{A_z(k), B_z(k)\}$ can be obtained and $x(k+1|k)$ can be predicted exactly as:

$$x(k+1|k) = A_z(k)x(k|k) + B_z(k)u(k|k) \quad (32)$$

Substituting(32) into(30), we can obtain

$$J_0^\infty(k) \le x(k|k)^T Q_0 x(k|k) + u(k|k)^T R_0 u(k|k) + [A_z(k)x(k|k) + B_z(k)u(k|k)]^T S_{zz}^{-1} \times [A_z(k)x(k|k) + B_z(k)u(k|k)] \quad (33)$$

Define a scalar γ and suppose

$$x(k|k)^T Q_0 x(k|k) + u(k|k)^T R_0 u(k|k) + [A_z(k)x(k|k) + B_z(k)u(k|k)]^T S_{zz}^{-1}[A_z(k)x(k|k) + B_z(k)u(k|k)] \le \gamma \quad (34)$$

Then minimizing the upper bound of $J_0^\infty(k)$ is equivalent to the minimization of γ, subject to (34).

By defining

$$\tilde{S}_{zz}^{-1} = \frac{S_{zz}^{-1}}{\gamma} \quad (35)$$

and using Schur complements [5],(34) can be expressed as:

$$\begin{bmatrix} 1 & * & * & * \\ A_z(k)x(k|k) + B_z(k)u(k|k) & \tilde{S}_{zz} & 0 & 0 \\ Q_0^{1/2}x(k|k) & 0 & \gamma I & 0 \\ R_0^{1/2}u(k|k) & 0 & 0 & \gamma I \end{bmatrix} \ge 0 \quad (36)$$

which is equivalent to(37) according to (25),

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z)w_j(z)n_{ij} \ge 0 \quad i,j = 1,2...r \quad (37)$$

Now we use slack matrices and collection matrices to relax the sufficient conditions for (37).

By applying (16), (17) and (18) , the left side of (37)

$$\begin{aligned} &= \sum_{i=1}^{r} w_i^2(z)n_{ii} + \sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(n_{ij} + n_{ji}) \ge \\ &\sum_{i=1}^{r} w_i^2(z)m_{ii} + \sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(m_{ij} + m_{ji}) \\ &= \begin{bmatrix} w_1(z)I & w_2(z)I & \cdots & w_r(z)I \end{bmatrix} \\ &\quad \times M \begin{bmatrix} w_1(z)I & w_2(z)I & \cdots & w_r(z)I \end{bmatrix}^T > 0 \end{aligned} \quad (38)$$

Therefore, (37) holds.

Part 2(stability constraint): With the non-PDC law:

$$u(k+s|k) = Y_z G_z^{-1} x(k+s|k), s > 0 \quad (39)$$

the predictive closed-loop fuzzy system can be described as:

$$\begin{aligned} x(k+s+1|k) &= A_z x(k+s|k) + B_z Y_z G_z^{-1} x(k+s|k) \\ &= (A_z + B_z Y_z G_z^{-1}) x(k+s|k) \end{aligned} \quad (40)$$

Substituting (35), (39) , (40) into (28) and noticing that $(G_z - \tilde{S}_{zz})^T \tilde{S}_{zz}^{-1}(G_z - \tilde{S}_{zz}) > 0 \Rightarrow G_z^T \tilde{S}_{zz}^{-1} G_z \ge G_z^T + G_z - \tilde{S}_{zz}$, the stability constraint (28) is satisfied if:

$$\begin{aligned} &G_z^T + G_z - \tilde{S}_{zz} - (A_z G_z + B_z Y_z)^T \tilde{S}_{zz+}^{-1}(A_z G_z + B_z Y_z) - \\ &G_z^T \frac{Q_0}{\gamma} G_z - Y_z^T \frac{R_0}{\gamma} Y_z > 0 \end{aligned} \quad (41)$$

(41) can be expressed by the LMI below:

$$\begin{bmatrix} G_z + G_z^T - \tilde{S}_{zz} & * & * & * \\ (A_z G_z + B_z Y_z) & \tilde{S}_{zz+} & 0 & 0 \\ G_z & 0 & \gamma Q_0^{-1} & 0 \\ Y_z & 0 & 0 & \gamma R_0^{-1} \end{bmatrix} > 0 \quad (42)$$

which is equivalent to (43) according to (23):

$$\sum_{k=1}^{r}\sum_{l=1}^{r} w_k(z_+)w_l(z_+)\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(z)w_j(z)r_{ij}^{kl} > 0 \quad i,j,k,l = 1,2,...,r \quad (43)$$

Next, combined with the approach in [13], we use slack matrices and collection matrices to obtain less conservative sufficient condition for (43).

