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A Retrospective Correction Filter for Discrete-Time Adaptive Control of Nonmiminum-Phase Systems

Mario A. Santillo and Dennis S. Bernstein

Department of Aerospace Engineering,
The University of Michigan,
Ann Arbor, MI 48109-2140
{santillo, dsbaero}@umich.edu

***Abstract*— We present a discrete-time adaptive control law that is effective for systems that are unstable, MIMO, and/or nonminimum phase. The adaptive control algorithm provides guidelines concerning the modeling information needed for implementation. This information includes a sufficient number of Markov parameters to capture the sign of the high-frequency gain as well as the nonminimum-phase zeros. No additional information about the poles or zeros need be known. We present numerical examples to illustrate the algorithm's effectiveness in handling nonminimum-phase zeros.**

I. Introduction

Unlike robust control, which fixes the control gains based on a prior, fixed level of modeling uncertainty, adaptive control algorithms tune the feedback gains in response to the true plant and exogenous signals, that is, commands and disturbances. Generally speaking, adaptive controllers require less prior modeling information than robust controllers, and thus can be viewed as highly parameter-robust control laws. The price paid for the ability of adaptive control laws to operate with limited prior modeling information is the complexity of analyzing and quantifying the stability and performance of the closed-loop system, especially since adaptive control laws, even for linear plants, are nonlinear.

Stability and performance analysis of adaptive control laws often entails restrictive assumptions on the dynamics of the plant. For example, a widely invoked assumption in adaptive control is passivity [1], which is restrictive and difficult to verify in practice. A related assumption is that the plant is minimum phase [2, 3], which may entail the same difficulties. In fact, sampled-data control may give rise to nonminimum-phase zeros whether or not the continuous-time system is minimum phase [4]. Beyond these assumptions, adaptive control laws are known to be sensitive to unmodeled dynamics and sensor noise [5, 6], which motivates robust adaptive control laws [7].

In addition to these basic issues, adaptive control laws may entail unacceptable transients during adaptation, which may be exacerbated by actuator limitations [8–10]. In fact, adaptive control under extremely limited modeling information such as uncertainty in the high-frequency gain [11, 12] may yield a transient response that exceeds the practical limits of the plant. Therefore, the type and quality of the available modeling information as well as the speed of adaptation must be considered in the analysis and implementation of adaptive control laws. These issues are discussed in [13].

Adaptive control laws have been developed in both continuous time and discrete time. In the present paper we consider discrete-time adaptive control laws since these control laws can be implemented directly in embedded code without requiring an intermediate discretization step with potential loss of phase margin. Although discrete-time adaptive control laws are less developed than their continuous-time counterparts, the literature is substantial and growing [2, 14–18].

The goal of the present paper is to present a discrete-time adaptive control law that is effective for nonminimum-phase systems. In [2], a discrete-time adaptive control law with stability guarantees was developed under a minimum-phase assumption. Extensions given in [3] based on internal model control [19] and Lyapunov analysis also invoke this assumption. To circumvent the minimum-phase assumption, the zero annihilation periodic control law [20] uses lifting to move all of the plant's zeros to the origin.

The present paper is motivated by the adaptive control laws given in [3] and [21]. The control law given in [21] lacks a proof of stability, but is known numerically to be effective on nonminimum-phase plants without recourse to lifting. Accordingly, we present an adaptive control law based on [3] and [21] for systems that are unstable, MIMO, and/or nonminimum phase. The adaptive control algorithm provides guidelines concerning the modeling information needed for implementation. This information includes a sufficient number of Markov parameters to capture the sign of the high-frequency gain as well as the nonminimum-phase zeros. Except for an estimate of the plant order, no additional information about the plant need be known.

The novel feature of this adaptive control law is the use of a retrospective correction filter (RCF). The RCF provides an inner loop to the adaptive control law by modifying the sensor measurements based on the difference between the actual past control inputs and the recomputed past control inputs based on the current control law. This technique is inherent in [21] in the use of the estimated performance variable but is more fully developed in the present paper.

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The goal of the present paper is to develop the RCF adaptive control algorithm and demonstrate its effectiveness in handling nonminimum-phase zeros. Extensive numerical examples given in [22] illustrate the response of the algorithm under conditions of uncertainty in the relative degree and Markov parameters, measurement noise, and actuator and sensor saturation. Basic command following, disturbance rejection, and model reference adaptive control examples are also included in [22]. These numerical studies show that the RCF adaptive control algorithm is effective for handling nonminimum-phase zeros under minimal modeling assumptions. These studies also provide guidance into the choice of the learning rate α for stable response and acceptable transient behavior. This guidance can provide the basis for future Lyapunov-based stability and performance analysis.

