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A study on the use of virtual sensors in vehicle control

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Abstract— **The design of linear virtual sensors to estimate yaw rate for vehicle stability control systems is investigated. Standard model-based virtual sensor design techniques are compared to novel direct virtual sensor (DVS) design methodologies. The obtained DVS is stable and it can be used in a large range of operating conditions. It is shown how the use of virtual sensors derived directly from data and a suitable choice of the measured variables in sensor design improves the estimation and control accuracy. The effectiveness of the proposed DVS design is demonstrated by its employment in an existing yaw rate feedback loop, based on an Active Front Steering actuator and designed using Internal Model Control techniques. Robust stability is guaranteed in the presence of model uncertainty and of the DVS. In particular, the presented study shows that the DVS technology can be conveniently taken into account to replace physical sensors to obtain low cost stability control solutions for application on A and B segment cars.**

I. INTRODUCTION

Stability control systems are able to significantly enhance safety and handling properties ([1]), modifying vehicle dynamics by means of a wide range of technical solutions (see [2]-[8]). A feedback control structure is usually employed, where the controlled variable is the yaw rate, which is related to vehicle trajectory and can be measured with reasonably low production cost on medium and high class cars. The effectiveness of vehicle stability control motivates the actual trends of both car manufacturers and road regulations to apply these devices also on A and B segment cars, leading to large-scale production. In this context, economical advantages can be achieved by substituting an already low-cost solution with an even cheaper one: thus, the idea of replacing a physical yaw rate sensor with a virtual one is of great interest, since it would allow the application of yaw control also on economic cars. Virtual Sensors (VS, see e.g. [9]) are software algorithms which exploit a set of available measurements to compute an estimate of a physical quantity of interest. In the automotive context, a quite extensive literature can be found regarding yaw rate estimation using the measures of wheel speeds (already available due to the presence of Anti-lock Braking Systems), steering angle (measured for electric power steering systems) and/or lateral acceleration (whose sensor is less expensive than yaw rate), see e.g. [10], [11], [12]. The common approach to obtain a VS is to design a linear or nonlinear observer based on a simplified vehicle model. Linear observers like Kalman filters are simple to derive and to implement and they have guaranteed stability, but they are accurate only in a restricted range of operating conditions. Nonlinear observers may be able to give good estimates in a larger range of operating conditions, but their computational cost for on-line implementation may be high and stability of the estimation error is much harder to guarantee. Moreover, when the estimated variable is employed in a control structure, robust closed loop stability has to be guaranteed for a reasonably large scenario for obvious safety reasons.

A new approach to derive VS has been introduced in [13] for the nonlinear case and in [14] for the linear case. In particular, with such approach the VS is derived directly from data (i.e. Direct Virtual Sensor, DVS), using a one–step procedure which avoids the use of a model of the system. As a matter of fact the use of nonlinear DVS has been successfully employed in [15] for the open loop prediction of the vehicle sideslip angle. In this paper, a new DVS approach is proposed to design linear yaw rate virtual sensors to be employed in an existing vehicle stability control loop. The obtained DVS is stable and the estimation error is low also in nonlinear vehicle operating conditions. Furthermore, it is shown that the fact that the DVS operates in an already designed feedback control loop can be taken into account using closed-loop data in the virtual sensor design, improving the performance with respect to the use of open-loop data and obtaining quite good results with a reduced number of measured variables with respect to the case of open-loop DVS derivation. A comparison with a Kalman filter approach is introduced too, in order to show the possible advantages of the use of the one-step DVS procedure in the considered context.

An Active Front Steering device (AFS, see [6]) has been chosen as actuator, in the considered control structure, for its safety properties since, contrary to steer by wire systems, the driver intervention on the steering angle is always kept through a mechanical link. The feedback controller design is carried out using a linear vehicle model and robust Internal Model Control (IMC, [16]) techniques based on H_∞ optimization, which have been already successfully applied in stability control problems ([5], [8]). The robust stability properties are checked a posteriori in the presence of the considered VS. The effectiveness of the proposed approach is tested through simulation tests using a detailed 14 degrees of freedom (d.o.f.) nonlinear vehicle model, which proved to give good description of vehicle behavior with respect to real measured data.