Applying (9)-(12), yields:
The left side of (43)

$$\begin{aligned} &\ge \sum_{k=1}^{r} w_k^2(z_+)[\sum_{i=1}^{r} w_i^2(z)Q_{ii}^{kk} + \sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(Q_{ij}^{kk} + Q_{ji}^{kk})] + \\ &\sum_{k=1}^{r}\sum_{l>k}^{r} w_k(z_+)w_l(z_+)[\sum_{i=1}^{r} w_i^2(z)(Q_{ii}^{kl} + Q_{ii}^{lk}) + \\ &\quad \sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(Q_{ij}^{kl} + Q_{ji}^{kl} + Q_{ij}^{lk} + Q_{ji}^{lk})] \end{aligned} \quad (44)$$

Let

$$\sum_{i=1}^{r} w_i^2(z)Q_{ii}^{kk} + \sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(Q_{ij}^{kk} + Q_{ji}^{kk}) \ge \Theta_{kk} \quad (45)$$

$$\begin{aligned} &\sum_{i=1}^{r} w_i^2(z)(Q_{ii}^{kl} + Q_{ii}^{lk}) + \\ &\sum_{i=1}^{r}\sum_{j>i}^{r} w_i(z)w_j(z)(Q_{ij}^{kl} + Q_{ji}^{kl} + Q_{ij}^{lk} + Q_{ji}^{lk}) \ge \Theta_{kl} + \Theta_{lk} \end{aligned} \quad (46)$$

and apply (13), we have:
The left side of (43)

$$\begin{aligned} &\ge \sum_{k=1}^{r} w_k^2(z_+)\Theta_{kk} + \sum_{k=1}^{r}\sum_{l>k}^{r} w_k(z_+)w_l(z_+)[\Theta_{kl} + \Theta_{lk}] \\ &= \begin{bmatrix} w_1(z_+)I & w_2(z_+)I & \cdots & w_r(z_+)I \end{bmatrix} \end{aligned}$$

$$\times \Psi\left[w_1(z_+)I \quad w_2(z_+)I \quad \cdots \quad w_r(z_+)I\right]^T > 0 \qquad (47)$$

Therefore, (43) holds.

Similarly, by applying (14), (15) and (24), we can show that (45) and (46) hold.

Part 3(input constraint): Since we split the input into free variables and future control moves determined by non-PDC law, we must constrain them accordingly.

For the free variables, we directly constrain them by the peak bound (19). For the future input moves, by using the approach in [5] and considering $(G_z^T \tilde{S}_{zz}^{-1} G_z)^{-1} \leq (G_z^T + G_z - \tilde{S}_{zz})^{-1}$, the constraint is satisfied if there exists a matrix U, such that the following LMIs are feasible:

$$\begin{bmatrix} U & Y_z \\ Y_z^T & G_z + G_z^T - \tilde{S}_{zz} \end{bmatrix} \geq 0 \,,\ U_{pp} \leq u_{p,\max}^2 \ ,\ p = 1,2...m \quad (48)$$

Similarly using the slack and collection matrices, we can show that (20)-(22) guarantee (48). Therefore, the input constraint is achieved.

Remark 1: We apply an extended-FLF to improve the stability condition of MPC. Note that if we impose $S_{i1} = S_{i2} = ... = S_{ir} = S_i$, the extended-FLF reduces to FLF; and if all Lyapunov matrices S_{ij} are imposed to be the same, the extended-FLF reduces to CLF. Therefore the extended-FLF leads to less conservatism in stability analysis and stabilization design.

Remark 2: The technique of slack and collection matrices is used in all LMIs, including minimization of the upper bound of infinite horizon objective function; stability constraint and input constraint. This technique can reduce the conservatism in that it collects the interactions among subsystems. Notice that, combined with the extended-FLF, we use the slack and collection matrices twice in stability constraint, thus less conservative result can be obtained. If all slack matrices are chosen to be zeros, the conventional result will be achieved. And since we use this technique in all LMIs, we can reduce the number of matrices to reduce the computational burden for some matrices appearing in the form of summations. Take $\tilde{S}_{ij}$ and $\tilde{S}_{ji}$, $(j > i)$ for example, since they always appear in the form of $\tilde{S}_{ij} + \tilde{S}_{ji}$,we can set $\tilde{S}_{ij} = \tilde{S}_{ji}$ $(j > i)$ to reduce the number of matrices.

