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Noniterative data-driven design of multivariable controllers

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Abstract— **In this paper, a data-driven technique is proposed to deal with multivariable fixed-order controller design. The method is based on the Virtual Reference Feedback Tuning (VRFT) philosophy and thus does not require any model of the plant. As far as the authors are aware, this is the first noniterative method that allows one to tune either tracking and decoupling terms of a MIMO controller. Unlike standard VRFT for SISO systems, extended instrumental variables and variance weighting are used to counteract the effect of noise and achieve consistent controller estimate with a single set of input-output data. The proposed strategy is validated on three banchmark examples.**

I. INTRODUCTION

Many engineering systems are equipped with several actuators that affect their static and dynamic behavior. In order to achieve a given desired operation, closed-loop control is generally employed and a suitable controller is designed based on a mathematical description of the system. The aim of closed-loop controllers for multivariable plants is typically either to track some reference signals and to eliminate any interaction between the outputs of the closed-loop system (see [16]).

Concerning multi-input/multi-output (MIMO) linear time-invariant (LTI) systems, one of the main drawbacks in controller tuning is that an assessment of structural properties of transfer matrices is required. This is sometimes difficult to obtain when plants are too complex and/or modeling cost is too high. Moreover, if the structure of the MIMO controller is fixed (*e.g.* PIDs), model reduction is required and several problems arise, see [2].

The modeling work can be skipped, by tuning the controller parameters with data collections. The first attempts to perform data-to-controller design date back to first papers on self-tuning regulation (STR) and model-reference adaptive control (MRAC) (see [1] for a complete overview). More recently, two iterative methods have been proposed to design reliable controllers off-line with a small number of closed-loop experiments on the plant.

The first method is the Iterative Feedback Tuning (IFT) approach (see [9]), where an unbiased estimate of the gradient is provided entirely from input/output (I/O) data collected on the actual closed-loop system. The main drawback of this method for MIMO plants (see [7]) is that the number of experiments needed to estimate the gradient increases with the number of plant inputs n_u and outputs n_y, specifically $n_y n_u + 1$ experiments are required.

The second technique is known as iterative Correlation-based Tuning (CbT) and has been introduced in [12] for single-input/single-output (SISO) plants. A MIMO extension is discussed in [13], where it has been shown that the correlation-based controller tuning provides better tracking performance in fewer experiments than IFT. However, one experiment for each iteration is still needed.

The aim of this work is to provide a noniterative method that allow to directly tune a multivariable controller by using a single set of open-loop I/O data collected on a stable MIMO LTI plant. Unlike the SISO case, for which many different design methods are available in the scientific literature (see *e.g.* [3] and [11]), a noniterative technique for MIMO controller tuning has not been studied yet.

It should be mentioned that in 2002, a MIMO version of Virtual Reference Feedback Tuning (VRFT) method has been proposed to cope with this problem (see [14]). The method presented therein is though only a straightforward extension of the existing SISO algorithm, whereas important issues as undermodeling and treatement of noise are not discussed. In this work, a VRFT-based solution for MIMO systems is provided in which the above-mentioned problems are taken into account. In detail, the case of controller underparameterization is analyzed in-depth and the extended instrumental-variable solution presented in [18] is employed to cope with the noise that necessarily corrupts the output measurements.

The remainder of the paper is organized as follows. Section II provides some notations and the basic definitions. The idea behind noniterative MIMO data-driven tuning is presented in Section III in a noiseless setting, while Section IV deals with the case of noisy data. Simulation results are given in Section V for three benchmark examples. Some concluding remarks end the paper.

