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Sampled-data Straight-line Path Following Control for Underactuated Ships

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***Abstract*— For sampled-data underactuated three degree-of-freedom ships, the design of discrete-time surge and yaw state feedback laws that make a ship track a desired straight-line path while maintaining a desired nonzero constant forward speed is considered. First a line-of-sight guidance algorithm is introduced and by using the Euler approximate model of tracking error dynamics, discrete-time surge and yaw state feedback laws are designed. Then by applying the nonlinear sampled-data control theory, the designed surge and yaw state feedback laws achieve sampled-data straight-line path following control. Experimental results are also given to show the efficiency of the design method.**

I. INTRODUCTION

Recently analysis and synthesis of the control problems for ships have been considered based on nonlinear continuous-time models and the design methods of continuous-time controllers have been mainly discussed (for details see [4] and references therein). For fully-actuated and underactuated ships, dynamic positioning control, trajectory tracking control, formation control and etc have been considered ([1]-[4]).

Practical and modern control systems usually use digital computers as discrete-time controllers with samplers (A/D converters) and zero-order holders (D/A converters) to control continuous-time systems. Such a control system is called a sampled-data system. Recently the framework to design controllers for nonlinear sampled-data systems based on discrete-time approximate models is proposed (for details see [6], [9], [10] and references therein). Several design methods have been also given to guarantee the stability of nonlinear sampled-data systems.

In this paper we consider sampled-data straight-line path following control for underactuated three degree-of-freedom (3DOF) ships, which have only two control inputs. We introduce a straight-line as a reference trajectory and a reference nonzero forward speed of a ship. Then the control objective is to design discrete-time state feedback laws which make a ship track a desired straight-line trajectory while maintaining a desired nonzero constant forward speed in the continuous-time sense. For a desired straight-line trajectory we introduce a line-of-sight (LOS) guidance algorithm [2]. Then we can define a cross-track error and tracking errors of the surge velocity and the yaw angle. First we consider two tracking error dynamics in surge and yaw and their Euler approximate models. We consider yaw and surge control, independently and we design uniformly globally asymptotically (UGA) stabilizing state feedback laws for each Euler approximate models. Next we consider the Euler approximate models of a cross-track error dynamics and the tracking error dynamics in yaw and surge. Since these Euler approximate models can be rewritten as a parameterized discrete-time cascade interconnection and the Euler approximate models in yaw and surge are UGA stabilized by the designed state feedback laws in the discrete-time sense, by [8] we can show that the Euler approximate model of a cross-track error dynamics is also UGA stabilized. Then by the results in [6], we can show that SPUA stability for the cascade interconnection of the continuous-time cross-track error dynamics and the tracking error dynamics in yaw and surge is achieved. Hence the designed discrete-time state feedback laws achieve sampled-data straight-line path following of underactuated ships in the continuous-time SPUA stable sense. By experimental results, we show the efficiency of the proposed design method.

Notation: A function $\alpha : \mathbf{R}_{\geq 0} \to \mathbf{R}_{\geq 0}$ is of class $\mathcal{K}$ ($\alpha \in \mathcal{K}$) if it is continuous, zero at zero and strictly increasing. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A function β: $\mathbf{R}_{\geq 0} \times \mathbf{R}_{\geq 0} \to \mathbf{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for any fixed $t \geq 0$, $\beta(\cdot, t) \in \mathcal{K}$ and for each fixed $s \geq 0$, $\beta(s, \cdot)$ is decreasing to zero as its argument tends to infinity [5]. A function γ: $\mathbf{R}_{\geq 0} \to \mathbf{R}_{\geq 0}$ is of class $\mathcal{N}$ if $\gamma(\cdot)$ is continuous and nondecreasing [8].

II. PRELIMINARY RESULTS

Consider the nonlinear sampled-data system

$$\dot{x}_c = f(x_c, u_c),\ x_c(0) = x_0 \tag{1}$$

where $x_c \in \mathbf{R}^n$ is the state, $u_c \in \mathbf{R}^m$ is the control input realized through a zero-order hold, i.e., $u_c(t) = u(k)$ for any $t \in [kT, (k+1)T)$ and $T > 0$ is a sampling period. Here we assume that for each initial condition and each constant control, there exists a unique solution of (1) defined on some bounded interval of the form $[0, s)$. We also assume that the sampling period is a design parameter and can be assigned arbitrarily. Let $x_e(k) = x_c(kT)$. Then the difference equations corresponding to the exact discrete-time model and the Euler approximate model of (1) are given by

$$\begin{aligned} x_e(k+1) &= F_T^e(x_e(k), u(k)),\ x_e(0) = x_0, && (2) \\ \xi(k+1) &= F_T^{Euler}(\xi(k), u(k)),\ \xi(0) = x_0 && (3) \end{aligned}$$

respectively, where $F_T^e(x_e(k), u(k)) = x_e(k) + \int_{kT}^{(k+1)T} f(x_c(s), u(k)) ds$ and $F_T^{Euler}(\xi(k), u(k)) =$