II. VEHICLE YAW CONTROL

Vehicle active control systems aim to enhance handling and comfort characteristics ensuring stability in critical maneu-

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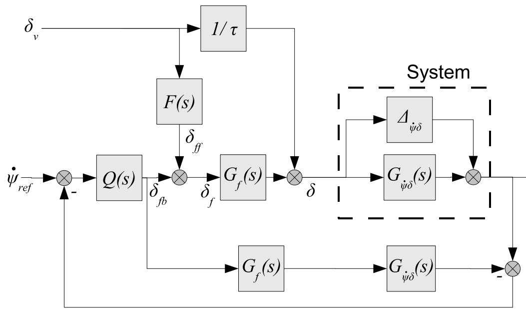

Fig. 1. The employed control structure

vering situations. Most of these systems employ a feedback control structure where the controlled variable is the yaw rate $\dot{\psi}(t)$, since it is strictly related to vehicle trajectory. Indeed, the control input should be able to modify the vehicle dynamics exploiting appropriate combinations of longitudinal and/or lateral tyre forces. Among the different solutions proposed in the literature (see e.g. [2]-[8]), in this paper, an approach similar to AFS systems (see [6]) is adopted, where the steering angle issued by the driver is modified using an electromechanical device. Such a solution is quite interesting from a safety point of view as, contrary to steer by wire systems, driver intervention on the steering action is always kept (see [6] for details).

A. Control structure and model description

A control structure based on IMC methodology is used as it has been proved to be quite effective in the context of vehicle stability as shown in [5] and [8] (see Fig. 1). In the adopted scheme, the handwheel angle $\delta_v(t)$ acts the steer via the transmission ratio τ, together with the superimposed steering angle provided by the AFS system, described by $G_f(s)$, on the basis of the control input $\delta_f(t)$. The latter is the sum of the two contributions $\delta_{\text{ff}}(t)$ and $\delta_{\text{fb}}(t)$: $\delta_{\text{ff}}(t)$ is the feedforward action from the driver steering angle $\delta_v(t)$ computed through the filter $F(s)$, while $\delta_{\text{fb}}(t)$ is the feedback action of the IMC controller $Q(s)$. For the control design, vehicle dynamics are described by the transfer function $G_{\dot{\psi}\delta}(s)$ between the steering angle $\delta(t)$ and yaw rate $\dot{\psi}(t)$ computed, for a given value of the speed $v(t)$, on the basis of the following single track model (see e.g. [1]) which takes into account the dynamic generation mechanism of tyre forces:

$$
\begin{aligned}
&mv\dot{\beta}(t) + mv\dot{\psi}(t) = F_{yf}(t) + F_{yr}(t) \\
&J_z\ddot{\psi}(t) = aF_{yf}(t) - bF_{yr}(t) \\
&F_{yf}(t) + l_f/v\dot{F}_{yf}(t) = -c_f(\beta(t) + a\dot{\psi}(t)/v - \delta(t)) \\
&F_{yr}(t) + l_r/v\dot{F}_{yr}(t) = -c_r(\beta(t) - b\dot{\psi}(t)/v)
\end{aligned} \tag{1}
$$

In (1), m is the vehicle mass, J_z is the moment of inertia around the vertical axis, β is the side-slip angle, a and b are the distances between the center of gravity and the front and rear axles respectively; the front and rear tyre relaxation lengths are indicated as l_f and l_r, while the symbols c_f and c_r stand for the front and rear axle cornering stiffnesses. F_{yf} and F_{yr} are the front and rear axle lateral forces.

In order to take into account the uncertainty induced by the different operating conditions on the nominal model (1), an additive model set of the form (2) has been employed in the control design:

$$\mathcal{G}_{\dot{\psi}\delta}(G_{\dot{\psi}\delta},\Gamma) = \{(G_{\dot{\psi}\delta}(s) + \Delta_{\dot{\psi}\delta}(s)) : |\Delta_{\dot{\psi}\delta}(j\omega)| \leq \Gamma(\omega)\} \tag{2}$$

$\Delta_{\dot{\psi}\delta}(s)$ is the unstructured additive uncertainty (see e.g. [17]) and $\Gamma(\omega)$ is an upper bound of the magnitude of $\Delta_{\dot{\psi}\delta}(j\omega)$.

B. Control design

In the control structure of Fig. 1, the yaw rate reference $\dot{\psi}_{\text{ref}}$ is provided by a static map that uses as inputs the current values of $\delta_v(t)$ and $v(t)$. The values of $\dot{\psi}_{\text{ref}}$ are computed in order to improve the vehicle maneuverability and increase the lateral acceleration limit. For a detailed description on the criteria followed in the map construction, see [5].