II. PRELIMINARIES AND NOTATION

Consider the unknown LTI multivariable plant $G(q^{-1})$, where q^{-1} denotes the backward shift operator, with n_u inputs and n_y outputs. The objective of the model-reference control problem is to design a linear, fixed-order controller $K(q^{-1}, \rho)$, parameterized through ρ, for which the *output* complementary sensitivity function matches the user-defined stable strictly proper reference model $M(q^{-1})$ (see Figure 1). More formally, the aim is to find the vector of parameters that minimizes the two-norm of the difference between the

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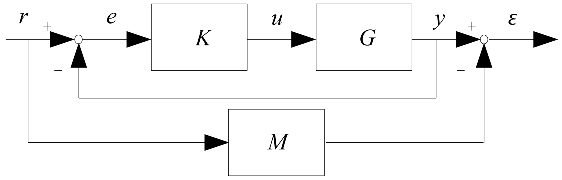

Fig. 1. Model reference control problem.

reference model and the achieved closed-loop system:

$$J_{MR}(\rho) = \left\| M - (I + GK(\rho))^{-1} GK(\rho) \right\|_2^2, \quad (1)$$

where I is the $(n_y \times n_y)$ identity matrix. Notice that (1) is non-convex with respect to the controller parameters ρ. A convex approximation of (1) can be found by means of two additional assumptions, *i.e.*:

Assumption 1: the desired sensitivity function $I - M$ is close to the actual one $(I + GK(\hat{\rho}))^{-1}$ in the minimum $\hat{\rho}$.

Assumption 2: the controller is linearly parameterized and a fixed known denominator is common to all transfer functions.

Without loss of generality, a MIMO FIR structure with integral action will be used in the rest of the paper. The n-th order control law is then defined as

$$u(t) = u(t-1) + \sum_{k=0}^{n} B_k e(t-k), \quad (2)$$

where $B_k \in \mathbb{R}^{n_u \times n_y}$ are matrices containing the unknown parameters such that ρ is defined as

$$\rho = [vec^T(B_0) \dots vec^T(B_n)]^T. \quad (3)$$

Notice that this parameterization includes all P-PI-PID-like controller structures.

The above assumptions lead to the following approximation of the model reference criterion:

$$J(\rho) = \| M - (I - M) GK(\rho) \|_2^2 \quad (4)$$

and to the subsequent definition of "optimal controller':

Definition 1: The parameters ρ_o of the n-th order optimal controller $K(\rho_o)$ in the given class (2) are defined as the optimum of the convex optimization problem

$$\rho_o = \arg\min_{\rho} J(\rho).$$

Some remarks are due:

- $K(\rho_o)$ generally does not correspond to the controller that makes $J(\rho) = 0$. The latter might be of very high order since it depends on the unknown and possibly high-order plant $G(q^{-1})$ and it might also be non-causal.
- The criterion $J(\rho)$ is a good approximation of $J_{mr}(\rho)$ only if the difference between $K(\rho)$ and the ideal controller that makes $J(\rho) = 0$ can be made small. However, it should be mentioned that Assumption 1 is widely used in identification for control, data-driven tuning and $\mathcal{H}_2$ model-reduction (see [8] for an overview).

Consider now that an open-loop collection of I/O data $\{u(t), y(t)\}_{t=1,\dots,N}$ is available. In standard "indirect" data-driven approaches, minimization of (4) can be achieved by identifying from data a model $\hat{G}$ of the plant and solving Problem 1 for $J(\rho)$ evaluated in $\hat{G}$. Unfortunately, this approach is very sensitive to modeling errors and the closed-loop performance is subsequently limited by the quality of the model.

In the rest of the paper, a novel approach for direct identification of controller parameters from data is proposed. The method is based on the Virtual Reference philosophy first proposed in [6] and [15] with the name of Virtual Reference Direct Design (VRD^2) and fixed and extended in [3], [5] and [4], respectively for LTI, LPV and nonlinear SISO systems. However, the idea presented herein is not only an extension of an existing method but also proposes different solution for noise rejection and an *ad-hoc* discussion on optimal filtering for dealing with undermodeling.

III. NONITERATIVE DATA-DRIVEN MULTIVARIABLE CONTROLLER TUNING

The main idea to solve (4) without identifying $G(q^{-1})$ is to build a "virtual" closed-loop system, where the input and output signals are equal to $u(t)$ and $y(t)$ and the closed-loop transfer function is assumed to correspond to $M(q^{-1})$. From the above loop, the so-called "virtual reference" $r_V(t)$ and "virtual error" $e_V(t)$ signals can be computed as

$$r_V(t) = M^{-1}(q^{-1})y(t)\ ,\ e_V(t) = r_V(t) - y(t).$$

The control design issue in Definition 1 is then reduced to an identification problem and the optimization procedure is still convex if the controller is chosen as in (2). The cost index to be carried out is then, in the noiseless case,

