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Safe LPV Controller Switching

Klaus Trangbaek

Abstract— **Before switching to a new controller it is crucial to assure that the new closed loop will be stable. In this paper it is demonstrated how stability can be checked with limited measurement data available from the current closed loop.**

The paper extends an existing method to linear parameter varying plants and controllers. Rather than relying on frequency domain methods as done in the LTI case, it is shown how to use standard LPV system identification methods.

It is furthermore shown how to include model uncertainty to robustify the results. By appropriate filtering, it is only necessary to evaluate the worst case gain of the additive uncertainty, making the analysis relatively simple.

I. Introduction

Changes to plant behaviour due to reconfiguration or gradual wear may lead to a desire to reconfigure the control system. In many cases it is not desirable to remove the current controller in order to obtain the data needed for identifying a new plant model. Thus we are faced with deciding on a new control strategy based on only closed loop data.

There exist iterative methods for improving controller performance using closed loop data, see e.g. [1] and references, but in order to assure stability of the new closed loop, it is necessary to obtain a full model of either the current closed loop or of the plant, and the requirements on the data can therefore be greater than what was required for the controller design.

In [2], [3], a method is devised for analysing the stability of the new closed loop by looking at the inverse transfer function from an excitation signal to a filtered output. Due to the particular choice of filters, it is possible to analyse stability looking only at a limited frequency range, thereby making the identification problem more tractable. In [4] the method was extended to assess performance also.

Since the above papers use frequency domain methods it cannot be directly translated to time varying systems. In [5], extensions are made to handle nonlinearities by conditions on the maximum gain of these, but for linear time-varying systems that would be an unnecessarily conservative approach.

In this paper we show how the same basic scheme can be applied to linear time varying systems, in particular linear parameter varying (LPV) systems. It is demonstrated that by focusing on the same closed loop operator as in [2], a more reliable stability assessment is achieved compared to using a model of the plant.

Furthermore, in contrast to the above papers, the application of standard system identification methods is treated in some detail.

After reviewing some basic notions on LPV systems in Section II, the concept of safe switching is discussed in more detail in Section III. In order to get robust results from the analysis, it is necessary to take the uncertainty of the identified models into account. Data driven generation of uncertainty models is still an undeveloped subject for LPV systems. In Section IV it is shown how a simple filtering scheme can reduce the problem to estimating the worst case gain of an additive uncertainty. Finally, in Section V, the presented method is demonstrated through a simulation example.

II. LPV systems

This section provides background knowledge on LPV systems. We consider discrete-time linear parameter-varying (LPV) systems G_θ with a minimal state space realisation given by matrix functions $A_\theta \in \mathbb{R}^{n\times n}, B_\theta \in \mathbb{R}^{n\times m}, C_\theta \in \mathbb{R}^{p\times n}$ and $D_\theta \in \mathbb{R}^{p\times m}$, mapping an input signal vector $u \in \mathbb{R}^m$ to an output measurement signal $y \in \mathbb{R}^p$. Specifically, we deal with systems of the form

$$G_\theta: \quad x_{k+1} = A_{\theta(k)}x_k + B_{\theta(k)}u_k \tag{1}$$

$$y_k = C_{\theta(k)}x_k + D_{\theta(k)}u_k \tag{2}$$

where $\theta(k) \in \mathbb{R}^q$ is a *scheduling parameter*, which is allowed to vary as a function of time but not as a function of the system states x. Since we only allow θ to depend on k, we will simply write θ rather than $\theta(k)$ in the following. As is common in LPV control, we assume that θ is not known a priori, but that it is available online and can be used for controller scheduling.

For notational convenience, we will use the shorthand

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & D_\theta \end{array}\right]$$

for the LPV system (1)–(2).[1]

If $D_{T,\theta}$ is nonsingular, i.e., $D_{T,\theta}^{-1}$ exists, for all θ, the LPV operator $T_\theta = \left[\begin{array}{c|c} A_{T,\theta} & B_{T,\theta} \\ \hline C_{T,\theta} & D_{T,\theta} \end{array}\right]$ has an inverse operator

$$T_\theta^{-1} = \left[\begin{array}{c|c} A_{T,\theta} - B_{T,\theta}D_{T,\theta}^{-1}C_{T,\theta} & B_{T,\theta}D_{T,\theta}^{-1} \\ \hline D_{T,\theta}^{-1}C_{T,\theta} & D_{T,\theta}^{-1} \end{array}\right]$$

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Department of Electronic Systems, Automation and Control, Aalborg University, Fr. Bajers Vej 7C, 9220 Aalborg, Denmark; ktr@es.aau.dk

[1] Please note that this notation should not be confused with "transfer functions"; we strictly consider operators defined in state space, as given by (1)–(2), with $x_0 = 0$ unless otherwise noted.

in the sense that $T_\theta T_\theta^{-1} = T_\theta^{-1} T_\theta = I$ for any trajectory of θ. In the following, this inversion will only need to be applied to operators that are invertible by construction.