The IMC controller $Q(s)$ is designed to optimize vehicle performance, while guaranteeing robust stability in the presence of the model uncertainty generated by the wide range of operating conditions. In particular, $Q(s)$ can be computed by means of the following optimization problem (see e.g. [16]):

$$Q(s) = \arg \min_{\left\|Q(s)G_f(s)\bar{\Gamma}(s)\right\|_\infty < 1} \left\|W_S^{-1}(s)\,S(s)\right\|_\infty \tag{3}$$

where $\bar{\Gamma}(s)$ is a suitable rational function with real coefficients, stable, whose magnitude strictly overbounds $\Gamma(\omega)$ and $W_S(s)$ is a weighting function introduced to take into account a given specification on the nominal sensitivity $S(s) = 1 - G_{\dot{\psi}\delta}(s)G_f(s)Q(s)$.

Remark - As discussed in [5] and [8], enhanced IMC structures can be employed in order to improve performance in the presence of the control input saturation, while still guaranteeing robust stability.

The feedforward contribution $\delta_{\text{ff}}(t)$ has been added to improve the dynamic response characteristic. In particular, the filter $F(s)$ is designed to match the open loop yaw rate behavior given by $G_{\dot{\psi}\delta}(s)$ with the one described by an objective transfer function $T^{\text{des}}(s)$:

$$\dot{\psi}(s) = T^{\text{des}}(s)\delta_v(s) \tag{4}$$

Therefore, since the control input $\delta_{\text{ff}}(t)$ should be actuated by the AFS system, the feedforward filter $F(s)$ is computed as:

$$F(s) = \left(\frac{T^{\text{des}}(s)}{G_{\dot{\psi}\delta}(s)} - 1/\tau\right)\frac{1}{G_f(s)} \tag{5}$$

Moreover, since the feedforward controller aims to enhance the transient response only, its contribution should be deactivated in steady state conditions. This is achieved when the dc-gains of $T^{\text{des}}(s)$ and $G_{\dot{\psi}\delta}(s)/\tau$ are the same.

III. Vehicle yaw control using virtual sensors

The operation of the control structure presented in Section II implies the measurement of the yaw rate. However, as discussed in the Introduction, a cheaper solution for stability control on A and B segment cars could be obtained using an estimate of the yaw rate given by a virtual sensor, VS for short, which relies on measurements usually available on passenger cars. A VS is a dynamic system whose output is an estimate $\hat{z}$ of the unmeasured output z of a process. The inputs of the virtual sensor are a subset of the manipulated inputs u and of the measured outputs y of the process. In this Section, two alternative design procedures of linear virtual sensors for yaw control are presented.

A. Model based virtual sensors

The standard methodology to design a virtual sensor is a two step procedure: on a first step, a process model is built, either from first principles or from an identification experiment. Then, an optimal estimator, in a minimum variance sense, is designed for the model. When the available model is linear and disturbances and noises are characterized as white random processes, the optimal VS is a full state observer tuned as a steady state Kalman filter.

In particular, consider a process S with inputs $u \in \mathbb{R}^m$ and $d \in \mathbb{R}^d$, outputs $\tilde{y} \in \mathbb{R}^q$ and $z \in \mathbb{R}$. Assume d is a white random process with covariance matrix Q_d. Output $\tilde{y}$ is corrupted by white noise v with covariance matrix Q_y. A steady state Kalman filter is a dynamic system $H^K(s)$ that takes u and $y = \tilde{y} + v$ and gives and estimate $\hat{z}^K$:

$$\hat{Z}^K(s) = H_U^K(s)U(s) + H_Y^K(s)Y(s),$$

such that:

$$E\{(z - \hat{z}^K)^2\} = \min_{H(s)\in\mathcal{H}_\infty} E\{(z - \hat{z})^2\}$$

where $E\{\cdot\}$ is the expected value operator.

B. Direct Virtual Sensors

An alternative approach to the virtual sensor design problem, proposed in [13], is to derive the filter from suitable process input-output measured data. Such information is employed to identify a linear direct virtual sensor (DVS) in a one-step procedure, in which the DVS structure is not fixed a priori. Depending on the data employed in the design, the obtained linear DVS is able to give good estimation performance also in nonlinear process operating conditions, when the linear models used in classic approaches suffers from under-modeling (for a complete comparison, see [14]).

