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Graphical Observer Design Suitable for Large-scale DAE Power Systems

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***Abstract*— In this paper we discuss an intuitive observer design approach that is suitable for large-scale systems, such as power systems. In arriving at this approach we drew inspiration from the field of linear structured systems, where qualitative statements (e.g., solvability of the fault detection and identification (FDI) problem [1], controllability) about the system are made by analyzing the structure of the system. Our emphasis is design and we use information regarding the structure of the system as well as the actual values of specific entries in the system matrices to design observer-based monitors. These monitors can estimate the state of the system or detect and identify specific occurrences of faults in the system, all in the presence of disturbances and uncertainty. In this paper we demonstrate our design approach by designing observer-based monitors for electromechanical dynamics on a small 3-bus and an intermediate-sized 179-bus power system model.**

***Index Terms*— System-Wide Monitoring, Observers, State Estimation, Fault Detection and Isolation**

I. INTRODUCTION

In this paper we are concerned with designing real-time observer-based monitors for: dynamic state estimation; and centralized FDI for large-scale power systems. Interconnected power systems can span continents, with numerous power generators, loads/consumers, transmission and distribution systems.

In light of the lack of economically viable large-scale energy storage, system operators have to assure that an electricity-supply chain remains balanced (i.e., supply equals demand) in real-time. In order to maintain this balance a system operator relies heavily on local-level protection and control, system-wide state estimation, and human operators that take supervisory control actions. Power system state estimation in the traditional sense has mainly focused on *static* estimation from *redundant* measurements [2] using Weighted Least Squares. There also exists literature in the power system field on dynamic-state estimation, which deals with: *recursive* processing of measurements, but with *no dynamics* in the state [3]; or *slow-speed* state dynamics induced by *load* variations, and these dynamics are estimated on-line in various ways using load forecasting ideas [3], [4].

In the examples presented in this paper we confine our attention to the so-called "swing-model" (i.e., electromechanical dynamics) of a power system. The swing model is Newton's second law in rotational format, i.e., every generator in the network has an input mechanical power and it supplies electrical power to the loads through the network. If there is a mismatch between these two powers at a generator bus the machine will accelerate or decelerate.

Various FDI schemes are already employed in power systems. These schemes rely on extensive placement of local monitoring devices such as relays that provide information to devices such as switches to help in isolating the fault or protect a piece of equipment. These relays are designed to act in real time, however this local information is generally not communicated to a centralized control center where system-wide state estimation is executed. In this paper we illustrate how the same observer structure can be used either as a state estimator or as a FDI filter, located at a centralized location. We do not propose that the existing and important power system FDI schemes be discarded, our aim is rather to illustrate the potential of realizing power system dynamic state estimation and real-time FDI at a centralized location using similar observer-based filters.

Observer design for large-scale systems can be very challenging. In [5] DAE-H_∞ observers were designed for state estimation using Linear Matrix Inequalities. In this works it was discussed how computational intensive such a design is for an intermediate-sized power systems. This challenge prompted us to devise an alternative design approach, which is the focus of this paper.

In the next few sections we will show how one can design unknown-input observers by creating a desired directed graph of the associated linear-structured[1] error dynamics. What makes our novel (to the best of our knowledge) observer-design approach attractive is the fact that the designed gain matrix can easily be updated by *extracting* select values from linearized system matrices, making it feasible to realize an on-line Linear-Parameter-Varying observer (observer's model is linearized along the observer's system trajectory).

The rest of the paper is organized as follows. In Section II, we briefly discuss the swing model of a power system, as well as the two monitoring tasks mentioned earlier. Section III discusses the novel (to the best of our knowledge) graphical observer design technique. In Section IV, we illustrate the working of our observer-based monitors. In Section V, we summarize our results and discuss additional design issues.

II. OBSERVER-BASED MONITORS FOR POWER SYSTEM SWING MODELS

The linearized form of a power system's swing model can be expressed as the following structure-preserving Differen-

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[1]A linear system is considered structured when each entry of the system matrices is either a fixed zero, a free parameter, or a fixed nonzero parameter [1], [6].

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tial Algebraic Equation model

$$M\dot{x} = Ax + Bu + Ew \quad (1)$$
$$y = Cx + Du, \quad (2)$$

with: internal variables x; measurements y; known inputs u; and subject to unknown inputs w. The differential states of the system are mainly attributed to the generation sites and the algebraic internal variables are associated with the loads in the system. The singular M contains the inertia information of the synchronous generators, and A contains the network information.

We propose to use an observer of the form

$$M\dot{\widehat{x}} = A\widehat{x} + Bu + L(y - \widehat{y}) \quad (3)$$
$$\widehat{y} = C\widehat{x} + Du, \quad (4)$$

where $\widehat{x}$ is the estimate of x, L is the designed-observer gain, and $\widehat{y}$ is the estimated version of y. Defining $e = x - \widehat{x}$ the resulting DAE-error system for this estimation problem has the form

$$M\dot{e} = (A - LC)e + Ew \quad (5)$$
$$r = QCe, \quad (6)$$

where r is defined as residuals and we use $(M, A-LC, E, I)$ as shorthand notation for the DAE-error system, and $(M, A-LC, E, QC)$ for the DAE-residual system.

