

Low-order control design for chatter suppression in high-speed milling

N.J.M. van Dijk, N. van de Wouw and H. Nijmeijer

Abstract—Chatter is an instability phenomenon in machining processes which must be avoided at all times. The occurrence of chatter can be predicted using stability lobes diagrams. In this paper a low-order active chatter control design is proposed, which enables dedicated shaping of the chatter stability boundary such that working points of higher machining productivity become feasible while avoiding chatter. The control design problem is cast into a nonsmooth optimization problem, which is solved using bundle methods. A static output feedback controller is designed for an illustrative example, which illustrates the power of the proposed methodology.

I. INTRODUCTION

The productivity in the manufacturing industry is limited by the occurrence of a dynamic instability phenomenon called chatter. Chatter results in an inferior workpiece quality due to heavy vibrations of the cutter. The occurrence of chatter can be visualized in stability lobes diagrams (SLD), where the stability boundary between a stable cut and an unstable cut is visualized in terms of spindle speed and depth of cut. To increase the number of chatter-free working points, and therewith the productivity of the milling process, it is desired to alter the chatter stability boundary by means of control.

The chatter stability boundary can be altered by actively adapting the machine dynamics. Active chatter control in milling has mainly been focused on active damping of the machine dynamics [1], [2]. Damping the machine dynamics results in a uniform increase of the stability boundary for all spindle speeds.

To enable more dedicated shaping of the stability boundary (e.g. lifting the SLD locally around a specific spindle speed), the regenerative effect, which is the root cause for chatter, should be taken into account during chatter controller design as presented in [3], [4].

Except for the work in [2], [4], all research on active chatter control is limited to low spindle speeds (i.e. below 5000 rpm). Moreover, all aforementioned research either does not include the regenerative effect during controller design or utilizes high-order finite-dimensional approximations of the milling model for controller design yielding high-order controllers which is disadvantageous from an implementation perspective.

This paper presents a controller design methodology, which can guarantee chatter-free milling operations in an a priori defined range spindle speed and depth of cut. The

proposed approach takes into account the regenerative effect, responsible for chatter, in the control design, which yields models in terms of a set of delay differential equations (DDEs) and allows for dedicated shaping of the SLD. Moreover, we refrain from employing finite-dimensional approximations of the delay yielding high-order models and high-order controllers as proposed in [4]. Instead, we propose a design for low-order active chatter controllers for the milling process using an infinite-dimensional model of the milling process. The proposed methodology, firstly, will allow the machinist to define a desired working range (in spindle speed and depth of cut) and correspondingly shapes the SLD locally in a dedicated fashion and, secondly, lets the user directly impose the order of the controller.

In fixed-structure or fixed-order controller synthesis for time delay systems, results are often obtained using a Lyapunov-based approach, see e.g. [5]. Lyapunov-based approaches allow the incorporation of a more general class of uncertainties, such as time-varying uncertainties. However, the resulting optimization problems are in the form of bilinear matrix inequalities where the number of unknown variables in general grows quadratically with the number of states [6] which may lead to computational issues. Moreover, generally the application of a Lyapunov approach leads to conservative results. The usage of an eigenvalue based approach can overcome these disadvantages as explained in [7]. Therefore, we employ such an approach in this paper.

The paper is organized as follows. Section II presents a the model of the milling process. The problem setting is described in Section III. The problem will be cast into a generalized plant formulation, which is discussed in Section IV. Section V presents the low-order controller design procedure. Results for an illustrative example are presented in Section VI. Finally, conclusions are drawn in Section VII.

II. THE MILLING PROCESS

This section presents a comprehensive model of the milling process and discusses (chatter-related) stability properties of the model.

A. A comprehensive milling model

In Figure 1, a schematic representation of the milling process is given. A block diagram of the milling process, with controller, is given in Figure 2. As can be seen from the block diagram, the milling process is a closed-loop position-driven process. The setpoint of the milling process is the predefined motion of the tool with respect to the workpiece, given in terms of the static chip thickness $h_{j,\text{stat}}(t) = f_z \sin \phi_j(t)$, where f_z is the feed per tooth and $\phi_j(t)$ the rotation angle of the j -th tooth of the tool with respect to the y (normal) axis (see Figure 1). However, the total chip thickness $h_j(t)$ also depends on the interaction between the cutter and the

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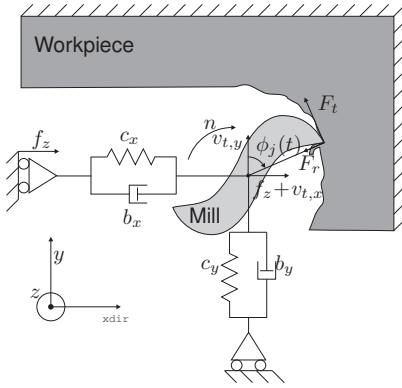


Fig. 1: Schematic representation of the milling process.

