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Modeling and control of VTOL UAVs interacting with the environment

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Abstract— **In this paper we focus on the problem of modeling and controlling a certain configuration of UAV (Unmanned Aerial Vehicle) considering explicitly the interaction with the environment. This innovative problem is particularly interesting in order to employ unmanned aircrafts in tasks and operations which may require explicitly or implicitly contacts between the UAV itself and the environment such as manipulation of remote objects or indoor flight. For a class of VTOL (Vertical Take-Off and Landing) aircrafts we start studying the problem of safe take-off from hostile terrains and the control of the vehicle when in contact with vertical fixed surfaces.**

I. INTRODUCTION

The aim of the paper is to present preliminary studies regarding modeling and control of a particular configuration of VTOL UAVs in an innovative scenario in which interaction with the environment, in term of desired or unpredictable contacts, is possible and may be a desired control goal. This problem is particularly interesting for the employment of unmanned flying machines in operations which may require explicitly, such as remote manipulation, or implicitly, such as indoor flight with reduced measurement information, contacts between the UAV itself and the environment, qualifying the aircraft as a real unmanned flying robot. In all these possible situations, the control action which is necessary even to maintain the stability does not come straightforward from standard control laws which are usually synthesized considering the *free-flight* dynamics. For this reason we start investigating the dynamics of the system considering explicitly the constraints deriving from the interaction with the environment and we synthesize a control strategy to robustly achieve a desired control goal in a particular configuration. The problem has been addressed considering the approximate planar dynamics of a ducted-fan MAV (Miniature Aerial Vehicle) (see also [12]). This particular configuration of VTOL UAV have been recently considered in literature (see [6], [15], [10]), for its interesting flight properties, which render this system similar to a helicopter. The ducted-fan aerial vehicle (compactly denoted as DFAV) on which we concentrate is a miniature unmanned aircraft characterized by a very simple mechanical structure which is composed of two main subsystems: the propulsive subsystem, composed by a propeller and an electric motor, and the attitude control subsystem, composed by a set of profiled flaps actuated by miniature servo controllers. Both the propeller and the flaps are protected by the presence of a cylindrical fuselage, the duct, which can also be designed in order to improve the flying qualities (see also [6]). The main reason of looking at this kind of configuration is that, being compact and with the propeller safely separated from the environment thanks to the presence of the fuselage, it appears suitable for operations in which interactions with environment and eventually human being are possible and may be desired tasks.

We start addressing two different control scenario, the *take-off* and *set-stand* to vertical fixed surfaces. The first scenario is representative of all the situations in which the system has to interact with horizontal fixed surfaces, like the ground or possibly more hostile terrains, while the second one includes all possible situations in which the system can come into contact with vertical fixed surfaces such as a wall. In particular in section II different dynamical models in order to describe the system both in the free flight condition and in case of contacts with rigid surfaces are derived; moreover a hybrid dynamical model of the overall dynamics is presented. In section III the take-off maneuver is take into account: the problem of performing this task over hostile terrains is considered and a control strategy is derived to avoid undesired overturning (see figure 1 (a)). In section IV the interaction with a vertical surface is considered (see figure 1 (b)), a co-design hint is pointed out and a control strategy to perform a tracking task when the DFAV is in contact with a wall is designed under some conditions depending on the trajectory to be tracked and on the initial conditions related to the movement of the DFAV when it is flying freely and is approaching the wall. In section V conclusions and future works are explained.

(a) (b)

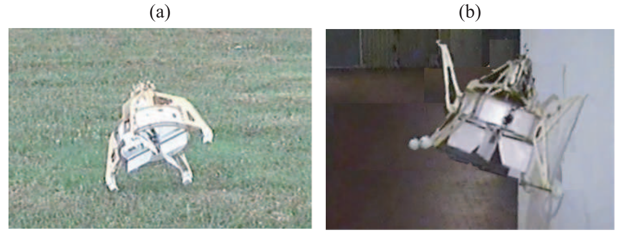

Fig. 1. The DFAV overturning during a take-off maneuver (a) and interacting with a wall (*set-stand*) (b).

II. DFAV MODELING

In the following we will consider different dynamical models in order to describe the system both in the free flight condition and in case of contacts with rigid surfaces. For each dynamical model we derive the conditions on the

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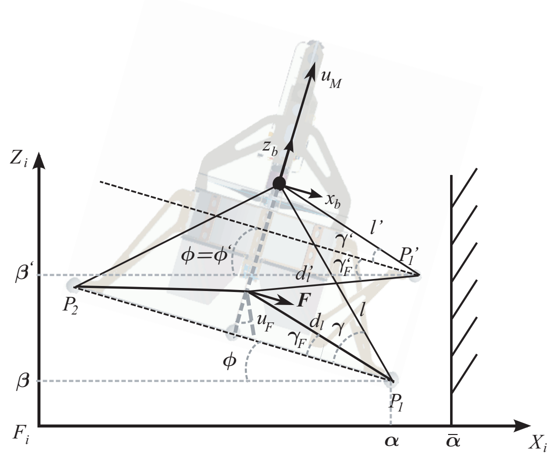

Fig. 2. The DFAV on the configuration manifold $S_1 \times I\!R^2$.

state of the system and on the control inputs to define an hybrid dynamical model of the overall dynamics. For the sake of simplicity, we will concentrate on a simplified model (see figure 2) considering only the "planar dynamic" on the configuration manifold $S_1 \times I\!R^2$.