Sources of Unknown Signals w: Various perturbations and uncertainties in the power system can be modeled in the form Ew. Perturbations such as load/generation changes, generator outages, line flow perturbations (e.g., line outages or short circuits) as well as uncertainties of line parameters and generator inertias.

An unknown line change occurring on line h will impact the entries in A associated with the contributions of line h. In [5], we show that each line change (or uncertain line parameter) will have an associated rank one perturbation matrix $\widetilde{A}_h$ in (1), for which we can model $\widetilde{A}_h x = Ew$.

Another source of w can be changes in adjacent unmodeled power systems. In this case our study area is given by (1) and the influences from the rest of the network on our study area can be thought of as w signals. We can thus make our estimator insensitive to changes in adjacent networks, or make it sensitive to indicate that a neighbor is experiencing problems.

Before discussing our observer design method we highlight the types of monitors studied in this paper.

State-Estimation Monitor: The purpose of the state-estimation monitor is to have $\widehat{x}$ be an estimate of x, forcing e to tend to zero in the presence of nonzero unknown signals w and for nonzero initial conditions $e(0) = x(0) - \widehat{x}(0)$. In this paper we assume that $e(0) = 0$ and we are interested in forcing as many entries in $G_{ew}(s) = I(sM - A + LC)^{-1}E$ to zero as s tends to zero. Here $G_{ew}(s)$ is the transfer function matrix from unknown signals w to errors e. In order to achieve this requirement we have L available for design. Observer design in the presence of unknown signals is challenging, but the design method we introduce in the next section provides an intuitive approach to accomplish this task.

FDI Monitor: We partition w to distinguish between faults α (to identify) and disturbances β (to attenuate), and accordingly form E_α and E_β from E. The objectives of the FDI monitor are to have have $G_{r\alpha}(s) = QC(sM - A + LC)^{-1}E_\alpha$ be upper triangular — to aid with FDI — and have $G_{r\beta}(s) = QC(sM - A + LC)^{-1}E_\beta$ be identical to zero for all s [1]. This will insure that the filter's residuals r will be significantly nonzero when α is nonzero, and r will not be impacted by nonzero β's. In [1], the authors provide necessary and sufficient conditions the system $(M = I, A, E, C)$ has to satisfy in order for the FDI problem to be generically solvable.

III. Graphical Observer Design

For the linear structured system $(M, A - LC, E, QC)$ given by Equations (5) and (6) we have $e \in \mathbb{R}^n$; $w \in \mathbb{R}^m$; and $r \in \mathbb{R}^p$. We can associate a directed graph $\mathbf{G}(\mathbf{V}, \mathbf{Z})$ with this system. $\mathbf{V}$ denotes the set of vertices of the directed graph and is obtained by forming $\mathbf{V} = \mathbf{E} \cup \mathbf{W} \cup \mathbf{R}$, where $\mathbf{E}, \mathbf{W}, \mathbf{R}$ are the sets $\{e_1, e_2, \cdots, e_n\}$, $\{w_1, w_2, \cdots, w_m\}$, $\{r_1, r_2, \cdots, r_p\}$ respectively. Hence the variables of the system description form the vertices of the directed graph.

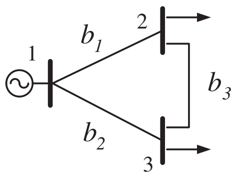
The arc set $\mathbf{Z}$ is obtained by forming the union of $\mathcal{E}_w = \{(w_i, e_j) | E_{ji} \neq 0\}$, $\mathcal{E}_e = \{(e_i, e_j) | A_{ji} \neq 0\}$, $\mathcal{E}_m = \{(e_i, e_j) | M_{ji} \neq 0\}$, $\mathcal{E}_r = \{(e_i, r_j) | Q_{j,:}C_{:,i} \neq 0\}$, and $\mathcal{E}_l = \{(e_i, e_j) | L_{j,:}C_{:,i} \neq 0\}$ (where $Q_{j,:}$ and $L_{j,:}$ represents rows j of Q and L respectively, and $C_{:,i}$ represents column i of C). The above edge weights are obtained from the system matrices, except for $\mathcal{E}_l$, which we will design. In the studies discussed in [1], [6], the authors view the edge weights of the arcs as free parameters and are not concerned with the quantitative information contained in these weights. We will use these weights, which are a function of the current operating point, for observer design.

Reinschke [6] investigated generic controllability of linear structured DAE systems and from his work it is evident that additional arcs are associated with entries in the M matrix. For the swing model considered in this thesis, these additional arcs help us to distinguish between differential and algebraic variables, but do not provide us with additional insight during the observer design process. We will elaborate on this observation when we introduce an example.