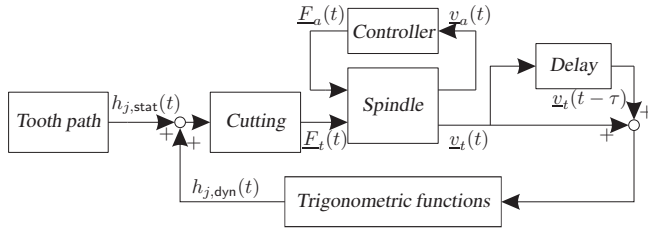


Fig. 2: Block diagram of the milling process.

workpiece. This leads to cutter vibrations resulting in a dynamic displacement $\underline{v}_t(t) = [v_{t,x}(t) \ v_{t,y}(t)]^T$ of the tool, see Figure 1, which is superimposed on the predefined tool motion. This results in a wavy surface on the workpiece. The next tooth encounters the wavy surface, left behind by the previous tooth, and generates its own waviness. This is called the regenerative effect and results in the block *Delay* in Figure 2, see [8]. The difference between the current and previous wavy surface is denoted as the dynamic chip thickness $h_{j,dyn}(t) = [\sin \phi_j(t) \ \cos \phi_j(t)] (\underline{v}_t(t) - \underline{v}_t(t - \tau))$ with $\tau = 60/(zn)$ the delay, with z the number of teeth and n the spindle speed in revolutions per minute (rpm). Hence, the total chip thickness of tooth j , $h_j(t)$, is the sum of the static and dynamic chip thickness: $h_j(t) = h_{j,stat}(t) + h_{j,dyn}(t)$.

The cutting force model (indicated by the *Cutting* block in Figure 2) relates the total chip thickness to the forces acting at the tool tip of the machine spindle. The forces in tangential and radial direction for a single tooth j are described by the following exponential cutting force model:

$$\begin{aligned} F_{t,j}(t) &= g_j(\phi_j(t)) K_t a_p h_j(t)^{x_F}, \\ F_{r,j}(t) &= g_j(\phi_j(t)) K_r a_p h_j(t)^{x_F}, \end{aligned} \quad (1)$$

where $0 < x_F \leq 1$ and $K_t, K_r > 0$ are cutting parameters which depend on the workpiece material, and a_p is the axial depth of cut. The function $g_j(\phi_j(t))$ describes whether a tooth is in or out of cut:

$$g_j(\phi_j(t)) = \begin{cases} 1, & \phi_s \leq \phi_j(t) \leq \phi_e \wedge h_j(t) > 0, \\ 0, & \text{else,} \end{cases} \quad (2)$$

where ϕ_s and ϕ_e are the entry and exit angle of the cut, respectively. Via trigonometric functions, the cutting force can easily be converted to x (feed)- and y (normal)-direction. Hence, cutting forces in x - and y -direction can be obtained

by summing over all z teeth:

$$\begin{aligned} \underline{F}_t(t) &= a_p \sum_{j=0}^{z-1} g_j(\phi_j(t)) \left(\left(h_{j,stat}(t) \right. \right. \\ &\quad \left. \left. + [\sin \phi_j(t) \ \cos \phi_j(t)] (\underline{v}_t(t) - \underline{v}_t(t - \tau)) \right)^{x_F} \right. \\ &\quad \left. \mathbf{S}(t) \begin{bmatrix} K_t \\ K_r \end{bmatrix} \right), \end{aligned} \quad (3)$$

where

$$\mathbf{S}(t) = \begin{bmatrix} -\cos \phi_j(t) & -\sin \phi_j(t) \\ \sin \phi_j(t) & -\cos \phi_j(t) \end{bmatrix}.$$

The cutting force interacts with the spindle and tool dynamics (block *Spindle*) in Figure 2. The machine dynamics are modeled via a linear multi-input-multi-output (MIMO) state-space model,

$$\begin{aligned} \dot{\underline{x}}(t) &= \mathbf{A} \underline{x}(t) + \mathbf{B}_t \underline{F}_t(t) + \mathbf{B}_a \underline{F}_a(t), \\ \underline{v}_t(t) &= \mathbf{C}_t \underline{x}(t), \quad \underline{v}_a(t) = \mathbf{C}_a \underline{x}(t), \end{aligned} \quad (4)$$

where $\underline{x}(t)$ is the state (the order of this model primarily depends on the order of the spindle-tool dynamics model) and cutting forces $\underline{F}_t(t) = [F_{t,x}(t) \ F_{t,y}(t)]^T$ are given in (3), where $F_{t,x}(t)$ and $F_{t,y}(t)$ are the cutting forces in x - and y -direction, respectively. The control forces are given by $\underline{F}_a(t) = [F_{a,x}(t) \ F_{a,y}(t)]^T$, where $F_{a,x}(t)$ and $F_{a,y}(t)$ are the control forces acting in x - and y -direction, respectively. Moreover, $\underline{v}_t(t)$ and $\underline{v}_a(t)$ are the displacements of the cutter and the measured displacements available for feedback, respectively.