A. Aerodynamic Simplifications

Assuming low translational speed, the overall relative wind vector for the control flaps is represented only by the *propeller downwash* V_i which, according to [16] and [17], is approximated by $V_i = \sqrt{2u_M/\rho S_{disk}}$ in which u_M denotes the thrust of the propeller, ρ is the air density and S_{disk} is the area of the propeller (the influence of the duct in our setup is negligible). For a control flap of area S_{flap} we denote with F the aerodynamic lift force, which is given by

$$F = \frac{1}{2}\rho S_{flap} C_L V_i^2 = C_L \frac{S_{flap}}{S_{disk}} u_M$$

where $C_L = c_L u_F$, in which c_L is a constant coefficient which depends on the geometry of the flap, and u_F is the angle of attack with respect to the propeller downwash, which is the control input used to manipulate the flap force. As an approximation we neglect the aerodynamic drag forces. We will assume that the thrust u_M and the flap's angle of attack u_F are the available control inputs.

B. Free Flight Model

In order to derive a dynamical model to describe the ducted-fan UAV in free flight, according to [12], we make use of Newton-Euler equations of motion of a rigid body evolving on the configuration manifold $S_1 \times I\!R^2$. By fixing an inertial coordinate frame $F_i = \{O_i, \overrightarrow{i}_i, \overrightarrow{j}_i\}$ and a body frame $F_b = \{O_b, \overrightarrow{i}_b, \overrightarrow{j}_b\}$ in the center of mass of the aircraft, and, for sake of simplicity, by parameterizing the S_1 manifold with the angle $\phi \in I\!R$, we obtain the following equations

$$\begin{aligned} m\ddot{x}^i &= u_M \sin\phi + k_F u_M u_F \cos\phi \\ m\ddot{z}^i &= u_M \cos\phi - k_F u_M u_F \sin\phi - mg \\ J\ddot{\phi} &= -k_\tau u_M u_F \end{aligned} \quad (1)$$

in which m and J are respectively the vehicle mass and inertia, the control inputs $u = [u_M\ u_F]^T \in U$, i.e. $u_M \in [0, \bar{u}_M] \subset I\!R_{\geq}$ and $u_F \in [-\bar{u}_F, \bar{u}_F] \subset I\!R$, are given respectively by the propeller thrust and the flap's control angle, $k_F = c_L S_{flap}/\ S_{disk}$, and finally $k_\tau = c_L d S_{flap}/\ S_{disk}$, with d the moment arm of the flap's forces with respect to the center of mass of the vehicle.

Property 1 ***(Small body forces)*** [7] **-** The effects of the flaps forces on the translational dynamics are small (and indeed negligible) compared to the effect of the propeller's thrust, i.e. $k_F \bar{u}_F \ll 1$. □

Small body forces property holds for this kind of systems: it comes from a mechanical property of miniature aircraft of having a very small inertia J compared to the mass m.

Property 2 ***(Attitude controllability)*** **-** The attitude subsystem of (1) is *controllable* through the input u_F if and only if $u_M \neq 0$ (which means $u_M > 0$). □

C. Tracking for the approximate dynamics

We consider for the dynamics (1), approximated by *small body forces* properties, the problem of tracking a desired position $[x^i_{\rm r}\ y^i_{\rm r}]^T \in I\!R^2$ (see also [13], [7]). By exploiting the property of the approximate system of being *flat* for the above choice of the outputs (see [3], [11]), we compute the inputs which are necessary to achieve the invariance of the *zero-error manifold*

$$u_{M_{\rm r}} = \frac{m(g + \ddot{z}^i_{\rm r})}{\cos\phi_{\rm r}}\,, \qquad u_{F_{\rm r}} = \frac{-J\ddot{\phi}_{\rm r}\cos\phi_{\rm r}}{k_\tau m(g + \ddot{z}^i_{\rm r})}$$

where $\phi_{\rm r} = \arctan(\ddot{x}^i_{\rm r}/g + \ddot{z}^i_{\rm r})$. The *attitude controllability property* is satisfied by requiring $u_{M_{\rm r}} > \epsilon_u$ with $\epsilon_u > 0$.

D. Constrained Model

In the following sections we consider the dynamics of the system when the terminal points of the landing gear are into contact with a horizontal or vertical surface. These two scenario are representative for all the type of interactions that we are considering in this work. For sake of simplicity we will limit the analysis only to the case in which P_1 may be into contact to a fixed surface, observing that the analogous results can be extended to P_2 exploiting the symmetries of the system.