A. Stability

Stability analysis of an LPV system is slightly more complicated than for LTI systems since not only the range of θ but also known limitations on its trajectories, e.g. rate limits, can be exploited to assure stability. In this paper we will assume that no such restrictions are known.

Stability is usually checked by means of Lyapunov candidate functions. Another complication stems from having to check these for all possible values of θ which leads to an infinite number of inequalities. If the test is formulated as LMIs and the system is polytopic, i.e. Θ is a polytope and the system matrices depend affinely on θ, then only the vertices need checking, leading to a finite number of equations.

A discrete time LPV system $x_{k+1} = A_{\theta,k} x_k$ is *quadratically stable* (QS) iff there exist a matrix $P_Q > 0$ such that

$$A_\theta^T P_Q A_\theta < P_Q,\ \forall \theta \in \Theta. \tag{3}$$

Then $V_Q(x) = x^T P_Q x$ is a Lyapunov function.

In discrete time, the boundedness of θ results in an implicit rate limit, which can be exploited to give a less conservative stability assessment:

Lemma 1: Let an LPV system be described by $x_{k+1} = \sum_i \xi_{i,k} A_i x_k$, $\sum_i \xi_i = 1$, $\xi_i \geq 0$. If there exist G_i and symmetric P_i such that

$$\begin{bmatrix} G_i^T + G_i - P_j & G_i A_i \\ A_i^T G_i^T & P_i \end{bmatrix} > 0 \tag{4}$$

for all i, j, then we will say that the system is *parameter dependent Lyapunov stable* (PDLS) and $V_{PDLS}(x_k) = x_k^T \sum_i \xi_{i,k} P_i x_k$ is a Lyapunov function.

Proof: The lemma is a specialisation of the observer design in [6].

It is possible to obtain even less conservatism by using *polyhedral Lyapunov functions* [7], [8], but the analysis is computationally heavy and non-convex.

B. Factorisation

Here we will provide en explicit doubly coprime factorisation, assuming that the plant G_θ is strictly proper, i.e.

$$G_\theta = \left[\begin{array}{c|c} A_\theta & B_\theta \\ \hline C_\theta & 0 \end{array}\right] \tag{5}$$

and that it can be stabilised by an observer-based LPV controller of the form

$$K_{i,\theta} = \left[\begin{array}{c|c} A_\theta + B_\theta F_{i,\theta} + L_\theta C_\theta & -L_{i,\theta} \\ \hline F_{i,\theta} & 0 \end{array}\right] \tag{6}$$

for all $\theta \in \Theta$, where $F_{i,\theta}$ and $L_{i,\theta}$ are such that $\bar{x}_{k+1} = (A_\theta + B_\theta F_{i,\theta})\bar{x}_k$ and $\hat{x}_{k+1} = (A_\theta + L_{i,\theta} C_\theta)\hat{x}_k$ are stable.

Any G_θ that satisfies the above assumption for any trajectory of $\theta \in \Theta$, can be written as a right, respectively left, coprime factorisation of the form: [9], [10], [11]

$$G_\theta = N_\theta M_\theta^{-1} = \tilde{M}_\theta^{-1} \tilde{N}_\theta \tag{7}$$

where $N_\theta, M_\theta, \tilde{M}_\theta$ and $\tilde{N}_\theta$ are LPV stable operators of a specific form given below. Correspondingly, $K_{i,\theta}$ can be factorised as

$$K_{i,\theta} = U_{i,\theta} V_{i,\theta}^{-1} = \tilde{V}_{i,\theta}^{-1} \tilde{U}_{i,\theta} \tag{8}$$

with LPV stable $U_{i,\theta}, V_{i,\theta}, \tilde{U}_{i,\theta}, \tilde{V}_{i,\theta}$. The factors are given as