Substitution of (3) into (4) yields the nonlinear, non-autonomous delay differential equations (DDE) describing the milling process:

$$\begin{aligned} \dot{\underline{x}}(t) &= \mathbf{A} \underline{x}(t) + \mathbf{B}_t a_p \sum_{j=0}^{z-1} g_j(\phi_j(t)) \left(\left(h_{j,stat}(t) + \right. \right. \\ &\quad \left. \left. [\sin \phi_j(t) \ \cos \phi_j(t)] \mathbf{C}_t (\underline{x}_t(t) - \underline{x}_t(t - \tau)) \right)^{x_F} \right. \\ &\quad \left. \mathbf{S}(t) \begin{bmatrix} K_t \\ K_r \end{bmatrix} \right) + \mathbf{B}_a \underline{F}_a(t), \quad \underline{v}_a(t) = \mathbf{C}_a \underline{x}(t). \end{aligned} \quad (5)$$

B. Stability of the milling process

In the milling process the static chip thickness is periodic with period time $\tau = 60/(zn)$. In general, the milling model (5) has a periodic solution $\underline{x}^*(t)$ with period time τ . When no chatter occurs, the periodic solution is (at least locally) asymptotically stable and when chatter occurs it is unstable. Therefore, the chatter stability boundary can be found by studying the (local) stability properties of the periodic solution. To this end, the milling model is linearized about the periodic solution $\underline{x}^*(t)$ for zero control input (i.e. $\underline{F}_a(t) = \underline{0}$) which yields the following linearized dynamics in terms of perturbations $\tilde{\underline{x}}(t)$ ($\underline{x}(t) = \underline{x}^*(t) + \tilde{\underline{x}}(t)$):

$$\begin{aligned} \dot{\tilde{\underline{x}}}(t) &= \mathbf{A} \tilde{\underline{x}}(t) + a_p \mathbf{B}_t \sum_{j=0}^{z-1} \mathbf{H}_j(t) \mathbf{C}_t (\tilde{\underline{x}}(t) - \tilde{\underline{x}}(t - \tau)) \\ &\quad + \mathbf{B}_a \underline{F}_a(t), \quad \tilde{\underline{v}}_a(t) = \mathbf{C}_a \tilde{\underline{x}}(t), \end{aligned} \quad (6)$$

where

$$\mathbf{H}_j(t) = g_j x_F (f_z \sin \phi_j)^{x_F-1} \mathbf{S}(t) \begin{bmatrix} K_t \\ K_r \end{bmatrix} \begin{bmatrix} \sin \phi_j & \cos \phi_j \end{bmatrix}. \quad (7)$$

As can be seen from (6) the linearized model is a *nonautonomous* DDE. The focus in this work lies on full immersion cuts, where the full width of the cutter is used for cutting. As described in [9], for full immersion cuts it is sufficient to average the dynamic cutting forces $\sum_{j=0}^{z-1} \mathbf{H}_j(t)$ over the tool path such that the milling model becomes an *autonomous* (time-invariant) DDE model. Since the cutter is only cutting when $\phi_s \leq \phi \leq \phi_e$ the averaged cutting forces are given by

$$\bar{\mathbf{H}} = \frac{z}{2\pi} \int_{\phi_s}^{\phi_e} \sum_{j=0}^{z-1} \mathbf{H}_j(\phi) d\phi. \quad (8)$$

Then, the linear time-invariant model of the milling process is obtained by combining (6) with $\mathbf{H}_j(t) = \bar{\mathbf{H}}$ and $\bar{\mathbf{H}}$ given in (8), such that:

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= (\mathbf{A} + a_p \mathbf{B}_t \bar{\mathbf{H}} \mathbf{C}_t) \tilde{\mathbf{x}}(t) - a_p \mathbf{B}_t \bar{\mathbf{H}} \mathbf{C}_t \tilde{\mathbf{x}}(t - \tau) \\ &\quad + \mathbf{B}_a \underline{\mathbf{F}}_a(t), \quad \tilde{\mathbf{v}}_a(t) = \mathbf{C}_a \tilde{\mathbf{x}}(t). \end{aligned} \quad (9)$$

The characteristic equation of the (uncontrolled) linear DDE (9) is then given as

$$\det(\mathbf{I} - a_p \mathbf{G}_{tt}(i\omega) \bar{\mathbf{H}} (1 - e^{-i\omega\tau})) = 0, \quad (10)$$

where $\mathbf{G}_{tt}(i\omega) = \mathbf{C}_t(i\omega \mathbf{I} - \mathbf{A})^{-1} \mathbf{B}_t$ represents the frequency response function (FRF) from cutting forces at the tooltip to tooltip displacements. By solving (10) for given depth-of-cut a_p and delay τ , as e.g. discussed in [9], the stability lobes diagram (SLD) can be obtained. Based on an SLD a machine operator can select working points in terms of spindle speed and depth of cut for which a chatter free milling operation can be performed.

III. PROBLEM STATEMENT

As discussed in the introduction, the aim of this paper is to design a low-order, finite-dimensional linear controller $\mathbf{K}$ to generate control inputs $\underline{\mathbf{F}}_a$ based on measurements $\tilde{\mathbf{v}}_a$, which guarantees:

- robust stability of $\tilde{\mathbf{x}} = \mathbf{0}$ in (9) for uncertainties in depth of cut a_p and time delay τ ;
- performance by minimizing the total amount of actuator energy needed to stabilize the milling process.