E. Interaction with a horizontal fixed surface

With an eye at figure 2, we assume that the ducted-fan can move horizontally over the surface characterized by $\beta = 0$ and rotate around the contact point P_1. Let us consider the generalized forces acting on the mass m with respect to the generalized coordinates $\alpha = x^i + l\cos\theta$ and $\theta = \phi + \gamma$

$$\begin{aligned} \mathcal{F}_\alpha &= k_F u_M u_F \cos\phi + u_M \sin\phi - \lambda_H \dot{\alpha} \\ \mathcal{F}_\theta &= k_F d_l \sin\gamma_F u_M u_F + l\cos\gamma u_M \end{aligned} \quad (2)$$

in which l, d_l, γ and γ_F are defined in figure 2 and λ_H is the viscous friction coefficient of the horizontal surface. The Lagrangian function of the system, considering kinetic and potential energies, is then given by:

$$\mathcal{L} = \frac{1}{2}m(\dot{\alpha}^2 - 2l\dot{\alpha}\dot{\theta}\sin(\theta) + l^2\dot{\theta}^2) - mgl\sin\theta\,. \quad (3)$$

Simple calculations allow to write the dynamical model

$$\begin{aligned} m\ddot{\alpha} + ml\ddot{\theta}\sin\theta + ml\dot{\theta}^2\cos\theta &= \mathcal{F}_\alpha \\ ml\ddot{\alpha}\sin\theta + ml^2\ddot{\theta} + mgl\cos\theta &= \mathcal{F}_\theta \end{aligned} \quad (4)$$

F. Interaction with a vertical fixed surface

With an eye at figure (2), we assume that the ducted-fan can move along a vertical surface characterized by $\alpha = \bar{\alpha}$ and rotate around the contact point P_1. Let us consider the generalized forces acting on the mass m with respect to the generalized coordinates $\beta = z^i - l\sin\theta$ and $\theta = \phi + \gamma$

$$\begin{aligned} \mathcal{F}_\beta &= -k_F u_M u_F \sin\phi + u_M\cos\phi - \lambda_V\dot{\beta} \\ \mathcal{F}_\theta &= k_F d_l \sin\gamma_F u_M u_F + l\cos\gamma u_M \end{aligned} \quad (5)$$

in which λ_V is the viscous friction coefficient of the vertical surface. The Lagrangian function of the system, considering kinetic and potential energies, is then given by:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{\beta}^2 + 2l\cos\theta\dot{\beta}\dot{\theta} + l^2\dot{\theta}^2\right) - mg(\beta + l\sin\theta) \quad (6)$$

and the dynamical model is given by

$$\begin{aligned} m\ddot{\beta} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 + mg &= \mathcal{F}_\beta \\ ml\ddot{\beta}\cos\theta + ml^2\ddot{\theta} + mgl\cos\theta &= \mathcal{F}_\theta \end{aligned} \quad (7)$$

G. An Hybrid Dynamical Model of the Overall Dynamics

In order to define the conditions under which the system dynamics are described by respectively the *free flight model* (1) or by the *constrained models* (4) and (7) we define the *model domains* by introducing appropriate conditions on system state and inputs. Let us first consider the two following force contribution

$$\begin{aligned} F_{Z^i} &= u_M\cos\phi - k_F u_M u_F \sin\phi - mg \\ F_{X^i} &= u_M\sin\phi + k_F u_M u_F\cos\phi \end{aligned}$$

where F_{Z^i} denotes the resultant vertical force directed along the inertial z^i axis, whereas F_{X^i} denotes the resultant force directed towards the vertical fixed surface and then along the inertial x^i axis. To denote a certain macro-state of the system we introduce a discrete variable q, where $q \in \{L, FF, SS, ToL\}$ with the following meaning

- L: *landed*. Both P_1 and P_2 are in contact with a horizontal surface (*ground*).
- FF: *free flight*. Neither P_1 nor P_2 are constrained by a contact with a vertical or horizontal surface. The system dynamics are described by (1).
- SS: *set-stand* to a vertical surface. $P_1 \oplus P_2$ are into contact with a vertical fixed surface. The system dynamics are described by (7). ($\oplus$ is the XOR logical operator).
- ToL: *take-off and landing*. $P_1 \vee P_2$ are into contact with a horizontal fixed surface. The system dynamics are described by (4). ($\vee$ is the OR logical operator).

More clearly, we define the two sets $C_h = \{p^i \in I\!R^2 : h(p) = 1\}$ and $C_v = \{p^i \in I\!R^2 : v(p) = 1\}$ where $h : I\!R^2 \mapsto \{0, 1\}$ and $v : I\!R^2 \mapsto \{0, 1\}$ are functions which define if a point in the frame F_i belong to respectively a horizontal or vertical fixed surface, and we consider the following constraints (see also [8], [1])

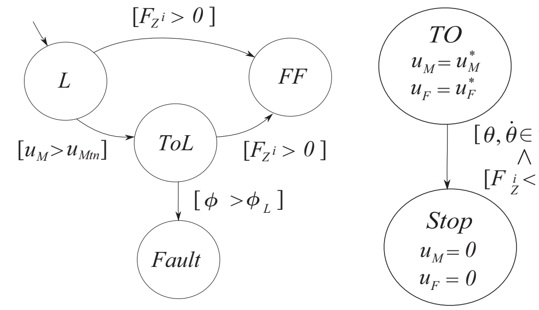

Fig. 3. The hybrid automata which model the dynamics during a take-off (on the left) and the control hybrid automata (on the right). The guard conditions are denoted with $[\cdot]$.