$$J_{VR}^N(\rho) = \frac{1}{N} \sum_{t=1}^{N} \| u_{L_u}(t) - K(\rho) e_{L_e}(t) \|_2^2, \quad (5)$$

where $u_{L_u}(t) = L_u(q^{-1})u(t)$, $e_{L_e}(t) = L_e(q^{-1})e(t) = L_e(q^{-1})(M^{-1} - I)GL_y u$, and L_u, L_e, L_y are suitable data prefilters. Such filters are required in case the controller that leads the cost function to zero is not in the controller set (see [3]), as minimizers of (5) and (4) could not coincide. Optimal filter selection is defined by the following Proposition.

Proposition 1: If data prefilters in (5) are selected as

$$L_u = M\Phi_{uu}^{-1/2},\ L_e = K^{-1}(\rho)M,\ L_y = K(\rho)\Phi_{uu}^{-1/2}, \quad (6)$$

where Φ_{uu} is the power spectral density of u and $\Phi_{uu}^{1/2}$ denotes a spectral factor of Φ_{uu}, then the minimizers of (5) and (4) aymptotically coincide.

Proof: Write the frequency-wise counterpart of $J(\rho)$ by means of Parseval's theorem as

$$J(\rho) = \frac{1}{2\pi} \int_{-\pi}^{\pi} tr\left[M - (I - M) GK(\rho)\right] \times \quad (7)$$

$$\times \left[M - (I - M)\,GK(\rho)\right]^H \; d\omega$$

(the superscript H indicates the hermitian conjugate of a complex expression). The asymptotic value of $J_{VR}^N(\rho)$ (*i.e.* as N goes to infinity) is instead

$$J_{VR}(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi} tr\left[L_u - K(\rho)L_e\,(I-M)\,GL_y\right] \times \quad (8)$$

$$\times \Phi_{uu}\left[L_u - K(\rho)L_e\,(I-M)\,GL_y\right]^H \; d\omega.$$

Expression (7) is obtained by simply substituting filters (6) in (8).

■

Since (6) are ρ-dependant, a suboptimal but feasible solution will be implemented next.

Choose the filters above as $L_u = L_e = L = M$ and $L_y = I$. This choice is still optimal in case $K(\rho)$, M and G can commute (*e.g.* for SISO systems), but it can always be given a systemic interpretation. Actually, the above filter selection makes the asymptotic frequency-wise expression of $J_{VR}^N(\rho)$ equal to a new cost function $\tilde{J}_{VR}(\rho)$

$$\tilde{J}_{VR}(\rho) = \frac{1}{2\pi}\int_{-\pi}^{\pi} tr\left[M - K(\rho)G\,(I-M)\right] \times \quad (9)$$

$$\times \Phi_{uu}\left[M - K(\rho)G\,(I-M)\right]^H \; d\omega,$$

Notice that (9) is also the frequency-wise convex approximation of the model-reference cost

$$\tilde{J}_{MR}(\rho) = \left\| \left(M - (I + K(\rho)G)^{-1} K(\rho)G\right) \Phi_{uu}^{1/2} \right\|_2^2 . \quad (10)$$

Some concluding reasoning can be made:

- the main difference between the suboptimal filter selection and the optimal one is that in (10), $K(\rho)$ is chosen such to make the *input* complementary sensitivity function as close as possible to M;
- in the suboptimal solution, $\Phi_{uu}^{1/2}$ can be used as a frequency-weighting function;
- the optimal filter and the one employed in [3] are different if the proposed method is applied on SISO systems. This fact is actually not contradictory, as is due to filter derivation procedure. In [3], Assumption 1 is used only after having computed the optimal filter, whereas herein the same Assumption is used to refer to a convex approximation of (1) at the beginning of the discussion.

In Section V, simulations will show that this setting guarantees good matching results even if the desired behavior is formulated via *output* complementary sensitivity function and the filter is derived via slightly different mathematical steps.