Herewith, chatter-free cutting operations can be guaranteed in an a priori defined range of spindle speeds n and depth of cut a_p , and, moreover, the actuator forces will be limited during the controller design, which is an important practical performance requirement. Note that the perturbation vibrations $\tilde{\mathbf{v}}_a$ are used for feedback. In practice, $\tilde{\mathbf{v}}_a(t)$ can be obtained by using a chatter detection algorithm based on a parametric model of the milling process, as presented in [10].

It is assumed that the fixed structure controller $\mathbf{K}$, with measured output $\tilde{\mathbf{v}}_a \in \mathbb{R}^2$ and control input $\underline{\mathbf{F}}_a \in \mathbb{R}^2$, has the following state-space description:

$$\begin{aligned} \dot{\underline{\xi}}(t) &= \mathbf{A}_c \underline{\xi}(t) + \mathbf{B}_c \tilde{\mathbf{v}}_a(t), \\ \underline{\mathbf{F}}_a(t) &= \mathbf{C}_c \underline{\xi}(t) + \mathbf{D}_c \tilde{\mathbf{v}}_a(t). \end{aligned} \quad (11)$$

Herein, $\underline{\xi} \in \mathbb{R}^{n_c}$, $\mathbf{A}_c \in \mathbb{R}^{n_c \times n_c}$, $\mathbf{B}_c \in \mathbb{R}^{n_c \times 2}$, $\mathbf{C}_c \in \mathbb{R}^{2 \times n_c}$ and $\mathbf{D}_c \in \mathbb{R}^{2 \times 2}$ with n_c the order of the controller.

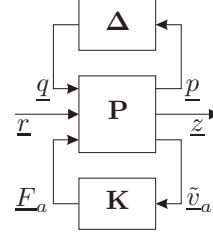


Fig. 3: Generalized plant interconnection.

IV. GENERALIZED PLANT FORMULATION

In order to solve the problem stated in the previous section, the model of the milling process will be extended with uncertainties in depth of cut a_p and spindle speed n . Hereto, the control goal will be cast into the generalized plant framework. Figure 3 shows the configuration of this framework. The generalized plant $\mathbf{P}$ is a given system with three sets of inputs and three sets of outputs. The signal pair $\underline{p}, \underline{q}$ denote the in-/outputs of the uncertainty channel. The signal $\underline{r}$ represents an external input in which possible disturbances, measurement noise and reference inputs are stacked. The signal $\underline{\mathbf{F}}_a$ is the control input. The output $\underline{z}$ can be considered as a performance variable while $\tilde{\mathbf{v}}_a$ denotes the measured outputs used for feedback. To derive the generalized plant formulation, consider the linearized time-invariant model of the milling process (9). Based on the discussion above let us define the following uncertainty sets:

$$a_p = \frac{1}{2} \bar{a}_p (1 + \delta_{a_p}), \tau = \tau_0 + \delta_\tau, \quad (12)$$

where $\bar{a}_p$ is the maximal depth of cut for which stable cutting is desired, $\delta_{a_p} \in \mathbb{C}$, with $|\delta_{a_p}| \leq 1$, $\tau_0 = \frac{\bar{\tau} + \underline{\tau}}{2}$ and $\delta_\tau \in \frac{\bar{\tau} - \underline{\tau}}{2} [-1, 1]$ with $0 < \underline{\tau} \leq \bar{\tau}$. Moreover, as described in Section III, it is desired to limit the amount of actuator forces. Therefore, the performance output is chosen as the weighted control input $\underline{z}(s) = \mathbf{W}_{KS}(s) \underline{\mathbf{F}}_a(s)$, $s \in \mathbb{C}$, where $\mathbf{W}_{KS}$ is a stable weighting filter with the following state-space realization:

$$\begin{aligned} \dot{\underline{\mathbf{x}}}_{KS}(t) &= \mathbf{A}_{KS} \underline{\mathbf{x}}_{KS}(t) + \mathbf{B}_{KS} \underline{\mathbf{F}}_a(t), \\ \underline{\mathbf{z}}(t) &= \mathbf{C}_{KS} \underline{\mathbf{x}}_{KS}(t) + \mathbf{D}_{KS} \underline{\mathbf{F}}_a(t). \end{aligned} \quad (13)$$

Substituting (12) in (9) and by adding the performance channel in-/output to the system and rearranging terms, the state-space representation of the generalized plant $\mathbf{P}$ is given as follows:

$$\begin{aligned} \dot{\underline{\mathbf{x}}}_P(t) &= \mathbf{A}_{P,0} \underline{\mathbf{x}}_P(t) + \mathbf{A}_{P,1} \underline{\mathbf{x}}_P(t - \tau_0) + \mathbf{B}_P \underline{\mathbf{u}}_P(t) \\ \underline{\mathbf{v}}_P(t) &= \mathbf{C}_{P,0} \underline{\mathbf{x}}_P(t) + \mathbf{C}_{P,1} \underline{\mathbf{x}}_P(t - \tau_0) + \mathbf{D}_P \underline{\mathbf{u}}_P(t) \end{aligned} \quad (14)$$

with the state vector $\underline{\mathbf{x}}_P(t) = [\tilde{\mathbf{x}}^T(t) \underline{\mathbf{x}}_{KS}^T(t)]^T$, input vector $\underline{\mathbf{u}}_P(t) = [\underline{q}^T(t) \underline{r}^T(t) \underline{\mathbf{F}}_a^T(t)]^T$, output vector $\underline{\mathbf{v}}_P(t) = [\underline{p}^T(t) \underline{z}^T(t) \tilde{\mathbf{v}}_a^T(t)]^T$. The uncertainty channel input $\underline{p}(t)$ and output $\underline{q}(t)$ are defined as

$$\underline{p}(t) = [\underline{p}_1^T(t) \underline{p}_2^T(t)]^T, \underline{q}(t) = [\underline{q}_1^T(t) \underline{q}_2^T(t)]^T.$$

