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A New Approach to Integrated Damping Parameter and Control Design in Structural Systems

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Abstract— **The paper presents a Linear Matrix Inequality (LMI)-based approach for the simultaneous optimal design of output feedback control gains and damping parameters in structural systems with collocated actuators and sensors. The proposed integrated design is based on simplified $\mathcal{H}^2$ and $\mathcal{H}^\infty$ norm upper bound calculations for collocated structural systems. Using these upper bound results, the combined design of the damping parameters of the structural system and the output feedback controller to satisfy closed-loop $\mathcal{H}^2$ or $\mathcal{H}^\infty$ performance specifications is formulated as an LMI optimization problem with respect to the unknown damping coefficients and feedback gains. Numerical examples motivated from structural and aerospace engineering applications demonstrate the advantages and computational efficiency of the proposed technique for integrated structural and control design. The effectiveness of the proposed integrated design becomes apparent, especially in very large scale structural systems where the use of classical methods for solving Lyapunov and Riccati equations associated with $\mathcal{H}^2$ and $\mathcal{H}^\infty$ designs are time-consuming or intractable.**

I. Introduction

The traditional design optimization of a plant and the controller usually follows a sequential strategy, where the plant is designed first, followed by the controller design. However, this design strategy will not lead to an optimal performance since the plant and the controller optimization problems are coupled. In fact, in the above two-step design methodology, the full design freedom is not utilized to obtain the optimal total system. It is known that the overall system performance can be significantly improved if the design process of the plant and the control system is integrated [13], [8], [9]. The integrated design strategy corresponds to a simultaneous optimization of the design parameters of both the plant and the controller to satisfy desired design specifications and to optimize the performance of the closed-loop system. Past research work has verified that the integrated strategy provides closed-loop systems with improved performance compared to the sequential method of design. However, the integrated plant/controller design optimization problem is a complex nonlinear nonconvex optimization problem and does not guarantee convergence to the global optimum of the design variables [13], [7], [14], [1]. This makes the integrated design strategy computationally very challenging. Recently, several integrated $\mathcal{H}^\infty$ plant/controller design approaches have been proposed using a Linear Matrix Inequality (LMI) formulation of the control problem to take advantage of systematic LMI approaches for robust control design [10], [6]. In principle, these formulations result in Bilinear Matrix Inequality (BMI) problems even if we assume that the coefficient matrices of the plant state space form are linear functions of the structural design parameters. In past attempts, the BMI formulation of the integrated plant/controller design is solved as an iterative LMI-based optimization problem. However, these iterative methods are also unable to guarantee convergence to the optimum solution, and they are computationally intensive.

The control of structural systems with collocated sensors and actuators has been shown to provide great advantages from a stability, passivity, robustness and an implementation perspective. For example, collocated control can easily be achieved in a space structure when an attitude rate sensor is placed at the same location as a torque actuator [3], [5]. Collocation of sensors and actuators leads to externally symmetric transfer functions. Several other classes of engineering systems, such as circuit systems, chemical reactors and power networks, can be modeled as systems with symmetric transfer functions.

This paper presents an effective and computationally tractable approach to integrate the structural and control design in collocated structural systems using $\mathcal{H}^2$ or $\mathcal{H}^\infty$ norm closed-loop performance criteria. The objective of the paper is to determine the optimal values of the damping parameters of the structure and to simultaneously design optimal output feedback gains such that an upper-bound of the closed-loop system norm (either in the $\mathcal{H}^2$ or the $\mathcal{H}^\infty$ setting) from the disturbance input signals to the desired outputs is minimized.

In the present paper, we use recently developed control-oriented algebraic tools to formulate the simultaneous damping and the control parameter design problem as an LMI optimization problem. Such problems are convex and can be readily solved using efficient interior-point optimization solvers. By exploiting the particular structure of collocated structural systems, explicit upper bound expressions for the $\mathcal{H}^2$ and $\mathcal{H}^\infty$ norm of such systems can be obtained. Subsequently, our simultaneous damping and output feedback controller parameter design problem is formulated as a convex optimization problem of minimizing the $\mathcal{H}^2$ and $\mathcal{H}^\infty$ norm bounds with respect to the unknown damping and control variables subject to constraints on the damping parameters and the feedback control gain norm. Note that, unlike past approaches, the proposed method for integrated design is not based on an iterative procedure to determine the design parameters. The benefits of the proposed integrated design become obvious when dealing with very large scale structural systems, where the use of past iterative methods

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for integrated design becomes intractable or even fails.