II. Problem Formulation

Consider the MIMO discrete-time system

$$x(k+1) = Ax(k) + Bu(k) + D_1 w(k), \tag{1}$$

$$y(k) = Cx(k) + D_2 w(k), \tag{2}$$

$$z(k) = E_1 x(k) + E_0 w(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^{l_y}$, $z(k) \in \mathbb{R}^{l_z}$, $u(k) \in \mathbb{R}^{l_u}$, $w(k) \in \mathbb{R}^{l_w}$, and $k \geq 0$. Our goal is to develop an adaptive output feedback controller under which the performance variable z is minimized in the presence of the exogenous signal w. Note that w can represent either a command signal to be followed, an external disturbance to be rejected, or both. For example, if $D_1 = 0$ and $E_0 \neq 0$, then the objective is to have the output $E_1 x$ follow the command signal $-E_0 w$. On the other hand, if $D_1 \neq 0$ and $E_0 = 0$, then the objective is to reject the disturbance w from the performance measurement $E_1 x$. The combined command following and disturbance rejection problem is addressed when D_1 and E_0 are block matrices. More precisely, if $D_1 = \begin{bmatrix} \hat{D}_1 & 0 \end{bmatrix}$, $E_0 = \begin{bmatrix} 0 & \hat{E}_0 \end{bmatrix}$, and $w(k) = \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$, then the objective is to have $E_1 x$ follow the command $-\hat{E}_0 w_2$ while rejecting the disturbance w_1. Lastly, if D_1 and E_0 are empty matrices, then the objective is output stabilization, that is, convergence of z to zero.

Model reference adaptive control (MRAC) is a special case of (1)–(3), where the performance variable z is the difference between the measured output of the plant and the output of the reference model. For MRAC, the exogenous command w is available to the controller as an additional measurement variable, as shown in Figure 1.

III. Controller Construction

In this section we formulate an adaptive control algorithm for the general control problem represented by (1)–(3). We use a strictly proper time-series controller of order n_c, such that the control $u(k)$ is given by

$$u(k) = \sum_{i=1}^{n_c} M_i(k) u(k-i) + \sum_{i=1}^{n_c} N_i(k) y(k-i), \tag{4}$$

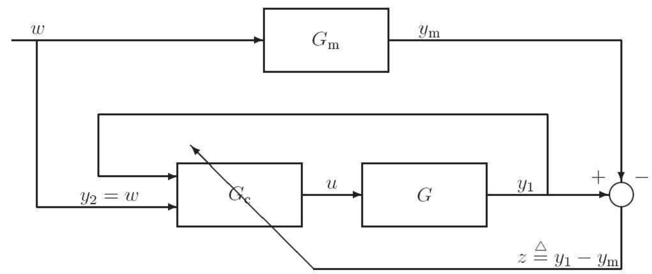

Fig. 1. Model reference adaptive control problem.

where, for all $i = 1, \ldots, n_c$, $M_i \in \mathbb{R}^{l_u \times l_u}$ and $N_i \in \mathbb{R}^{l_u \times l_y}$ are given by the adaptive law presented below. The control can be expressed as

$$u(k) = \theta(k)\phi(k), \tag{5}$$

where

$$\theta(k) \triangleq \begin{bmatrix} N_1(k) & \cdots & N_{n_c}(k) & M_1(k) & \cdots & M_{n_c}(k) \end{bmatrix}$$

is the *controller parameter block matrix*, and the *regressor vector* $\phi(k)$ is given by

$$\phi(k) \triangleq \begin{bmatrix} y(k-1) \\ \vdots \\ y(k-n_c) \\ u(k-1) \\ \vdots \\ u(k-n_c) \end{bmatrix} \in \mathbb{R}^{n_c(l_u+l_y)}. \tag{6}$$

For positive integers p and μ, we define the *extended measurement vector* $Y(k)$, the *extended performance vector* $Z(k)$, and the *extended control vector* $U(k)$ by

$$Y(k) \triangleq \begin{bmatrix} y(k) \\ \vdots \\ y(k-p+1) \end{bmatrix}, \quad Z(k) \triangleq \begin{bmatrix} z(k) \\ \vdots \\ z(k-p+1) \end{bmatrix},$$

$$U(k) \triangleq \begin{bmatrix} u(k) \\ \vdots \\ u(k-p_c+1) \end{bmatrix}, \tag{7}$$

where $p_c \triangleq \mu + p$.

From (5), it follows that the extended control vector $U(k)$ can be written as

$$U(k) \triangleq \sum_{i=1}^{p_c} L_i \theta(k-i+1)\phi(k-i+1), \tag{8}$$

where

$$L_i \triangleq \begin{bmatrix} 0_{(i-1)l_u \times l_u} \\ I_{l_u} \\ 0_{(p_c-i)l_u \times l_u} \end{bmatrix} \in \mathbb{R}^{p_c l_u \times l_u}. \tag{9}$$

We define the *surrogate performance vector* $\hat{Z}(\hat{\theta}(k), k)$ by

$$\hat{Z}(\hat{\theta}(k), k) \triangleq Z(k) - \bar{B}_{zu}\left(U(k) - \hat{U}(k)\right), \tag{10}$$

where $\hat{U}(k) \triangleq \sum_{i=1}^{p_c} L_i \hat{\theta}(k)\phi(k-i+1)$, $\hat{\theta}(k) \in \mathbb{R}^{l_u \times n_c(l_u+l_y)}$ is the *surrogate controller parameter block* *matrix*, and the block-Toeplitz *surrogate control matrix* $\bar{B}_{zu}$ is constructed below.