The definition of the state-space matrices of the generalized plant can be found in the appendix of the paper. In the following discussion also the transfer function description

of the generalized plant will be used. The transfer function description of the generalized plant $\mathbf{P}$ is given as follows:

$$\mathbf{P}(s) = (\mathbf{C}_{P,0} + \mathbf{C}_{P,1}e^{-s\tau_0})[s\mathbf{I} - \mathbf{A}_{P,0} - \mathbf{A}_{P,1}e^{-s\tau_0}]^{-1}\mathbf{B}_P + \mathbf{D}_P, \quad (15)$$

$s \in \mathbb{C}$. The (structured) uncertainty channel is then given as follows:

$$\underline{q}_1(t) = (\mathcal{D}_\tau - 1)\underline{p}_1(t), \quad \underline{q}_2(t) = \delta_{a_p}\underline{p}_2(t), \quad (16)$$

where the operator $\mathcal{D}_\tau$ is defined as $\mathcal{D}_\tau \underline{x}(t) = \underline{x}(t - \tau)$. Let $\Delta(s)$ denote the Laplace transform of the uncertainty term (16), such that $\underline{q}(s) = \Delta(s)\underline{p}(s)$ with

$$\Delta(s) = \text{diag}((e^{-s\delta_\tau} - 1)\mathbf{I}_2, \delta_{a_p}\mathbf{I}_2) \quad (17)$$

with identity matrix $\mathbf{I}_n \in \mathbb{R}^{n \times n}$. It can be seen that the uncertainty term, given above, depends on the frequency. In [11] it is shown that the delay uncertainty $e^{-s\delta_\tau} - 1$ can be upper bounded by a (non-rational) frequency-dependent upper bound $\kappa(\omega)$, given as follows

$$\kappa(\omega) = \begin{cases} 2 \sin \frac{\delta_\tau \omega}{2}, & \forall \omega, 0 \leq \omega \leq \pi/\delta_\tau \\ 2, & \forall \omega \geq \pi/\delta_\tau. \end{cases} \quad (18)$$

To reduce conservatism in the controller synthesis procedure, which will be discussed in the next section, a rational overapproximation $\rho(s)$ of the upper bound $\kappa(\omega)$, such that $\kappa(\omega) \leq |\rho(i\omega)|$, as discussed in [11], will be incorporated in the generalized plant. As discussed in [4], for the high-speed milling process, a tight over approximation in the frequency range ω , $0 \leq \omega \leq \pi/\delta_\tau$ is desired. Hence, $\rho(s)$ is chosen as follows:

$$\rho(s) = \delta_\tau s / ((\delta_\tau / 3.456)s + 1). \quad (19)$$

Let us define $\mathbf{L}(s) = \text{diag}(\rho(s)\mathbf{I}_2, \mathbf{I}_2)$. Then the (scaled) generalized plant and uncertainty term is obtained as follows:

$$\tilde{\mathbf{P}}(s) = \text{diag}(\mathbf{L}(s), \mathbf{I}_2, \mathbf{I}_2)\mathbf{P}(s), \quad \tilde{\Delta} = \Delta(s)\mathbf{L}^{-1}(s).$$

V. LOW-ORDER ACTIVE CHATTER CONTROL

Based on the discussion in the previous section, it can be seen that the control problem is actually a robust performance problem. It is well known from robust control theory, that a robust performance problem can be easily transformed to a robust stability problem by adding an additional uncertainty block $\Delta_P \in \mathbb{C}^{2 \times 2}$, $\|\Delta_P\|_\infty \leq 1$, associated with the performance channel, to the problem, see [12]. Then the design of a controller, guaranteeing robust stability, requires the following optimization problem to be solved:

$$\min_{\mathbf{K}} \sup_{\omega \in \mathbb{R}} \mu_{\tilde{\Delta}}(\mathbf{N}), \quad \text{subject to } \Psi(\mathbf{K}) < 0, \quad (20)$$

with $\mu_{\tilde{\Delta}}$ the structured singular value of $\mathbf{N}$ with respect to expanded uncertainty set $\tilde{\Delta} = \text{diag}(\tilde{\Delta}, \Delta_P)$, $\mathbf{N}$ the lower fractional transformation (LFT) of $\tilde{\mathbf{P}}$ and fixed structure controller $\mathbf{K}$ (see [12] for the definition of LFTs), and $\Psi(\mathbf{K})$ the spectral abscissa function of the closed-loop system defined as:

