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Modified Pseudo-Inverse Method with Generalized Linear Quadratic Regulator for Fault Tolerant Model Matching with Prescribed Stability Degree

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Abstract— **When faults turn the nominal operation to an unstable post-fault regime, the accommodation procedure must recover stability and as much performance of the reference model as possible, ideally attaining perfect Model Matching (MM). In this paper, the authors develop the Modified Pseudo-Inverse Method (MPIM) with the Generalized Linear Quadratic Regulator (GLQR) stabilization using the weighting matrices derived from the Robust Optimal Model Matching (ROMM) approach and show that the MPIM-GLQR technique is the best candidate to solve the MM problem extended with the prescribed stability degree property, as opposed to the classical MPIM with simple LQR stabilization using arbitrary penalties, as was proposed in the original method. The theoretical development presented in the paper is illustrated by an aeronautical example, namely the longitudinal motion control of the B747 aircraft impaired by a structural and an actuator fault.**

I. INTRODUCTION

Fault Tolerant Control (FTC) refers to the approach by which the controlled system is able to exhibit desired properties, e.g., Model Matching / Model Following (MM/MF), both in normal operation and in the presence of faults [1], [2], [3].

Model Matching assumes that the closed-loop matrix of the actual system is identical with the closed-loop one of the reference model, while in Model Following the identity should hold for the state trajectories, in both cases, the matching / following conditions being mathematically expressed by zeroing the corresponding matrix- / vector- norms of the error [4], [5]. Unfortunately, MM/MF stresses an idealistic objective in relation with the FTC design, because achieving exactly the same closed-loop performance as in the nominal case after the system impairment is most often impossible and approximate quality has to be accepted [6].

In this paper, the Modified Pseudo-Inverse Method (MPIM) [4] with the Generalized Linear Quadratic Regulator (GLQR) stabilization using the weighting matrices derived from the Robust Optimal Model Matching (ROMM) approach [7] is developed.

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The authors show that the MPIM-GLQR technique is the only pertinent approach to solving the MM problem extended with the Prescribed Degree of Stability (PDS) property, contrary to the classical MPIM with simple LQR stabilization using arbitrary penalties, as suggested in the original MPIM-LQR method.

Moreover, two fault accommodation schemes under the form of the MPIM-G/LQR algorithms are proposed - also, the authors improve at the theoretical level and organize better the previous version of the paper [8], and provide it with an application example from aeronautics for the numerical and graphical illustration of the advantage of using MPIM-GLQR to solving MM-PDS, contrary to the MPIM-LQR which fails.

The paper is organized as follows: Section II addresses the perfect MM problem with the Pseudo-Inverse Method (PIM) solution, recalling its instability drawback, while Section III tackles the approximate MM problem with the Modified Pseudo-Inverse Method (MPIM), recalling its inability to satisfy the PDS property and proposes the MPIM-GLQR solution. Also, an application example for the fault tolerant model matching of the B747 longitudinal motion control is provided in Section IV and the paper is concluded in Section V. In the end, Appendix I provides the reader with the particular solution given by the ROMM procedure.

II. PSEUDO-INVERSE MODEL MATCHING

A. Nominal Operation

Consider the continuous linear time-invariant (LTI) deterministic system whose nominal operation is modeled by

$$M_n : \dot{x}(t) = A_n x(t) + B_n u_n^*(t) \,, \text{ with } x(t_0 = 0) = x_0 \,, \quad (1)$$

where $A_n \in \mathbb{R}^{n \times n}$ and $B_n \in \mathbb{R}^{n \times m}$ are the nominal system and control matrices, $x(t) \in \mathbb{R}^n$ and $u_n^*(t) \in \mathbb{R}^m$ are the state and control vectors, and $x_0 \in \mathbb{R}^n$ is the initial condition.

In nominal operation, the linear state-feedback control law

$$u_n^*(t) = -K_n^* x(t) \,, \quad (2)$$

conducts to the closed-loop system

$$\hat{M}_n^* : \dot{x}(t) = \hat{A}_n^* x(t) \,, \text{ with } \hat{A}_n^* = A_n - B_n K_n^* \,, \quad (3)$$

which is stable and provides the ideal dynamic performance, where by $\alpha_n^* \triangleq |\lambda(\hat{A}_n^*)|$ the maximum absolute value of the real-part of the eigenvalues of $\hat{A}_n^*$ is denoted.