Taking the vec of (10) yields

$$\hat{Z}(\hat{\theta}(k),k) = f(k) + D(k)\text{vec }\hat{\theta}(k), \tag{11}$$

where $f(k) \triangleq Z(k) - \bar{B}_{zu}U(k)$ and $D(k) \triangleq \sum_{i=1}^{p_c} \phi^{\mathrm{T}}(k-i+1) \otimes \bar{B}_{zu}L_i$. Note that

$$\hat{U}(k) = \sum_{i=1}^{p_c} L_i\hat{\theta}(k)\phi(k-i+1) = M(k)\text{vec }\theta, \tag{12}$$

where $M(k) \triangleq \sum_{i=1}^{p_c} \phi^{\mathrm{T}}(k-i+1) \otimes L_i$.

Now, consider the *surrogate cost function*

$$\begin{aligned}\hat{J}(k) \triangleq\ & \hat{Z}^{\mathrm{T}}(\hat{\theta}(k),k)R_1(k)\hat{Z}(\hat{\theta}(k),k) \\ &+ \hat{U}^{\mathrm{T}}(\hat{\theta}(k),k)R_2(k)\hat{U}(\hat{\theta}(k),k) \\ &+ \text{tr}\left[\left(\hat{\theta}(k)-\theta(k)\right)^{\mathrm{T}} R_3(k)\left(\hat{\theta}(k)-\theta(k)\right)\right],\end{aligned} \tag{13}$$

where $R_1(k) = R_1^{\mathrm{T}}(k) > 0, R_2(k) \geq 0$, and $R_3(k) = R_3^{\mathrm{T}}(k) > 0$. Substituting (11) and (12) into (13) yields

$$\hat{J}(k) = c(k) + b^{\mathrm{T}}(k)\text{vec }\hat{\theta}(k) + \left(\text{vec }\hat{\theta}(k)\right)^{\mathrm{T}} A(k)\text{vec }\hat{\theta}(k),$$

where (14)

$$\begin{aligned}A(k) \triangleq\ & D^{\mathrm{T}}(k)R_1(k)D(k) + M^{\mathrm{T}}(k)R_2(k)M(k) \\ &+ R_3(k) \otimes I_{n_c(lu+ly)},\end{aligned} \tag{15}$$

$$b(k) \triangleq 2D^{\mathrm{T}}(k)R_1(k)f(k) - 2R_3(k) \otimes I_{n_c(lu+ly)}\text{vec }\theta(k), \tag{16}$$

$$c(k) \triangleq f^{\mathrm{T}}(k)R_1(k)f(k) + \text{tr}\left[\theta^{\mathrm{T}}(k)R_3(k)\theta(k)\right]. \tag{17}$$

Since $A(k)$ is positive definite, $\hat{J}(k)$ has the strict global minimizer

$$\hat{\theta}(k) = -\frac{1}{2}\text{vec}^{-1}(A^{-1}(k)b(k)). \tag{18}$$

The update law is given by

$$\theta(k+1) = \hat{\theta}(k). \tag{19}$$

For all future discussion, we specialize (15)–(17) with

$$R_1(k) \triangleq I_{pl_z}, \quad R_2(k) \triangleq 0_{p_cl_u}, \quad R_3(k) \triangleq \alpha(k)I_{l_u}, \tag{20}$$

where $\alpha(k) > 0$ is a scalar, yielding

$$A(k) = D^{\mathrm{T}}(k)D(k) + \alpha(k)I, \tag{21}$$

$$b(k) = 2D^{\mathrm{T}}(k)f(k) - 2\alpha(k)\text{vec }\theta(k), \tag{22}$$

$$c(k) = f^{\mathrm{T}}(k)f(k) + \alpha(k)\text{tr}\left[\theta^{\mathrm{T}}(k)\theta(k)\right]. \tag{23}$$

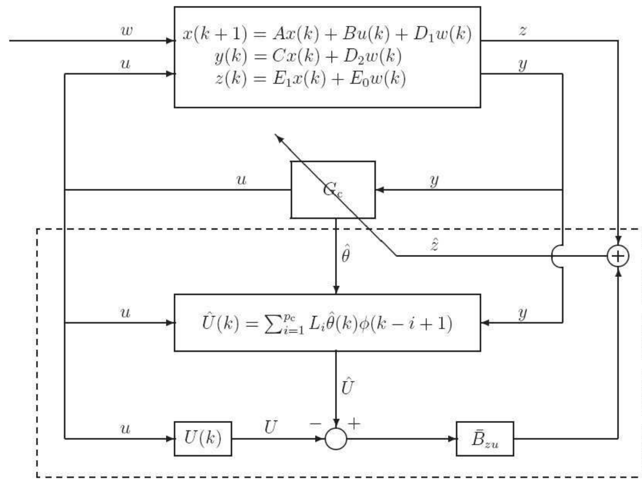

Fig. 2. Closed-loop system including adaptive control algorithm with the retrospective correction filter (dashed box) for $p = 1$.

IV. Markov Parameter-Based Update

The novel feature of the adaptive control algorithm (5), (18), and (19) is the use of the retrospective correction filter (RCF) (10), as shown in Figure 2 for $p = 1$. The RCF provides an inner loop to the adaptive control law by modifying the performance variable $Z(k)$ based on the difference between the actual past control inputs $U(k)$ and the recomputed past control inputs $\hat{U}(k)$ based on the current control law.