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$\xi(k) + Tf(\xi(k), u(k))$. To define the stability of the parameterized discrete-time models, we first consider the following discrete-time system

$$x(k+1) = F_T(x(k)),\ x(0) = x_0. \tag{4}$$

Definition 2.1: ([6], [9]) 1) The parameterized discrete-time system (4) is semiglobally practically uniformly asymptotically stable (SPUAS) if there exists $\beta \in \mathcal{KL}$ such that for any pair of strictly positive real numbers (D, d), there exists $T^* > 0$ such that $\| x(k) \| \leq \beta(\| x_0 \|, kT) + d$ for all $k \geq 0$, $x_0 \in \mathbf{R}^n$ with $\| x_0 \| \leq D$ and $T \in (0, T^*)$.
2) The parameterized discrete-time system (4) is uniformly globally asymptotically stable (UGAS) if there exists $\beta \in \mathcal{KL}$ such that there exists $T^* > 0$ such that $\| x(k) \| \leq \beta(\| x_0 \|, kT)$ for all $k \geq 0$, $x_0 \in \mathbf{R}^n$ and $T \in (0, T^*)$.
3) The parameterized discrete-time system (4) is uniformly globally bounded (UGB) if there exist $\alpha \in \mathcal{K}_\infty$ and $c \geq 0$ such that there exists $T^* > 0$ such that $\| x(k) \| \leq \alpha(\| x_0 \|) + c$ for all $x_0 \in \mathbf{R}^n$ and $T \in (0, T^*)$.
4) The parameterized discrete-time system (4) is Lyapunov-UGAS if there exist $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\alpha_3 \in \mathcal{K}$, $L \in \mathcal{N}$, $T^* > 0$ and a continuous function $V_T : \mathbf{R}^n \to \mathbf{R}_{\geq 0}$ for each $T \in (0, T^*)$ such that for all x, $y \in \mathbf{R}^n$ and $T \in (0, T^*)$

$$\alpha_1(\| x \|) \leq V_T(x) \leq \alpha_2(\| x \|), \tag{5}$$

$$V_T(F_T(x)) - V_T(x) \leq -T\alpha_3(\| x \|), \tag{6}$$

$$|V_T(x) - V_T(y)| \leq L \max\{\| x \|, \| y \|\} \| x - y \| . \tag{7}$$

Now we consider the design of stabilizing state feedback laws $u_T(x_e)$ for the exact discrete-time model (2). By [9] the Euler approximate model (3) with $u(k) = u_T(\xi(k))$ is one-step consistent with the exact discrete-time model (2) with $u(k) = u_T(x_e(\xi))$. Moreover, if $f(x, u)$ and a parameterized state feedback law $u_T(x)$ are locally Lipschitz for any $T \in (0, T^*)$, the Euler approximate model (3) with $u(k) = u_T(\xi(k))$ is multi-step consistent with the exact discrete-time model (2) with $u(k) = u_T(x_e(k))$.

Theorem 2.1: ([6], [9]) If the Euler approximate model (3) with $u(k) = u_T(\xi(k))$ is multi-step consistent with the exact discrete-time model (2) with $u(k) = u_T(x^e(k))$ and the Euler approximate model (3) with $u(k) = u_T(\xi(x))$ is UGAS, then the exact discrete-time model (2) with $u(k) = u_T(x^e(k))$ is SPUAS.

Remark 2.1: If F_T^e is locally Lipschitz, then there exists $T^* > 0$ such that for any $T \in (0, T^*)$, u_T which SPUA stabilizes the exact discrete-time model (2), SPUA stabilizes (1), i.e., there exists $\beta \in \mathcal{KL}$ such that for any strictly positive real numbers (D, d), there exists $T^* > 0$ such that for any $T \in (0, T^*)$ and any $x_c(0) \in \mathbf{R}^n$ satisfying $\| x_c(0) \| \leq D$, a solution $x_c(t)$ of the system $\dot{x}_c = f(x_c, u_T(x_c(kT))$ for any $t \in [kT, (k+1)T)$ satisfies $\| x_c(t) \| \leq \beta(\| x_c(0) \|, t) + d$ ([6], [9], [10]). In this case we say that the system $\dot{x}_c = f(x_c, u_T(x_c(kT)))$ for any $t \in [kT, (k+1)T)$ is SPUAS in the continuous-time sense or $x_c(t) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense.

Consider the parameterized discrete-time cascade interconnected system

$$x(k+1) = f_T(x(k), z(k)), \tag{8}$$

$$z(k+1) = l_T(z(k)) \tag{9}$$

which corresponds to a closed-loop system of the exact discrete-time model or the Euler approximate model of

$$\dot{x}_c = f(x_c, z_c), \quad \dot{z}_c = l(z_c, u_c) \tag{10}$$

and a state feedback law $u_c(t) = u_T(z_c(kT))$ for any $t \in [kT, (k+1)T)$ where x, $x_c \in \mathbf{R}^n$, z, $z_c \in \mathbf{R}^{\bar{n}}$. For the system (8), we assume

A1: There exist $\gamma_2 \in \mathcal{N}$, $\gamma_1, \gamma_3 \in \mathcal{K}_\infty$ and $T^* > 0$ such that for all $\xi = [x^T\ z^T]^T \in \mathbf{R}^{n+\bar{n}}$ and $T \in (0, T^*)$ we have $\| f_T(x, z) \| \leq \gamma_1(\| \xi \|)$ and $\| f_T(x, z) - f_T(x, 0) \| \leq T\gamma_2(\| x \|)\gamma_3(\| z \|)$.

Theorem 2.2: ([8]) Assume **A1**. Then the system (8) and (9) is UGAS if the following conditions hold
1) The system $x(k+1) = f_T(x(k), 0)$ is Lyapunov-UGAS,
2) The system (9) is UGAS,
3) The system (8) and (9) is UGB.

The sufficient condition for UGB of the system (8) and (9) is given by the following result.