Model $\hat{M}_n^*$ is taken as the reference model whose dynamics is to be matched in any other operating conditions different from the ideal one, while α_n^* indicates the minimum distance imposed between the eigenvalues position to the imaginary axis in the complex plane, namely the Prescribed Degree of Stability (PDS) [9].

B. Post-Fault Operation

Whether parametric faults are considered, assuming that an LTI model is still able to capture its behavior, the system to be controlled is described by the pair (A_f, B_f) provided by an on-line Fault Detection, Isolation, and Identification / Estimation (FDIE) module [10].

That is, after the fault estimation time t_f, the post-fault system operation is modeled by

$$M_f \,:\, \dot{x}(t) = A_f x(t) + B_f u_f(t)\,. \tag{4}$$

In post-fault operation, the Exact Model Matching (EMM) technique with state-feedback aims at designing the linear control law

$$u_f(t) = -K_f x(t)\,, \tag{5}$$

such that the resulting closed-loop behavior matches that of the reference model from (3) (full state is assumed to be available and known after the fault - otherwise, state estimation must be performed before controller redesign).

The solution to the EMM problem is obtained by solving the matrix equation

$$\hat{A}_f \triangleq A_f - B_f K_f = \hat{A}_n^*\,, \tag{6}$$

whose necessary and sufficient condition for a solution to exist is expressed as (see [11])

$$\mathrm{Im}\left(A_f - \hat{A}_n^*\right) \subseteq \mathrm{Im}\left(B_f\right)\,, \tag{7}$$

and the solution is given by

$$(K_f^{\mathrm{PIM}} \triangleq)\, K_f = B_f^{\dagger}(A_f - \hat{A}_n^*)\,, \tag{8}$$

where $(.)^{\dagger}$ stands for the pseudo-inverse matrix.

C. Pseudo-Inverse Method

Regarding the pseudo-inverse control (8), condition (7) will obviously hold only for very particular faults and therefore no exact solution will exist in most fault cases; for this reason, approximate solutions rather than exact ones might be of interest.

When exact model matching is impossible $\left(\hat{A}_f \neq \hat{A}_n^*\right)$, an approximate control solution may be computed as $\tilde{K}_f = \arg\min_{K_f} J_f(K_f)$ by minimizing the criterion (see [4])

$$J_f(K_f) = \|\hat{E}_f\|_F\,, \tag{9}$$

with the impairment matrix

$$\hat{E}_f = \hat{A}_n^* - (A_f - B_f K_f)\; (\Leftrightarrow \hat{A}_f = \hat{A}_n^* - \hat{E}_f)\,, \tag{10}$$

where $\|.\|_F$ is the Frobenius matrix-norm.

Unfortunately, for certain fault cases, PIM encounters instability problems (see also [12]); besides that, using this approach, no action to make the prescribed degree of stability tangible can be taken.

III. Modified Pseudo-Inverse Model Matching

A. Modified Pseudo-Inverse Method without Stabilization and without Correction

The improved PIM, namely the Modified Pseudo-Inverse Method (MPIM), counteracts the above instability drawback, that is it restricts the computation of K_f such that to guarantee the stability of the post-fault system $\Lambda(\hat{A}_f) \subseteq \mathbb{C}^-$ while achieving as much of the closed-loop nominal performance as possible, i.e., $\min_{K_f} J_f(K_f)$, where $\Lambda(.)$ denotes the spectrum of the matrix-argument.

Unfortunately, there is no closed-form solution working for general multi-input multi-output (MIMO) systems but just for the single-input multi-output (SIMO) case, that is in the case when $m = 1$.

This is the situation that will be tackled in the paper - though $B_f \in \mathbb{R}^{n\times 1}$, it still is appropriate for the control of the longitudinal motion of an aircraft, classically done using just the elevator input, which will be the application example.