Assume that an initial experiment can be performed, where the variable z is measured. Denote with u_k, y_k and z_k the sampled values of u, y and z respectively, corresponding to any sampling instant $k \in \mathbb{N}$, with fixed sampling period T_s. Assume that a number N of measurements are collected in the initial experiment, corresponding to sampling instants kT_s, $\forall k \in [1, N]$. In the following, these N values of u_k, y_k, z_k are denoted as the *data set*. In the design phase, the DVS is expressed as a discrete time FIR filter that uses present and past values of u_k and y_k to give an estimate $\hat{z}_k^{DVS}$ of z_k, that is:

$$\hat{z}_k^{DVS} = \sum_{j=0}^{n_u} \alpha_j u_{k-j} + \sum_{j=0}^{n_y} \beta_j y_{k-j} \qquad (6)$$

where n_u, n_y are design parameters which define the structure of the DVS. Assuming that z is observable from y, it can be shown that the estimation error $\epsilon_k = z_k - \hat{z}_k^{DVS}$ is bounded for any bounded input sequence (see [14]). On the basis of (6), the DVS can be designed by minimizing a weighted p-norm of the estimation error on the collected data set, i.e. on the collected values of u_k, y_k and z_k for any $k \in [\underline{k}, N]$, where $\underline{k} = \max(n_u, n_y)$:

$$[\hat{\alpha}_0, \ldots, \hat{\alpha}_{n_u}, \hat{\beta}_0, \ldots, \hat{\beta}_{n_y}] = \arg\min \left(\sum_{k=\underline{k}}^{N} \left|w_k^{-1}\epsilon_k\right|^p\right)^{1/p}$$

such that

$$\begin{cases} \epsilon_k = z_k - \sum_{j=0}^{n_u} \alpha_j u_{k-j} - \sum_{j=0}^{n_y} \beta_j y_{k-j}, k \in [\underline{k}, \underline{k}+1, \ldots, N] \\ ||\alpha_j||_\infty \le L_u \rho^j,\ j \in [0, 1, \ldots, n_u] \\ ||\beta_j||_\infty \le L_y \rho^j,\ j \in [0, 1, \ldots, n_y] \end{cases} \qquad (7)$$

where $L_u > 0$, $L_y > 0$ and $0 < \rho < 1$. This convex optimization problem with linear constraints can be efficiently solved. L_u, L_y and ρ bound the decay rate of the DVS impulse response and can be tuned to minimize the estimation error of the filter. Larger bounds, with ρ close to 1, lead to lower errors for the identification data and longer impulse responses, but also cause poor performances on new data. By suitably selecting the weights w_k, it is possible to consider noise measures dependent on k, for example relative measurement errors. Details on how to tune this parameters are shown in [18].

Regardless of the used norm, solution to problem (7) is usually a high order FIR filter. Then, model order reduction techniques are used to fit the identified impulse responses with a stable and causal IIR filter of a prefixed order n. Finally, the resulting estimator is mapped to the s domain using a bilinear transformation, in order to evaluate its effects on the IMC control loop. Thus, the form of the DVS considered in the following is:

$$\hat{Z}(s)^{DVS} = H_U^{DVS}(s)U(s) + H_Y^{DVS}(s)Y(s) \qquad (8)$$

C. VS for yaw rate

In the automotive context, several low-cost sensors can be used to provide the measurements y used by a VS to compute an estimate of the unmeasured variable of interest z, i.e. the vehicle yaw rate. In particular, in this paper it is assumed that the measures of the differences between the left and right wheel speeds of the front and rear axles, $\Delta\omega_f$ and $\Delta\omega_r$ respectively, and of the lateral acceleration a_y can be employed. Therefore, the considered output y is composed by a suitable subset of the variables a_y, $\Delta\omega_f$, $\Delta\omega_r$, depending on the specific measurements chosen in the VS design. The other variables introduced in Section III-B are $u = \delta$ and $z = \dot{\psi}$. As a first approximation, the values of $\Delta\omega_f$ and $\Delta\omega_r$ are related to vehicle yaw rate through the kinematic equations ([1]):

$$\begin{aligned} \Delta_{\omega_f} &= \omega_{fl} - \omega_{fr} = \dot{\psi} d_f / R_w \\ \Delta_{\omega_r} &= \omega_{rl} - \omega_{rr} = \dot{\psi} d_r / R_w \end{aligned} \qquad (9)$$

where R_w is the nominal wheel radius and d_f, d_r are the distances between the wheels of the front and rear axle respectively. Moreover, according to the single track model, the lateral acceleration is linked to lateral forces through the following dynamic equilibrium ([1]):

$$a_y(t) = (F_{yf} + F_{yr})/m \qquad (10)$$

Equations (9) and (10) and the model (1) are used to verify that, at least when the underlying assumptions are satisfied,