Proposition 2.1: ([7], [8]) Consider the system (8) with input z. Suppose that there exist $\tilde{\alpha}_1, \tilde{\alpha}_2, \varphi \in \mathcal{K}_\infty$, $\tilde{\gamma}_1, \tilde{\gamma}_2 \in \mathcal{N}$, $T^* > 0$, $c \geq 0$ and for each $T \in (0, T^*)$ there exists $\tilde{V}_T : \mathbf{R}^n \to \mathbf{R}_{\geq 0}$ such that for all $x \in \mathbf{R}^n$, $z \in \mathbf{R}^{\bar{n}}$ and $T \in (0, T^*)$ we have

$$\tilde{\alpha}_1(\| x \|) \leq \tilde{V}_T(x) \leq \tilde{\alpha}_2(\| x \|) + c, \tag{11}$$

$$\tilde{V}_T(f_T(x, z)) - \tilde{V}_T(x) \leq T\tilde{\gamma}_1(\| z \|)\varphi(\tilde{V}_T(x)) + T\tilde{\gamma}_2(\| z \|), \tag{12}$$

$$\int_1^\infty \frac{ds}{\varphi(s)} = \infty. \tag{13}$$

If, furthermore, the solutions of (9) satisfy the summability condition

$$T \sum_{k=0}^{\infty} \mu(\| z(k) \|) \leq \rho(\| z(0) \|) \tag{14}$$

with some $\rho \in \mathcal{K}_\infty$ and $\mu(s) = \tilde{\gamma}_1(s) + \tilde{\gamma}_2(s)/\varphi(1)$, then the system (8) and (9) is UGB.

III. Sampled-data Straight-line Path Following Control for Underactuated Ships

A. Model of a Ship and a Problem Formulation

We first introduce notation to describe the equation of motion of a ship. Let $[x_c \quad y_c]^T$ and ψ_c be the inertial position and the yaw angle (orientation) of a ship, respectively in Cartesian coordinate system (Figure 1) and let u_c, v_c and r_c be the linear velocities in surge, sway and the angular velocity in yaw, respectively, decomposed in the body-fixed coordinate system [4]. Let $\xi_c = [x_c \quad y_c \quad \psi_c]^T$ and $\nu_c = [u_c \quad v_c \quad r_c]^T$. Consider the following simplified three degree-of-freedom (3DOF) model of a ship:

$$\dot{\xi}_c = R(\psi_c)\nu_c, \tag{15}$$

$$M\dot{\nu}_c + C(\nu_c)\nu_c + D\nu_c = f \tag{16}$$

where $f = [f_u \quad f_v \quad f_r]^T$, f_u is the control force in surge, f_v is the rudder force in sway, f_r is the rudder moment in yaw, $R(\psi_c) = \begin{bmatrix} \bar{R}(\psi_c) & 0 \\ 0 & 1 \end{bmatrix}$ with $\bar{R}(\psi_c) = \begin{bmatrix} \cos\psi_c & -\sin\psi_c \\ \sin\psi_c & \cos\psi_c \end{bmatrix}$ is the rotation matrix in yaw, $M = \begin{bmatrix} m_{11} & 0 \\ 0 & M_2 \end{bmatrix} > 0$ with $M_2 = \begin{bmatrix} m_{22} & m_{23} \\ m_{23} & m_{33} \end{bmatrix}$ is the inertia matrix including hydrodynamic added inertia, $D = \begin{bmatrix} d_{11} & 0 \\ 0 & D_2 \end{bmatrix}$ with $D_2 = \begin{bmatrix} d_{22} & d_{23} \\ d_{32} & d_{33} \end{bmatrix}$ is the linear damping matrix and

$$C(\nu_c) = \begin{bmatrix} 0 & 0 & -m_{22}v_c - m_{23}r_c \\ 0 & 0 & m_{11}u_c \\ m_{22}v_c + m_{23}r_c & -m_{11}u_c & 0 \end{bmatrix}$$

is the Coriolis-centripetal matrix. Here $f_v = f_v(\nu_c, \delta_c)$ and $f_r = f_r(\nu_c, \delta_c)$ are functions of ν_c and δ_c where δ_c is the rudder deflection. The model (15) and (16) is equivalently rewritten as or equivalently

$$\begin{aligned} \dot{x}_c &= u_c\cos\psi_c - v_c\sin\psi_c, & (17)\\ \dot{y}_c &= u_c\sin\psi_c + v_c\cos\psi_c, & (18)\\ \dot{\psi}_c &= r_c, & (19)\\ \dot{u}_c &= d_u(\nu_c) + \tau_c, & (20)\\ \dot{v}_c &= d_v(\nu_c) + \lambda_{v1}(\nu_c)\delta_c + \lambda_{v2}(\nu_c), & (21)\\ \dot{r}_c &= d_r(\nu_c) + \lambda_{r1}(\nu_c)\delta_c + \lambda_{r2}(\nu_c) & (22) \end{aligned}$$

where $[\tau_c \quad \lambda_{v1}(\nu_c)\delta_c + \lambda_{v2}(\nu_c) \quad \lambda_{r1}(\nu_c)\delta_c + \lambda_{r2}(\nu_c)]^T = M^{-1}f$, $[d_u(\nu_c) \quad d_v(\nu_c) \quad d_r(\nu_c)]^T = -M^{-1}[C(\nu_c) + D]\nu_c$ and λ_* satisfy $\lambda_*(0) = 0$, $* = v1,\ v2,\ r1$ and $r2$. Without loss of generality we can assume $\lambda_{r1}(\nu_c) \neq 0$ for any ν_c.