$$\begin{bmatrix} M_{i,\theta} & U_{i,\theta} \\ N_{i,\theta} & V_{i,\theta} \end{bmatrix} = \left[\begin{array}{c|cc} A_\theta + B_\theta F_{i,\theta} & B_\theta & -L_{i,\theta} \\ \hline F_{i,\theta} & I & 0 \\ C_\theta & 0 & I \end{array}\right] \tag{9}$$

$$\begin{bmatrix} \tilde{V}_{i,\theta} & -\tilde{U}_{i,\theta} \\ -\tilde{N}_{i,\theta} & \tilde{M}_{i,\theta} \end{bmatrix} = \left[\begin{array}{c|cc} A_\theta + L_{i,\theta} C_\theta & -B_\theta & L_{i,\theta} \\ \hline F_{i,\theta} & I & 0 \\ C_\theta & 0 & I \end{array}\right] \tag{10}$$

Then, it is possible to check that

$$\begin{aligned} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} &= \begin{bmatrix} \tilde{V}_{i,\theta} & -\tilde{U}_{i,\theta} \\ -\tilde{N}_{i,\theta} & \tilde{M}_{i,\theta} \end{bmatrix} \begin{bmatrix} M_{i,\theta} & U_{i,\theta} \\ N_{i,\theta} & V_{i,\theta} \end{bmatrix} \\ &= \begin{bmatrix} M_{i,\theta} & U_{i,\theta} \\ N_{i,\theta} & V_{i,\theta} \end{bmatrix} \begin{bmatrix} \tilde{V}_{i,\theta} & -\tilde{U}_{i,\theta} \\ -\tilde{N}_{i,\theta} & \tilde{M}_{i,\theta} \end{bmatrix} \end{aligned} \tag{11}$$

holds; this equation is referred to as the *double Bezout identity*.

C. Identification

System identification of LPV systems is in general more complicated than for LTI systems. Not only must there be sufficient excitation at the input, but it is also necessary to obtain data with the time varying parameter covering the whole range. One of the simplest methods is the LPV ARX method found in e.g. [12] and [13]. Here, the state vector simply consists of delayed outputs and inputs.

PBSIDopt, suitable for both open and closed loop identification, which is presented in an LPV version in [14] uses a subspace method to construct the state estimates, and consequently requires a lot of computational power.

There is a range of other methods available [15], [16], [17], [18], [19], but some research into the most appropriate choice for safe switching still remains.

Most importantly, the scheme presented below can be used with any standard identification method for open loop problems.

III. Safe switching

The following situation is considered: given a control system with an LPV plant P and a stabilising LPV controller K_0, we would like to switch to new controller K_1, which we expect to give better performance. However, the design of K_1 is based on an uncertain plant model, so the question is, will switching to the new controller K_1 still provide a stable closed loop $\mathscr{CL}(P, K_1)$?

(Note that this is fundamentally different from what is called safe switching in e.g. [20], where the aim is to design a new controller ensuring stability.)

Facing this problem, it is desirable to devise an analysis method that

- does not require a plant model,
- requires only closed-loop data
- with only a small amount of external excitation,

- uses standard identification methods,
- and gives as few incorrect analysis results as possible.

Note that incorrect results can come both in the form of false negatives, i.e. rejecting a controller that is actually stabilising, and false positives, i.e. indicating stability for a controller that is not actually stabilising.

In [2], the experiment setup in Figure 1 is presented for LTI systems. ($\tilde{U}_0$, $\tilde{V}_0$) is a coprime factorisation of the current controller K_0. ($\tilde{U}_1$, $\tilde{V}_1$) is a coprime factorisation of the new controller K_1. Note that the current closed loop is left intact, except for an excitation signal r being added in K_0. The output z is obtained simply by filtering closed loop measurements of plant input u and output y. With the closed loop $\mathscr{CL}(P,K_0)$ being stable, it is shown that stability of $\mathscr{CL}(P,K_1)$ is equivalent to the stability of T_{zr}^{-1}, the inverse of the transfer function from r to z.

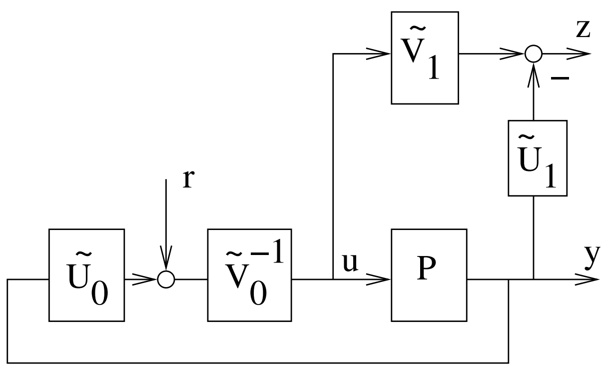

Fig. 1. Experiment setup.