1) $q = L \Leftrightarrow (P_1 \in C_h \wedge P_2 \in C_h) \wedge (F_{Z^i} < 0)$
2) $q = Tol \Leftrightarrow (P_1 \in C_h \vee P_2 \in C_h) \wedge (F_{Z^i} < 0)$
3) $q = SS \Leftrightarrow (P_1 \in C_v \wedge F_{X^i} > 0) \oplus (P_2 \in C_v \wedge F_{X^i} < 0)$
4) $q = FF$ otherwise. ($\wedge$ is the AND logical operator).

For each macro-state q, it is possible to design a control law such that the system evolves inside the domains defined above. At the same time we consider situations in which the desired control goal implies the transition from one macro-state to another one. In particular this happens during the *take-off* maneuvers and from FF to SS.
In order to model the transition between FF and SS or ToL configurations, we make use of momentum conservation to define the initial conditions for the state of the system.

III. TAKE OFF ANALYSIS

Let us now focus to the study of a particular situation: the DFAV must perform a take-off maneuver but, as the horizontal surface is not completely smooth, we will consider some approximations on the dynamics (4), assuming that the viscous friction between the contact point and the surface is large; neglecting the translational dynamics we obtain

$$ml^2\ddot{\phi} = l\cos\gamma u_M + k_F d_l \sin\gamma_F u_M u_F - mgl\cos(\phi + \gamma). \quad (8)$$

Our goal is to assure a safe take-off for the DFAV, avoiding undesired overturning.

Def. ***(Turn Thrust)*** **-** The *turn* thrust $u_{M_{tn}}$ is the smallest value of u_M, which depends on control input u_F, such that the angular acceleration is greater than zero, i.e. such that $\forall[u_M\ u_F]^T : u_M \geq u_{M_{tn}}(u_F) \Rightarrow \ddot{\phi} \geq 0$,

$$u_{M_{tn}} = mgl\frac{\cos(\phi + \gamma)}{l\cos\gamma + k_F d_l \sin\gamma_F u_F}. \quad (9)$$

Def. ***(Take-Off Thrust)*** **-** The *take-off* thrust $u_{M_{to}}$ is the smallest value of u_M, which depends on the input u_F, such that $\forall u_M \geq u_{M_{to}} \Rightarrow F_{Z^i} \geq 0$,

$$u_{M_{to}} = \frac{mg}{\cos\phi - k_F\sin\phi u_F}. \quad (10)$$

It is straightforward to see that the choice of $u_M > u_{M_{to}}$ guarantees a safe take-off preventing the system from being overturned. For a real system anyway, the propeller thrust cannot be changed instantaneously, but represent the state of a low-level dynamics. In particular for fixed pitch propellers, the thrust is proportional to the square of the angular velocity, and the dynamics $\dot{u}_M$ depends on the engine total power and, for electric motors, from the battery charge. Let us now consider the case in which $u_{M_{turn}} < u_{M_{to}}$. This happens for all initial conditions in which $\phi(0) > 0$, a likely condition when the horizontal surface is not ideal and the DFAV is not perfectly horizontal when the take-off maneuver starts. In this condition we have that, performing a take-off maneuver increasing the thrust value u_M, it reaches the Turn Thrust before the Take-Off Thrust value; this can lead to an undesirable overturn the overall system (see figure 1 (a)). The dynamics of the take-off are represented by the hybrid automata in figure 3, in which the $Fault$ configuration represent the undesired overturn of the system.

Proposition 1 *Given the system (8) with inputs* $u := [u_M\ u_F] = [0\ 0]^T$, *the point of the state space* $\phi = \phi^L$, $\dot{\phi} = 0$, *where* $\phi^L = \pi/\ 2 - \gamma$ *is denoted as the* ***limit angle****, corresponds to an unstable equilibrium point. Moreover, assuming that* $l \cos\gamma + k_F d_l \sin\gamma_F \bar{u}_F > 0$ ***(small-body torques)****, the equilibrium point cannot be robustly stabilized by feedback unless a structural property holds for the system.*

Proof: Considering the linear approximation of (8), we have that the eigenvalues of the state transition matrix are $\pm\sqrt{g/\ l}$; converse Lyapunov theorem can be applied to show that the equilibrium point is unstable. Considering the change of coordinates $\phi \mapsto \tilde{\phi} := \phi - \phi^L$, we rewrite (8) as

$$ml^2\ddot{\tilde{\phi}} = l\cos\gamma u_M + k_F d_l \sin\gamma_F u_M u_F + mgl\sin\tilde{\phi} \quad (11)$$