Due to the diagonal structure of M the set $\mathcal{E}_m$ will consists of self-cycles at the differential variable vertices [6]. These additional loops do not provide us with extra insight into observer design for the swing model, and they will not be drawn in the directed graph example that will follow. Also, for the figures illustrated in this paper we we will not explicitly draw the $\mathcal{E}_r$ edges.

We will demonstrate our design approach on a three-bus power system example, which consists of one generator and two loads as shown in Figure 1. This system has three bus angles, one generator speed and $M = diag\left(\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}\right)$.

We assume that the system is subject to an unknown load change at bus 3, i.e., $E = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}'$, and



$$e_1 = \theta_1 - \widehat{\theta}_1$$
$$e_2 = \omega_1 - \widehat{\omega}_1$$
$$e_3 = \theta_2 - \widehat{\theta}_2$$
$$e_4 = \theta_3 - \widehat{\theta}_3$$

Fig. 1. Three-bus system and the definition of the elements e_i in $\mathbf{E}$.

that we measure the voltage angle at bus 2, i.e., $C = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array}\right]$. The system matrix A is given as

$$A = \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ -b_1 - b_2 & -d & b_1 & b_2 \\ b_1 & 0 & -b_1 - b_3 & b_3 \\ b_2 & 0 & b_3 & -b_2 - b_3 \end{array}\right].$$

The unassigned observer gain is given as $L = \left[\begin{array}{cccc} l_1 & l_2 & l_3 & l_4 \end{array}\right]'$, and we can now write

$$A - LC = \left[\begin{array}{cc|c|c} 0 & 1 & -l_1 & 0 \\ -b_1 - b_2 & -d & b_1 - l_2 & b_2 \\ b_1 & 0 & -b_1 - b_3 - l_3 & b_3 \\ b_2 & 0 & b_3 - l_4 & -b_2 - b_3 \end{array}\right].$$

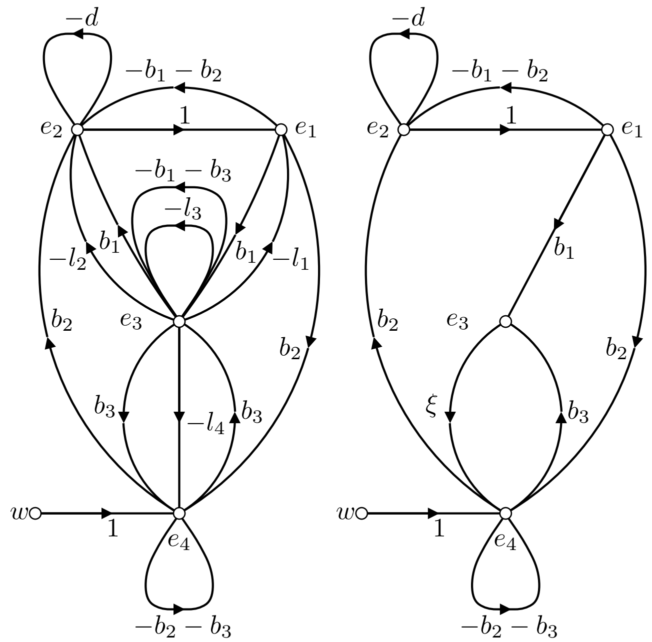
Design Approach: We highlight our design approach here, and the interested reader is referred to [5] for more details:

1) Draw $\mathbf{G}(\mathbf{V}, \mathbf{Z})$ of the error dynamical system $(M, A - LC, E, I)$. See Figure 2(a) for the design setup.
2) Eliminate forward paths (by choosing the values of L appropriately) in $\mathbf{G}(\mathbf{V}, \mathbf{Z})$ *from* the e-vertex where the $\mathcal{E}_l$ arcs originate (i.e., the measured variable), **except** the forward path to the e-vertex directly impacted by w. (This latter edge has a weight of $\xi = b_3 - l_4$.) For our example we choose $l_1 = 0$, $l_2 = b_1$, and $l_3 = -b_1 - b_3$. See Figure 2(b) for the design result. From this figure we notice that some signal flow paths in $\mathbf{G}(\mathbf{V}, \mathbf{Z})$ were eliminated.
3) ξ can now be chosen in order to realize the two types of monitors for:
   - state estimation by choosing ξ to be large in order to attenuate the effect of **disturbance** w. $G_{ew}(s) = \left[\begin{array}{cccc} 0 & 0 & \frac{1}{\xi} & 0 \end{array}\right]'$.
   - FDI by choosing ξ small in order to amplify the effect of **fault** w. $G_{rw}(s) = \frac{1}{\xi}$.

The above design approach does not cover all possible cases. For instance, when we are dealing with a line-flow perturbation (i.e., two e vertices are dependently affected by the same perturbation), the two high-gain arcs we insert between the measurement site and these two e-vertices will necessarily end up being related to one another. One such example will be considered at the end of this section.

Next we state a more general result than what the above design approach and examples suggest. The dual problem, disturbance rejection using full-state feedback control, is discussed by Reinschke in [7]. Our theorem is not the dual extension of Reinschke's work, although his work served as inspiration.