Therefore, in MPIM, the control gain K_f^{MPIM} (for $j = 1,2,\ldots,n$) computed as

$$k_{f_j}^{\mathrm{MPIM}} = k_{f_j}^{\mathrm{PIM}}\,,\ \text{if}\ |k_{f_j}^{\mathrm{PIM}}| \leq \tilde{\delta}_f^{\mathrm{PIM}}\,, \tag{11a}$$

$$k_{f_j}^{\mathrm{MPIM}} = \mathrm{sgn}\left(k_{f_j}^{\mathrm{PIM}}\right)\tilde{\delta}_f^{\mathrm{PIM}}\,,\ \text{otherwise}\,, \tag{11b}$$

solves the reconfigurable control problem with stability guarantees, where $\tilde{\delta}_f^{\mathrm{PIM}}$ represents the truncated robust stability-margin, computed as $\tilde{\delta}_f^{\mathrm{PIM}} = \delta_f^{\mathrm{PIM}} - \varepsilon_f^{\mathrm{PIM}}$, for some small $\varepsilon_f^{\mathrm{PIM}}$, and $\mathrm{sgn}(.)$ stands for the sign-function of the real-argument (for details on the computation of δ_f^{PIM}, and of $\delta_f^{\text{PIM-G/LQR}}$ used later in the next subsection, see below in this subsection).

This comes from the fact that if the post-fault closed-loop system designed with PIM, described as

$$\dot{x}(t) = \hat{A}_f^{\mathrm{PIM}} x(t) = (A_f - B_f K_f^{\mathrm{PIM}})x(t)\,,$$

is unstable, then the pseudo-inverse control must be redesigned with the modified-pseudo inverse method such that the MPIM closed-loop system, described as

$$\begin{aligned}\dot{x}(t) = \hat{A}_f^{\mathrm{MPIM}} x(t) &= (A_f - B_f K_f^{\mathrm{MPIM}})x(t)\\ &= (A_f - \delta_f^{\mathrm{PIM}} B_f K_f^{\mathrm{PIM}})x(t)\,,\end{aligned}$$

becomes stable, from where $K_f^{\mathrm{MPIM}} = f(K_f^{\mathrm{PIM}})$.

The idea is to see the closed-loop matrix in the post-fault case $\hat{A}_f^{\mathrm{MPIM}}$ as a perturbed matrix $\hat{A}_f^{\mathrm{MPIM}} = A_f - B_f K_f^{\mathrm{MPIM}} \triangleq A_f - E_f^{\mathrm{MPIM}}$, where the perturbation matrix, denoted by E_f^{MPIM}, is taken as $E_f^{\mathrm{MPIM}} \triangleq B_f K_f^{\mathrm{MPIM}} = B_f f(K_f^{\mathrm{PIM}})$, and assumed to be bounded by δ_f^{PIM}, from where it will follow that $K_f^{\mathrm{MPIM}} = \delta_f^{\mathrm{PIM}} K_f^{\mathrm{PIM}}$.

Thus, choosing the stability bound to be $\delta_f^{\mathrm{PIM}} = \delta_{f_Y}$ [13], the right-hand side of (12),

$$|k_{f_j}| < \frac{1}{\sup_{\omega\geq 0}\rho\left((j\omega I_n - A_f)^{-1}E_f\right)} \triangleq \delta_{f_Y}\,,\ \text{for}\ j = 1,2,\ldots,n\,, \tag{12}$$

which involves the evaluation of the resolvent-matrix at the limit but the bound is less restrictive than those proposed in other results [14] (see also [15]), where $\rho(X)$ denotes the spectral-radius of the matrix-argument, precisely the magnitude of the largest eigenvalue of X, condition

$$|k_{f_j}^{\text{MPIM}}| \leq \tilde{\delta}_f^{\text{PIM}}\,,\ \text{for}\ j = 1,2,\ldots,n\,, \tag{13}$$

guarantees the stability of $\hat{A}_f$ in the impaired operation.

B. Modified Pseudo-Inverse Method with Stabilization and with Correction

Unfortunately, one of the deficiencies of MPIM is that the robust stability margin cannot be always used but in the case in which matrix A_f remains stable after the fault, i.e., $\Lambda(A_f) \subseteq \mathbb{C}^-$, otherwise the system must first be stabilized using another method.