II. Symmetric Output Feedback Control of Collocated Systems

Consider the following vector second-order representation of a structural system with collocated actuators and sensors

$$\begin{aligned} M\ddot{q}(t) + D\dot{q}(t) + Kq(t) &= Fu(t) + Ew(t) \\ y(t) &= F^T\dot{q}(t) \\ z(t) &= E^T\dot{q}(t) \end{aligned} \quad (1)$$

where $q(t) \in \mathbb{R}^n$ is the generalized coordinate vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $w(t) \in \mathbb{R}^k$ is the vector of disturbance inputs, $y(t) \in \mathbb{R}^m$ is the measured output vector, and $z(t) \in \mathbb{R}^k$ is the output performance vector. The matrices M, D and K are symmetric positive definite matrices that represent the structural system mass, damping and stiffness distribution, respectively. The above finite-dimensional representation is often encountered in the dynamics of structural systems resulting from a finite element approximation of distributed parameter structural systems. It is noted that velocity feedback as in (1) is common in the collocated control of structural systems through a velocity sensor, a displacement sensor with a derivative controller or an accelerometer with an integral controller. In smart structures with piezoelectric sensors velocity feedback can be readily achieved through direct strain rate feedback [4].

The symmetric static output feedback control synthesis problem is to design a symmetric static feedback gain G such that the output feedback control law

$$u(t) = -Gy(t) \quad (2)$$

renders the closed-loop system stable with appropriate closed-loop performance.

The closed-loop system has a state-space realization as follows

$$\begin{aligned} \dot{x} &= Ax + Bw \\ z &= Cx \end{aligned} \quad (3)$$

with

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}(D + FGF^T) \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}, \quad C = \begin{bmatrix} 0 & E^T \end{bmatrix} \quad (4)$$

where $x^T = [q^T \quad \dot{q}^T]$. Notice that the transfer function $H(s)$ of the system (3)-(4) determined to be

$$H(s) = sE^T(Ms^2 + (D + FGF^T)s + K)^{-1}E$$

is symmetric, *i.e.*, $H(s) = H^T(s)$.

A. $\mathcal{H}^\infty$ and $\mathcal{H}^2$ Norm Upper Bounds for Collocated Systems

Recall that the $\mathcal{H}^\infty$ norm of the system (3) is given by

$$||H||_\infty = \sup_{\omega \in \mathbb{R}} \sigma_{\max}\{H(j\omega)\} \quad (5)$$

where $H(s) = C(sI - A)^{-1}B$ is the transfer function of the system. The $\mathcal{H}^\infty$ norm of a single input-single output (SISO) system is the peak magnitude of its frequency response function. In a time-domain interpretation, the $\mathcal{H}^\infty$ norm corresponds to the energy (or $\mathcal{L}_2$ norm) gain of the system from the input w to the output z. Hence, in this setting the $\mathcal{H}^\infty$ norm defines a disturbance rejection property of the system. It is well known that for a stable LTI system, its $\mathcal{H}^\infty$ norm can be approximated iteratively, *e.g.* using a bisection method.

The collocated $\mathcal{H}^\infty$ control synthesis problem is to design a symmetric control law (2) that stabilizes the closed-loop system and guarantees an $\mathcal{H}^\infty$ norm less than a prescribed bound $\gamma > 0$. The authors in [2] have shown that for an open-loop vector second-order realization (3)-(4) (*i.e.* with $G = 0$), an upper bound on its $\mathcal{H}^\infty$ norm can be computed using a simple explicit formula. The explicit bound of [2] can be obtained using the Bounded Real Lemma (BRL) characterization of the $\mathcal{H}^\infty$ norm of a system.

Lemma 1: A stable system with a state-space realization (3) has an $\mathcal{H}^\infty$ norm from w to z less than or equal to γ if and only if there exists a matrix $P \geq 0$ satisfying

$$\begin{bmatrix} A^TP + PA & PB & C^T \\ B^TP & -\gamma I & 0 \\ C & 0 & -\gamma I \end{bmatrix} \leq 0 \quad (6)$$

The $\mathcal{H}^2$ norm of a stable continuous-time system with transfer function $H(s) = C(sI - A)^{-1}B$ is defined as the root-mean-square (rms) of its impulse response, or equivalently

$$\|H\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} trace(H^H(j\omega)H(j\omega))d\omega}$$

In the following, an LMI formulation for computing the $\mathcal{H}^2$ norm of a system using its state-space data is recalled. This formulation enables us to use the efficient LMI solvers to solve for the Lyapunov matrix and compute the $\mathcal{H}^2$ norm μ.