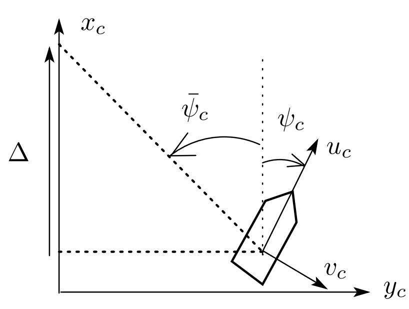

Fig. 1. A coordinate system and the LOS angle

We consider sampled-data straight-line path following control of underactuated ships where the control inputs are realized through a zero-order hold, i.e., $\tau_c(t) = \tau(k)$, $\delta_c(t) = \delta(k)$ for any $t \in [kT, (k+1)T)$ and T is a sampling period. We set the x-axis as a desired straight-line trajectory and the y-axis is chosen to complete the right-handed coordinate system (Figure 1). In this case, the y-position of a ship is the minimal distance between the position of a ship and the straight-line trajectory, which is called a cross-track error [2]. Let $\bar{u} > 0$ be a desired constant forward speed and assume that the states of a ship at each sampling time, i.e., $\xi_c(kT)$, $\nu_c(kT)$, $k = 0, 1, 2, ...$ are available to control a ship. Then the control objective is to design discrete-time state feedback laws which make $(y_c(t), \psi_c(t), u_c(t) - \bar{u}, v_c(t), r_c(t)) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense.

B. Sampled-data Straight-line Path Following Control

In this subsection we consider sampled-data straight-line path following control based on the line-of-sight (LOS) guidance algorithm and the nonlinear sampled-data control theory. The LOS guidance is often used for path control of ships [4]. We pick a point that lies a constant distance $\Delta > 0$ ahead of a ship, along the desired straight-line trajectory. The line-of-sight is the line joining a ship and a selected point. The angle describing the orientation of the line-of-sight is called the LOS angle and $\Delta > 0$ is called the look-ahead distance (Figure 1). The LOS angle is given by

$$\bar{\psi}_c(t) = \tan^{-1}\left(-\frac{y_c(t)}{\Delta}\right) \qquad (23)$$

([1], [2]). We introduce the following assumptions [2]:
B1: The surge velocity $u_c(t)$ satisfies $0 < U_{min} \leq u_c(t) \leq U_{\max}$ for any $t \geq 0$ where U_{min} and U_{max} are the minimal and the maximal surge velocities, respectively.
B2: For some $C_v > 0$ the sway velocity $v_c(t)$ satisfies $|v_c(t)| \leq \min\{U_{max}, C_v U_{max}|r_c(t)|\}$ for any $t \geq 0$.
B3: $\Delta > \max\{C_v U_{max}, |\lambda_{v1}(\nu_c)/\lambda_{r1}(\nu_c)|\}$ for any ν_c.
The assumption **B1** is needed to make the system controllable in the sway direction. The assumption $|v_c(t)| \leq U_{max}$ in **B2** is valid for most underactuated ships, since the hydrodynamic damping in the sway direction is usually much larger than the hydrodynamic damping in the surge direction. The assumption $|v_c(t)| \leq C_v U_{max}|r_c(t)|$ in **B2** implies that the convergence of the angular velocity to zero gives the convergence of the sway velocity to zero. This assumption is valid in an ideal situation of no natural disturbances such as winds and currents. Due to natural disturbances an offset of the sway velocity remains even if the angular velocity converges to zero in practical situations (see Figure 4 in Section IV).

Let $\tilde{u}_c(t) = u_c(t) - \bar{u}$ and $\tilde{\psi}_c(t) = \psi_c(t) - \bar{\psi}_c(t)$. Then using (18)-(23), we have

$$\begin{aligned} \dot{\tilde{u}}_c &= d_u(\nu_c) + \tau_c, & (24)\\ \dot{\tilde{\psi}}_c &= r_c - \bar{r}_c - \hat{k}_\psi\tilde{\psi}_c = -\hat{k}_\psi\tilde{\psi}_c + \tilde{r}_c, & (25)\\ \dot{\tilde{r}}_c &= d_r(\nu_c) + \lambda_{r2}(\nu_c) - \kappa(y_c, \psi_c, \nu_c, \bar{\psi}_c) & \\ &\quad + \Gamma(y_c)\tau_c\sin\psi_c & \\ &\quad + [\lambda_{r1}(\nu_c) + \lambda_{v1}(\nu_c)\Gamma(y_c)\cos\psi_c]\delta_c & (26) \end{aligned}$$

where $\tilde{r}_c = r_c - \bar{r}_c$, $\Gamma(y_c) = \Delta(\Delta^2 + y_c^2)^{-1}$,

$$\begin{aligned} \bar{r}_c &= -\hat{k}_\psi\tilde{\psi}_c + \dot{\bar{\psi}}_c = -\hat{k}_\psi\tilde{\psi}_c - \Gamma(y_c)\dot{y}_c, & (27)\\ \kappa &= -\hat{k}_\psi\dot{\tilde{\psi}}_c + \frac{2}{\Delta}\Gamma^2 y_c(u_c\sin\psi_c + v_c\cos\psi_c)^2 & \\ &\quad -\Gamma\{d_u\sin\psi_c + [d_v + \lambda_{v2}]\cos\psi_c & \\ &\qquad + (u_c\cos\psi_c - v_c\sin\psi_c)r_c\} & \end{aligned}$$

and $\hat{k}_\psi > 0$ is a design parameter which is assigned later.