1) Fault Accommodation Control with LQR Stabilization and MPIM Correction: Hence, for certain instability cases, in the classical paper on MPIM [4], it was suggested to use first an efficient method to get the stabilizing gain, e.g., through solving the corresponding Continuous Algebraic Riccati Equation (CARE) associated with the post-fault pair (A_f, B_f), that is

$$\mathit{CARE}_f : \tag{14}$$
$$A_f^T X_f^{\text{CARE}} + X_f^{\text{CARE}} A_f - X_f^{\text{CARE}} B_f R_f^{-1} B_f^T X_f^{\text{CARE}} + Q_f = 0\,,$$

where the solution to CARE_f is computed as

$$K_f^{\text{CARE}} = R_f^{-1} B_f^T X_f^{\text{CARE}}\,, \tag{15}$$

and then to make a control correction as

$$K_f^{\text{MPIM-LQR}} = K_f^{\text{CARE}} + \tilde{\delta}_f^{\text{PIM-LQR}} \Delta K_f^{\text{PIM-LQR}}\,, \tag{16}$$

where the control accommodation difference $\Delta K_f^{\text{PIM-LQR}}$ is obtained as

$$\begin{aligned}\Delta K_f^{\text{PIM-LQR}} &= B_f^{\dagger}\big(\hat{A}_f^{\text{LQR}} - \hat{A}_n^*\big)\\ &= B_f^{\dagger}\big((A_f - B_f K_f^{\text{CARE}}) - \hat{A}_n^*\big)\,;\end{aligned} \tag{17}$$

as no restrictions are indicated in that paper, see that the penalty matrices $Q_f^T = Q_f = \bar{Q}_f^T \bar{Q}_f \geq 0$ and $R_f^T = R_f > 0$ might be chosen as in (32a)-(32b).

For this situation, the fault accommodation control redesign scheme with simple LQR stabilization and MPIM correction becomes a two step procedure as follows [4].

Algorithm 1: MPIM-LQR Scheme

(1-1) Stabilize the post-fault system using the classical LQR procedure of (14) with the weighting matrices Q_f, R_f as in (32a)-(32b), provide K_f^{CARE} from (15).

(1-2) Make the appropriate control correction using the difference matrix $\Delta K_f^{\text{PIM-LQR}}$ as in (17), provide $K_f^{\text{MPIM-LQR}}$ from (16).

However, again, look that there is no touch onto the imposed degree of stability using this approach.

2) Fault Accommodation Control with GLQR Stabilization and MPIM Correction: Contrary to the simple LQR stabilization from the original approach and with direct regard on the model matching problem with prescribed stability degree, in this paper, the authors propose to obtain the initial stabilizing post-fault control matrix through solving the Generalized Continuous Algebraic Riccati Equation (GCARE) (similar to (30)) associated with the faulty pair (A_f, B_f), that is

$$\mathit{GCARE}_f : \tag{18}$$
$$\tilde{A}_f^T X_f^{\text{GCARE}} + X_f^{\text{GCARE}} \tilde{A}_f - X_f^{\text{GCARE}} B_f R_f^{-1} B_f^T X_f^{\text{GCARE}} + \tilde{Q}_f = 0\,,$$

where the solution to GCARE_f is computed as

$$K_f^{\text{GCARE}} = R_f^{-1}(B_f^T X_f^{\text{GCARE}} + S_f)\,, \tag{19}$$

and then to make a control correction as

$$K_f^{\text{MPIM-GLQR}} = K_f^{\text{GCARE}} + \tilde{\delta}_f^{\text{PIM-GLQR}} \Delta K_f^{\text{PIM-GLQR}}\,, \tag{20}$$

where the control accommodation difference $\Delta K_f^{\text{PIM-GLQR}}$ is obtained as

$$\begin{aligned}\Delta K_f^{\text{PIM-GLQR}} &= B_f^{\dagger}\big(\hat{A}_f^{\text{GLQR}} - \hat{A}_n^*\big)\\ &= B_f^{\dagger}\big((A_f - B_f K_f^{\text{GCARE}}) - \hat{A}_n^*\big)\,,\end{aligned} \tag{21}$$

again after an appropriate choice of the penalty matrices Q_f, R_f, S_f as in (32) (for details on the computation of Q_f, R_f, S_f, see Appendix I, Review on Robust Optimal Model Matching).

For this situation, the fault accommodation control redesign scheme with generalized GLQR stabilization and MPIM correction becomes a two step procedure as follows.

Algorithm 2: MPIM-GLQR Scheme

(2-1) Stabilize the post-fault system using the generalized GLQR procedure of (18) with the weighting matrices Q_f, R_f, S_f as in (32), provide K_f^{GCARE} from (19).

(2-2) Make the appropriate control correction using the difference matrix $\Delta K_f^{\text{PIM-GLQR}}$ as in (21), provide $K_f^{\text{MPIM-GLQR}}$ from (20).