IV. THE NOISY CASE

Suppose that there exist a LTI stable transfer matrix $H(q^{-1})$ and a white noise $w(t)$ with unitary variance such that the output of the system G can be written as $y(t) = Gu(t) + Hw(t)$. When $y(t)$ is noisy, $e_L(t) = (I-M)Gu + (I-M)Hw(t)$ and therefore also the input signal in controller identification is affected by noise. Clearly, in these case, simple minimization of (5) would yield biased results (see [17]). In this Section, the instrumental variable technique proposed in [18] will be employed to make the minima of noisy and noiseless cost criteria be coincident.

Rewrite the controller (2) in a linear regression form as follows:

$$u(t) = u(t-1) + B_0 e_V(t) + \ldots + B_n e_V(t-n) =$$

$$= u(t-1) + [e_V^T(t) \otimes I \ldots e_V^T(t-n) \otimes I]\rho = u(t-1) + \varphi_V^T(t)\rho,$$

where the last equality defines $\varphi_V(t)$ and $\otimes$ denotes the Kronecker matrix product.

Introduce now the extended instrumental variable $\zeta(t)$ (see [18]) as

$$\zeta(t) = \begin{bmatrix} u(t+l) \\ \vdots \\ u(t-l) \end{bmatrix},$$

where l is a sufficiently large integer, and define the *decorrelation* cost function as

$$J_d^N(\rho) = (r - R\rho)^T \hat{W}^{-1} (r - R\rho), \quad (11)$$

where

$$R = \frac{1}{N}\sum_{t=1}^{N} \zeta_L(t) \otimes \varphi_L(t), \quad (12)$$

$$r = \frac{1}{N}\sum_{t=1}^{N} \zeta_L(t) \otimes u_L(t), \quad (13)$$

$$\varphi_L(t) = [e_L^T(t) \otimes I \ldots e_L^T(t-n) \otimes I]^T,$$

$\zeta_L(t) = L\zeta(t)$, $u_L = Lu(t)$ and $\hat{W}$ is a consistent estimate of the residual covariance matrix

$$W = \mathbb{E}\left[(r - R\rho)(r - R\rho)^T\right].$$

In a noiseless setting, if l is large (at limit when it tends to infinity), $\rho = \rho_o$ is guaranteed to asymptotically imply $R\rho - r = 0$. It follows that the minima of (11) and (5) coincide and are given by

$$\hat{\rho} = \left(R^T W^{-1} R\right)^{-1} \left(R^T W^{-1} r\right). \quad (14)$$

At the same time, when data are collected in open-loop operation, input and noise signals are uncorrelated and the effect of noise becomes negligible as $N \to \infty$ (see [18] for further details). Moreover, use of $\hat{W}$ allows one to weight the variance of the parameter estimate and thus increase the statistical efficiency of the method. This is not possible, *e.g.*, with "repeated experiment" procedures like the one used in SISO VRFT, see Example 3 in Section V.

Concerning the design of the identification experiment, it should be recalled that for eliminating the influence of an input on a particular output, the experiment has usually to be designed such the other outputs are constant while that input is excited (see discussion in [13]). Intuitively, other input signals act as additional disturbances for the tuning of each decoupling term. That is, the addition of other references deteriorates the signal-to-noise ratio and increases the variances of the other elements of the controller transfer function. Nevertheless, separate excitation is not in principle mandatory for MIMO VRFT.

Notice also that unlike SISO VRFT, in the above approach only one experiment is needed to make the estimate insensitive to the effect of noise. Moreover, by using the estimate of the residual covariance matrix W, the overall statistical efficiency can be easily improved (ongoing work is being dedicated to iterative estimation of W and ρ). The only drawback is that an additional tuning knob l has to be tuned. Nevertheless, the choice of l is not too critical and for the estimate to be good it is sufficient that l is large enough to make R an accurate sample-based estimate of the correlation matrix of $\zeta_L(t)$ and $\varphi_L(t)$.
The final MIMO VRFT algorithm can then be summarized as follows.