Pick now an $\epsilon_\phi < \pi/\ 2$; no matter what is the input $u = \mathrm{col}(u_M, u_F) \in U$, we cannot find a positive small $\delta_\phi > 0$ such that if $\|\tilde{\phi}(0)\| \le \delta_\phi$ then $\|\tilde{\phi}(t)\| \le \epsilon_\phi$. In fact observe that, by the hypothesis on the control inputs and since $\cos(\tilde{\phi} + \pi/\ 2) < 0$ if $\tilde{\phi} > 0$, if we start the system with initial state $\tilde{\phi}_1(0) := \delta_{\phi_1} > 0$ the system (11) could be seen as a double integrator driven by a positive input and the trajectories diverge from the origin. This proves the proposition. □

Def. ***(limit angular velocity)*** **-** For any $\phi' < \phi^L$ we can find an angular velocity $\dot{\phi}^L(\phi') > 0$ such that for all $[\phi(0)\ \dot{\phi}(0)]^T = [\phi'\ \dot{\phi}']^T$ with $\dot{\phi}' < \dot{\phi}^L(\phi')$, the trajectory $\phi(t)$ of (8) with $u := [u_M\ u_F] = [0\ 0]^T$ is such that $|\phi(t)| < \phi_L$ for all $t \ge 0$.

The *limit angular velocity* can be derived numerically solving the system equations (8) with $u := [u_M\ u_F] = [0\ 0]^T$ for different initial conditions.

A. Hybrid take-off controller

Let us introduce an hybrid take-off controller able to avoid undesired overturning; the hybrid control law is represented by the second automata in figure 3. Let us consider the case in which $\phi(0) = \phi^\star > 0$. We design the control inputs u_M, u_F as open loop signals

- $u_M = u_M^\star > u_{M_{to}}(\phi^\star, u_F^\star)$
- $u_F = u_F^\star$ with $u_F^\star$ solution of the following optimization problem $min\ u_{M_{to}} + 1/u_{M_{tn}}$, $u_F \in [-\bar{u}_F, \bar{u}_F]$ which for $\phi > 0$ has the solution $u_F^\star = -\bar{u}_F$.

Then we define the unsafe domain $\mathcal{D}_{us} = \left\{[\phi\ \dot{\phi}]^T : \dot{\phi} > \dot{\phi}^L(\phi) - \epsilon_\phi\right\}$ with ϵ_ϕ a positive value. Once the state of the system is inside $\mathcal{D}_{us}$, if the actual input u_M has not reached, because of delays of the internal dynamics, the $u_{M_{to}}$ value, then the controller turn off the inputs to prevent the system from being overturned.

IV. Interaction with rigid vertical surfaces

In this section we consider the DFAV interacting with a vertical surface: this issue reveals to be very interesting as modern UAVs are requested to be not only capable to attain flight tasks, but even to perform tasks while interacting with the environment such as remote manipulation or indoor flight with reduced measurement information. In the next sections we are going to study firstly the mechanical characteristics that the DFAV must satisfy to be suitable to perform tasks while interacting with a wall and then a control law able to maintain the DFAV in SS configuration while performing a tracking task for the contact point.

A. Equilibrium analysis and design hints

Let us consider a DFAV as showed in figure 2 when its point $P1$ is in contact with the wall placed in $\alpha = \bar{\alpha}$. The system dynamics are described by (5), (7). Consider a generic equilibrium point $(\hat{\theta} = \hat{\phi} + \gamma, \hat{\beta})$; the control actions that should be performed at the equilibrium are

$$\begin{aligned}\hat{u}_M &= \frac{mg(\hat{d}_l \sin\gamma_F + \cos\hat{\theta}\sin\hat{\phi})}{(\hat{d}_l \sin\gamma_F \cos\hat{\phi} + \cos\gamma\sin\hat{\phi})} \\ \hat{u}_F &= \frac{-\sin\hat{\theta}\sin\hat{\phi}}{k_F(\hat{d}_l\sin\gamma_F + \cos\hat{\theta}\sin\hat{\phi})},\end{aligned} \quad (12)$$

where $\hat{d}_l = d_l/l$.

As our goal is to maintain the DFAV in contact with the wall (i.e. being able to perform tasks in SS configuration), we should investigate that the guard condition on the hybrid state transition in not verified while the system is performing its task. Let us calculate the constraining reaction of the wall, i.e. the resulting force acting along the X_i axis in equilibrium. To assure that the DFAV remains in contact with the wall this force should be positive (the discrete variable q defining the hybrid state of the system is constrained to be equal to SS). The resulting force $\hat{\mathcal{F}}_{X_i}$ is

$$\hat{\mathcal{F}}_{X_i} = mg\frac{\sin\hat{\phi}\sin(\gamma_F - \gamma)}{\cos\gamma\sin(\gamma_F + \hat{\phi})}. \quad (13)$$