From Algorithms 1 and 2, it appears clear now that only following the steps of the second one, namely MPIM-GLQR, the model matching problem with the imposition of prescribed stability degree is solved, as this property is inherently assured by the GLQR stabilization and it is not affected by the MPIM correction - this is also to be proven numerically in the case study for the impaired model of the aircraft longitudinal motion.

C. Interpretation of MPIM-G/LQR Control Actions and Future Research

The authors have showed in [8] that, after making use of the weighting matrices Q_f, R_f from (32a)-(32b) for the simple LQR stabilization respectively of Q_f, R_f, S_f from (32) for the generalized GLQR stabilization, the PIM-LQR and the PIM-GLQR control accommodation matrices $\Delta K_f^{\text{PIM-LQR}}$ and $\Delta K_f^{\text{PIM-GLQR}}$ from (17) and (21) get special forms.

Following that development, the corresponding control matrices become

$$K_f^{\text{MPIM-LQR}} = \tilde{\delta}_f^{\text{PIM-LQR}} K_f^{\text{PIM}} + (1-\tilde{\delta}_f^{\text{PIM-LQR}}) K_f^{\text{CARE}}, \quad (22)$$

$$K_f^{\text{MPIM-GLQR}} = K_f^{\text{PIM}} + (1-\tilde{\delta}_f^{\text{PIM-GLQR}}) K_f^{\text{CARE}}, \quad (23)$$

where the MPIM-LQR control provides to have somehow a bi-objective form, and remark the clear difference between the two which is proved through the fact that in the MPIM-GLQR control the PIM component is penalized no more.

Moreover, see that if the stability-bounds $\tilde{\delta}_f^{\text{PIM-G/LQR}}$ are much smaller than unity, their influence becomes negligible, such that (22) and (23) get the approximate forms

$$K_f^{\text{MPIM-LQR}} \simeq K_f^{\text{CARE}}, \quad (24)$$

$$K_f^{\text{MPIM-GLQR}} \simeq K_f^{\text{PIM}} + K_f^{\text{CARE}}; \quad (25)$$

in any case, having $\tilde{\delta}_f^{\text{PIM-G/LQR}} \ll 1$ is not always the case.

Also, if the fault is severe and the ideal performance imposed through $\hat{A}_n^*$ is too demanding, the control effort spent for stabilization will conduct to such K_f^{CARE} that might be several orders of magnitude bigger than K_f^{PIM} in norm, which will imply in (25) that the PIM correction becomes insignificant regarding the GLQR stabilization, that is

$$K_f^{\text{MPIM-GLQR}} \approx K_f^{\text{MPIM-LQR}}; \quad (26)$$

of course, this is a too qualitative analysis and further investigation on the magnitude of the corresponding Frobenius-norms of both control accommodation matrices must be done.

In the sequel, the design of a controller for the longitudinal model of the B747 civil aircraft is considered, with the aim of proving the advantages of using the modified pseudo-inverse method with generalized linear quadratic stabilization and post-fault penalty matrices provided by the robust optimal model matching approach for better fault tolerant model matching performance with prescribed stability degree over the method with simple linear quadratic stabilization, after a structural and an actuator fault.

IV. Application of MPIM-G/LQR Controls for the B747 Aircraft Longitudinal Model

A. Flight Condition

Consider an aircraft Boeing 747 in gradual maneuvering without precision tracking (Class I cruise flight, Category B flight phase) and assume a straight and constant level flight at 40000 ft fixed altitude and 0.8 Mach - the corresponding steady-state speed is 774 fps; the aircraft mass is 19792 *slug*.

B. Nominal System

1) Open-Loop Model: The aircraft longitudinal dynamics is described by the state vector $x = \begin{bmatrix} u & w & q & \theta \end{bmatrix}^T$, where u (ft/sec) and w (ft/sec) represent the inertial velocities in the x- and z- directions of F_B reference frame; (F_B denotes body-axis reference); also, q (rad/sec) and θ (rad) represent respectively the pitch rate and the pitch angle. The control input δ_E (rad) is the elevator deflection.