Consider the time-series representation of (1)–(3) from u to z, given by

$$z(k) = \sum_{i=1}^{n} -\alpha_i z(k-i) + \sum_{i=d}^{n} \beta_i u(k-i), \tag{24}$$

where $\alpha_1,\ldots,\alpha_n \in \mathbb{R}$, $\beta_d,\ldots,\beta_n \in \mathbb{R}^{l_z\times l_u}$, and the *relative degree* d is the smallest positive integer i such that the ith Markov parameter $H_i \triangleq E_1A^{i-1}B$ is nonzero. The transfer function matrix $G_{zu}(\mathbf{z})$ from u to z can be equivalently represented by

$$G_{zu}(\mathbf{z}) \triangleq \frac{1}{p(\mathbf{z})}\left(\beta_d\mathbf{z}^{n-d} + \beta_{d+1}\mathbf{z}^{n-d-1} + \cdots + \beta_n\right), \tag{25}$$

where $p(\mathbf{z}) \triangleq \mathbf{z}^n + \alpha_1\mathbf{z}^{n-1} + \cdots + \alpha_n$. Note that $\beta_d = H_d$.

Replacing k with $k-1$ in (24) and substituting the resulting relation back into (24) yields the *2-MARKOV* model. Repeating this procedure $\mu-1$ times yields the μ-MARKOV model from u to z of (1)–(3)

$$\begin{aligned}z(k) = & \sum_{i=1}^{n} \alpha_{\mu,i} z(k-\mu-i+1) + \sum_{i=d}^{\mu} H_i u(k-i) \\ &+ \sum_{i=2}^{n} \beta_{\mu,i} u(k-\mu-i+1),\end{aligned} \tag{26}$$

where, for $i = 1,\ldots,n$, the coefficients $\alpha_{\mu,i} \in \mathbb{R}$ and $\beta_{\mu,i} \in$

$\mathbb{R}^{l_z \times l_u}$ are given by

$$\begin{aligned} \alpha_{1,i} &\triangleq -\alpha_i, & \beta_{1,i} &\triangleq \beta_i, \\ \alpha_{\mu,i} &\triangleq \alpha_{\mu-1,1}\alpha_{1,i} & \beta_{\mu,i} &\triangleq \alpha_{\mu-1,1}\beta_{1,i} \\ &\quad +\alpha_{\mu-1,i+1}, & &\quad +\beta_{\mu-1,i+1}, \\ \alpha_{\mu,n} &\triangleq \alpha_{\mu-1,1}\alpha_{1,n}, & \beta_{\mu,n} &\triangleq \alpha_{\mu-1,1}\beta_{1,n}. \end{aligned} \tag{27}$$

Note that $H_\mu = \beta_{\mu,1}$.

Equation (26) can be equivalently represented as the μ-*MARKOV transfer function*

$$G_{\mu,zu}(\mathbf{z}) = \frac{1}{p_\mu(\mathbf{z})} \tag{28}$$
$$\cdot \left(H_d \mathbf{z}^{\mu+n-d-1} + \cdots + H_{\mu-1}\mathbf{z}^n + \beta_{\mu,1}\mathbf{z}^{n-1} + \cdots + \beta_{\mu,n}\right),$$

where $p_\mu(\mathbf{z}) \triangleq \mathbf{z}^{\mu+n-1} + \alpha_{\mu,1}\mathbf{z}^{n-1} + \cdots + \alpha_{\mu,n}$. This system representation is nonminimal, overparameterized, has order $n+\mu-1$, and the coefficients of the terms $\mathbf{z}^{n+\mu-2}$ through $\mathbf{z}^n$ in the denominator are zero.

The Laurent series expansion of $G_{zu}(\mathbf{z})$ about $\mathbf{z} = \infty$ is

$$G_{zu}(\mathbf{z}) = \sum_{i=d}^{\infty} \mathbf{z}^{-i} H_i. \tag{29}$$

Truncating the numerator and denominator of (28) is equivalent to the truncated Laurent series expansion of $G_{zu}(\mathbf{z})$ about $\mathbf{z} = \infty$, given by

$$\bar{G}_{\mu,zu}(\mathbf{z}) \triangleq \sum_{i=d}^{\mu} \mathbf{z}^{-i} H_i. \tag{30}$$

Finally, the *surrogate control matrix* $\bar{B}_{zu} \in \mathbb{R}^{pl_z \times p_c l_u}$ is

$$\bar{B}_{zu} \triangleq \left[\begin{array}{cccccccccc} 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} & H_d & \cdots & \cdots & H_\mu & 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} \\ 0_{l_z\times l_u} & \ddots & & \ddots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \ddots & & \ddots & 0_{l_z\times l_u} & \\ 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} & 0_{l_z\times l_u} & \cdots & \cdots & 0_{l_z\times l_u} & H_d & \cdots & H_\mu \end{array}\right]. \tag{31}$$

The leading zeros in the first row of $\bar{B}_{zu}$ account for the nonzero relative degree d. The advantage in constructing $\bar{B}_{zu}$ using the Markov parameters H_i, $i = d, \ldots, \mu$, as opposed to using all of the numerator coefficients of $G_{\mu,zu}$ is faster convergence and ease of identification. The algorithm places no constraints on either the value of d or the rank of H_d or $\bar{B}_{zu}$. In the particular case $z = y$, using the *surrogate performance variable* $\hat{z}$ in place of y in the regressor vector (6) results in faster convergence.