Noniterative MIMO data-driven controller tuning

1) collect a set of I/O data from an open-loop experiment on the multivariable plant;
2) set the data prefilter $L = M$ and compute $u_L(t) = Lu(t)$ and $e_L(t) = Le_V(t) = L(M^{-1} - I)y(t)$;
3) compute R and r as indicated in (12) and (13);
4) set $W = I$;
5) compute $\hat{\rho}$ as in (14);
6) *(optional)* compute a sample-based estimate of W and go to point 5.

V. NUMERICAL EXAMPLES

In this Section, three different benchmark control problem will be addressed to show the effectiveness of the proposed methodology. Specifically:

- **Example 1** investigates the behavior of MIMO VRFT in case of full and reduced parameterization, when data are affected by additive noise; a comparison with SISO VRFT and model-based design is also provided;
- **Example 2** compares the proposed method with the other main direct data-driven techniques, *i.e.* CbT and IFT. The same example has been used to present multivariable CbT and IFT in [13] and [7], respectively;
- **Example 3** shows how the technique behaves when used for SISO systems; as the proposed flexible transmission system is the same used in [3], the comparison with SISO VRFT is straightforward and therefore a fair evaluation of different filtering and statistical efficiency can be made.

Example 1.
Consider the discrete-time multivariable LTI system introduced in [10]

$$G(q^{-1}) = \begin{bmatrix} \frac{0.09516q^{-1}}{1-0.9048q^{-1}} & \frac{0.03807q^{-1}}{1-0.9048q^{-1}} \\ \frac{-0.02974q^{-1}}{1-0.9048q^{-1}} & \frac{0.04758q^{-1}}{1-0.9048q^{-1}} \end{bmatrix} \quad (15)$$

and the reference model

$$M(q^{-1}) = \begin{bmatrix} \frac{0.1148q^{-1}-0.0942q^{-2}}{1-1.79q^{-1}+0.8106q^{-2}} & 0 \\ 0 & \frac{0.1148q^{-1}-0.0942q^{-2}}{1-1.79q^{-1}+0.8106q^{-2}} \end{bmatrix}.$$

A perfect model-following can be achieved by using 4 PI, 2 for reference tracking (main diagonal of the controller matrix) and 2 for decoupling the different outputs (anti-diagonal terms). Figure 2 shows that this is actually achieved by the MIMO VRFT controller, even if the parameters are estimated using noisy data and Signal-to-Noise Ratio $SNR = 10$. In the example, the tuning knob l is found by trial-and-errors and the final value is 15. In the same figure, the behavior of 2 SISO PI tuned via standard VRFT is shown. As obvious, additional decoupling terms increase the value of the controller in terms of simultaneous reference following.
Figure 3 shows an analogous comparison on the same system for the case where only integral controllers are available, and thus underparameterization occurs. Also in this case, $SNR = 10$ and $l = 15$ and advantages of using MIMO structure is evident.
Recall also that in both full and underparameterization cases, 2 experiments are required to achieve unbiased parameters estimate for SISO VRFT, whereas a single experiment is needed by MIMO VRFT.

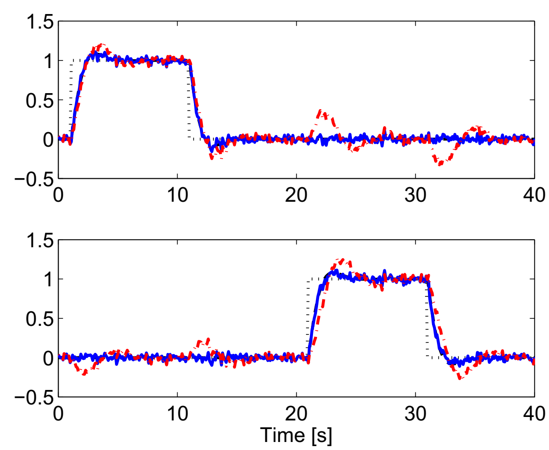

Fig. 2. Closed-loop responses to step reference excitation (dotted): M (dashed), closed-loop with VRFT SISO controller (dash-dot) and closed-loop with VRFT MIMO controller (solid). Notice that dashed and solid line are almost overlapped.