Of course, this transfer function is also uncertain, since it depends on P, but it is shown that it is only necessary to identify the phase up to a certain frequency. This makes the identification problem easier, although the practicality is a bit unclear.

A. Procedure for time varying systems

The necessary part of the proof in [2] does not rely on time-invariance, but for completeness, we will sketch a time-varying version here using a Youla-parametrisation approach, that may be useful for later developments.

Theorem 1: Let $\mathscr{CL}(P_\theta, K_{0,\theta})$ be stable and $\hat{P}_\theta$ be a nominal model stabilised by both $K_{0,\theta}$ and $K_{1,\theta}$. Factorise $\hat{P}_\theta = N_\theta M_\theta^{-1} = \tilde{M}_\theta^{-1}\tilde{N}_\theta$, $K_{i,\theta} = U_{i,\theta}V_{i,\theta}^{-1} = \tilde{V}_{i,\theta}^{-1}\tilde{U}_{i,\theta}$, $i = 0,1$, such that (11) holds for $i = 0,1$.

Then stability of $\mathscr{CL}(P,K_{1,\theta})$ is equivalent to the inverse stability of T_{zr}, the operator from r to z in Figure 1.

Proof: Consider the setup in Figure 2, which is the same as in Figure 1 with an additional path to generate $h = Q_\theta w$, where $Q_\theta = \tilde{V}_{0,\theta}U_{1,\theta} - \tilde{U}_{0,\theta}V_{1,\theta}$.

From the stability of $\mathscr{CL}(P_\theta, K_{0,\theta})$, we know that there exist a stable S_θ such that the top dashed box is exactly P_θ [10], [11]. The fact that the signals into and out of S_θ are equal to r and w follows by repeated use of the Bezout identities [21].

First observe that

$$Q_\theta w = (\tilde{V}_{0,\theta}U_{1,\theta} - \tilde{U}_{0,\theta}V_{1,\theta})(\tilde{M}_\theta y - \tilde{N}_\theta u) = (U_{1,\theta} - \tilde{U}_{0,\theta})y + (V_{0,\theta} - \tilde{V}_{1,\theta})u = r - z,$$

i.e. $Q_\theta = T_{rw} - T_{zw}$.

Closing the loop by setting $r = h$, we would have

$$\begin{aligned}
&(\tilde{V}_{0,\theta} + Q_\theta\tilde{N}_\theta)u = (\tilde{U}_{0,\theta} + Q_\theta\tilde{M}_\theta)y \Leftrightarrow \\
&(\tilde{V}_{0,\theta} + \tilde{V}_{0,\theta}U_{1,\theta}\tilde{N}_\theta - \tilde{U}_{0,\theta}V_{1,\theta}\tilde{N}_\theta)u = \\
&(\tilde{U}_{0,\theta} - \tilde{U}_{0,\theta}V_{1,\theta}\tilde{M}_\theta + \tilde{V}_{0,\theta}U_{1,\theta}\tilde{M}_\theta)y \Leftrightarrow \\
&(\tilde{V}_{0,\theta}M_\theta\tilde{V}_{1,\theta} - \tilde{U}_{0,\theta}N_\theta\tilde{V}_{1,\theta})u = \\
&(-\tilde{U}_{0,\theta}N_\theta\tilde{U}_{1,\theta} + \tilde{V}_{0,\theta}M_\theta U_{1,\theta})y \Leftrightarrow \\
&\tilde{V}_{1,\theta}u = \tilde{U}_{1,\theta}y \Leftrightarrow u = K_{1,\theta}y
\end{aligned}$$

i.e. the stability of $\mathscr{CL}(P_\theta, K_{1,\theta})$ is in fact equivalent to the stability of $\mathscr{CL}(S_\theta, Q_\theta)$. Due to the stability of S_θ and Q_θ this is again equivalent to the inverse stability of $I - S_\theta Q_\theta = I - (T_{rw} - T_{zw})T_{wr} = T_{zr}$. ■