Assumptions: We will first consider the case where the number of measurements, p, is larger than the number



(a) Design Setup (b) Design Result

Fig. 2. Graphical Observer Design with $C = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array}\right]$.

of unknown inputs, m, (i.e., $p \geq m$). We assume that the e-vertices of system are enumerated in order to yield $E = \left[\begin{array}{c} 0 \\ F \end{array}\right]$, where $F \in \mathbb{R}^{q \times m}$ and q is defined as the number of e-vertices that are one hop away from an unknown input. Initially we assume that $q \geq m$. The case where $q < m$ is discussed in [5] and will not be elaborated on here. We will focus on systems for which C may be written as $C = \left[\begin{array}{c} C_E \\ \hline C_\lambda \end{array}\right] = \left[\begin{array}{ccc} 0 & C_{E,b} & 0 \\ \hline C_{\lambda,a} & C_{\lambda,b} & C_{\lambda,c} \end{array}\right]$, where $C_E \in \mathbb{R}^{m \times n}$ and $C_\lambda \in \mathbb{R}^{(p-m) \times n}$. We have that $\left[\begin{array}{cc} C_{\lambda,a} & C_{\lambda,c} \end{array}\right] \in \mathbb{R}^{(p-m) \times (n-m)}$ and $C_{E,b} \in \mathbb{R}^{m \times m}$. The observer gain matrix is given by $L = \left[\begin{array}{c|c} L_E & L_\lambda \end{array}\right] = \left[\begin{array}{c|c} L_{0,E} & L_{0,\lambda} \\ L_{1,E} & L_{1,\lambda} \end{array}\right]$, where $L_E \in \mathbb{R}^{n \times m}$, $L_\lambda \in \mathbb{R}^{n \times (p-m)}$, $L_{0,E} \in \mathbb{R}^{(n-q) \times m}$, $L_{1,E} \in \mathbb{R}^{q \times m}$, $L_{0,\lambda} \in \mathbb{R}^{(n-q) \times (p-m)}$, and $L_{1,\lambda} \in \mathbb{R}^{q \times (p-m)}$.

Let $\mathcal{A} = sM - A + LC$, which we split up into $\mathcal{A} = \mathcal{A}_E + \mathcal{A}_\lambda$. We define $\mathcal{A}_E = sM - A + L_E C_E$ and accounting for the partitioning of E and C_E, we can express

$$\mathcal{A}_E = \left[\begin{array}{ccc} \mathcal{A}_{0,a} & \mathcal{L}_{0,b} & \mathcal{A}_{0,c} \\ \mathcal{A}_{1,a} & \mathcal{L}_{1,b} & \mathcal{A}_{1,c} \end{array}\right].$$

Here $\mathcal{L}_{0,b} = (sM_{0,b} - A_{0,b} + L_{0,E} C_{E,b}) \in \mathbb{R}^{(n-q) \times m}$ and $\mathcal{L}_{1,b} = (sM_{1,b} - A_{1,b} + L_{1,E} C_{E,b}) \in \mathbb{R}^{q \times m}$, where $M = \left[\begin{array}{ccc} M_{0,a} & M_{0,b} & M_{0,c} \\ M_{1,a} & M_{1,b} & M_{1,c} \end{array}\right]$ and $A = \left[\begin{array}{ccc} A_{0,a} & A_{0,b} & A_{0,c} \\ A_{1,a} & A_{1,b} & A_{1,c} \end{array}\right]$. This choice of $\mathcal{A}_E$ leads to

$$\mathcal{A}_\lambda = L_\lambda C_\lambda = \left[\begin{array}{ccc} L_{0,\lambda} C_{\lambda,a} & L_{0,\lambda} C_{\lambda,b} & L_{0,\lambda} C_{\lambda,c} \\ L_{1,\lambda} C_{\lambda,a} & L_{1,\lambda} C_{\lambda,b} & L_{1,\lambda} C_{\lambda,c} \end{array}\right].$$

The general idea is that the blocks $\mathcal{L}_{0,b}$ and $\mathcal{L}_{1,b}$ will be used to attenuate the impact of the unknown inputs on the

e-variables. The matrix $\mathcal{A}_\lambda$ will be used to move the poles of the closed-loop error system, after achieving the desired level of unknown input attenuation, and hence $\mathcal{A}_\lambda$ provides us with extra degrees of freedom. We can now state the following theorem.