As our objective will be likely to control the DFAV with small positive attitude angles, it is possible to remark that, if $\hat{\phi} \in]0; \pi/2 - \gamma[$, then $\hat{\mathcal{F}}_{X_i} < 0$. Hence the DFAV showed in figure 2 is not adequate to perform tasks in SS configuration as, already in equilibrium, resulting forces are trying to move the DFAV away from the wall, switching to the FF configuration (in this case $q^+ = FF$). This property is mainly due to the position of the flaps with respect to

the contact point P1: term $(d_l \sin\gamma_F - l\sin\gamma)$ in (13) is structurally negative as the aerodynamic force F is applied between the center of mass and the projection of P1 on the symmetry axis. In order to make the DFAV perform tasks in SS configuration, it should be possible to consider the equilibrium points characterized by $\hat{\phi} \in]-\pi/2+\gamma; 0[$; unfortunately this solution contrasts with the usual approach maneuver as the DFAV directs toward the wall with positive angle ϕ (and a controller able to switch from a configuration to another should be really difficult to implement) and with the small body forces property as this equilibrium should relay on the relevant use of force F in order to generate a positive F_{X_i} force. Nevertheless this issues represent an interesting future work to investigate. On the other hand the problem of maintain the DFAV in contact with the wall when particular tasks in SS configuration are requested can be solved with an accurate design of the mechanical structure of the DFAV: it is possible to design the system such that the aerodynamic lift force F acts below the contact point. In figure 2 a possible solution is depicted considering P1$'$ as the contact point (see also figure 1 (b)). Note that the model (5), (7) remains true just substituting γ with γ', l with l', d_l with d'_l and γ_F with $\gamma'_F < 0$. In order to improve the readability of the paper, in the following we consider P1$'$ as the contact point but continue to use variables name without the suffix $'$; then from now on $\gamma := \gamma'$, $l := l'$, $d_l := d'_l$ and $\gamma_F := \gamma'_F < 0$. It is possible to show that in this new configuration, considering an equilibrium point $(\hat{\phi}\,,\,\hat{\beta})$, the resulting force along the X_i axis $\hat{\mathcal{F}}_{X_i} > 0$ if $\hat{\phi} \in]0; |\gamma_F|[$.
All those considerations leads to an explicit mechanical design for a DFAV able to perform tasks in SS configuration: the contact point P1 should have its projection on the symmetry axis as much as possible near to the center of mass and away from the attitude control subsystem (maximizing $|\gamma_F|$ and hence the set of admissible attitude angles when the system is in SS configuration).

B. Tracking control law

In this section we are going to introduce a control law able to make the DFAV track a suitably defined trajectory in SS configuration. In particular we consider the system designed as in figure 2 with P1$'$ as contact point with the vertical surface (i.e. $\gamma_F < 0$). Let us introduce two new "virtual" control variables $\mathcal{F}_1$ and $\mathcal{F}_2$ as

$$\begin{aligned} \frac{\mathcal{F}_\theta}{l} &= ml\mathcal{F}_1 + m\cos\theta\mathcal{F}_2 \\ \mathcal{F}_\beta + \lambda_V\dot{\beta} &= ml\cos\theta\mathcal{F}_1 + m\mathcal{F}_2\,. \end{aligned} \tag{14}$$

It is now possible to write the system model (5) and (7) as

$$\begin{aligned} \ddot{\theta} &= \mathcal{F}_1 - \frac{\cos\theta\dot{\theta}^2}{\sin\theta} + \frac{\lambda_V\dot{\beta}\cos\theta}{ml\sin^2\theta} \\ \ddot{\beta} &= \mathcal{F}_2 + \frac{l\dot{\theta}^2}{\sin\theta} - g - \frac{\lambda_V\dot{\beta}}{m\sin^2\theta}\,. \end{aligned} \tag{15}$$

Remark 1 *If system trajectories fulfill* $0 < \phi(t) < \gamma_F\,,\,\forall\, t \geq 0$*, then real control actions* u_F*,* u_M *are univocally defined by* $\mathcal{F}_1$*,* $\mathcal{F}_2$ *and* θ*.*

As model (15) identifies the system when it is in SS configuration, if we want to perform a certain task in this configuration $\forall t > 0$, it must be assured that in this period $\mathcal{F}_{X_i} > 0$. Writing again $\mathcal{F}_{X_i}$ with respect to the new control inputs $\mathcal{F}_1$ and $\mathcal{F}_2$

$$\mathcal{F}_{X_i} = \frac{ml\mathcal{F}_1\bar{a}_1 + m\mathcal{F}_2\bar{a}_2}{\sin(\phi+\gamma_F)\cos\gamma} \tag{16}$$

with $\bar{a}_1 = \cos\gamma_F - \cos\gamma\cos(\phi+\gamma_F)\cos\theta$ and $\bar{a}_2 = \cos\theta\cos\gamma_F - \cos\gamma\cos(\phi+\gamma_F)$, it is possible to show that, suitably designing angles γ and γ_F, if $\phi \in]0; |\gamma_F|[$, $\bar{a}_1 > 0$ and $\bar{a}_2 \leq 0$.