The collocated $\mathcal{H}^2$ control synthesis problem is to design a symmetric static feedback gain G such that the output feedback control law (2) stabilizes the closed-loop system and guarantees an $\mathcal{H}^2$ norm less than a prescribed level $\mu > 0$.

Lemma 2: [15] Suppose that the system (3) is asymptotically stable, and let $H(s) = C(sI - A)^{-1}B$ denote its transfer function. Then the following statements are equivalent:

- $\|H\|_2 \leq \mu$
- There exist symmetric nonnegative definite matrices P and Z such that

$$\begin{bmatrix} PA + A^TP & PB \\ B^TP & -I \end{bmatrix} \leq 0 \quad (7)$$
$$\begin{bmatrix} P & C^T \\ C & Z \end{bmatrix} \geq 0 \quad (8)$$
$$trace(Z) \leq \mu^2 \quad (9)$$

The following lemma recalls the $\mathcal{H}^2$ norm calculation based on the solution of a Lyapunov equation.

Lemma 3: The $\mathcal{H}^2$ norm of the system (3) is given by

$$\|H\|_2 = [trace(CPC^T)]^{\frac{1}{2}} \quad (10)$$

where P is determined by solving the following Lyapunov equation.

$$AP + PA^T + BB^T = 0 \tag{11}$$

Computation of the $\mathcal{H}^2$ norm of a system using the LMI formulation (7)-(9) or the Lyapunov equation approach (10)-(11), and consequently designing an $\mathcal{H}^2$ control law, can be very intensive, especially for large scale systems. The LMI problem (7)-(9) has a polynomial-time complexity with respect to the number of decision variables, while solution of Lyapunov equations is of quadratic complexity with respect to flops and storage requirements. Consequently, the use of these tools for performance analysis and control of large scale systems is prohibitive. The authors in [11] provide a simple analytical explicit expression for an upper bound on the $\mathcal{H}^2$ norm of a collocated structural system. Numerical examples in [11] demonstrate the validity and computational efficiency of the proposed analytical bound on the $\mathcal{H}^2$ norm of collocated systems. The authors in [11] also present an explicit parametrization of the suboptimal output feedback control gains that achieve a desired level of closed-loop $\mathcal{H}^2$ performance.

III. Integrated Damping and Control Design Using the Analytical Bound Approach

We will consider the integrated design problem of simultaneously designing the damping parameters and the output feedback control gain of the collocated structural system (1)-(2) to satisfy $\mathcal{H}^\infty$ or $\mathcal{H}^2$ norm closed-loop specifications. For lumped parameter systems, the damping matrix D can be expressed in terms of the elemental damping coefficients as follows

$$D = \sum_{i=1}^{l} c_i \mathfrak{T}_i \tag{12}$$

where c_i denotes the viscous damping constant of the ith damper and $\mathfrak{T}_i$ represents the distribution matrix of the corresponding damper in the structural system. The distribution matrices $\mathfrak{T}_i$ are known symmetric matrices with elements 0, 1 and -1 that define the structural connectivity of the damping elements in the structure. Our objective is to formulate the $\mathcal{H}^2$ and $\mathcal{H}^\infty$ integrated damping parameter and control gain design problems as LMI optimization problems.

Practical structural system design specifications impose upper bound constraints on the values of the damping coefficients, that is

$$0 \le c_i \le c_{i\max} \quad , \quad i = 1, \ldots, l \tag{13}$$

Also, often an upper bound on the total available damping resources is enforced, that is

$$\sum_{i=1}^{l} c_i \le c_{\text{cap}}. \tag{14}$$

Another constraint that is needed in the proposed output feedback control design is a bound on the norm of the feedback gain matrix. This restriction is placed to constrain the amount of control effort required by the controller. For this purpose, we include the following constraint in the integrated design problem.

$$\|G\| \le g_{bound} \tag{15}$$

We assume that $c_{i\max}$, c_{cap} and g_{bound} are given scalar bounds determined by the physical constraints of the design problem.