Theorem 1: For a system $(M, A - LC, E, I)$ satisfying all the assumptions given in the preceding paragraphs, the following structure can be enforced on $G_{ew}(0)$:

$$G_{ew}(0) = I(sM - A + LC)^{-1}E|_{s=0} = \begin{bmatrix} 0 \\ G_b(0) \\ 0 \end{bmatrix}, \quad (7)$$

if we have:
(a) $\mathcal{L}_{0,b}|_{s=0}G_b(0) = 0$;
(b) $\ker \begin{bmatrix} \mathcal{A}_{0,a} + L_{0,\lambda}C_{\lambda,a} & \mathcal{A}_{0,c} + L_{0,\lambda}C_{\lambda,c} \\ \mathcal{A}_{1,a} + L_{1,\lambda}C_{\lambda,a} & \mathcal{A}_{1,c} + L_{1,\lambda}C_{\lambda,c} \end{bmatrix}|_{s=0} = \emptyset$;
(c) $\begin{bmatrix} L_{0,\lambda} \\ L_{1,\lambda} \end{bmatrix} C_{\lambda,b} = 0$.
(d) $\mathcal{L}_{1,b}|_{s=0}G_b(0) = F$.

Proof: Pre-multiplying both sides of the expression $G_{ew}(s) = I\mathcal{A}^{-1}E$ by $\mathcal{A} = sM - A + LC$ yields the following set of linear equations:

$$\mathcal{A}G_{ew}(s) = E. \quad (8)$$

With the partitioning $G_{ew}(s) = \begin{bmatrix} G'_a(s) & G'_b(s) & G'_c(s) \end{bmatrix}'$, and developing the submatrix multiplications in the preceding equation the following sets of linear equations are obtained:

$$\underbrace{\begin{bmatrix} \mathcal{A}_{0,a} + L_{0,\lambda}C_{\lambda,a} & \mathcal{A}_{0,c} + L_{0,\lambda}C_{\lambda,c} \end{bmatrix}}_{\mathcal{B}_0} \underbrace{\begin{bmatrix} G_a(s) \\ G_c(s) \end{bmatrix}}_{G_0(s)} + \left(\mathcal{L}_{0,b} + L_{0,\lambda}C_{\lambda,b}\right) G_b(s) = 0; \quad (9)$$

$$\underbrace{\begin{bmatrix} \mathcal{A}_{1,a} + L_{1,\lambda}C_{\lambda,a} & \mathcal{A}_{1,c} + L_{1,\lambda}C_{\lambda,c} \end{bmatrix}}_{\mathcal{B}_1} \underbrace{\begin{bmatrix} G_a(s) \\ G_c(s) \end{bmatrix}}_{G_0(s)} + \left(\mathcal{L}_{1,b} + L_{1,\lambda}C_{\lambda,b}\right) G_b(s) = F, \quad (10)$$

where $\mathcal{B}_0 \in R^{(n-q)\times(n-m)}$ and $\mathcal{B}_1 \in R^{(q)\times(n-m)}$. We group (9) and (10) together to form:

$$\begin{bmatrix} \mathcal{B}_0 \\ \mathcal{B}_1 \end{bmatrix} G_0(s) + \begin{bmatrix} \mathcal{L}_{0,b} \\ \mathcal{L}_{1,b} \end{bmatrix} G_b(s) + \begin{bmatrix} L_{0,\lambda} \\ L_{1,\lambda} \end{bmatrix} C_{\lambda,b}G_b(s) = \begin{bmatrix} 0 \\ F \end{bmatrix} \quad (11)$$

Investigating conditions (a), (c) and (d), we notice that we need to evaluate (11) at $s = 0$ due to our restriction that the observer gain elements remain real.

Accounting for conditions (a), (c) and (d) from the theorem statement and evaluating Equation (11) at $s = 0$ yields:

$$\begin{bmatrix} \mathcal{B}_0 \\ \mathcal{B}_1 \end{bmatrix}|_{s=0}G_0(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (12)$$

and from condition (b) we conclude that $G_0(0) = 0$, implying that $G_{ew}(0) = \begin{bmatrix} 0 & G'_b(0) & 0 \end{bmatrix}'$. ■

Discussion: Requirement (a) in the theorem statement can be satisfied if we set $\mathcal{L}_{0,b}|_{s=0} = (sM_{0,b} - A_{0,b} + L_{0,E}C_{E,b})|_{s=0} = 0$. This implies that we eliminate all the entries in this matrix and thus set $L_{0,E}C_{E,b} = A_{0,b}$. Interpreting this action graphically on the directed graph of $(M, A - LC, E, I)$ translates to cutting the forward paths from the measurement site to all the e-vertices that are not directly influenced by unknown inputs (i.e., the 0 block of E).

To obtain $\mathcal{L}_{1,b}$ we partition $F = \begin{bmatrix} F_R \\ F_Z \end{bmatrix}$ and correspondingly $\mathcal{L}_{1,b} = \begin{bmatrix} \mathcal{L}_R \\ \mathcal{L}_Z \end{bmatrix}$. The R subscript implies that the matrix is in $\mathbb{R}^{m\times m}$ and the Z subscript implies $\mathbb{R}^{(q-m)\times m}$. From condition (d) of Theorem 1 we have $\mathcal{L}_R|_{s=0}G_b(0) = F_R$, and by defining the desired level of attenuation for the elements of $G_b(0)$ we can find the elements of $\mathcal{L}_R|_{s=0}$ (note that $G_b(s)$ is invertible). To find $\mathcal{L}_Z|_{s=0}$ we use the relationship $\mathcal{L}_Z|_{s=0}G_b(s)|_{s=0} = F_Z$, and for F_R invertible we have $\mathcal{L}_Z|_{s=0} = F_Z F_R^{-1}\mathcal{L}_R|_{s=0}$.