Let $z_c = \frac{1}{2}y_c^2$. By the assumption **B2**, (27) and the definition of $\tilde{r}_c$, we have

$$|v_c| \le C_v U_{max}\left(|\tilde{r}_c| + \hat{k}_\psi|\tilde{\psi}_c| + \frac{1}{\Delta}|\dot{y}_c|\right)$$

and by (18) we obtain

$$\dot{z}_c \le y_c u_c \sin\psi_c + C_v U_{max}|y_c|\left(|\tilde{r}_c| + \hat{k}_\psi|\tilde{\psi}_c|\right) + \frac{C_v U_{max}}{\Delta}|\dot{z}_c|.$$

Since

$$u_c \sin\psi_c = \bar{u}\sin\bar{\psi}_c + \tilde{u}_c \sin(\tilde{\psi}_c + \bar{\psi}_c) + \bar{u}\left\{\cos\bar{\psi}_c \frac{\sin\tilde{\psi}_c}{\tilde{\psi}_c} + \sin\bar{\psi}_c \frac{\cos\tilde{\psi}_c - 1}{\tilde{\psi}_c}\right\}\tilde{\psi}_c,$$

we have

$$\dot{z}_c \le y_c \bar{u}\sin\bar{\psi}_c + |y_c|q(\zeta_c) + \frac{C_v U_{max}}{\Delta}|\dot{z}_c|$$

where $\zeta_c = [\,\tilde{u}_c \quad \tilde{\psi}_c \quad \tilde{r}_c\,]^T$ and

$$q(\zeta_c) = |\tilde{u}_c| + (2\bar{u} + \hat{k}_\psi C_v U_{max})|\tilde{\psi}_c| + C_v U_{max}|\tilde{r}_c|.$$

Using $\sin\bar{\psi}_c = -y_c/\sqrt{\Delta^2 + y_c^2}$, we obtain

$$\dot{z}_c \le -\frac{\bar{u}}{\sqrt{\Delta^2 + y_c^2}}y_c^2 + |y_c|q(\zeta_c) + \frac{C_v U_{max}}{\Delta}|\dot{z}_c|$$

and

$$\left\{1 - \frac{C_v U_{max}}{\Delta}\mathrm{sgn}(\dot{z}_c)\right\}\dot{z}_c \le -\frac{2\bar{u}}{\sqrt{\Delta^2 + 2z_c}}z_c + \sqrt{2z_c}\,q(\zeta_c)$$

where $\mathrm{sgn}(z)$ is a sign function. Let $\Pi_1 = (1 + C_v U_{max}/\Delta)^{-1}$ and $\Pi_2 = (1 - C_v U_{max}/\Delta)^{-1}$. Then by **B3**, $\Pi_2 > \Pi_1 > 0$ and we obtain

$$\dot{z}_c \le -\frac{2\bar{u}\Pi_1}{\sqrt{\Delta^2 + 2z_c}}z_c + \Pi_2\sqrt{2z_c}\,q(\zeta_c). \quad (28)$$

By the comparison lemma [5], a solution $\eta_c(t)$ of the differential equation

$$\dot{\eta}_c = -\frac{2\bar{u}\Pi_1}{\sqrt{\Delta^2 + 2\eta_c}}\eta_c + \Pi_2\sqrt{2\eta_c}\,q(\zeta_c) \quad (29)$$

satisfies $0 \le z_c(t) \le \eta_c(t)$ if $0 \le z_c(0) \le \eta_c(0)$. Hence if control inputs (τ_c, δ_c) stabilize the system (24)-(26) and (29) with $\eta_c(0) = \frac{1}{2}y_0^2$, then (τ_c, δ_c) stabilize the system (18) and (24)-(26) with $y_c(0) = y_0$, since $z_c = \frac{1}{2}y_c^2$ and $0 \le z_c(t) \le \eta_c(t)$ for any $t \ge 0$. Summing up we have the following result.

Lemma 3.1: Assume **B1-B3**. Then if the control inputs $(\tau_c(t), \delta_c(t)) = (\tau(k), \delta(k))$ for any $t \in [kT, (k+1)T)$ SPUA stabilize the system (24)-(26) and (29) with $\eta_c(0) = \frac{1}{2}y_0^2$, then $(\tau_c(t), \delta_c(t)) = (\tau(k), \delta(k))$ for any $t \in [kT, (k+1)T)$ SPUA stabilize the system (18) and (24)-(26) with $y_c(0) = y_0$.

Since $(\tau_c(t), \delta_c(t)) = (\tau(k), \delta(k))$ for any $t \in [kT, (k+1)T)$, the Euler approximate model of the system (24)-(26) and (29) is given by

$$\eta(k+1) = f_T(\eta(k), \zeta(k)), \quad (30)$$
$$\tilde{u}(k+1) = \tilde{u}(k) + T[d_u(\nu(k)) + \tau(k)], \quad (31)$$
$$\tilde{\psi}(k+1) = (1 - T\hat{k}_\psi)\tilde{\psi}(k) + T\tilde{r}(k), \quad (32)$$
$$\tilde{r}(k+1) = \tilde{r}(k) + T\{d_r(\nu) + \lambda_{r2}(\nu) - \kappa(y, \psi, \nu, \bar{\psi}) + \Gamma\tau\sin\psi + [\lambda_{r1}(\nu) + \lambda_{v1}(\nu)\Gamma(y)\cos\psi]\delta\}(k) \quad (33)$$

where $\zeta = [\,\tilde{u} \quad \tilde{\psi} \quad \tilde{r}\,]^T$, $f_T(\eta, \zeta) = F_T(\eta) + TG(\eta, \zeta)$, $G(\eta, \zeta) = G_1(\eta, \zeta)G_2(\zeta)$, $G_1(\eta, \zeta) = \sqrt{2\eta}q^{\frac{1}{2}}(\zeta)$, $G_2(\zeta) = \Pi_2 q^{\frac{1}{2}}(\zeta)$ and $F_T(\eta) = (1 - T\cdot 2\bar{u}\Pi_1/\sqrt{\Delta^2 + 2\eta})\eta$. For the system (31)-(33) we consider the following parameterized surge and yaw discrete-time state feedback laws