A. *Integrated Damping and Controller Design for $\mathcal{H}^\infty$ Specifications*

Using the above formulation, the solution of the integrated design of the damping parameters and the output feedback controller to satisfy closed-loop $\mathcal{H}^\infty$ specifications is obtained by the following result.

Theorem 1: Consider the collocated structural system (1) with the damping distribution (12). For a given positive scalar γ, the $\mathcal{H}^\infty$ norm of the closed-loop system of the collocated structural system (1) and the output feedback controller (2) is less than γ if the following matrix inequalities with respect to the controller gain G and the damping coefficients c_i are feasible.

$$\begin{bmatrix} \sum_{i=1}^{l} c_i \mathfrak{T}_i + FGF^T & E \\ E^T & \gamma I \end{bmatrix} \ge 0 \tag{16a}$$

$$0 \le c_i \le c_{i\max} \quad , \quad i = 1, \ldots, l \tag{16b}$$

$$\sum_{i=1}^{l} c_i \le c_{cap} \tag{16c}$$

$$\|G\| \le g_{bound} \tag{16d}$$

Proof. Consider the closed-loop interconnection of the collocated structural system (1) and the output feedback law (2). Taking the Lyapunov matrix P into account as

$$P = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \tag{17}$$

along with substituting the closed-loop system matrices (4) into the BRL condition (6) and using the Schur complement formula [15] results in the following inequality

$$\begin{bmatrix} D + FGF^T & E \\ E^T & \gamma I \end{bmatrix} \ge 0 \tag{18}$$

Substitution of the damping matrix expansion (12) results in the inequality (16a). The constraints (16b)-(16d) represent the physical constraints of the design problem as discussed earlier. ■

The above conditions establish an LMI feasibility problem with respect to the unknown damping coefficients c_i and the controller gain G. The optimum damping coefficients c_i and the controller gain G to minimize the $\mathcal{H}^\infty$ norm bound of the closed-loop system can be obtained by solving the following LMI optimization problem

$$\begin{cases} \min_{c_i, G} & \gamma \\ \text{subject to} & (16a) - (16d) \end{cases} \tag{19}$$

B. Integated Damping and Controller Design for $\mathcal{H}^2$ Specifications

Based on our $\mathcal{H}^2$ upper bound results, the integrated design of the damping parameters of a collocated structural system and the output feedback control law to satisfy closed-loop $\mathcal{H}^2$ norm specifications is formulated as follows.

Theorem 2: Consider the collocated structural system (1) with the damping distribution (12). For a given positive scalar μ, the $\mathcal{H}^2$ norm of the closed-loop system of the collocated structural system (1) and the output feedback controller (2) is less than μ if the following matrix inequalities with respect to the controller gain G, the damping coefficients c_i, and the positive scalar α are feasible.

$$-2(\sum_{i=1}^{l} c_i \mathfrak{T}_i + FGF^T) + \alpha FF^T \le 0 \qquad (20a)$$

$$\begin{bmatrix} \alpha M & F \\ F^T & Z \end{bmatrix} \ge 0 \qquad (20b)$$

$$trace(Z) \le \mu^2 \qquad (20c)$$

$$0 \le c_i \le c_{i\max} \quad , \quad i = 1, \ldots, l \qquad (20d)$$

$$\sum_{i=1}^{l} c_i \le c_{cap} \qquad (20e)$$

$$\|G\| \le g_{bound} \qquad (20f)$$

Proof. We now consider a Lyapunov matrix P as follows

$$P = \alpha \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \qquad (21)$$

where α is a positive scalar. Substituting the matrix P and the closed-loop system matrices (4) into the $\mathcal{H}^2$ inequality conditions (7)-(9) results in the following inequalities

$$\begin{bmatrix} -2\alpha(D + FGF^T) & \alpha F \\ \alpha F^T & -I \end{bmatrix} \le 0 \qquad (22a)$$

$$\begin{bmatrix} \alpha M & F \\ F^T & Z \end{bmatrix} \ge 0 \qquad (22b)$$

$$trace(Z) \le \mu^2 \qquad (22c)$$

The scalar α in the selected Lyapunov matrix (21) is an unknown parameter that can be used as an additional degree of freedom in our formulation in order to reduce the conservativeness of the $\mathcal{H}^2$ norm bound. Note that due to the cross product of α and D in (22a), this inequality is not an LMI. However, application of Schur complement formula to (22a) yields

$$-2(D + FGF^T) + \alpha FF^T \le 0$$

which is an LMI with respect to α, G, and D. Substitution of the damping matrix expansion (12) completes the results. ■