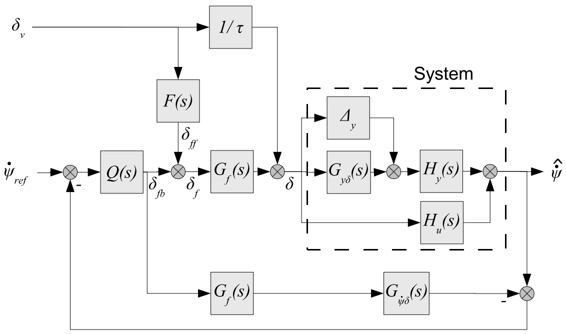

Fig. 2. Control structure using a virtual sensor.

the considered system is stable and observability of yaw rate is guaranteed from any of the measured outputs. Thus, the hypotheses for the application of virtual sensors are fulfilled.

Remark - A VS that provides a yaw rate estimate $\hat{\dot{\psi}}$ can therefore be designed using one or more of the possible available measurements. As it will be shown in Section IV, the use of all the three measures of a_y, $\Delta\omega_f$ and $\Delta\omega_r$ does not necessarily give the best estimation accuracy. Indeed, figuring out a priori which measured variables should be used to obtain good accuracy appears to be a hard task: physical insight and trial-and-error procedures can be used to practically establish the best combination of measurements to be employed.

D. Yaw control using VS

As already pointed out in the Introduction, the use of a yaw rate estimate for feedback control must also be addressed from the point of view of robust stability. On the basis of the small gain theorem (see e.g. [17]), a robust stability condition is derived, which can be checked a posteriori to assess if the control strategy already designed according to (3) is still robustly stable in presence of the VS. To this end, equations (1), (9) and (10) are employed to derive a nominal transfer matrix $G_{y\delta}$ from input δ to the considered output y, together with an additive model set of the form:

$$\mathcal{G}_{y\delta}(G_{y\delta},\Gamma) = \{G_{y\delta}(s) + \Delta_y(s) : \bar{\sigma}\left(\Delta_y(j\omega)\right) \le \Gamma_y(\omega)\} \quad (11)$$

where $\Delta_y(s)$ is the additive uncertainty associated to the transfer matrix $G_{y\delta}(s)$ and the symbol $\bar{\sigma}(\cdot)$ stands for the maximum singular value. When the yaw rate virtual sensor is used for control purposes instead of a physical sensor, the IMC loop becomes that of Fig. 2. Defining the functions:

$$G_{VS}(s) = (H_y(s)G_{y\delta}(s) + H_u(s))G_f(s) \quad (12)$$

$$\Delta_{VS}(s) = H_y(s)\Delta_y(s)G_f(s) \quad (13)$$

$$C(s) = Q(s)(1 - G_{\dot{\psi}\delta}(s)G_f(s)Q(s))^{-1} \quad (14)$$

The scheme of Fig. 2 can be seen as a classical feedback structure, where the robust stability condition is:

$$\left\|\Gamma_{VS}(s)C(s)(1 + C(s)G_{VS}(s))^{-1}\right\|_\infty < 1 \quad (15)$$

where $\Gamma_{VS}(s)$ is a stable rational transfer function of real coefficients, such that $\bar{\sigma}\left(\Delta_{VS}(j\omega)\right) \le \Gamma_{VS}(j\omega)$. If condition (15) is satisfied, robust closed loop stability in front of model uncertainty and in presence of the VS is guaranteed. Note that the evaluation of (15) is significantly simplified by the use of a linear VS. If (15) is not satisfied, a new VS design should be performed.

IV. Simulation results

A. Control design

The control design has been performed using transfer function $G_{\dot{\psi}\delta}(s)$, evaluated at a nominal speed v = 100 km/h = 27.77 m/s and with the following values for the other model parameters:
m = 1715 kg, J_z = 2697 kgm^2, a = 1.07 m, b = 1.47 m,
l_f = 1 m, l_r = 1 m, c_f = 89733 Nm/rad, c_r = 114100 Nm/rad
R_w = 0.303 m, d_f = 1.48 m, d_r = 1.35 m
Model sets (2) and (11) have been constructed by considering variations of the vehicle nominal speed between 70 and 130 km/h, increments of vehicle mass up to a $+20\%$, with consequent inertial and geometrical changes, independent variations of rear and front tire cornering stiffness between $[-30\%, +10\%]$. Moreover, $\pm 10\%$ variations of the tire radius have been considered when constructing the model set associated to the transfer functions between steering angle and wheel angular speeds (see [5] for details). For simplicity, in this study, $G_f(s) = 1$ has been considered. The AFS steering contribution δ_f is supposed to be limited such that $|\delta_f| \le 5°$. The IMC controller has been obtained as the solution to (3) using the weighting function:

$$W_S(s) = 1.06\frac{s}{s+19}$$

The following transfer function $T^{\text{des}}(s)$ is employed in the feedforward filter $F(s)$ design (see (5)):

$$T^{\text{des}}(s) = \frac{1}{(1+s/2)(1+s/20)}.$$

B. Virtual sensors design and performance

Kalman filter (KF) design Model equations (1), have been used as process model to design minimum variance filters. All possible combinations of the three available measurements, i.e. a_y, $\Delta\omega_f$ and $\Delta\omega_r$, have been considered: thus, seven different filters have been designed. The considered input noise variance is $Q_d = 2.5 \cdot 10^{-5}\,\text{rad}^2$ and the output noise variances are 0.01 (m/s^2)^2 for a_y and $2.5 \cdot 10^{-3}$rad/s^2 for Δ_{ω_f} and Δ_{ω_r}.

Direct virtual sensor (DVS) identification A 14 d.o.f. nonlinear vehicle model, which proved to give an accurate description of the vehicle dynamics with respect to real measurements (see [5], [8]), has been used to generate the identification data and to validate the virtual sensors and the control strategies. White noise with the same variance characteristics used for Kalman filter design has been employed to corrupt the identification data. Two data sets have been obtained through two different experiments: in the first scenario, the vehicle has been driven in open loop by imposing a suitably designed handwheel course, composed of quick ramps and constant intervals plus a pseudo-random binary signal. The second experiment has been performed in closed loop fashion using a similar handwheel input and employing the controllers designed in Section IV-A. The experimental tests lasted 90 s each and the employed vehicle speeds were 70 km/h between 0 s and 30 s, 100 km/h from 30 s to 60 s and

130 km/h for the remaining 30 s. Identification algorithm (7), using Euclidean norm and unitary weights, has been applied to the collected open and closed loop data sets. All the possible combinations of the three available measurements have been considered and in each case, different decay rate constraints and filter parameters have been considered and those offering lower estimation error, while satisfying the robust stability condition (15), have been selected.

At this point, the performance of the yaw rate control system, when the physical sensor is replaced by a VS has to be evaluated. In particular, as remarked in III-C, a suitable analysis has to be performed in order to select an appropriate sensor able to ensure performance and safety for the widest possible range of the vehicle operating conditions. To this end the following maneuvers have been considered:

- constant speed steering pad at 100 km/h, the handwheel angle is increased slowly (1°/s) while the vehicle is kept at constant speed to evaluate the steady state tracking behavior.
- steer reversal test with handwheel angle of 5° and 50°, performed at 90 km/h to evaluate steady state and transient vehicle performance in linear and non linear operating conditions.

The root mean squared error (RMSE), defined as:

$$E_{\text{rms}} = \sqrt{\frac{1}{t_{end} - t_0} \int_{t_0}^{t_{end}} (\dot{\psi}_{\text{ref}}(t) - \dot{\psi}(t))^2 dt}$$

where t_0 and t_{end} are the starting and final test time instants, has been used as index to evaluate the average system operation in terms of safety and performance. Tables I - III, present the performance of the control loops using the yaw rate estimate obtained with Kalman filters (KF) and direct virtual sensors, identified using either open loop data (DVS_{OL}) or closed loop data (DVS_{CL}). Missing values, denoted as "–", mean that the control loop shows too poor performance (lightly damped dynamics) during the maneuver. It can be noticed that for the Kalman filter based control loops, only the one using the a_y measure offers acceptable performances for a large range of maneuvers. The best tracking and estimation accuracy for the DVS identified from the open loop data, is obtained with the filter using all the available measures, while for the DVS identified from closed loop data, the filter using a_y and $\Delta\omega_f$ only shows the best performance (see the bold face values in Tables I-III).