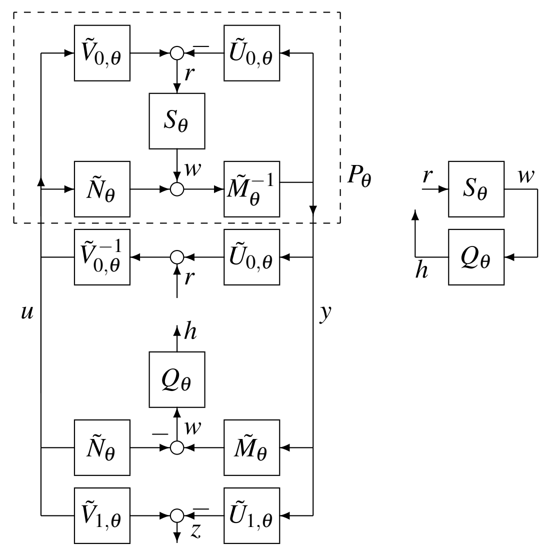

Fig. 2. Left: Plant-controller interconnection represented by dual Youla and Youla parametrisations. Right: Corresponding internal loop.

The procedure for stability testing in [2] relies on frequency domain properties and can therefore not be directly translated to LPV systems. We can however still gain something from checking the stability of the operator T_{zr}^{-1}, as will be demonstrated in Section V. The point is that looking at T_{zr} focuses the identification problem on parameters that are essential to establishing stability.

Remark 1 The proof reveals the close connection to the Hansen Scheme of identifying the dual Youla parameter S_θ in closed loop [22], [23], which is known to have good properties when identifying in closed loop. In particular, if a decent nominal model is used in the factorisation, then the knowledge built into that model will improve the identification [10], [11], e.g. if the model used for control design is used, then the safe switching identification will be able to focus on the modelling errors. ◁

Remark 2 Nothing in the above requires the controller to be on observer-based form (6). As long as coprime factors can be established, the method can be used for general LPV controllers, such as in [24]. However, the construction of such factors is still an object of research, see e.g. [25]. ◁

In order to evaluate the novel procedure, we compare it with the straight-forward method

DI: (direct identification)

1) Apply excitation through r.
2) Identify a model $\hat{P}$ directly from u to y.
3) Test stability of $\mathscr{CL}(\hat{P}, K_1)$.

Inspired by the scheme from [2], the following procedure is suggested:

FI: (forward identification)

1) Apply excitation through r.
2) Compute z by filtering u and y.
3) Identify a model $\hat{T}_{zr}$ from r to z.
4) Test stability of $\hat{T}_{zr}^{-1}$.

For exact model structures, persistent excitation and no noise, all these methods would give the same (correct) answer, but in more realistic settings the result may differ significantly. T_{zr} is in theory a more complex (higher order) operator than P, but by identifying a closed loop operator that is so closely related to the stability of the object we are interested in we can expect to get more accurate results in spite of the increased complexity. It should also be noted that the identifications in **FI** is open loop problems, whereas **DI** has a more difficult closed loop character.

Some care must be taken in the choice of LPV model structures in the respective identifications. For **DI**, the structure of $\hat{P}$ must be such that the resulting structure of $\mathscr{CL}(\hat{P}, K_1)$ can be checked for stability.

When identifying $\hat{T}_{zr}$ it is essential to note that the coprime factors can always be chosen so that its D-matrix is identity. This means that rather than identifying a full model, we can identify the operator from r to $\tilde{z} = z - r$ with $D = 0$. It also means that it is relatively simple to choose the model structure in a way that results in affine parameter dependence in $\hat{T}_{zr}^{-1}$, leading to a simple stability analysis.

IV. Robustness

In order to make the results robust, we need to obtain an estimate of the model uncertainty. Luckily, the structure of the setup lets us do this in a simple way.

Lemma 2: Let $T = \hat{T} + \Delta$ be stable and assume that T and $\hat{T}$ are invertible and that $\hat{T}^{-1}$ is stable. Then

$$||\hat{T}^{-1}\Delta||_{i2} < 1 \Rightarrow T^{-1} \text{ is stable.}$$

Proof: Stability of $T^{-1} = (\hat{T} + \Delta)^{-1} = (I + \hat{T}^{-1}\Delta)^{-1}\hat{T}^{-1}$ follows from the small gain theorem.