$$\Psi(\mathbf{K}) := \sup\{\text{Re}(\lambda) : \det(\lambda\mathbf{I} - \tilde{\mathbf{A}}_0 - \tilde{\mathbf{A}}_1 e^{-\lambda\tau_0}) = 0\}, \quad (21)$$

where

$$\tilde{\mathbf{A}}_0 = \begin{bmatrix} \mathbf{A} + \frac{1}{2}\bar{a}_p\mathbf{B}_t\bar{\mathbf{H}}\mathbf{C}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{D}_c & \mathbf{C}_c \\ \mathbf{B}_c & \mathbf{A}_c \end{bmatrix} \begin{bmatrix} \mathbf{C}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix},$$

$$\tilde{\mathbf{A}}_1 = \begin{bmatrix} -\frac{1}{2}\bar{a}_p\mathbf{B}_t\bar{\mathbf{H}}\mathbf{C}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

The constraint on the objective function, defined above, is a necessary condition to guarantee the existence of the $\mathcal{H}_\infty$ -norm of $\mathbf{N}$ along with stability of the closed-loop feedback system, see also [12].

It is well known that, in general, it is difficult to calculate $\mu_{\tilde{\Delta}}$. However, an upper bound on $\mu_{\tilde{\Delta}}$ can be obtained by calculating the scaled $\mathcal{H}_\infty$ norm of $\mathbf{N}$ [12]. Since the uncertainties are modeled by complex uncertainties, a reasonable approach to solve the problem is to apply D-K-iteration, see [12].

For a given controller $\mathbf{K}$, the problem of finding the scaling matrix $\mathbf{D}$, with the structure of $\mathbf{D}$ chosen such that $\mathbf{D}\tilde{\Delta} = \tilde{\Delta}\mathbf{D}$ holds, can be turned into convex optimization problem which is generally solved pointwise in the frequency domain. Since in this paper fixed structure controllers are considered, the problem of finding $\mathbf{K}$, for a given $\mathbf{D}$, in general results in a nonconvex, nonsmooth, constrained optimization problem, given as follows:

$$\min_{\mathbf{K}} f(\mathbf{K}), \quad \text{subject to } \Psi(\mathbf{K}) < 0. \quad (22)$$

with $f(\mathbf{K}) := \sup_{\omega \in \mathbb{R}} \bar{\sigma}(\mathbf{D}(i\omega)\mathbf{N}\mathbf{D}(i\omega)^{-1})$. Herein, $\bar{\sigma}$ denotes the largest singular value. The nonsmooth dependence of the objective function $f(\mathbf{K})$ on the controller parameters of $\mathbf{K}$ typically occurs when the maximum of the objective function is located at two (or more) different frequencies. Due to the nonsmoothness of (22), standard optimization algorithms cannot be used to determine the parameters of controller $\mathbf{K}$, since they tend to switch about a nonsmooth surface of the objective function $f(\mathbf{K})$. Instead, nonsmooth optimization techniques, based on bundle methods, will be used. The key assumption supporting the application of bundle methods is that the continuous objective function is differentiable *almost everywhere*. Burke *et al.* [13] present a gradient bundle method, called gradient sampling, where the user specifies, for given controller parameters of $\mathbf{K}$, the objective function $f(\mathbf{K})$ in (22) and the gradient $(\nabla f(\mathbf{K}))$, when the objective function is differentiable.

The gradient sampling algorithm can be used to locally minimize nonsmooth, nonconvex objective functions. In general, the gradient sampling algorithm is quite expensive per iteration. Therefore, Lewis and Overton have developed a hybrid algorithm, based on BFGS, for nonsmooth optimization (HANSO) [14]. HANSO can in general be applied to finite-dimensional systems with continuous objective functions. As shown in [7, Chp. 9], the $\mathcal{H}_\infty$ -norm of a system with time-delay exhibits continuity properties and is differentiable almost everywhere which allows the application of HANSO for the present system. From (22), it can be seen that the problem of finding a fixed structure controller which guarantees robust performance of the milling process is actually a *constrained* optimization problem. However, HANSO is only able to deal with unconstrained optimization problems. Therefore, the constrained optimization problem is converted

TABLE I: Milling model parameters.

Parameter	Value	Parameter	Value
$m_{t,x} = m_{t,y}$	0.015 kg	K_t	462 [N/mm ^(1+x_F)]
$m_{a,x} = m_{a,y}$	0.14 kg	K_r	38.6 [N/mm ^(1+x_F)]
$\omega_{t,x} = \omega_{t,y}$	2350 Hz	x_F	1 [-]
$\omega_{a,x} = \omega_{a,y}$	1400 Hz	ϕ_s	0 [rad]
$\zeta_{t,x} = \zeta_{t,y}$	0.05 [-]	ϕ_e	π [rad]
$\zeta_{a,x} = \zeta_{a,y}$	0.12 [-]	f_z	0.2 mm/tooth
z	4 [-]		

to an unconstrained optimization problem using a penalty method, see [15]. The constrained optimization problem (22) is replaced with the following unconstrained (nonsmooth) optimization problem:

$$\min_{\mathbf{K}} \bar{f}(\mathbf{K}), \quad (23)$$

where $\bar{f}(\mathbf{K}) = f(\mathbf{K}) + \gamma \max(0, \Psi(\mathbf{K}))$, where γ is a positive constant. The value of γ is in general iteratively chosen, see [16] for rules on how to choose γ .