The weighting parameter α introduced in (20) is called the *learning rate* since it affects convergence speed of the adaptive control algorithm. As α is increased, a higher weight is placed on the difference between the previous control coefficients and the current control coefficients, and, as a result, convergence speed is lowered. Likewise, as α is decreased, converge speed is raised.

V. SMITH-MCMILLAN-BASED UPDATE

If information about the plant's nonminimum-phase zeros is known, an alternative construction for $\bar{B}_{zu}$ is available. We first represent G_{zu} in Smith-McMillan form. We then define the surrogate transfer function matrix $\hat{G}_{zu}$ to be identical to G_{zu} in Smith-McMillan form except that the minimum-phase transmission zeros of G_{zu} are replaced by transmission zeros at the origin. Thus $\hat{G}_{zu}$ has the form

$$\hat{G}_{zu}(\mathbf{z}) \triangleq \frac{1}{p(\mathbf{z})}\left(\bar{\beta}_d \mathbf{z}^{n-d} + \bar{\beta}_{d+1}\mathbf{z}^{n-d-1} + \cdots + \bar{\beta}_n\right), \tag{32}$$

where $\bar{\beta}_d, \ldots, \bar{\beta}_n \in \mathbb{R}^{l_z \times l_u}$ are the *surrogate numerator coefficients*. We redefine $p_c \triangleq n + p$, and $\bar{B}_{zu}$ is given by

$$\bar{B}_{zu} \triangleq \left[\begin{array}{cccccccccc} 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} & \bar{\beta}_d & \cdots & \cdots & \bar{\beta}_n & 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} \\ 0_{l_z\times l_u} & \ddots & & \ddots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \ddots & & \ddots & 0_{l_z\times l_u} \\ 0_{l_z\times l_u} & \cdots & 0_{l_z\times l_u} & 0_{l_z\times l_u} & \cdots & \cdots & 0_{l_z\times l_u} & \bar{\beta}_d & \cdots & \bar{\beta}_n \end{array}\right]. \tag{33}$$

In the SISO case, the construction of $\bar{B}_{zu}$ requires knowledge of the relative degree d, the first nonzero Markov parameter H_d (which includes knowledge of the sign of the high-frequency gain), and the number and location of nonminimum-phase zeros, if any. The MIMO case is more subtle. The advantage in using the surrogate numerator coefficients $\bar{\beta}_d, \ldots, \bar{\beta}_n$ of $\hat{G}_{zu}$ as opposed to the actual numerator coefficients $\beta_d, \ldots, \beta_n$ of G_{zu} is faster convergence.

For all future discussion, we will use the Markov-parameter-based construction of $\bar{B}_{zu}$ given by (31).

VI. NUMERICAL EXAMPLES

We now present numerical examples to illustrate the response of the RCF adaptive control algorithm. We consider a sequence of examples of increasing complexity, ranging from SISO, minimum-phase plants to MIMO, nonminimum-phase plants, including stable and unstable cases. Each plant can be viewed as a sampled-data discretization of a continuous-time plant sampled at $T_s = 0.01$ sec. All examples assume $z = y$.

For simplicity, each example is taken to be a disturbance rejection simulation, that is, $E_0 = 0$, with unknown sinusoidal disturbance given by

$$w(k) = \begin{bmatrix} \sin 2\pi\nu_1 k T_s \\ \sin 2\pi\nu_2 k T_s \end{bmatrix}, \tag{34}$$

where $\nu_1 = 5$ Hz and $\nu_2 = 13$ Hz. The RCF adaptive control algorithm requires no information about w. With each plant realized in controllable canonical form, we take $D_1 = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$, and, therefore, the disturbance is not matched.

Example 6.1 (SISO, Nonminimum Phase, FIR, Stable): Consider an FIR plant of order $n = 8$ and zeros

$\{0.3 \pm 0.7\jmath, -0.7 \pm 0.3\jmath, 2 \pm 0.5\jmath\}$. We take $n_\mathrm{c} = 15$, $p = 2$, $\mu = 8$, and $\alpha = 25$. The closed-loop response is shown in Figure 3. The control is turned on at $t = 2\,\mathrm{sec}$, and the performance variable reduces to zero within $3\,\mathrm{sec}$. ■

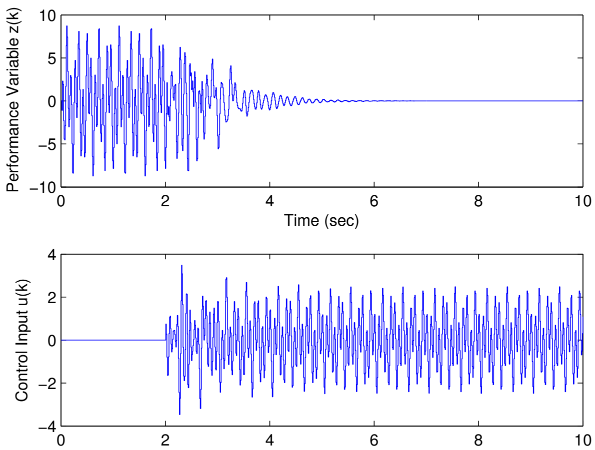

Fig. 3. Closed-loop disturbance rejection response for an FIR, nonminimum phase, SISO plant. The control is turned on at $t = 2\,\mathrm{sec}$. The controller order is $n_\mathrm{c} = 15$ with parameters $p = 2, \mu = 8, \alpha = 25$.