The above conditions establish an LMI feasibility problem with respect to the damping coefficients c_i, the controller gain G, and the positive scalar α. Given the scalar bounds $c_{i\max}$, c_{cap} and g_{bound}, the optimum values of the damping coefficients and the controller gain that minimize the $\mathcal{H}^2$ norm bound can be obtained by solving the following LMI optimization problem

$$\begin{cases} \min_{\alpha, c_i, G} & \mu^2 \\ \text{subject to} & (20a) - (20f) \end{cases} \qquad (23)$$

Remark 1: The control gain matrix norm bound condition in (16d) and (20f) can be written in an LMI form as follows

$$\begin{bmatrix} g_{bound}^2 I & G^T \\ G & I \end{bmatrix} \ge 0 \qquad (24)$$

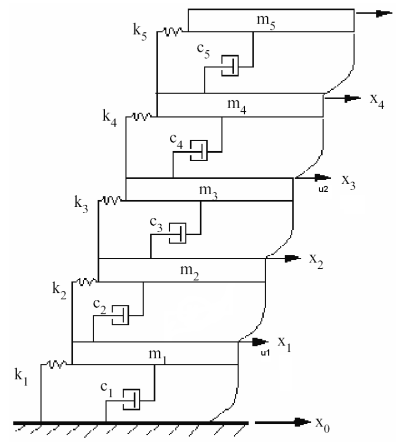

Fig. 1. Lumped model schematic of a 5-story structure.

IV. Numerical Examples

In this section, we validate the proposed integrated damping parameter and control design methodology using computational examples borrowed from the structural engineering and the aerospace engineering fields.

Example 1:

As a first application example, we consider the model of a 5-story base isolated building structure as shown in Figure 1. The mass of each floor, including that of the base, is assumed to be 6×10^5 kg. The stiffness of the structure varies in steps of $5 \times 10^7 N/m$ between floors from $7 \times 10^8 N/m$ for the first floor to $9 \times 10^8 N/m$ for the fifth floor. The design objective is to optimize the values of the damping coefficients c_i, $i = 1, ..., 5$, as well as, the output feedback control gain G such that the $\mathcal{H}^\infty$ norm of the closed-loop system consisting of the collocated structure and the output feedback from the disturbance forces $w_1(t)$ and $w_2(t)$ to the velocities of the masses m_1 and m_3 is minimized.

The damping distribution matrix D of this system is given by (12), where the distribution matrices $\mathfrak{T}_i$ are known. To examine integrated design trade-offs, let us consider a family of optimal designs using the result of Theorem 1. We examine two different scenarios. First, we fix the upper bound on the feedback control gain matrix norm $g_{bound} = 3$. We consider different designs corresponding to different values of the total damping capacity c_{cap} ranging from $1\ Ns/m$ to $10^5\ Ns/m$. The results of the optimal integrated designs using the results of Theorem 1 are shown in Figures 2 and 3. Figure 2 shows the profile of the $\mathcal{H}^\infty$ norm bound obtained from solving the convex optimization problem (19), as well as, the exact $\mathcal{H}^\infty$ norm that corresponds to each design as the total damping capacity c_{cap} changes. This figure illustrates the accuracy of the closed-loop $\mathcal{H}^\infty$ norm bound of the collocated structural system achieved by solving the LMI optimization problem of Theorem 1. We also show in Figure 3 the values of the optimal structural damping parameters, as well as, the closed-loop damping ratios (in %) corresponding to each design. It should be noted that the damping ratios corresponding to the last two floors are comparable to those of the first three floors even though the damping parameters c_4 and c_5 are very small. Indeed, c_4 and c_5 remain very small (close to zero) and c_1, c_2 and c_3 are becoming significantly larger as c_{cap} increases. The reason for this behavior is the location of the sensors (and actuators) on the first and third floors. It is indeed expected that the first three dampers will be the dominant ones to damp the structure's velocity response to the sensors on the first and third floors.