The linearized model of the system is given by the nominal pair of system and control matrices (A_n, B_n) as (see [16], for the aircraft longitudinal model)

$$A_n = \begin{bmatrix} -0.0069 & 0.0139 & 0 & -32.2000 \\ -0.0904 & -0.3147 & 773.9766 & 0 \\ 0.0001 & -0.0010 & -0.4284 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix},$$

$$B_n = \begin{bmatrix} -0.0002 & -18.0610 & -1.1577 & 0 \end{bmatrix}^T.$$

2) Closed-Loop Reference Model: Consider the reference model $\hat{A}_n^* (\equiv \hat{A}_n)$ as

$$\hat{A}_n^* = \begin{bmatrix} -0.0069 & 0.0139 & -0.0006 & -32.2006 \\ 0.0469 & -0.3996 & 720.6443 & -54.2101 \\ 0.0089 & -0.0064 & -3.8470 & -3.4748 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix},$$

computed with the full state-feedback control gain

$$K_n^* = \begin{bmatrix} 0.0076 & -0.0047 & -2.9529 & -3.0015 \end{bmatrix},$$

which gives the stable spectrum

$$\Lambda(\hat{A}_n^*) = \{-0.0685 \pm 0.0899\imath, -2.0582 \pm 2.2066\imath\},$$

for the nominal closed-loop matrix $\hat{A}_n^* = A_n - B_n K_n^*$.

The reference model $\hat{A}_n^*$ provides ideal dynamic performance for the natural modes of the longitudinal motion of the aircraft, precisely for the short-period (SP) mode eigenvalues $\lambda_{1,2}^{SP} = -2.0582 \pm 2.2066\imath$ this provides the pair of natural undamped frequency (rad/sec) and damping coefficient $(\omega^{SP}, \zeta^{SP}) \approx (3, 0.6)$ and for the phugoid (PH) mode eigenvalues $\lambda_{1,2}^{PH} = -0.0685 \pm 0.0899\imath$ this gives the pair $(\omega^{PH}, \zeta^{PH}) \approx (0.1, 0.6)$ [17].

C. Impaired System

1) Post-Fault Model: Consider an actuator fault defined by the loss-of-effectiveness with 50% in the elevator deflection; that can be modeled in the post-fault system by multiplying the control matrix with a factor $(1-\tau_f) = 0.5$, for $\tau_f = 0.5$, i.e., $B_f' = (1-\tau_f)B_n$. Also, assume a structural fault modeled in the system matrix as $A_f' = A_n + \Delta A_f \times I_4$, with $\Delta A_f = \sigma_n^{\max} \times 10^{-3}$, where $\sigma_n^{\max} = 773.9774$ is the maximum singular value of the nominal matrix A_n. Furthermore, consider also that the actual post-fault matrices provided by the FDIE module are $A_f = (1-\gamma_f)A_f'$ and $B_f = (1-\gamma_f)B_f'$, with $\gamma_f = 0.1$, thus denoting a 10% structured identification error (see [18], for the aircraft fault models).

2) Pseudo-Inverse Solution: The Pseudo-Inverse Method (PIM) provides the control gain

$$K_f^{\text{PIM}} = \begin{bmatrix} 0.0158 & -0.0997 & 2.9162 & -6.6700 \end{bmatrix},$$

which is unacceptable, as it gives an unstable spectrum

$$\Lambda(\hat{A}_f^{\text{PIM}}) = \{0.6811 \pm 0.0655\imath, 0.7292 \pm 6.3188\imath\},$$

for the closed-loop matrix $\hat{A}_f^{\text{PIM}} = A_f - B_f K_f^{\text{PIM}}$.

3) Modified Pseudo-Inverse Solution: The Modified Pseudo-Inverse Method (MPIM) provides the control gain

$$K_f^{\text{MPIM}} = \left[\begin{array}{cccc} 0.0158 & -0.0195 & 0.0195 & -0.0195 \end{array}\right],$$

which still is unacceptable, as it is not able to provide a stable spectrum

$$\Lambda(\hat{A}_f^{\text{MPIM}}) = \{0.2966 \pm 2.7770\imath, 0.6847 \pm 0.1016\imath\},$$

for the closed-loop matrix $\hat{A}_f^{\text{MPIM}} = A_f - B_f K_f^{\text{MPIM}}$; this is due to the fact that matrix A_f is unstable, with the spectrum

$$\Lambda(A_f) = \{0.3619 \pm 0.7887\imath, 0.6937 \pm 0.0592\imath\}.$$

D. Accommodated System

1) Post-Fault Weighting Matrices: The penalty matrices derived as in the Robust Optimal Model Matching procedure, described in Appendix I, are identified as