Example 6.2 (SISO, Minimum Phase, IIR, Stable): Consider a plant with poles $\{0.5 \pm 0.5\jmath, -0.5 \pm 0.5\jmath, \pm 0.9, \pm 0.7\jmath\}$ and zeros $\{0.3 \pm 0.7\jmath, -0.7 \pm 0.3\jmath, 0.5\}$. We take $n_\mathrm{c} = 15$, $p = 1$, $\mu = 3$, and $\alpha = 25$. The closed-loop response is shown in Figure 4. The control is turned on at $t = 2\,\mathrm{sec}$, and the performance variable reduces to zero within $1\,\mathrm{sec}$. The control algorithm converges to an internal model controller with high gain at the disturbance frequency, as seen in Figure 5. ■

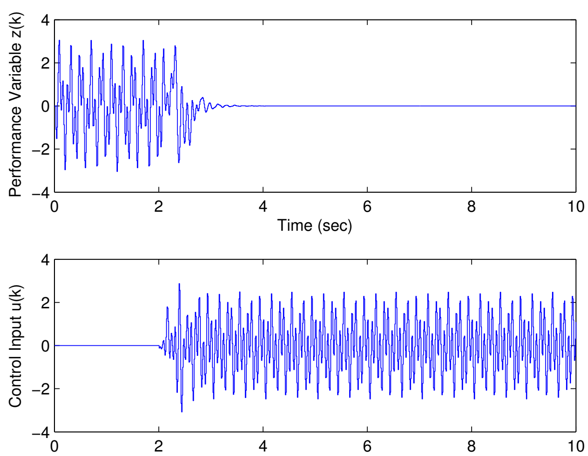

Fig. 4. Closed-loop disturbance rejection response for an IIR, stable, minimum phase, SISO plant. The control is turned on at $t = 2\,\mathrm{sec}$. The controller order is $n_\mathrm{c} = 15$ with parameters $p = 1, \mu = 3, \alpha = 25$.

Example 6.3 (SISO, Nonminimum Phase, IIR, Stable): Consider a plant with poles $\{0.5 \pm 0.5\jmath, -0.5 \pm 0.5\jmath, \pm 0.9, \pm 0.7\jmath\}$ and zeros $\{0.3 \pm 0.7\jmath, -0.7 \pm 0.3\jmath, 2\}$. We take $n_\mathrm{c} = 15$, $p = 2$, $\mu = 8$, and $\alpha = 25$. The closed-loop response is shown in Figure 6. The control is turned on at $t = 2\,\mathrm{sec}$, and the performance variable reduces to zero within $2\,\mathrm{sec}$. ■

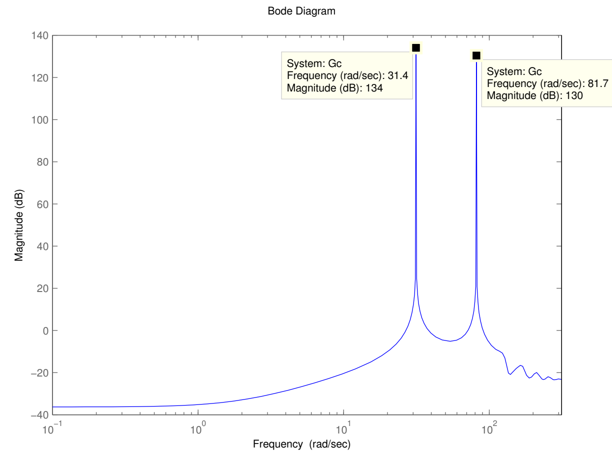

Fig. 5. Bode magnitude plot of the adaptive controller at $t = 10$ sec. The adaptive controller places poles at the disturbance frequencies $\nu_1 = 5$ Hz and $\nu_2 = 13$ Hz. The controller magnitude $|G_\mathrm{c}(e^{\jmath\omega T_\mathrm{s}})|$ is plotted for ω up to the Nyquist frequency $\omega_\mathrm{Nyq} = \frac{\pi}{T_\mathrm{s}} = 314$ rad/sec.

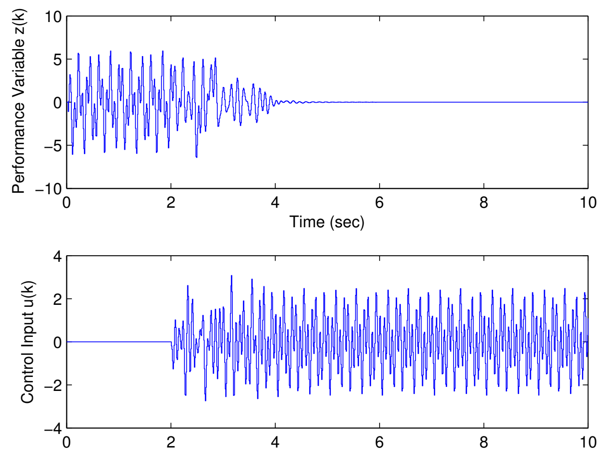

Fig. 6. Closed-loop disturbance rejection response for an IIR, stable, nonminimum phase, SISO plant. The control is turned on at $t = 2\,\mathrm{sec}$. The controller order is $n_\mathrm{c} = 15$ with parameters $p = 2, \mu = 8, \alpha = 25$.