As a second design scenario, we consider a given bound for the total damping capacity $c_{cap} = 100\ Ns/m$, and we minimize the $\mathcal{H}^\infty$ norm of the closed-loop system. We compare different designs obtained by varying the upper bound on the controller gain norm g_{bound}. Figure 4 depicts the optimal $\mathcal{H}^\infty$ norm bound obtained from solving the optimization problem of Theorem 1, as well as, the exact $\mathcal{H}^\infty$ norm corresponding to each design versus g_{bound}. Note that for calculating the actual $\mathcal{H}^\infty$ norm of the closed-loop system, the feedback interconnection of the open-loop structure and the controller is considered, where the structure includes the designed values of the damping parameters c_i, and the controller is constructed using the feedback control gain G obtained from the solution of the optimization problem (19).

Example 2:

As a second example, we apply the proposed integrated design method for the damping parameter and control gain design of a large scale collocated structural system. We consider the finite element structural model for the assembly phase 8A-OBS of the International Space Station (ISS) with collocated control and Rayleigh damping. This example follows the state-space model in (3) and (4) with 216 states. For this example, we are interested in the optimum design of the damping parameters and the output feedback control gain from the closed-loop $\mathcal{H}^2$ norm performance viewpoint. We assume given bounds on the total damping capacity and the norm of the controller gain as follows: $c_{cap} = 1\ Ns/m$ and $g_{bound} = 5$. Solving the LMI optimization problem of Theorem 2 for the unknown damping parameters and control gains results in an optimal closed-loop $\mathcal{H}^2$ norm bound of $\mu = 0.42706$. The actual $\mathcal{H}^2$ norm of the system for the designed parameters is $\mu = 0.42696$. The frequency responses of the undamped open-loop system, the open-loop system damped by optimized damping parameters, and the damped closed-loop system designed using the $\mathcal{H}^2$ upper bound approach are shown in Figure 5. Notice that

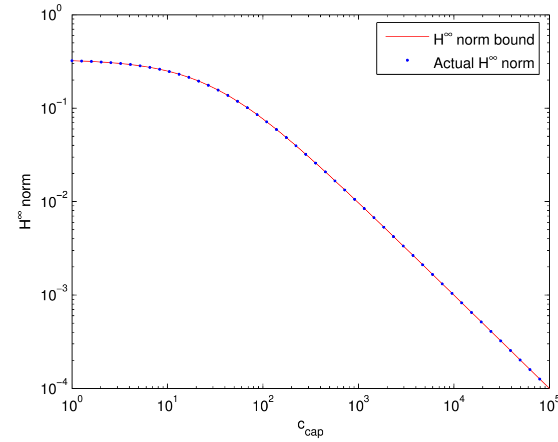

Fig. 2. Profiles of the $\mathcal{H}^\infty$ norm upper bound and the actual $\mathcal{H}^\infty$ norm for the optimized structure vs. the total damping capacity.

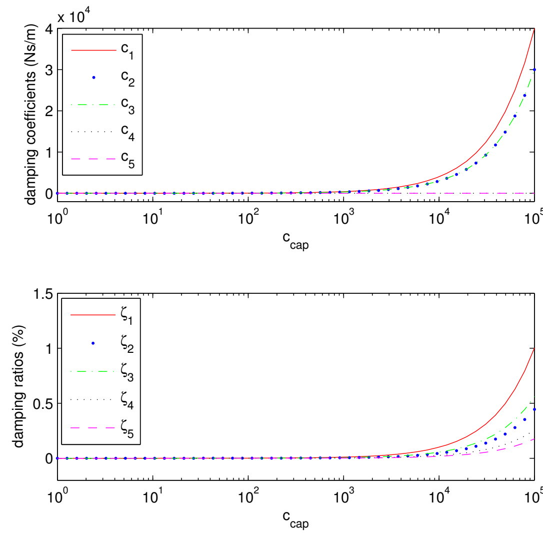

Fig. 3. Values of the optimized structural damping coefficients (top plot) and the optimized closed-loop damping ratios (bottom plot) vs. the total damping capacity.