During an optimization step, in order to evaluate the objective function (23) for given $\mathbf{K}$ and $\mathbf{D}$, the (scaled) $\mathcal{H}_\infty$ -norm of $\mathbf{DND}^{-1}$ as well as spectral abscissa $\Psi(\mathbf{K})$, defined in (21), need to be calculated. Since, in this case, the system is infinite-dimensional (due to the presence of the time-delay), standard Hamiltonian approaches to calculate the $\mathcal{H}_\infty$ -norm cannot be used. Hence, here the $\mathcal{H}_\infty$ -norm will be determined pointwise across a grid of frequencies $\underline{\omega} = [\omega_1, \omega_2, \dots, \omega_N]^T$. The spectral abscissa is determined using the DDE-BIFTOOL [17] software package, which can be used to determine the right-most characteristic roots of a linear time-invariant system with time-delays. More information about computation of characteristic roots for time-delay systems can be found in [7].

VI. RESULTS

In this section, a static output feedback controller ($n_c = 0$, in (11)) will be designed for the uncertain time-delay system (9). The goal of this section is to illustrate the effectiveness of the low-order control design approach. Hereto, consider the parameters of the milling process as given in Table I. The spindle dynamics is modeled by two decoupled subsystems consisting of a two mass-spring-damper model in order to capture the inherent compliance between the actuator/sensor system (denoted by subscript a , with undamped eigenfrequency ω_a and dimensionless damping ratio ζ_a) and the cutting tool (denoted by subscript t , with undamped eigenfrequency ω_t and dimensionless damping ratio ζ_t). Here, a linear cutting model is considered (i.e. $x_F = 1$). The structure of the controller matrix $\mathbf{D}_c \in \mathbb{R}^{2 \times 2}$ is chosen such that it has a similar structure as the averaged cutting force matrix $\bar{\mathbf{H}}$ which, for $x_F = 1$, can be written as the sum of a diagonal matrix $k\mathbf{I}$ and a skew-symmetric matrix, see also [9, page 107]. Therewith, only two controller parameters need to be synthesized, i.e. the controller matrix structure is given as

$$\mathbf{D}_c = \begin{bmatrix} k_1 & -k_2 \\ k_2 & k_1 \end{bmatrix}. \quad (24)$$

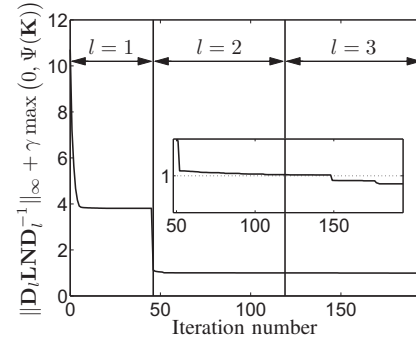


Fig. 4: Objective function in K-step of algorithm as function of iteration number.

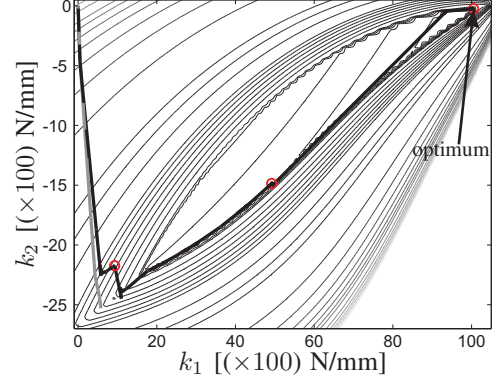


Fig. 5: Contour plot of objective function along with optimization result using nonsmooth optimization and standard BFGS (gray) during three D-K-iterations of the algorithm. The circles indicate the end of each K-step.

We chose to present such a case since it allows for a graphical representation of the results.

The static output feedback controller is designed such that it stabilizes milling operations between $n \in [36000, 38000]$ rpm, for a depth of cut which is as large as possible given the performance requirement on the weighted controller output, where $W_{KS} = K_p$, with $K_p = 1 \cdot 10^{-6}$ mm/N. Moreover, the parameter of the penalty function is set to $\gamma = 100$ and the initial sampling radius of the gradient sampling algorithm $\epsilon_s = 0.1$. Starting at the initial controller parameters $k_1 = k_2 = 0$, the fixed structure controller synthesis algorithm aims to find a minimizer for the constrained optimization problem (22) by iterating over $\mathbf{D}$ and $\mathbf{K}$. The algorithm converges after three D-K steps resulting in $\sup_{\omega \in \mathbb{R}} \bar{\sigma}(\mathbf{D}(i\omega)\mathbf{N}(i\omega)\mathbf{D}^{-1}(i\omega)) = 0.9911$ there- with guaranteeing robust performance for milling operations between $n \in [36000, 38000]$ rpm up to a depth of cut of $\bar{a}_p = 2.4375$ mm. In Figure 4, the values of the objective function during the K-step, i.e. $\sup_{\omega \in \mathbb{R}} \bar{\sigma}(\mathbf{D}_l \mathbf{N} \mathbf{D}_l^{-1}) + \gamma \max(0, \Psi(\mathbf{K}))$ with $l = 1, 2, 3$ the index of the corresponding D-scale matrix functions $\mathbf{D}_l$, are given as function of iteration number. In Figure 5, a contour plot is depicted where the upper bound on $\sup_{\omega \in \mathbb{R}} \mu_{\hat{\Delta}}(\mathbf{N}(i\omega))$ is calculated for several values of k_1 and k_2 . Moreover, the path of the fixed structure controller synthesis algorithm in the feedback gain parameter space is given, where the end points of each K-step in the D-K-iteration process are indicated by a circle. The obtained feedback gains that guarantee robust perfor-