The lemma can be interpreted as the interconnections in Figure 3. Thus, if we can obtain an estimate $\hat{T}_{zr}$ of T_{zr} along with an additive uncertainty model, then inverse stability can be assured by checking that the induced 2-norm $||\hat{T}_{zr}^{-1}\Delta||_{i2} < 1$ for all trajectories of θ.

This leads to the following procedure:

RFI: (robust forward identification)

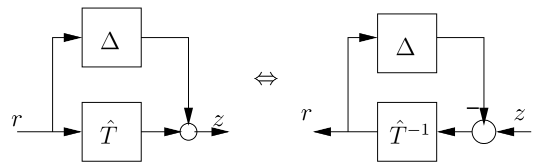

Fig. 3. Inversion of T as a feedback loop.

1) Obtain a model $\hat{T}_{zr}$ as in **FI**.
2) If $\hat{T}_{zr}$ is not inversely stable, then reject the controller.
3) Else, compute an additive uncertainty model and check $||\hat{T}^{-1}\Delta||_{i2} < 1$.

How to obtain such an uncertainty model from data is still an open problem, in particular since we can expect it to be parameter-dependent. This means that by using frequency-domain (or similar LTI) methods on the whole data set, we risk averaging the uncertainty over the whole parameter range, when what we are really looking for is the worst case gain.

Remark 3 Note that the robustness here deals with modelling errors in the identification of the operator $\hat{T}_{zr}$. Errors in the model used for control design of $K_{1,\theta}$ are only relevant through the effect they may have on the identification procedure. ◁

A. Filtering

Identifying uncertainty models for LPV systems is still an open research problem, but by using the following filtering procedure, the safe switching identification can be reduced to a relatively simple problem.

Since the main objective is to assess inverse stability rather than to obtain a model, we are free to filter z or r through a stable and inversely stable filter before doing the identification.

One possible approach is to iteratively filter with the obtained model:

FRFI: (filtered robust forward identification)

1) Obtain a model $\hat{T}_{zr}$ as in **FI**.
2) If $\hat{T}_{zr}$ is not inversely stable, then reject the controller.
3) Define $z_0 = z$, $\hat{T}_0 = \hat{T}_{zr}$,
4) and initialise the counter $k := 1$.
5) LOOP: Perform the filtering $z_k = \hat{T}_{k-1} z_{k-1}$.
6) Identify a model from r to z_k: $z_k \approx \hat{T}_k r$.
7) If $\hat{T}_k$ is not inversely stable, then reject the controller.
8) Else, if $k < k_{max}$, set $k := k+1$ and go to LOOP
9) Evaluate inverse stability of $\hat{T}_k$ by the **RFI** method.

In most cases, as k increases, $\hat{T}_k$ will approach identity. We are then left with checking if $||\Delta|| < 1$ for all trajectories of θ, which should be a relatively simple task compared to analysing the interconnection of two parameter-dependent operators.

V. Simulation example

We consider the unstable system P:

$$\begin{aligned} x_{k+1} &= A_\theta x_k + Bu_k + K_e e_k, \\ y_k &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_k + e_k \end{aligned}$$

with

$$A_\theta = \begin{bmatrix} 1 & -0.5+0.1\theta & 0.4-0.3\theta \\ 0.5-0.3\theta & 0.7 & 0 \\ 0 & 0.3 & 1 \end{bmatrix}, \quad (12)$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, K_e = \begin{bmatrix} -0.1 \\ -0.1 \\ 0.1 \end{bmatrix}, \quad (13)$$

where $\theta \in [0;1]$ and e is white Gaussian noise with standard deviation 0.2. The parameter follows a transformed Brownian motion $\theta_k = 0.5 + 0.5 \sin w_k$, where $w_{k+1} = w_k + v_k$, where v is white Gaussian noise with standard deviation 0.4.

A first principles model

$$P_0 = \left[\begin{array}{c|c} \hat{A}_\theta & \hat{B} \\ \hline \hat{C} & 0 \end{array} \right] \quad (14)$$

has been obtained with $\hat{B} = B$ and $\hat{C} = C$, but due to some unmodelled phenomenon, we have

$$\hat{A}_\theta = \begin{bmatrix} 1 & -0.7+0.2\theta & 0.4-0.3\theta \\ 0.4-0.3\theta & 0.7 & 0 \\ 0 & 0.3 & 1 \end{bmatrix}. \quad (15)$$