Illustrative Example: We will investigate the 3-bus power system example we discussed at the beginning of this section. Let $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 0 & \kappa_1 & \kappa_2 \end{bmatrix}'$, hence we have $m = 1$, $q = 2$, and $p = 2$. The structure of E is indicative of a line-flow perturbation. We split $\mathcal{A}$ up into

$$\mathcal{A}_E = \left[\begin{array}{cc|c|c} s & -1 & l_{11} & 0 \\ b_1 + b_2 & s + d & -b_1 + l_{21} & -b_2 \\ \hline -b_1 & 0 & +b_1 + b_3 + l_{31} & -b_3 \\ -b_2 & 0 & -b_3 + l_{41} & b_2 + b_3 \end{array}\right], \text{ and}$$

$$\mathcal{A}_\lambda = \left[\begin{array}{cc|c|c} l_{12} & 0 & 0 & 0 \\ l_{22} & 0 & 0 & 0 \\ \hline l_{32} & 0 & 0 & 0 \\ l_{42} & 0 & 0 & 0 \end{array}\right].$$

In order to satisfy condition (a) of Theorem 1 we set $\mathcal{L}_{0,b}|_{s=0} = 0$, by choosing $l_{11} = 0$ and $l_{21} = b_1$.

Testing condition (b) of Theorem 1, we construct

$$\begin{bmatrix} \mathcal{B}_0 \\ \mathcal{B}_1 \end{bmatrix} = \left[\begin{array}{ccc} s + l_{12} & -1 & 0 \\ b_1 + b_2 + l_{22} & s + d & -b_2 \\ \hline -b_1 + l_{32} & 0 & -b_3 \\ -b_2 + l_{42} & 0 & b_2 + b_3 \end{array}\right],$$

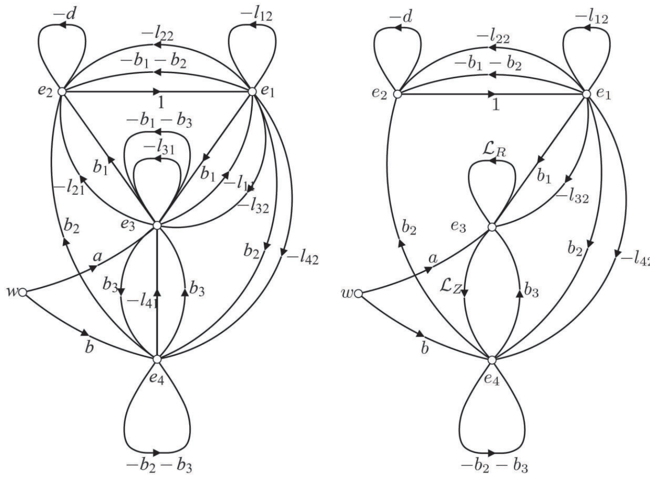
from which we see that the nullspace of the above matrix is empty at $s = 0$ when:

- $l_{12} \neq 0$, $l_{22} \neq -2b_2 - b_1$, $l_{32} \neq b_1 - b_3$, $l_{42} \neq 2b_2 + b_3$;
- $l_{12} \neq 0$, $l_{22} \neq -b_1$, $l_{32} \neq b_1 + b_3$, $l_{42} \neq -b_3$;
- or $l_{22} \neq -b_1 - b_2 - l_{12}d$, $l_{32} \neq b_1$, $l_{42} \neq b_2$.

Thus we have placed modest restrictions on the values that L_λ can take.

Condition (c) of is also satisfied and $L_\lambda C_{\lambda,b} = 0$.

In order to complete the unknown-input attenuation design we need to find $\mathcal{L}_{1,b}|_{s=0}$. By Setting $G_b(0) = \frac{\kappa_1}{b_1+b_3+l_{31}}$ we deduce that $\mathcal{L}_R = b_1 + b_3 + l_{31}$ and that $\mathcal{L}_Z = \kappa_2\kappa_1^{-1}(b_1 + b_3 + l_{31})$. However, $\mathcal{L}_Z = -b_3 + l_{41}$, from which we can back out what we need l_{41} to be as a function of l_{31} (our design parameter used to manipulate $G_b(0)$).



(a) Design Setup (b) Design Result

Fig. 3. Graphical Observer Design with $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.

Thus, in order to enforce $G_{ew}(s) = \begin{bmatrix} 0 & 0 & \frac{\kappa_1}{b_1+b_3+l_{31}} & 0 \end{bmatrix}$, we need to set $l_{11} = 0$, $l_{21} = b_1$, $l_{41} = \frac{b_1\kappa_2+b_3(\kappa_1+\kappa_2)+\kappa_2 l_{31}}{\kappa_1}$ (note we did not require $s = 0$, because we measured an algebraic variable). From this example we also note that we still have four degrees of freedom (l_{12}, l_{22}, l_{32} and l_{42}) to move the poles of $(M, A - LC)$, however, we have some restrictions on what these values can be.