Remark 3: Compared with the conventional fuzzy control, the proposed approach has the advantage that it can achieve stability in an optimal way; moreover, (28) guarantees the Lyapunov function of stable MPC to decrease faster than conventional fuzzy control ($\Delta V \leq 0$).

IV. Illustrative Example

In this section, an example is presented to show the advantage of the proposed stable MPC.

Consider the following discrete nonlinear model:

$$\begin{aligned} x_1(k+1) &= x_1(k) - x_1(k)x_2(k) + (5 + x_1(k))u(k) \\ x_2(k+1) &= -x_1(k) - 0.5x_2(k) + 2x_1(k)u(k) \end{aligned} \qquad (49)$$

Define x_1 as the premise variable, then (49) can be described by the TS fuzzy model:

$$R^i : IF\ x_1(\mathrm{k})\ \ is\ \ \mathrm{M}^i$$
$$THEN \quad x(k+1) = A_i x(k) + B_i u(k) \qquad i = 1,2$$

with membership functions:

$$\mathrm{M}^1(k) = \frac{1.65 + x_1(k)}{3.3} \ ; \ \ \mathrm{M}^2(k) = \frac{1.65 - x_1(k)}{3.3}$$

The system matrices are given by:

$$A_1 = \begin{bmatrix} 1 & -1.65 \\ -1 & -0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 6.65 \\ 3.3 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1 & 1.65 \\ -1 & -0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 3.35 \\ -3.3 \end{bmatrix}$$

We apply the approach in Theorem1 and approaches in [6], [9] and [16] to this model with the initial state $x(0)=\begin{bmatrix} 1.65 & -5 \end{bmatrix}^T$; controller parameters $Q_0 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.95 \end{bmatrix}$, $R_0 = 0.9$ and input constraint $|u| \leq 1.3$.

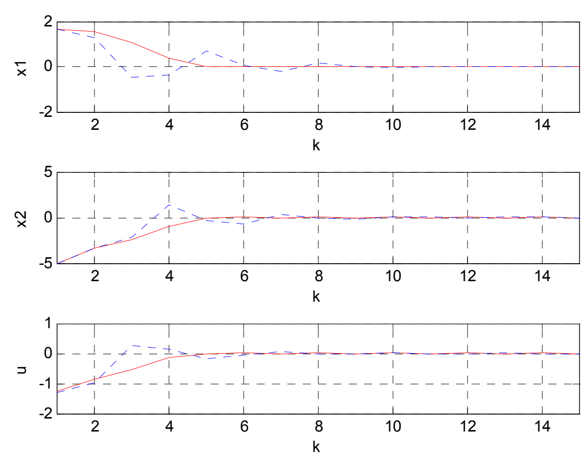

Fig.1. Closed-loop control performance for Theorem 1 and approach in [16]. (solid line: Theorem 1; dotted line: approach in [16])

Simulation results shows that there are no feasible solutions for the approach in [6] and [9] which are MPC based on CLF and FLF. While the other two approaches can find feasible solutions and their control results are shown in Fig.1. This clearly demonstrated the advantage of the extended-FLF. Approach in [16] is another MPC based on extended-FLF, however, since we use appropriate slack and collection matrices in all LMIs and take full advantage of extended-fuzzy Lyapunov function, the approach we propose has better performance.

V. Application to a Boiler-Turbine Coordinated System of Power Unit

In this section, the proposed controller is applied to the typical Bell-Åström boiler–turbine dynamic model [17]. The model is a 3rd order MIMO system which has the characteristics of highly nonlinearity.

The state variables in $x=[x_1,x_2,x_3]^T$ denote the drum pressure (kg/cm^2), the power output (MW) and the density of fluid in the system (kg/cm^3), respectively. The normalized input variables in $u=[u_1,u_2,u_3]^T$ denote the position of fuel flow valve, the position of steam control valve and the position of feedwater flow valve, respectively. All valve position variables are constrained to lie in the interval [0, 1]. The output variables in $y=[y_1,y_2,y_3]^T$ denote the drum pressure (kg/cm^2), the power output (MW) and the drum water level (m), respectively.