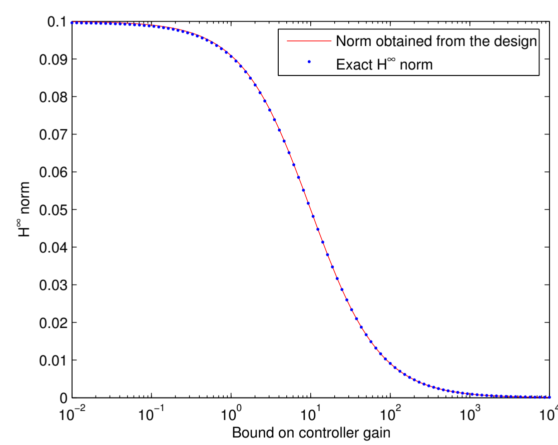

Fig. 4. Profiles of the closed-loop $\mathcal{H}^\infty$ norm obtained by solving the LMI optimization problem and the actual norm calculated based upon the optimized structure vs. bound on feedback control gain g_{bound}.

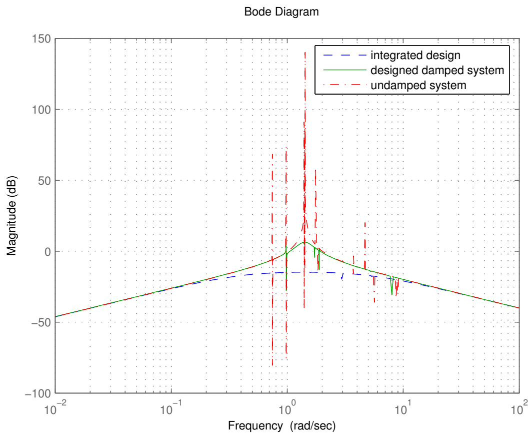

Fig. 5. Frequency responses of the undamped system, damped system constructed by designed damping parameters, and closed-loop system of the damped system and $\mathcal{H}^2$ controller, from $w_2(t)$ to $\dot{q}_2(t)$.

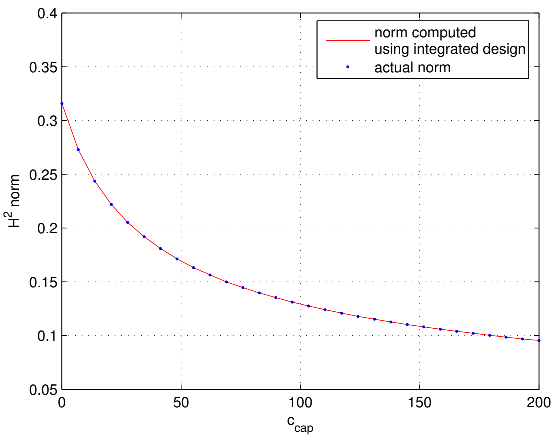

Fig. 6. Profiles of the $\mathcal{H}^2$ norm obtained by solving the optimization problem of Theorem 2 and the actual $\mathcal{H}^2$ norm of the closed-loop system of the optimized structure and feedback control vs. the total damping capacity.

the optimized damping parameter design of the open-loop system is obtained by setting $g_{bound} = 0$ and coincides with the result of [12]. The results demonstrate that the simultaneous design of structure and controller provides improved disturbance rejection compared to the approach of [12] that seeks to optimize only damping parameters. Note that using traditional methods for simultaneous control and damping parameter design of this system could easily become prohibitive due to the high dimensionality of the system.

Finally, Figure 6 shows the $\mathcal{H}^2$ norm bound obtained by solving the integrated damping parameters and control gain optimization problem presented in Theorem 2 for different values of the total damping capacity c_{cap} and the actual $\mathcal{H}^2$ norm of the structural system for each design. It is observed that the value of the $\mathcal{H}^2$ norm bound and the achievable $\mathcal{H}^2$ norm are extremely close.

V. Conclusion

The paper presents an efficient computational methodology for the simultaneous design of damping parameters and control gain of a collocated structural system with velocity feedback to satisfy closed-loop $\mathcal{H}^2$ or $\mathcal{H}^\infty$ performance specifications. The design approach is based on an LMI formulation of the integrated design problem that can be effectively solved for the design variables using available semidefinite programming optimization algorithms. Despite the fact that the method is based on an upper bound formulation of the $\mathcal{H}^\infty$ or $\mathcal{H}^2$ performance of the closed-loop system, computational examples illustrate that the bounds provide a close approximation of the actual gains of the system and are effective for structural parameter and control design. The proposed design method is especially suitable for very large scale systems where existing nonlinear optimization approaches to determine system parameters and control gains are computationally prohibitive.

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