The effectiveness of instrumental variable techniques and data prefilter can be further appreciated in the underparameterization case (integral controller) by looking at Table I, where (1) is evaluated using the same set of I/O noisy data for different design solutions, *i.e.*

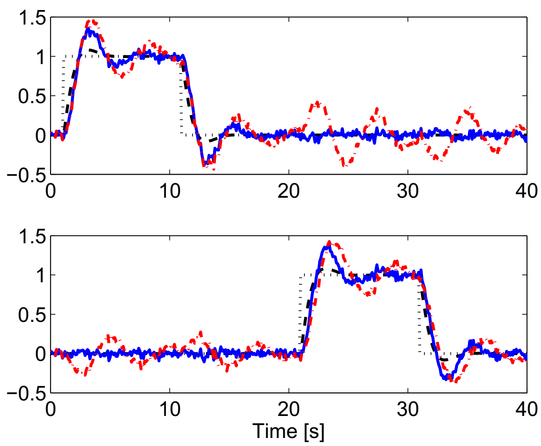

Fig. 3. Closed-loop responses to step reference excitation (dotted): M (dashed), closed-loop with VRFT SISO controller (dash-dot) and closed-loop with VRFT MIMO controller (solid).

TABLE I

ACHIEVED PERFORMANCE (1) FOR DIFFERENT VRFT METHODS AND COMPARISON WITH IDEAL MODEL-BASED DESIGN ($\hat{G} = G$).

Model-based	SISO	MIMO (no L no IV)
$0.816{\cdot}10^{-9}$	0.2472	0.3523
MIMO (no IV)	**MIMO (no L)**	**MIMO**
0.3832	0.0519	0.0214

- ideal model-based controller, that is the controller introduced in Definition 1 when additionally the available model of the plant $\hat{G} = G$: this case is not realistic in a data-driven setting but it is useful herein as it represents the ideal lower bound of any data-driven methodology;
- SISO VRFT tuning of two decoupled integral controllers;
- direct minimization of (5) from noisy data without filtering and with no instrumental variable;
- direct minimization of (5) from noisy data with L-prefiltering but without instrumental variable;
- MIMO VRFT tuning of integral controllers from unfiltered data;
- complete MIMO VRFT tuning of integral controllers.

From Table I, it is clear that in this example, the rejection of the noise effect is the key issue. Nonetheless, a better approximation of (4) via data-prefiltering further increases the closed-loop performance.

Example 2.
In this example, a comparison between existing direct data-driven procedures for MIMO controller tuning, *i.e.* IFT and CbT, is presented. The simulation study considers the tuning of a multivariable PI controller for a LV100 gas turbine engine (see [19]). The same example has been already employed in [13] for providing a comparison between CbT and IFT with simulation conditions given in [7]. The plant is described in continuous time and has 5 states: gas generator spool speed, the power output, temperature, fuel flow actuator level and variable area turbine nozzle actuator level. The input signals are the fuel flow and variable area turbine nozzle and the output signals are gas generator spool speed and temperature. The measurement noise is zero mean white noise with variance $0.0025I$ and the reference model reads

$$M(q^{-1}) = \begin{bmatrix} \frac{0.4q^{-1}}{1-0.6q^{-1}} & 0 \\ 0 & \frac{0.4q^{-1}}{1-0.6q^{-1}} \end{bmatrix}.$$

For this problem, [7] provides, after 6 iterations and 30 experiments, the IFT controller

$$K_{IFT}(q^{-1}) = \begin{bmatrix} \frac{0.248-0.03q^{-1}}{1-q^{-1}} & \frac{0.38-0.199q^{-1}}{1-q^{-1}} \\ \frac{16.47-15.91q^{-1}}{1-q^{-1}} & \frac{0.063-0.054q^{-1}}{1-q^{-1}} \end{bmatrix}$$

whereas [13] gives, after 8 iterations and 8 experiments, the CbT controller

$$K_{CbT}(q^{-1}) = \begin{bmatrix} \frac{0.3636-0.09866q^{-1}}{1-q^{-1}} & \frac{0.3653-0.2691q^{-1}}{1-q^{-1}} \\ \frac{18.69-18.16q^{-1}}{1-q^{-1}} & \frac{-3.453+2.652q^{-1}}{1-q^{-1}} \end{bmatrix}.$$