In Figure 3(a) the directed graph of $(M, A - LC, E, I)$ is shown before we assigned the values of L_E. In Figure 3(b) we show the directed graph of $(M, A - LC, E, I)$ after L_E was designed to achieve unknown-input attenuation. Notice that we cut the forward paths from e_3 to other e-vertices not directly influenced by w (i.e., forward paths from e_3 to e_1 and e_2). In this figure we show the high-gain arcs from e_3 to e_3, namely $\mathcal{L}_R$, and from e_3 to e_4, namely $\mathcal{L}_Z$.

IV. Monitor Examples for an 179-Bus System

In this section we study an 179-bus aggregated version of the Western States Coordinating Council (WSCC) power system[2]. The one-line diagram of this system is shown in Figure 4. The system consists of 29 generators and 150 load buses and the DAE model of form (1) for this system has 208 internal variables with 58 state variables and 150 algebraic variables. In Figure 4 we indicate the locations of angle measurements we will use for observer design: $y_1 = \theta_{78}$, $y_2 = \theta_{59}$, $y_3 = \theta_{83}$, and $y_4 = \theta_2$.

In Figure 4, we also indicate the locations where unknown power injections/extractions w_1 to w_4 can occur. In our example we assume that occurrences of these unknown inputs are in the form of pulses as described in Table I.

[2]We would like to thank Professors A. G. Phadke (VPISU), and V. Vittal at (ASU) for sharing the aggregated WSCC 179 bus model.

	$w_1 = \widetilde{P}_{67}$	$w_2 = \widetilde{P}_{41}$	$w_3 = \widetilde{P}_{100}$	$w_4 = \widetilde{P}_{85}$
Per Unit [p.u.]	-1	1	1.5	-1.5
t_d [s]	[1, 3]	[2.5, 5]	[0.001, 7]	[0.001, 7]

TABLE I

Amplitude and duration of perturbations and faults.

In this section we will illustrate the functioning of the two types of monitors we discussed in Section II.

In all of the following simulations we assume that the generators are uncontrolled. We introduce parametric model uncertainty into the system model, by assuming that the parameters of the system are randomly perturbed by 10% (using an uniform distribution) around their nominal values. We use these nominal values in the realization of the observer. In this paper we do not design the observer to be robust to these introduced parameter uncertainties, because we want to demonstrate how well the observer performs even in the presence of moderate parametric model uncertainty.

We also assume that $x(0)$ is equal to $\widehat{x}(0)$ in order to avoid solving the power flow problem for the perturbed system before commencing the dynamic simulation. This approach limits the impact of transients due to a large nonzero $e(0)$, which is not the included in the design objective of this paper. In practice we can limit the effect of nonzero $e(0)$ on the desired monitoring task by using additional measurements to move the eigenvalues of the $(M, A - LC)$ system to force transient trajectories of e, driven by nonzero $e(0)$, to decay to zero.

State Estimation: As part of the observer design we associate each measurement with an unknown signal w. In this example we associate y_i with w_i for $i \in [1, \cdots, 4]$. In this example all w_i's given in Table I are seen as disturbances, and each ξ_i (as defined in Section III) is set large (i.e., 1×10^4). We also found that by choosing the ξ_i's to be 1×10^4 we had $\|A\|_2 = \|A - LC\|_2$. The other nonzero entries in L are obtained from extracting entries from A as discussed in Section III. After constructing L we check whether $(M, A - LC)$ is stable, and if this is indeed the case we use L in the realization of (3).

In Figure 5, $\theta_{99} - \theta_{coi}$ is compared to its observer estimate $\widehat{\theta}_{99} - \widehat{\theta}_{coi}$[3]. This plot was selected by evaluating $\|\theta_i - \widehat{\theta}_i\|_2$ for each of the 179 bus angles and then choosing the one with the the maximum norm. Even in this instance we notice that the observer's estimate track the trend of the system trajectory satisfactorily.

FDI: For this example y_1 and y_2 are used to detect the occurrences of w_1 and w_2 respectively. We use y_3 and y_4 to attenuate the effect of nonzero w_3 and w_4 disturbances. From Section III we know that if w_k is a fault we want to detect, we set ξ_k to be very small, and if w_k is a disturbance, ξ_k is set very large. For our current example we set $\xi_1 = 50$, $\xi_2 = 50$, $\xi_3 = 1 \times 10^4$, and $\xi_4 = 1 \times 10^4$. After constructing L we check whether $(M, A - LC)$ is stable, and if so we use L in the realization of (3).

[3]In this figure the system angle θ_{99} is expressed relative to the evolution of the center-of-inertia angle θ_{coi}. The observer angles $\widehat{\theta}_{99}$ is expressed relative to the observer's $\widehat{\theta}_{coi}$.