Remark 2 *If we are able to design a control action assuring that, considering a suitable set* $\Omega^\star$ *of initial conditions* $(\theta(0), \dot{\theta}(0), \beta(0), \dot{\beta}(0)) \in \Omega^\star$*,* $\phi(t) \in]0; |\gamma_F|[\ \forall t \geq 0$ *and* $\mathcal{F}_1(t) < 0$*,* $\mathcal{F}_2(t) > 0\ \forall t \geq 0$ *then* $\mathcal{F}_{X_i}(t) > 0$ *and the system remains in* SS *configuration* $\forall t \geq 0$*.*

Now let us introduce the family of trajectories that the system is structurally able to track when constrained in SS configuration. When system (15) is forced to track a generic trajectory defined by $\theta^r(t)$, $\beta^r(t)$, asymptotically, "virtual" control inputs $\mathcal{F}_1$, $\mathcal{F}_2$ are

$$\begin{aligned} \mathcal{F}_1^r &= \ddot{\theta}^r + \frac{\cos\theta^r\dot{\theta^r}^2}{\sin\theta^r} - \frac{\lambda_V\dot{\beta}^r\cos\theta^r}{ml\sin^2\theta^r} \\ \mathcal{F}_2^r &= \ddot{\beta}^r + g - \frac{l\dot{\theta^r}^2}{\sin\theta^r} + \frac{\lambda_V\dot{\beta}^r}{m\sin^2\theta^r}\,. \end{aligned}$$

In order to be trackable, trajectories must be defined such that $\mathcal{F}_{X_i} > 0$; in our discussion we are interested in trajectories such that $\phi^r(t) \geq 0$ and in particular we are going to consider $0 < \phi^r(t) < \gamma_F\,,\forall t \geq 0$. Hence trajectories must be designed such that $\mathcal{F}_1^r < 0$ and $\mathcal{F}_2^r > 0$.
Given suitably defined trajectories $\theta^r(t)$ and $\beta^r(t)$, it is possible to design the "virtual" control input $\mathcal{F}_1$ and $\mathcal{F}_2$ as

$$\begin{aligned} \mathcal{F}_1 &= \frac{cos\theta\dot{\theta}^2}{\sin\theta} - \frac{\lambda_V\dot{\beta}\cos\theta}{ml\sin^2\theta} + \ddot{\theta}^r - \theta + \theta^r - \dot{\theta} + \dot{\theta}^r \\ \mathcal{F}_2 &= g - \frac{l\dot{\theta}^2}{\sin\theta} + \frac{\lambda_V\dot{\beta}}{m\sin^2\theta} + \ddot{\beta}^r - \beta + \beta^r - \dot{\beta} + \dot{\beta}^r\,. \end{aligned} \tag{17}$$

Defining $M := \sqrt{(\theta(0)-\theta^r(0))^2 + (\dot{\theta}(0)-\dot{\theta}^r(0))^2}$ and $N := \sqrt{(\beta(0)-\beta^r(0))^2 + (\dot{\beta}(0)-\dot{\beta}^r(0))^2}$, it is possible to state the following proposition.

Proposition 2 *If initial conditions and trajectories to be tracked are such that* $\forall t \geq 0$

$$M < \min\{\theta^r - \gamma\,,\,|\gamma_F| - \theta^r + \gamma\} \tag{18}$$

and

$$\begin{aligned} &\frac{\cos(\theta^r - M)}{\sin(\theta^r - M)}(M + \dot{\theta}^r)^2 + \\ &-\frac{\lambda_V(\dot{\beta}^r - N)\cos(\theta^r + M)}{ml\sin^2(\theta^r + M)} + \ddot{\theta}^r + 2M < 0 \end{aligned} \tag{19}$$

$$g - \frac{l(M+\dot{\theta}^r)^2}{\sin(\theta^r - M)} + \frac{\lambda_V(\dot{\beta}^r - N)}{m\sin^2(\theta^r + M)} + \ddot{\beta}^r - 2N > 0\,, \tag{20}$$

then $\forall t \geq 0 \ \phi \in \ ]0\,,|\gamma_F|[,\ \mathcal{F}_{X_i} > 0$ *and asymptotically* $\theta \to \theta^r$ *and* $\beta \to \beta^r$.