$$\tau_T(k) = -\frac{k_u}{T + c_u}\tilde{u}(k) - d_u(\nu(k)) \quad (34)$$

$$\delta_T(k) = \frac{1}{(\lambda_{r1}(\nu) + \lambda_{v1}(\nu)\Gamma(y)\cos\psi)(k)} \times[-d_r(\nu) - \lambda_{r2}(\nu) + \kappa(y, \psi, \nu, \bar{\psi}) - \frac{k_r}{T + c_r}\tilde{r} - \Gamma(y)\tau\sin\psi](k) \quad (35)$$

where k_u, k_r, c_u, $c_r > 0$ are design parameters which are assigned later. Let $\hat{k}_\psi = k_\psi/(T + c_\psi)$ where k_ψ, $c_\psi > 0$. Note that the control input $\delta_T(k)$ given by (35) is always calculated by the assumptions **B1** and **B3**. Then the closed-loop system (31)-(35) is given by

$$\zeta(k+1) = \begin{bmatrix} 1 - \frac{Tk_u}{T+c_u} & 0 & 0 \\ 0 & 1 - \frac{Tk_\psi}{T+c_\psi} & T \\ 0 & 0 & 1 - \frac{Tk_r}{T+c_r} \end{bmatrix}\zeta(k) \quad (36)$$

and we have the following result. A simple proof is omitted.

Lemma 3.2: There exists $T^* > 0$ such that the closed-loop system (36) is globally exponentially stable for any $T \in (0, T^*)$, k_u, k_ψ, $k_r \in (0, 2]$ and c_u, c_ψ, $c_r > 0$.

Now we shall show that the designed surge and yaw state feedback laws SPUA stabilize the sampled-data interconnected system (24)-(26) and (29) by Theorems 2.1 and 2.2. Proofs of the following lemma and theorem are given in Appendix.

Lemma 3.3: There exists $T^* > 0$ such that the closed-loop system (30) and (36) is globally asymptotically stable for any $T \in (0, T^*)$, k_u, k_ψ, $k_r \in (0, 2]$ and c_u, c_ψ, $c_r > 0$.

Since the system (17)-(22) and the designed control laws (34) and (35) are locally Lipschitz, by Lemmas 3.1-3.3 and Theorem 2.1, we have the following result.

Theorem 3.1: Assume **B1-B3**. There exists $T^* > 0$ such that for any $T \in (0, T^*)$ the surge and the yaw control laws

$$\tau_T(k) = -\frac{k_u}{T + c_u}\tilde{u}_c(kT) - d_u(\nu_c(kT)), \quad (37)$$

$$\delta_T(k) = \frac{1}{(\lambda_{r1}(\nu_c) + \lambda_{v1}(\nu_c)\Gamma(y_c)\cos\psi_c)(kT)} \times [(-d_r(\nu_c) - \lambda_{r2}(\nu_c) + \kappa(y_c, \psi_c, \nu_c, \bar{\psi}_c))(kT) \quad (38)$$

$$-\frac{k_r}{T+c_r}\tilde{r}_c(kT) - \Gamma(y_c(kT))\tau(k)\sin\psi_c(kT)]$$

with $k_u,\ k_\psi,\ k_r \in (0,2]$ and $c_u,\ c_\psi,\ c_r > 0$ achieve $(y_c(t), \psi_c(t), u_c(t) - \bar{u}, v_c(t), r_c(t)) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense for the sampled-data system (17)-(22) with $(\tau_c(t), \delta_c(t)) = (\tau_T(k), \delta_T(k))$ for any $t \in [kT, (k+1)T)$.

Remark 3.1: We can also show that an emulation of continuous-time state feedback laws

$$\tau(k) = -k_u\tilde{u}_c(kT) - d_u(\nu_c(kT)), \quad (39)$$

$$\delta(k) = \frac{1}{(\lambda_{r1}(\nu_c) + \lambda_{v1}(\nu_c)\Gamma(y_c)\cos\psi_c)(kT)} \times[(-d_r(\nu_c) - \lambda_{r2}(\nu_c) + \kappa(y_c, \psi_c, \nu_c, \bar{\psi}_c))(kT) -k_r\tilde{r}_c(kT) - \Gamma(y_c(kT))\tau(k)\sin\psi_c(kT)] \quad (40)$$

given by [1] and [2], achieve the sampled-data straight-line path following control of underactuated ships in the continuous-time SPUAS sense for sufficiently small $T > 0$.

The designed parameterized state feedback laws (37) and (38) are usually more useful than an emulation of continuous-time state feedback laws (39) and (40). For example, the choice of the design parameters k_u, $\hat{k}_\psi$ and k_r in (39) and (40) restricts the maximal sampling period T^* or depends on T^*, but the parameters k_u, k_ψ, k_r, c_u, c_ψ and c_r in (37) and (38) can be chosen independent of T^* and hence a longer sampling period can be usually used for the parameterized state feedback laws. Furthermore, the parameterized controllers usually give larger regions of attraction and (or) better performances than an emulation of continuous-time controllers ([6], [9]) and hence the proposed design method based on the Euler approximate models is more beneficial than a combination of an emulation and a standard continuous-time design method.