If a single set of 5000 data-points with the same SNR as above is available, the MIMO VRFT method yields the following controller:

$$K_{VRFT}(q^{-1}) = \begin{bmatrix} \frac{0.3309-0.04894q^{-1}}{1-q^{-1}} & \frac{0.4288-0.3368q^{-1}}{1-q^{-1}} \\ \frac{19-18.44q^{-1}}{1-q^{-1}} & \frac{-3.143+2.241q^{-1}}{1-q^{-1}} \end{bmatrix}.$$

A closed-loop experiment without noise is performed and illustrated in Figure 4 to highlight the main differences among the three controllers. As already noticed in [13], the IFT controller does not succeed in completely decoupling the closed-loop system and satisfying the model-following specification for the temperature loop. On the contrary, CbT guarantees an almost perfect matching of the required reference model. Concerning MIMO VRFT, the behavior is very similar to CbT, except for the decoupling effect of the second loop, that is worse for the proposed method. However, the result of this example is very satisfactory, as MIMO VRFT yields good performance with a single-set of I/O data collected in open-loop operation. Moreover, differences between the ideal and the actual tracking behavior are so small that, even in case they cannot be accepted for closed-loop operation, the VRFT controller could still constitute a useful tool to initialize the iterative CbT procedure.

Example 3.
In the last example, the MIMO VRFT is employed to tune a SISO controller to be compared with the standard VRFT algorithm introduced in [3]. Specifically, the flexible transmission system used to present the VRFT method in [3] is considered.

The main differences between the methods are the data prefilter and the instrumental variable employed to counteract the effect of noise. To compare the two methods, 100 Monte-Carlo simulation runs are performed (variance of closed-loop

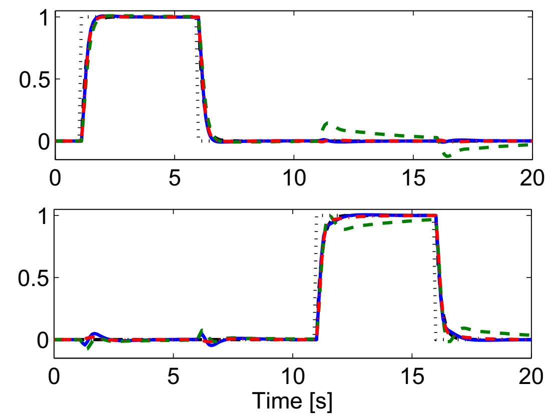

Fig. 4. Comparison between existing iterative methods and VRFT for LV100 control design in a noise-free context. Reference signals (dotted) and achieved responses: with IFT controller (light dashed), with CbT controller (dash-dot) and with MIMO VRFT controller (solid). The desired response is also shown (dark dashed) but it is almost completely overlapped with CbT.

TABLE II

ACHIEVED EXPECTED VALUE OF J_{MR} FOR 90/100 STABILIZING SISO VRFT CONTROLLERS AND FOR 100/100 STABILIZING MIMO VRFT CONTROLLERS.

	SISO	**MIMO**
$\mathbb{E}[J_{MR}]$	0.2200	0.0840

systems become visible) and the expected value of the final objective J_{MR} is computed. For MIMO VRFT, the output of the plant is perturbed in each run by a different realization of the white Gaussian sequence, whereas the input signal is for all runs the same white noise sequence of length 1000. For SISO VRFT, the same experimental conditions hold, except for the fact that the input sequence has length 500, since two sets of I/O data are required to use "repeated experiments" instrumental-variable technique (the total amount of data is therefore the same for both the cases, see [3]).

In Table II, it is evident how extended instrumental variables increase the statistical efficiency of the controller estimate. Moreover, in 10 of 100 runs of SISO VRFT, the controllers unstabilize the system, while all 100 controllers given by MIMO VRFT are stabilizing.

VI. CONCLUSIONS

In this paper, a direct method to design multivariable controllers from data without identifying a model of the plant is proposed and analyzed. Unlike other existing methods, the proposed technique is noniterative and requires a single set of I/O open-loop collected data. The algorithm is based on the Virtual Reference Feedback Tuning philosophy but employs an extended instrumental variable technique to counteract noise and a different prefilter is derived. The MIMO noniterative technique has been tested on three benchmark examples, already used to test and validate other data-driven methodologies. The examples have shown a very good behavior of the proposed technique even in noisy environment and with underparameterized controller structures. Moreover, the method shows higher statistical efficiency than standard VRFT if applied on a SISO system. The only drawback is that an additional tuning knob, *i.e.* the length of the instrumental variable vector, has to be chosen.