Example 6.4 (SISO, Minimum Phase, IIR, Unstable): Consider a plant with poles $\{0.5 \pm 0.5\jmath, -0.5 \pm 0.5\jmath, \pm 1.04, 0.1 \pm 1.025\jmath\}$ and zeros $\{0.3 \pm 0.7\jmath, -0.7 \pm 0.3\jmath, 0.5\}$. We take $n_\mathrm{c} = 15$, $p = 1$, $\mu = 10$, and $\alpha = 25$. The closed-loop response is shown in Figure 7. The control is turned on at $t = 2\,\mathrm{sec}$, and, after a transient, the performance variable reduces to zero. ■

Example 6.5 (MIMO, Nonminimum Phase, IIR, Stable): Consider a two-input, two-output plant with poles $\{-0.5 \pm 0.5\jmath, \pm 0.7\jmath, 0.3 \pm 0.7\jmath, -0.4, 0.9\}$ and transmission zeros $\{0.5, 2\}$. We take $n_\mathrm{c} = 15$, $p = 2$, $\mu = 8$, and $\alpha = 1$. The closed-loop response is shown in Figure 8. The control is turned on at $t = 2\,\mathrm{sec}$, and the performance variable reduces to zero. ■

Example 6.6 (MIMO, Nonminimum Phase, IIR, Unstable): Consider a two-input, two-output plant with poles $\{-0.5 \pm 0.5\jmath, \pm 0.7\jmath, 0.1 \pm 1.025\jmath, -0.4, 0.9\}$ and transmission zeros $\{0.5, 2\}$. We take $n_\mathrm{c} = 10$, $p = 1$, $\mu = 10$, and $\alpha = 1$. The closed-loop response is shown in Figure 9. The control is turned on at $t = 2\,\mathrm{sec}$, and, after a slight transient, the performance variable reduces to zero. ■

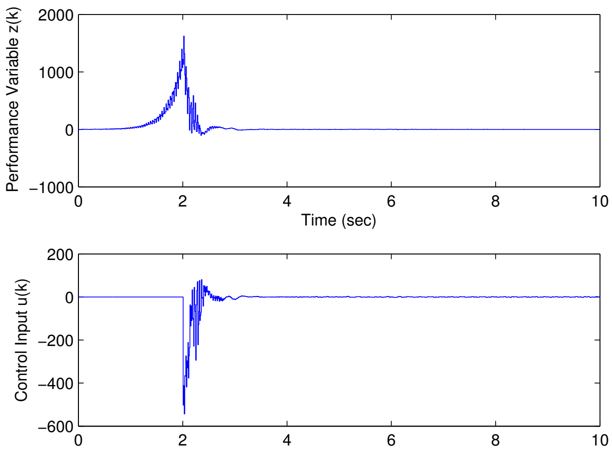

Fig. 7. Closed-loop disturbance rejection response for an unstable, minimum phase, SISO plant. The control is turned on at $t = 2\,\text{sec}$. The controller order is $n_c = 15$ with parameters $p = 1, \mu = 10, \alpha = 25$.

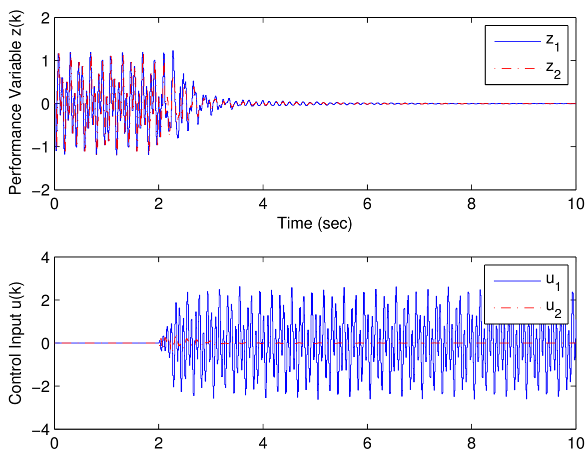

Fig. 8. Closed-loop disturbance rejection response for an IIR, stable, nonminimum phase, two-input two-output plant. The control is turned on at $t = 2\,\text{sec}$. The controller order is $n_c = 15$ with parameters $p = 2, \mu = 8, \alpha = 1$.