IV. Experimental Result

Fig. 2. A real ship (Yamaha Motor Co., Ltd) for experiments

We apply the designed state feedback laws to a real ship (Yamaha Motor Co., Ltd) with an electric thruster and a rudder in Figure 2. The size of the ship is about 3 meters long and 1 meter wide. An electric thruster and a rudder are attached at the stern. We have programmed the designed control laws by using MATLAB/Simulink and we have implemented them to dSPACE MicroAutoBox 1401/1501 (MABX). Real-Time Kinematic Global Positioning System (RTK-GPS), Inertial Measurement Unit and Magnetic Compass are used to measure the position and the velocity of the ship. Each measurement and control units are controlled by Electronic Control Units (ECUs) locally. The experiment is executed under a weak current and a strong wind of 6 (m/s).

We set a sampling period $T = 200$ (msec), a look-ahead distance $\Delta = 4.5$ (m) and $\bar{u} = 1$ (m/s) as a desired surge velocity. We apply the state feedback laws (37) and (38) with $k_u = 1$, $k_\psi = 0.5$, $k_r = 0.1$ and $c_u = c_\psi = c_r = 0.3$ to the ship. Let $\xi_c(0) = [0 \quad 10 \quad \pi/2]^T$ and $\nu_c(0) = [0.5 \quad 0 \quad 0]^T$ be an initial condition. Then the position of the ship with the desired straight-line trajectory (the red solid line) is shown in Figure 3 where symbols like a ship express the position and the attitude of the ship at every 4 (sec). The time responses of the surge, sway velocities and the angular velocity in yaw are shown in Figure 4 where the red line expresses the desired surge velocity.

The designed state feedback laws make the underactuated ship follow a straight-line path with a suitable attitude (Figure 3). Due to a strong wind which is facing to the ship and a weak current, small offsets of the error of the surge velocity, i.e., $u_c(t) - \bar{u}$ and the sway velocity remain (Figure 4). But the convergence of the angular velocity in yaw to zero is achieved. Hence the designed state feedback laws are enough useful for sampled-data straight-line path following control of underactuated ships from a practical point of view.

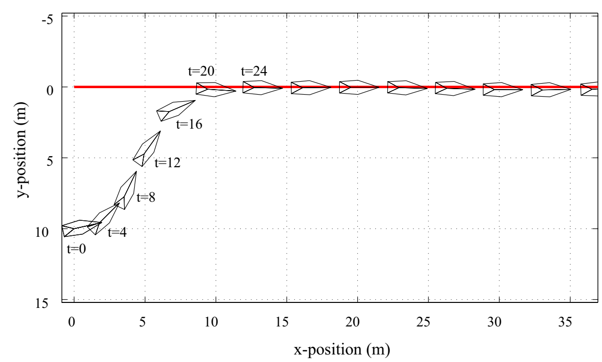

Fig. 3. The position and the attitude of the ship at every 4 (sec) with a desired straight-line

V. Conclusions

We have considered sampled-data straight-line path following control for underactuated 3DOF ships. We have used a LOS guidance algorithm to design control laws, which make a ship track a desired straight-line path while maintaining a desired nonzero constant forward speed. By using the Euler approximate models of the error dynamics in surge and yaw, we have designed discrete-time surge and yaw control laws, independently. Then by applying the nonlinear sampled-data control theory, we have shown that sampled-data straight-line path following control is achieved by the designed control laws. Experimental results have been also given to show the efficiency of the proposed design method.

Appendix

Proof of Lemma 3.3: It is enough to show that the assumption **A1** and the conditions 1)-3) in Theorem 2.2 are

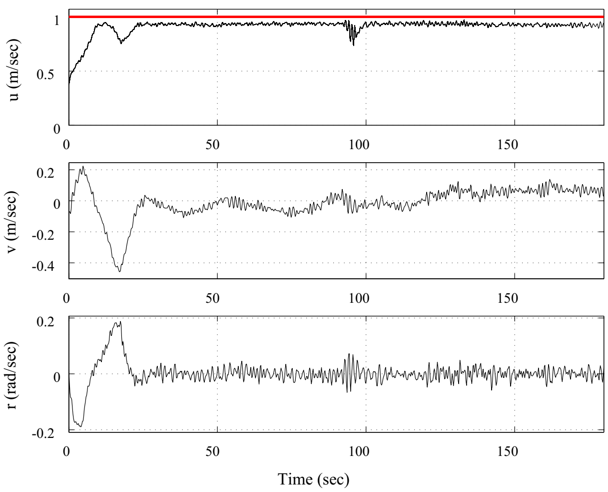

Fig. 4. Time responses of the surge and sway velocities u, v (m/s) and the angular velocity of yaw r (rad$/s$)

satisfied for the system (30) and (36). Let $T^* = \Delta/(2\bar{u}\Pi_1)$. Then we have $0 < T \cdot 2\bar{u}\Pi_1/\sqrt{\Delta^2+2\eta} < 1$ for any $T \in (0, T^*)$.

First we shall show that the assumption **A1** is satisfied for the system (30). In fact, $|f_T(\eta,\zeta)| \le |\eta| + T\Pi_2\sqrt{2\eta}q(\zeta) \le \gamma_1(\| \ \bar{\eta} \ \|)$ and $|f_T(\eta,\zeta) - f_T(\eta,0)| = T\Pi_2\sqrt{2\eta}q(\zeta) \le T\gamma_2(\eta)\gamma_3(\| \ \zeta \ \|)$ for any $\bar{\eta} = [\eta \quad \zeta^T]^T$ where $\gamma_3(s) = \Theta s$, $\gamma_2(s) = \Pi_2\sqrt{2s}$, $\gamma_1(s) = s + T^*\gamma_2(s)\gamma_3(s)$ and $\Theta = 1 + 2\bar{u} + (1+k_\psi)C_vU_{max}$. Obviously, γ_1, γ_2, $\gamma_3 \in \mathcal{K}_\infty$ and hence **A1** is satisfied for the system (30).