Future work will focus on variance analysis and optimal input design for MIMO VRFT. Moreover, use of iterative computation of variance weighting W and controller parameters ρ will be investigated to improve the quality of the closed-loop matching.

REFERENCES

[1] K.J. Astrom and B. Wittenmark. *Adaptive control*. Addison-Wesley Longman Publishing Co., Inc. Boston, MA, USA, 1994.

[2] A. Bunse-Gerstner, D. Kubalinska, G. Vossen, and D. Wilczek. H_2-norm optimal model reduction for large scale discrete dynamical MIMO systems. *Journal of computational and applied mathematics*, 233(5):1202–1216, 2010.

[3] M.C. Campi, A. Lecchini, and S.M. Savaresi. Virtual reference feedback tuning: a direct method for the design of feedback controllers. *Automatica*, 38(8):1337–1346, 2002.

[4] M.C. Campi and S.M. Savaresi. Direct nonlinear control design: The virtual reference feedback tuning (vrft) approach. *IEEE Transactions on Automatic Control*, 51(1):14–27, 2006.

[5] S. Formentin and S.M. Savaresi. Virtual Reference Feedback Tuning for linear parameter-varying systems. In *IFAC World Congress, Milan*, 2011.

[6] G.O. Guardabassi and S.M. Savaresi. Approximate feedback linearization of discrete-time non-linear systems using virtual input direct design. *Systems & Control Letters*, 32(2):63–74, 1997.

[7] H. Hjalmarsson. Efficient tuning of linear multivariable controllers using iterative feedback tuning. *International journal of adaptive control and signal processing*, 13(7):553–572, 1999.

[8] H. Hjalmarsson. From experiment design to closed-loop control. *Automatica 41 393438*, 2005.

[9] H. Hjalmarsson, M. Gevers, S. Gunnarsson, and O. Lequin. Iterative feedback tuning: theory and applications. *Control Systems Magazine, IEEE*, 18(4):26–41, 2002.

[10] A. Karimi, L. Miskovic, and D. Bonvin. Iterative correlation-based controller tuning. *International Journal of Adaptive Control and Signal Processing, 18(8):645664*, 2004.

[11] A. Karimi, K. Van Heusden, and D. Bonvin. Noniterative Data-driven Controller Tuning Using the Correlation Approach. In *European Control Conference*, 2007.

[12] L. Miškovic, A. Karimi, and D. Bonvin. Iterative controller tuning by minimization of a generalized decorrelation criterion. In *13th IFAC Symp. on System identification*, pages 1177–1182.

[13] L. Miskovic, A. Karimi, D. Bonvin, and M. Gevers. Correlation-based tuning of decoupling multivariable controllers. *Automatica*, 43(9):1481–1494, 2007.

[14] M. Nakamoto. An Application of the Virtual Reference Feedback Tuning Method for Multivariable Process Control. *Transactions of the Society of Instrument and Control Engineers*, 39(10):924–932, 2003.

[15] S.M. Savaresi and G.O. Guardabassi. Approximate I/O feedback linearization of discrete-time non-linear systems via virtual input direct design. *Automatica*, 34(6):715–722, 1998.

[16] S. Skogestad and I. Postlethwaite. *Multivariable feedback control: analysis and design*. Wiley New York, 1996.

[17] T. Soderstrom and P.G. Stoica. *Instrumental variable methods for system identification*. Springer-Verlag, New York, 1983.

[18] P. Stoica and M. Jansson. MIMO system identification: state-space and subspace approximations versus transfer function and instrumental variables. *Signal Processing, IEEE Transactions on*, 48(11):3087–3099, 2002.

[19] M. Yeddanapudi and A.F. Potvin. Nonlinear Control Design Blockset: User's Guide. Natick, MA: The Mathworks Inc, 1997.