TABLE I

AVERAGE VS PERFORMANCE FOR THE STEERING PAD MANEUVER

Inputs	KF	DVS_{OL}	DVS_{CL}
$[\delta, a_y]$	**0.0101**	0.0254	0.0543
$[\delta, \Delta\omega_f]$	–	0.0074	0.0087
$[\delta, \Delta\omega_r]$	–	0.0084	0.0095
$[\delta, a_y, \Delta\omega_f]$	–	0.0087	**0.0060**
$[\delta, a_y, \Delta\omega_r]$	–	0.0062	0.0098
$[\delta, \Delta\omega_f, \Delta\omega_r]$	–	0.0065	0.0078
$[\delta, a_y, \Delta\omega_f, \Delta\omega_r]$	–	**0.0062**	0.0067

TABLE II

AVERAGE VS PERFORMANCE FOR 5° STEER REVERSAL TEST

Inputs	KF	DVS_{OL}	DVS_{CL}
$[\delta, a_y]$	**0.0040**	0.0103	0.0122
$[\delta, \Delta\omega_f]$	0.0033	0.0049	0.0054
$[\delta, \Delta\omega_r]$	0.0031	0.0048	0.0053
$[\delta, a_y, \Delta\omega_f]$	0.0039	0.0043	**0.0041**
$[\delta, a_y, \Delta\omega_r]$	0.0039	0.0046	0.0038
$[\delta, \Delta\omega_f, \Delta\omega_r]$	0.0032	0.0044	0.0053
$[\delta, a_y, \Delta\omega_f, \Delta\omega_r]$	0.0039	**0.0041**	0.0043

TABLE III

AVERAGE VS PERFORMANCE FOR 50° STEER REVERSAL TEST

Inputs	KF	DVS_{OL}	DVS_{CL}
$[\delta, a_y]$	**0.0292**	0.0406	0.0562
$[\delta, \Delta\omega_f]$	–	0.0256	0.0236
$[\delta, \Delta\omega_r]$	–	0.0253	0.0236
$[\delta, a_y, \Delta\omega_f]$	0.0340	0.0234	**0.0210**
$[\delta, a_y, \Delta\omega_r]$	0.0433	0.0246	0.0226
$[\delta, \Delta\omega_f, \Delta\omega_r]$	–	0.0240	0.0235
$[\delta, a_y, \Delta\omega_f, \Delta\omega_r]$	–	**0.0232**	0.0218

C. Vehicle performance using virtual sensor inside the loop

A deeper performance analysis is carried out choosing for each of the considered VS typology (i.e. KF, DVS_{OL}, DVS_{CL}) the structure which reaches the best overall results. The following scheme illustrates such a choice

$$\begin{aligned} KF &\leftrightarrow [\delta, a_y] \\ DVS_{OL} &\leftrightarrow [\delta, a_y, \Delta\omega_f, \Delta\omega_r] \\ DVS_{CL} &\leftrightarrow [\delta, a_y, \Delta\omega_f] \end{aligned}$$

Considering the steer reversal maneuvers, Tables IV and V show the obtained results in terms of rise time t_r, maximum overshoot with respect to the steady state $\hat{s}$, of settling time t_s and of steady state errors e_{ref}:

$$t_r = t_2 - t_1 \quad t_2 : \dot{\psi}(t_2) = 0.9 \cdot \dot{\psi}_\infty \quad t_1 : \dot{\psi}(t_1) = 0.1 \cdot \dot{\psi}_\infty$$
$$\hat{s} = \frac{\dot{\psi}_{max}}{\dot{\psi}_\infty} \cdot 100$$
$$t_s : 0.99 \cdot \dot{\psi}_\infty \leq \dot{\psi}(t) \leq 1.01 \cdot \dot{\psi}_\infty \quad \forall t \geq t_s$$
$$e_{ref} = \frac{\dot{\psi}_\infty}{\dot{\psi}_{ref}} \cdot 100$$

where $\dot{\psi}_{max}$ is the maximum value assumed by the response, $\dot{\psi}_{ref}$ is the steady state value of the yaw rate reference and $\dot{\psi}_\infty$ is the steady state value of the yaw rate response. In particular, steady state error is linked to the estimation error, thus the more accurate is the estimated yaw rate the lower is the steady state tracking error. It can be noted that with a handwheel angle of 5°, the DVS perform better than Kalman filter, for all the considered indexes. With an handwheel angle of 50°, the performances obtained using the direct virtual sensors are very close to each other, but the DVS_{OL} needs 4 measures in input, while the DVS_{CL} requires only 3 measures, i.e. 3 physical sensors.