Note that the system $\eta(k+1) = F_T(\eta(k))$ is UGAS and $\eta(k) \ge 0$ for any $k \ge 0$ and $\eta(0) \ge 0$. Let $V_T(\eta) = \frac{1}{2}\eta^2$. Then we have

$$V_T(F_T(\eta)) - V_T(\eta) \le -T\frac{\bar{u}\Pi_1}{\sqrt{\Delta^2+2\eta}}\eta^2 \in \mathcal{K}_\infty.$$

We also have $|V_T(r) - V_T(s)| \le \frac{1}{2}\{|r|+|s|\}|r-s| \le \max\{|r|,|s|\}\,|r-s|$ and hence the system $\eta(k+1) = F_T(\eta(k))$ is Lyapunov UGAS.

By Lemma 3.2 it is obvious that the condition 2) in Theorem 2.2 is satisfied. By Proposition 2.1, we shall show that the condition 3) is satisfied. Let $\tilde{V}_T(\eta) = |\eta|$. Then the condition (11) is satisfied with $\tilde{\alpha}_1(s) = \tilde{\alpha}_2(s) = s$ and $c = 0$. We also have

$$\tilde{V}_T(f_T(\eta,\zeta)) - \tilde{V}_T(\eta) \le T\Theta \parallel \zeta \parallel [\tilde{V}_T(\eta) + \frac{1}{2}\Pi_2^2].$$

Let $\varphi(s) = s$, $\tilde{\gamma}_1(s) = \Theta s$ and $\tilde{\gamma}_2(s) = \frac{1}{2}\Pi_2^2\Theta s$. Then φ, $\tilde{\gamma}_1$, $\tilde{\gamma}_2 \in \mathcal{K}_\infty$ and we obtain (12). Since $\varphi(s) = s$, (13) is satisfied. Since $\mu(s) = \tilde{\gamma}_1(s) + \tilde{\gamma}_2(s)/\varphi(1) = (1+2\Pi_2^2)\Theta s$ is linear and the system (36) is globally exponentially stable for any $T \in (0, T^*)$, (14) is satisfied. Hence by Proposition 2.1, the condition 3) in Theorem 2.2 is satisfied.

Consequently by Theorem 2.2, the closed-loop system (30) and (36) is UGAS. ∎

Proof of Theorem 3.1: First note that there exists $T_1^* > 0$ such that the assumption **B1** is satisfied for any $u_c(0) \in [U_{min}, U_{max}]$, since the surge control law (37) SPUA stabilizes the surge dynamics (20) in the continuous-time sense. By Lemmas 3.1, 3.3 and Theorem 2.1, there exists $0 < T_2^* \le T_1^*$ such that for any $T \in (0, T_2^*)$ the surge and yaw control laws (37), (38) achieve $(y_c(t), \zeta_c(t)) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense for the sampled-data system (17)-(22). Then by (23), we also have $\bar{\psi}_c(t) \to 0$ (and hence $\psi_c(t) \to 0$) as $t \to \infty$ in the continuous-time SPUAS sense. Hence there exists $\beta_1 \in \mathcal{KL}$ such that for any positive numbers (D_1, d_1), there exists $0 < T^* \le T_2^*$ such that for any $\chi_c(0)$ with $\parallel \chi_c(0) \parallel \le D_1$ and any $T \in (0, T^*)$, $\parallel \chi_c(t) \parallel \le \beta_1(\parallel \chi_c(0) \parallel, t) + d_1$ where $\chi_c = [\, y_c \quad \tilde{u}_c \quad \psi_c \quad \tilde{r}_c \,]^T$.

To complete the proof, it is enough to show that $r_c(t)$, $v_c(t) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense. By (18) and (27), we have

$$|r_c| \le |\tilde{r}_c| + k_\psi|\tilde{\psi}_c| + \frac{1}{\Delta}[|u_c||\sin\psi_c| + |v_c|] \qquad (41)$$

and using (41) with the assumptions **B2** and **B3**, we obtain

$$\begin{aligned} |v_c| \quad \le \quad & \Pi_2C_vU_{\max}|\tilde{r}_c| + k_\psi\Pi_2C_vU_{\max}|\tilde{\psi}_c| \\ & + \frac{\Pi_2C_vU_{\max}}{\Delta}|u_c||\sin\psi_c|. \end{aligned} \qquad (42)$$

Applying $|\tilde{r}_c(t)| \le \beta_1(\parallel \xi_c(0) \parallel, t) + d_1$, $|u_c(t)| \le \bar{u} + \beta_1(\parallel \xi_c(0) \parallel, t) + d_1$ and $|\sin\psi_c(t)| \le |\psi_c(t)| \le \beta_1(\parallel \xi_c(0) \parallel, t) + d_1$ to (42), there exists $\beta \in \mathcal{KL}$ and $d > 0$ such that $|v_c(t)| \le \beta(\parallel \xi_c(0) \parallel, t) + d$ where $\beta(s,t) = \Pi_2C_vU_{\max}[1 + k_\psi + \frac{1}{\Delta}(2d_1 + \bar{u} + \beta_1(s,t))]\beta_1(s,t)$ and $d = \Pi_2C_vU_{\max}[1 + k_\psi + \frac{1}{\Delta}(\bar{u} + d_1)]d_1$. This implies that $v_c(t) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense. Similarly using (41) and (42), we can show $r_c(t) \to 0$ as $t \to \infty$ in the continuous-time SPUAS sense. Hence we have the assertion. ∎

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