Based on P_0 and the design method in [6], a range of observer-based controllers

$$K_{i,\theta} = \left[\begin{array}{c|c} \hat{A}_\theta + \hat{B}F_{i,\theta} + L_\theta \hat{C} & -L_\theta \\ \hline F_{i,\theta} & 0 \end{array} \right] \quad (16)$$

have been designed, with

$$L_\theta^T = -\begin{bmatrix} 0.48+0.14\theta & 1.42+-0.94\theta & 0.99 \end{bmatrix}$$

$$\begin{aligned} F_{0,\theta} &= -\begin{bmatrix} 1.5-0.5\theta & 0.5+0.3\theta & 1+0.2\theta \end{bmatrix} \\ F_{1,\theta} &= -\begin{bmatrix} 1.275-0.425\theta & 0.425+0.255\theta & 0.85+0.17\theta \end{bmatrix} \\ F_{2,\theta} &= -\begin{bmatrix} 1.05-0.35\theta & 0.35+0.21\theta & 0.7+0.14\theta \end{bmatrix} \\ F_{3,\theta} &= -\begin{bmatrix} 0.825-0.275\theta & 0.275+0.165\theta & 0.55+0.11\theta \end{bmatrix} \\ F_{4,\theta} &= -\begin{bmatrix} 0.6-0.2\theta & 0.2+0.12\theta & 0.4+0.08\theta \end{bmatrix} \end{aligned}$$

K_3 and K_4 do not stabilise the real plant, whereas all the others do (both PDLS and QS).

50 experiments are performed with simulations of 800 samples with K_0 stabilising P. In each of the 50 experiments, different sequences of e, v and r are used. We would now like to establish whether K_1-K_4 also stabilise the plant.

The excitation r is chosen as white Gaussian noise with deviation 1.

For each data set, the three tests **DI**, **FI** and **FRFI** described in Section III-A are performed. All identifications are performed with an ARX model structure with 5 delayed inputs and outputs. PDLS is used as the stability criterion (by testing feasibility of (4)).

K	DI	FI	FRFI
1	50	50	50
2	45	50	49
3	35	50	10
4	28	50	0

TABLE I

Results of stability analysis using ARX identification.

K	DI	FI	FRFI
1	46	50	50
2	45	50	49
3	40	50	4
4	34	50	0

TABLE II

Results of stability analysis using PBSIDopt.

To estimate the gain of the uncertainty model in **FRFI**, we follow the following procedure. Since $\hat{T}_k$ converges to identity, $z_k - r$ becomes a simulation error, and we need to establish whether the worst case gain from r to $z_k - r$ is larger than 1. Since r is white noise noise of variance 1, for an LTI system, we would merely have to check if the amplitude of $z_k - r$ at any frequency, for instance by doing a discrete Fourier transform (DFT). Here, we will use this approach for the LPV system also, even though this is obviously not a completely valid approach.

Table I shows how many times the respective methods predicted stability for the new controller. We would of course prefer the numbers to be 0 for K_3 and K_4 and 50 for K_1 and K_2.

The forward identification produces inversely stable models in all the experiments, so obviously here the model uncertainty must be taken into account. When doing so, the filtered robust method produces results that are clearly superior to the direct method. It is expected that a more appropriate uncertainty modelling will produce even more reliable results.

The performance of the direct identification method is quite bad, which is expected, since the ARX method is not really suitable for closed loop identification. It should also be noted that in order for the ARX model set to correspond to the state space model set, it is necessary for the parameter to have dynamical dependence, which is not included in the above experiment. However, the proposed scheme results in open loop identification problems, and the **FRFI** handles the mismatching model sets through robustness.

We perform the same experiment again, but this time using PBSIDopt with model order 3 and window length 5 (see [14] for details) in all identifications. Also, the number of samples in each experiment is increased to 2000. The results are shown in Table II.

In fact, the direct method did not improve significantly, whereas the **FRFI** gives very good results. In this experiment, the system behaviour is included in the **DI** model set, so the only source for bias is the finite window length. The

FI does not have the full model order needed to represent the closed loop system, but the robustness of the **FRFI** handles the bias as an uncertainty.

VI. Conclusions

This paper presents a method for assessing stability of a new LPV closed loop, given relatively few data compared to what would be needed for establishing a full model of the plant.

In addition to extending an existing LTI method to LPV systems, the connection to standard identification methods is treated in more detail.

What is presented here is only a first step on the way, as LPV uncertainty modelling needs further development. The iterative filtering method presented here is such that we only need to establish the worst case gain of an additive uncertainty, which should be a tractable problem.

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