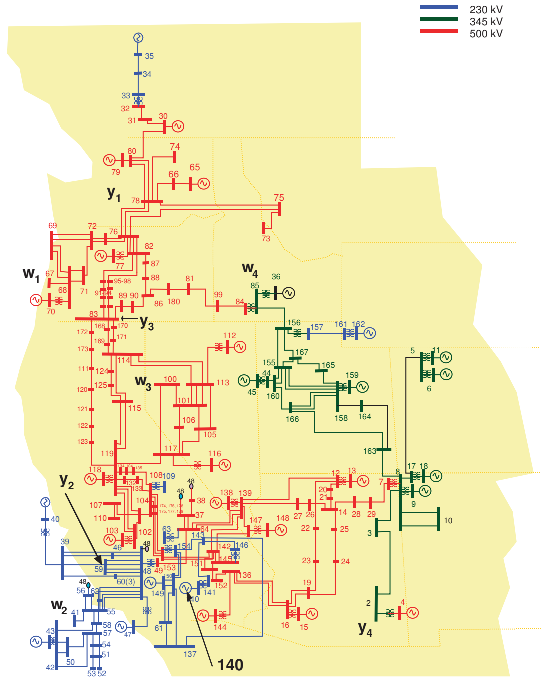


Fig. 4. One-line diagram of an aggregated WSCC network, illustrating fault, disturbance and measurement locations.

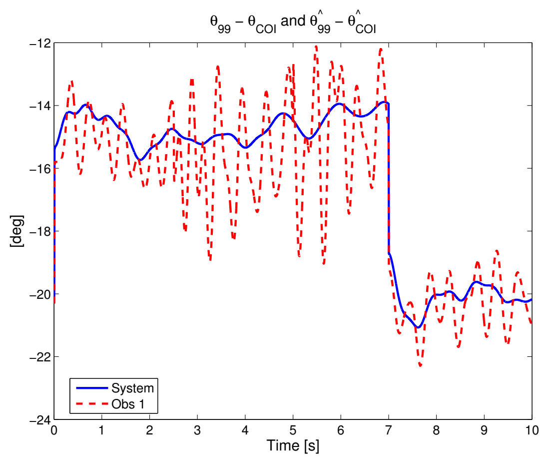


Fig. 5. $\theta_{99} - \theta_{coi}$ vs. $\widehat{\theta}_{99} - \widehat{\theta}_{coi}$

In Figure 6, the response of the designed observer-based monitor is shown in response to the events described in Table I. We note that the monitor attenuates the occurrence of disturbances w_3 and w_4, as well as *detects* and *isolates* the occurrences of w_1 and w_2 that occur separately as well as simultaneously. We notice that z_3 and z_4, which are driven by the disturbances, stay approximately zero and can be left out of the FDI monitor by choosing $Q = \begin{bmatrix} I_2 & 0 \end{bmatrix}$.

V. Conclusions

In this paper we demonstrated how observer-based monitors can readily be designed for an 179-bus power system model by using a novel graphical observer-design technique developed in [5]. We discussed the design of two types of observer-based monitors, i.e., the state-estimation monitor

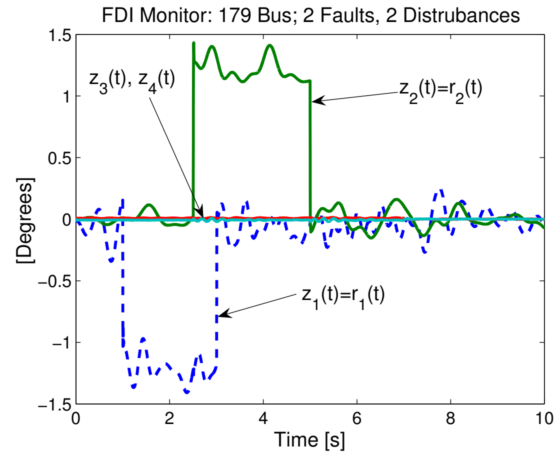


Fig. 6. Output of observer-based monitor in response to the events described in Table I. Here $z = y - \widehat{y}$ and $r = Qz$.

and the fault-detection-and-isolation monitor. We demonstrated the design and performance of a fault-detection-and-isolation monitor on an 179-bus power system model.

This graphical observer-design technique is powerful in designing the steady-state output of a monitor, but it provides no stability guarantees for the monitoring system. For a system where the number of measurements are equal to the number of disturbances plus faults then there is no degrees of freedom left to move the eigenvalues of $(M, A-LC)$. If the filter is unstable, extra measurements can be added in order to move the eigenvalues of the $(M, A-LC)$ system. For such situations we propose a dual design approach, where we first identify the unknown inputs, then use specific measurements to achieve the desired monitoring task. In the second step we use additional measurements for the sole purpose of moving the eigenvalues of the filter, to ensure that the monitor will be stable whenever the monitored system is stable.

In this paper we focussed on the swing dynamics of a power system, but the ideas presented here can be extended to include other power system dynamic phenomena (such as voltage dynamics).

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