$$Q_f = \left[\begin{array}{cccc} 0.0000 & -0.0000 & 0.0004 & -0.0001 \\ -0.0000 & 0.0000 & -0.0031 & 0.0007 \\ 0.0004 & -0.0031 & 5.1994 & -0.4034 \\ -0.0001 & 0.0007 & -0.4034 & 0.0695 \end{array}\right] \times 10^5, \quad (27a)$$

$$R_f = 66.3268, \quad (27b)$$

$$S_f = \left[\begin{array}{cccc} 1.0467 & -6.6100 & 193.4236 & -442.4004 \end{array}\right]^T, \quad (27c)$$

and the minimal decay rate $\alpha_f = 2.0$ is chosen according to its definition, precisely a little bit smaller than $\alpha_n^* = 2.05$.

2) MPIM-LQR Solution: The simple LQR stabilization, using just the two penalties Q_f, R_f from (27a)-(27b), with MPIM correction, the MPIM-LQR technique, provides the control gain

$$K_f^{\text{MPIM-LQR}} = \left[\begin{array}{cccc} 46.1534 & -1.4070 & -73.4641 & 636.2251 \end{array}\right],$$

which still is not acceptable, as it provides the stable spectrum

$$\Lambda(\hat{A}_f^{\text{MPIM-LQR}}) = \{-1.4017, -44.9486, -0.6211 \pm 0.1858\imath\},$$

for the closed-loop matrix $\hat{A}_f^{\text{MPIM-LQR}} = A_f - B_f K_f^{\text{MPIM-LQR}}$, but it does not manage to obey the superior limit for the negative values of the real-parts of the eigenvalues in its spectrum, i.e., -2.0.

3) MPIM-GLQR Solution: The generalized GLQR stabilization, using all the three penalties Q_f, R_f, S_f from (27), with MPIM correction, the MPIM-GLQR technique, provides the control gain

$$K_f^{\text{MPIM-GLQR}} = \left[\begin{array}{cccc} 0.0219 & -0.0016 & 0.0234 & 1.2047 \end{array}\right] \times 10^5,$$

which is finally acceptable, as it provides the stable spectrum

$$\Lambda(\hat{A}_f^{\text{MPIM-GLQR}}) = \{-2.6426, -49.0031, -5.2219 \pm 3.6603\imath\},$$

for the closed-loop matrix $\hat{A}_f^{\text{MPIM-GLQR}} = A_f - B_f K_f^{\text{MPIM-GLQR}}$, being the only approach that stabilizes the system and achieves the prescribed stability degree.

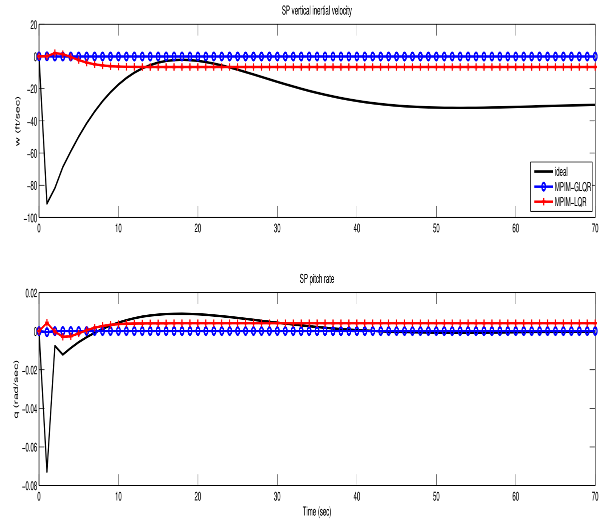

Fig. 1. MPIM-G/LQR Accommodation of the B747-SP Mode

4) MPIM-GLQR vs. MPIM-LQR Control: Unfortunately, both fault tolerant model matching techniques provide large values for the differences between the corresponding closed-loop matrices and the reference model one, but this is to be explained by the fact that both generalized / classical quadratic regulators provide large control gains.

However, there is one major advantage of the MPIM-GLQR approach over the MPIM-LQR in that it provides real-parts of the closed-loop eigenvalues smaller than $-\alpha_f$, which makes MPIM-GLQR the best candidate if, in the post-fault operation, the prescribed degree of stability is also imposed for the performance to be followed as well, as a plus to the primary minimum Frobenius-norm requirement.