Proof: To prove this proposition consider that control actions (17) decouple completely θ and β dynamics. Defining error variables as $\tilde{\theta} = \theta - \theta^r$ end $\tilde{\beta} = \beta - \beta^r$, select as Lyapunov function for $\tilde{\theta}$ dynamic $V_{\tilde{\theta}} = 1/2\tilde{\theta}^2 + 1/2\dot{\tilde{\theta}}^2$. It is possible to check that $\dot{V}_{\tilde{\theta}} \leq 0$ and by LaSalle invariance principle $\tilde{\theta} \to 0$ as well as $\dot{\tilde{\theta}} \to 0$. Moreover $\{\tilde{\theta}^2(t)\,,\dot{\tilde{\theta}}^2(t)\} \leq M^2\,\forall t \geq 0$. As $\tilde{\theta} = \theta - \theta^r = \phi - \phi^r$, algebraic calculations and condition (18) show that $0 < \phi(t) < \gamma_F$ as requested. For the $\tilde{\beta}$ dynamic let us choose the Lyapunov function $V_{\tilde{\beta}} = 1/2\tilde{\beta}^2 + 1/2\dot{\tilde{\beta}}^2$. Again it is simple to check that $\dot{V}_{\tilde{\beta}} \leq 0$ and by LaSalle invariance principle $\tilde{\beta} \to 0$ as well as $\dot{\tilde{\beta}} \to 0$. Moreover $\{\tilde{\beta}^2(t)\,,\dot{\tilde{\beta}}^2(t)\} \leq N^2\,\forall t \geq 0$. Equations (19), (20) assure that, in the worst cases, $\mathcal{F}_1 < 0$ and $\mathcal{F}_2 > 0$. Hence $\mathcal{F}_{X_i} > 0\ \forall t \geq 0$, proving the proposition. □

Remark 3 *The control designed to perform the tracking task relies on a suitable choice of the desired trajectory to be tracked and on a suitable definition of the initial conditions. These can be imposed by a suitable choice of the desired free flight state trajectories for* x^i, z^i *and* ϕ *(and hence for* β *and* α*). Since we consider inelastic hits, the momentum remains constant and initial conditions when the system switches from* FF *to* SS *are defined by*

$$\begin{aligned} \dot{\alpha}^+ + l\sin\theta^+\dot{\theta}^+ &= \dot{\alpha}^- + l\sin\theta^-\dot{\theta}^- \\ \dot{\beta}^+ + l\cos\theta^+\dot{\theta}^+ &= \dot{\beta}^- + l\cos\theta^-\dot{\theta}^- \end{aligned} \tag{21}$$

with $\dot{\alpha}^+ = 0$, $\theta^- = \theta^+$ *and* $\beta^- = \beta^+$*. Conditions (18), (19) and (20) could be verified not only choosing the task to be performed in* SS *configuration, but also with a suitable definition of the trajectories to be tracked in* FF *configuration when the DFAV is approaching the wall.*

Remark 4 *Some preliminary developments have been carried out to study robustness issues with respect to friction constant* λ_V*; the following adaptive algorithm assures the asymptotic stability of the system (see [4], [14]) but only preliminary and conservative conditions regarding the trajectories to be tracked and initial conditions have been derived.*

$$\begin{aligned} \mathcal{F}_1 &= \frac{\cos\theta\dot{\theta}^2}{\sin\theta} - \frac{\hat{\lambda}_V\dot{\beta}\cos\theta}{ml\sin^2\theta} + \ddot{\theta}^r + \\ &\quad -(k_1^2 + k_1 + 1)\tilde{\theta} - (k_1+1)\dot{\tilde{\theta}} \\ \mathcal{F}_2 &= g - \frac{l\dot{\theta}^2}{\sin\theta} + \frac{\hat{\lambda}_V\dot{\beta}}{m\sin^2\theta} + \ddot{\beta}^r + \\ &\quad -(k_2^2 + k_2 + 1)\tilde{\beta} - (k_2+1)\dot{\tilde{\beta}} \end{aligned} \tag{22}$$

with $k_1 \geq 1$, $k_2 \geq 1$ *and*

$$\dot{\hat{\lambda}}_V = -\frac{(\dot{\tilde{\beta}} + k_2\tilde{\beta})\dot{\beta}}{(m\sin^2\theta)} + \frac{(\dot{\tilde{\theta}} + k_1\tilde{\theta})\dot{\beta}\cos\theta}{ml\sin^2\theta}\,. \tag{23}$$

Further studies are surely required in order to assure non conservative conditions under which the system will remain in SS *configuration attaining the required task; with this regard, our belief is that saturated adaptive techniques (see [9] or [2]) will naturally help to obtain robust control algorithms.*

Some simulations have been carried out to point out the effectiveness of the control design and to enlighten the robustness of the adaptive extension. Due to reason of space, they can be found in [5].

V. Conclusion and Future Works

In this paper the problem of modeling and control of a particular configuration of ducted-fan aerial vehicle (DFAV) is taken into account considering explicitly the interaction with the environment. In particular two different scenario have been investigated: the first regards the study of safe take-off and is representative of all the situations in which the system has to interact with horizontal fixed surfaces. The second one includes all possible situations in which the system can come into contact with vertical fixed surfaces. In particular a controller able to perform a tracking task when the DFAV is in contact with a wall is designed under some conditions depending on the trajectory to be tracked and on the initial conditions related to the movement of the DFAV when it is flying freely and is approaching the wall. Robustness issues have been preliminarily taken into account and an adaptive algorithm to estimate system friction have been designed; nevertheless further studies are required in order to obtain less conservative results.

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