TABLE IV

STEER REVERSAL TEST WITH 5° HANDWHEEL

$Q(s)$	t_r (s)	$\hat{s}$ (%)	t_s (s)	e_{ref} (%)
DVS_{OL}	**0.25375**	1.0785	**3.0284**	**1.0841**
DVS_{CL}	0.29756	**1.0453**	3.3038	1.1546
KF	0.31047	1.079	3.1524	1.0994

TABLE V

STEER REVERSAL TEST WITH 50° HANDWHEEL

$Q(s)$	t_r (s)	$\hat{s}$ (%)	t_s (s)	e_{ref} (%)
DVS_{OL}	0.24003	**1.0443**	**2.8899**	**1.0034**
DVS_{CL}	**0.23882**	1.0568	3	1.0303
KF	0.27942	1.0977	3.1326	1.0489

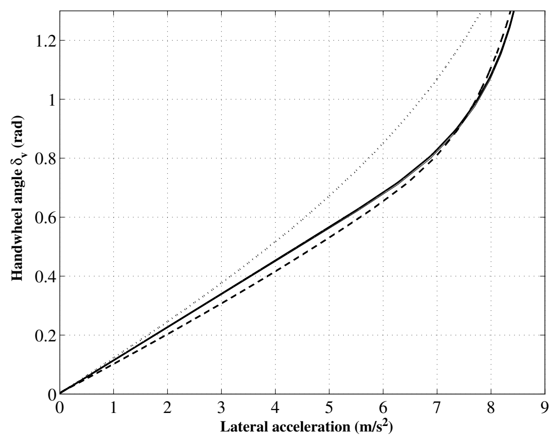

Fig. 3. Steering pad test at 100 km/h. Thin line: reference steering diagram. Comparison between the steering diagrams obtained with the uncontrolled vehicle (dotted) and with the controlled ones using either yaw rate sensor (solid) or DVS (dashed).

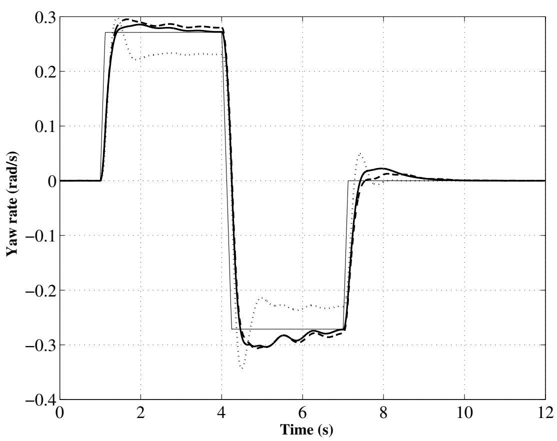

Fig. 4. 50° steer reversal test at 90 km/h. Thin line: reference yaw rate. Comparison between the yaw rate courses obtained with the uncontrolled vehicle (dotted) and with the controlled ones using either yaw rate sensor (solid) or DVS (dashed).

Employing the DVS_{CL} configuration, because of its simpler structure, steering pad and steer reversal maneuvers have been performed, corrupting the measures with white additive noise with the same variance considered for filter design. Fig. 3 shows the results of a steering pad maneuver at 100 km/h: it can be noted that when the DVS is employed the controlled vehicle is still able to reach a higher lateral acceleration value with respect to the uncontrolled case, thus improving vehicle handling, see [5]. Finally, performing a 50° steer reversal maneuver at 90 km/h, the obtained yaw rate course (Fig. 4) shows the improvements of the system damping properties and the controlled vehicle reaches a higher yaw rate value with respect to the uncontrolled one, thus showing improved maneuverability. In particular, the overall results obtained are quite close to the case of measured yaw rate feedback.

V. Conclusions and Future Developments

A new approach to design a yaw rate virtual sensor, to be employed in an existing vehicle stability control loop, has been presented. The VS is derived directly from the data collected in an initial experiment, without the use of a model of the considered vehicle. The obtained linear DVS is stable and the estimation error is low also in nonlinear vehicle operating conditions. Furthermore, the use of closed-loop data in the DVS design allows to obtain quite good results with a reduced number of measured variables with respect to the case of open-loop DVS derivation.

The considered stability system employs an AFS: the feedback controller design has been carried out using IMC techniques and a robust stability condition can be checked a posteriori to assure safety in presence of the both uncertainty and DVS. Simulation results, performed with an accurate 14 d.o.f. vehicle model, show that quite good estimation accuracy is obtained for a wide range of vehicle operating conditions. Significant improvements have been obtained with respect to a Kalman filter approach which has shown a performance degradation in nonlinear vehicle operating condition. As future developments, the design of a DVS with guaranteed closed loop stability will be investigated, as well as the computation of a robust controller which takes into account the presence of an already designed DVS in the feedback loop.

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