5) Closed-Loop Simulation: Due to the limited space in the paper, only the simulations for the short-period (SP) mode behavior (vertical inertial velocity and pitch rate) when applying the MPIM-G/LQR accommodating controls are shown in Fig. 1, during the post-fault period (restricted on the graphs to $t_f = 0$). At least from the graph of the pitch rate, there is to be seen that the MPIM-GLQR is a better follower than MPIM-LQR.

V. Conclusion

This paper interrogated the Modified Pseudo-Inverse Method (MPIM) used as an accommodation technique that recovers as much performance as before the faults in the situation of impairments turning the ideal nominal operation to an unstable post-fault regime.

The authors have developed the MPIM with the Generalized Linear Quadratic Regulator (GLQR) stabilization using the penalty matrices derived from the Robust Optimal Model Matching (ROMM) procedure and have shown that this is the only successful approach for the model matching problem extended with the prescribed stability degree property, as opposed to the classical MPIM used in conjuction with simple LQR stabilization.

Moreover, an application example for the fault tolerant model matching of the B747 longitudinal motion control is also provided.

APPENDIX I
REVIEW ON ROBUST OPTIMAL MODEL MATCHING

Let $\hat{A}_n^*$ be the ideal nominal closed-loop matrix from (3) and denote by $\alpha_n^* \triangleq |\lambda(\hat{A}_n^*)| \geq \alpha_f$ the maximum absolute value of the real-part of the eigenvalues of $\hat{A}_n^*$.

One of the solutions to the model matching problem with prescribed stability degree from (1)-(3) known in the literature, with some robustness and optimality properties attached, designs the post-fault state-feedback so to minimize the performance index (see [7])

$$J_f(\alpha_f, \hat{A}_n^*) = \int_{t_f}^{\infty} e^{2\alpha_f t}\left(\|\dot{x}(t) - \hat{A}_n^* x(t)\|_2^2 + \|\hat{A}_n^* x(t)\|_2^2\right) dt\,; \tag{28}$$

the exponent $\alpha_f \leq \alpha_n^*$ is defined as the desired minimal decay-rate of the post-fault closed-loop system, namely the exponential convergence rate of system state to the equilibrium point [19].

In this case, the optimal control $u_f(t) = -K_f x(t)$ which minimizes $J_f(\alpha_f, \hat{A}_n^*)$ from (28) subject to (A_f, B_f) is computed as

$$K_f = R_f^{-1}(B_f^T X_f + S_f)\,, \tag{29}$$

where the unique stabilizing symmetric positive-semidefinite matrix X_f represents the solution to the Generalized Continuous Algebraic Riccati Equation (GCARE)

$$GCARE_f\,:\, \tilde{A}_f^T X_f + X_f \tilde{A}_f - X_f B_f R_f^{-1} B_f^T X_f + \tilde{Q}_f = 0\,, \tag{30}$$

with the buffer matrices

$$\tilde{A}_f = A_f + \alpha_f I_n - B_f R_f^{-1} S_f\,,\; \tilde{Q}_f = Q_f - S_f^T R_f^{-1} S_f\,, \tag{31}$$

where the required post-fault weighting matrices from (30)-(31), are identified as (see also [20])

$$Q_f = A_f^T A_f - A_f^T \hat{A}_n^* - \hat{A}_n^{*T} A_f + 2\hat{A}_n^{*T}\hat{A}_n^*\,, \tag{32a}$$
$$R_f = B_f^T B_f\,, \tag{32b}$$
$$S_f = B_f^T(A_f - \hat{A}_n^*)\,, \tag{32c}$$

obtained after the reduction of the performance integral in (28) to the corresponding generalized form

$$J_f(\alpha_f) = \int_{t_f}^{\infty} e^{2\alpha_f t}\left(\|x(t)\|_{Q_f}^2 + \|u_f(t)\|_{R_f}^2 + 2u_f(t)^T S_f x(t)\right) dt\,, \tag{33}$$

for the squared Q_f- and R_f- state- and control- vectors norms, i.e., $\|x(t)\|_{Q_f}^2 = x(t)^T Q_f x(t)$ and $\|u_f(t)\|_{R_f}^2 = u_f(t)^T R_f u_f(t)$, and S_f the control-state coupling matrix.

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