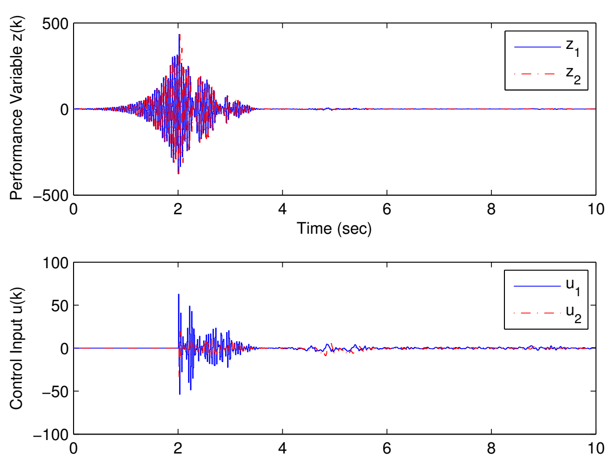

Fig. 9. Closed-loop disturbance rejection response for an unstable, nonminimum phase, two-input two-output plant. The control is turned on at $t = 2\,\text{sec}$. The controller order is $n_c = 10$ with parameters $p = 1, \mu = 10, \alpha = 1$.

VII. Conclusions

We presented the RCF adaptive control algorithm and demonstrated its effectiveness in handling nonminimum-phase zeros through numerical examples. These numerical studies serve as guidance with regard to the development of system identification algorithms that can estimate the required plant parameters with suitable accuracy. Future work includes the development of such identification algorithms as well as Lyapunov-based stability and robustness analysis of the RCF adaptive control algorithm.

References

[1] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, New Jersey: Prentice Hall, 1989.

[2] G. C. Goodwin, P. J. Ramadge, and P. E. Caines, "Discrete-time multivariable adaptive control," *IEEE Trans. Autom. Contr.*, vol. 25, pp. 449–456, 1980.

[3] J. B. Hoagg, M. A. Santillo, and D. S. Bernstein, "Discrete-time adaptive command following and disturbance rejection with unknown exogenous dynamics," *IEEE Trans. Autom. Contr.*, vol. 53, pp. 912–928, 2008.

[4] K. J. Åström, P. Hagander, and J. Sternby, "Zeros of sampled systems," *Automatica*, vol. 20, pp. 31–38, 1984.

[5] E. Bai and S. Sastry, "Persistency of excitation, sufficient richness and parameter convergence in discrete-time adaptive control," *Sys. Contr. Lett.*, vol. 6, pp. 153–163, 1985.

[6] C. Rohrs, L. Valavani, M. Athans, and G. Stein, "Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics," *IEEE TAC*, vol. 30, pp. 881–889, 1985.

[7] P. Ioannou and J. Sun, *Robust Adaptive Control*. Prentice Hall, 1996.

[8] F. Ohkawa and Y. Yonezawa, "A discrete model reference adaptive control system for a plant with input amplitude constraints," *Int. J. Contr.*, vol. 36, pp. 747–753, 1982.

[9] C. Zhang and R. Evans, "Amplitude constrained adaptive control," *Int. J. Contr.*, vol. 46, pp. 53–64, 1987.

[10] S. Karason and A. Annaswamy, "Adaptive control in the presence of input constraints," *IEEE TAC*, vol. 39, pp. 2325–2330, 1994.

[11] W. Lai and P. Cook, "A discrete-time universal regulator," *Int. J. Contr.*, vol. 62, pp. 17–32, 1995.

[12] A. Lindquist and V. Yakubovich, "Universal regulators for optimal tracking in discrete-time systems affected by harmonic disturbances," *IEEE Trans. Autom. Contr.*, vol. 44, pp. 1688–1704, 1999.

[13] B. Anderson, "Topical problems of adaptive control," in *Proc. European Contr. Conf.*, Kos, Greece, July 2007, pp. 4997–4998.

[14] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction, and Control*. Prentice Hall, 1984.

[15] R. Johansson, "Global Lyapunov stability and exponential convergence of direct adaptive control," *Int. J. Contr.*, vol. 50, pp. 859–869, 1989.

[16] I. Mareels and J. W. Polderman, *Adaptive Systems: An Introduction*. Boston, MA: Birkhäuser, 1996.

[17] T. Hayakawa, W. M. Haddad, and A. Leonessa, "A Lyapunov-based adaptive control framework for discrete-time non-linear systems with exogenous disturbances," *Int. J. Contr.*, vol. 77, pp. 250–263, 2004.

[18] S. Akhtar and D. S. Bernstein, "Lyapunov-stable discrete-time model reference adaptive control," *Int. J. Adaptive Contr. Signal Proc.*, vol. 19, pp. 745–767, 2005.

[19] J. B. Hoagg, M. A. Santillo, and D. S. Bernstein, "Internal model control in the shift and delta domains," *IEEE Trans. Autom. Contr.*, vol. 53, pp. 1066–1072, 2008.

[20] D. S. Bayard, "Extended horizon liftings for stable inversion of nonminimum-phase systems," *IEEE Trans. Autom. Contr.*, vol. 39, pp. 1333–1338, 1994.

[21] R. Venugopal and D. S. Bernstein, "Adaptive disturbance rejection using ARMARKOV/Toeplitz models," *IEEE Trans. Contr. Sys. Tech.*, vol. 8, pp. 257–269, 2000.

[22] M. A. Santillo and D. S. Bernstein, "Inherent robustness of minimal modeling discrete-time adaptive control to flight anomalies," in *Proc. Guid. Nav. Contr. Conf.*, Honolulu, HI, August 2008, AIAA-2008-7289.