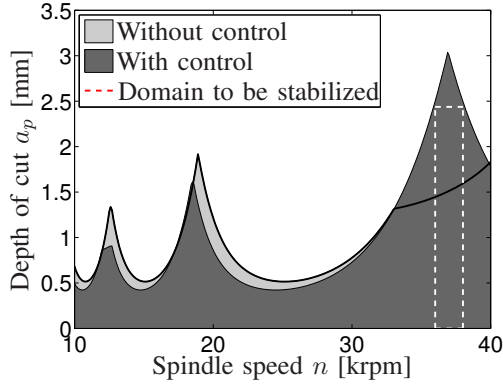


Fig. 6: Stability lobes diagram with and without static output feedback controller.

mance of the milling process for the desired uncertainties are given as $k_1 = 10072.7$ N/mm, $k_2 = -24.70908$ N/mm. From the contour plot, it can be seen that during the D-K-iteration the optimization converges to a (local) minimum, with $\sup_{\omega \in \mathbb{R}} \mu_{\Delta}(\mathbf{N}(i\omega)) < 1$, thereby guaranteeing robust performance of the milling process. During the second D-K-step, the optimization moves along a nonsmooth boundary of the objective function (a nonsmooth boundary can be distinguished from the nonsmoothness of a contour). In the same figure, the evaluation of the objective function by using the default BFGS algorithm for smooth functions (see [16, Chp. 6], invoked using `fminunc` from MATLAB) is given in gray. It can be seen that the standard BFGS algorithm gets stuck exactly at a nonsmooth boundary of the objective function. Stability lobes are determined for the case with and without the static output controller, using the technique as outlined in Section II-B. The SLDs are given in Figure 6. It can be seen that the controller synthesis algorithm has created a peak in the SLD exactly at the desired spindle speed range. Using the static output feedback controller, the maximal depth of cut indicated by the SLD $\bar{a}_{p,\max}$, in the considered spindle speed range, can be increased from $a_{p,\max} = 1.595$ mm to $a_{p,\max} = 3.037$ mm ($\bar{a}_p = 2.4375$ mm) which is an increase of more than 90%. Based on the discussion above, it can be concluded that the proposed controller synthesis strategy is able to effectively alter the SLD such that productivity is significantly increased. This is even accomplished for the least number of controller parameters.

VII. CONCLUSIONS

This paper proposes a control methodology to synthesize fixed structure controllers which guarantee robust stability and performance of the high-speed milling process (i.e. the avoidance of chatter in a predefined area of depth of cut a_p and spindle speed n and limitation of the required actuator forces). It is shown that the control synthesis problem can be cast into a nonsmooth constrained optimization problem which can be transformed to an unconstrained nonsmooth optimization problem using a penalty function. The unconstrained optimization problem is solved using D-K-iteration. The K-step is solved by utilizing a dedicated nonsmooth optimization algorithm based on bundle methods. The approach enables the design of relatively low-order controllers,

which is desirable from a practical point of view. The presented results illustrate the power of the proposed control methodology.

APPENDIX

The state-space matrices of the generalized plant, given in (14), are defined as follows:

$$\begin{aligned} \mathbf{A}_{P,0} &= \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{KS} \end{bmatrix}, \quad \mathbf{A}_{P,1} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \mathbf{B}_P &= \begin{bmatrix} -\frac{1}{2}\bar{a}_p \mathbf{B}_t \bar{\mathbf{H}} & \mathbf{B}_t \bar{\mathbf{H}} & \mathbf{0} & \bar{\mathbf{B}}_a \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{KS} \end{bmatrix}, \\ \mathbf{C}_{P,0} &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \frac{1}{2}\bar{a}_p \mathbf{C}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{KS} \\ \mathbf{C}_a & \mathbf{0} \end{bmatrix}, \quad \mathbf{C}_{P,1} = \begin{bmatrix} \mathbf{C}_t & \mathbf{0} \\ -\frac{1}{2}\bar{a}_p \mathbf{C}_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \mathbf{D}_P &= \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\frac{1}{2}\bar{a}_p \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{D}_{KS} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}, \end{aligned}$$

where $\mathbf{A}_0 := \mathbf{A} + \frac{1}{2}\bar{a}_p \mathbf{B}_t \bar{\mathbf{H}} \mathbf{C}_t$, $\mathbf{A}_1 := -\frac{1}{2}\bar{a}_p \mathbf{B}_t \bar{\mathbf{H}} \mathbf{C}_t$.

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