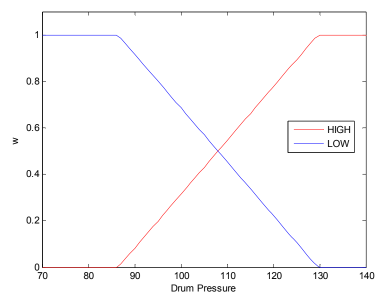

Fig. 2. Membership functions of the boiler-turbine fuzzy model.

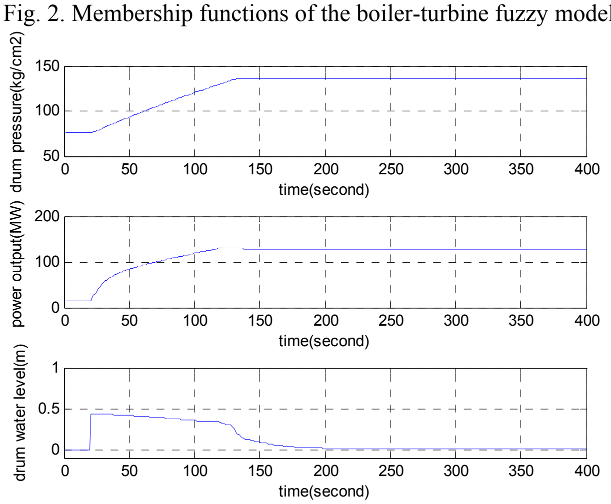

Fig. 3. Closed-loop control performance of the boiler-turbine coordinated system for theory 2 (output).

Linearizing the nonlinear system around heavy load operating point (120%) and low load operating point (80%) using Taylor's series approximation, then choosing the membership function as shown in Fig.2, we can easily obtain a two rules TS fuzzy model in form of (1) to represent the nonlinear BA model. Owing to space limitations, we shall omit the system matrices here.

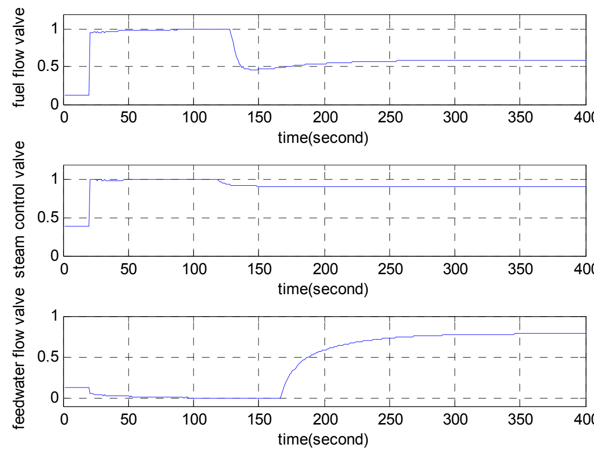

Fig. 4. Closed-loop control performance of the boiler-turbine coordinated system for theory 2 (input).

We now apply the proposed controller to the boiler-turbine coordinated system and consider the case of load change in a wide range. The control mission is tracking the expected operating points of drum pressure and output power while maintaining the drum water level. We assume that at t=20s, the desired operating point changes in step from 70% to 130%, and the setpoint of drum water lever maintains at zero. With controller parameters:

$$Q_0=\begin{bmatrix}1 & 0 & 0\\ 0 & 50 & 0\\ 0 & 0 & 1\end{bmatrix}; R_0=\begin{bmatrix}2 & 0 & 0\\ 0 & 100 & 0\\ 0 & 0 & 4\end{bmatrix}\times 10^3$$

the simulation results are shown in Fig. 3 and Fig. 4.

From the control results, we can see that when the load is increased, the drum pressure and power output respond rapidly, and then approaches to the expected operating points, while the drum water level jumps and then gradually return to zero after a period of fluctuation. The proposed fuzzy stable MPC can control the boiler-turbine coordinated system effectively.

VI. Conclusion

The combination of model predictive control and TS fuzzy model is an effective way to solve the control problem of nonlinear system, which motivates us to propose a stable fuzzy MPC based on extended-FLF. By solving a set of LMIs, both the stability of the closed-loop system and the satisfaction of input constraint can be achieved in an optimal way. The extended-FLF reduces the conservatism of CLF and FLF; and enlarges the feasible area of the predictive controller. Moreover, combined with some new results in fuzzy control, we use appropriate slack and collection matrices with extended-FLF to further reduce the conservatism. The simulations on numerical example and nonlinear boiler-turbine coordinated system of power plant demonstrate the advantage and effectiveness of the proposed approach.

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