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Dissipativity properties of detailed models of synchronous generators

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Abstract— **This paper studies the dissipativity properties of full order dynamic models of synchronous generators. It is shown that, under widely accepted assumptions, these models satisfy a balance between the internal storage of a generalized energy and a suitably defined power supply associated to the stator and excitation circuits. This property has particular relevance since it allows to incorporate the detailed model of synchronous machines to classical energy functions used to analyze power system stability. The dissipation inequality implies also that the linear model around the equilibrium point meet a convex condition in the frequency domain, able to be exploited in the stability analysis of interconnected systems. The impact of excitation control and resistive losses on these properties is studied through a numerical example.**

I. Introduction

The dynamic behavior of the synchronous generators plays a central role in power system stability. This machines have been extensively studied for decades and accurate and well established dynamic models are available, see [1], [10]. However, the complexity of power system dynamics has stimulated the seek for analysis tools which take advantage of structural dynamic properties of these systems and, consequently, of the synchronous generators.

Notable antecedents of this search are the references [2], [4], [16], [19]. This research line provide us with a set of techniques– also named direct methods–based in the energy function that have been used in the stability analysis of power systems. Its applications ranges from estimation of stability domains and critical clearing times to online techniques for the detection of loss of synchronism [3], [14], [17]. However, significant difficulties have been faced with the model of the synchronous machine. Typically, generators have been modeled with the classical constant voltage, second order model [3], [17], and with the third order models [19], [16], [12]. Some authors have included Automatic Voltage Regulators (AVR) circuits, at the expense of the inclusion of path-dependent terms to the energy function [14].

More recently, some progresses have been reported by applying a more fundamental concept: the theory of dissipative dynamical systems. This concept was originally stated by Willems in a seminal paper [20] and it was later extended and explored [8]. Basically, a dissipative system satisfies a balance between the storage of a generalized internal energy and a suitably defined supply rate function that describes the interchanges of the system with its environment. This fundamental ideas are strongly related with concepts like passivity and finite gain, see [20], [21], and constitute a fundamental basis of the development of the robustness analysis [11]. References [6], [13], [15] report applications of the dissipativity ideas on power systems.

This paper considers the conventional full order synchronous machine model [1], [10] which includes saliency and three damping circuits. It is shown that, under widely accepted simplifying assumptions–no resistive statorical losses, no "$p\psi$" terms, constant mechanical torque– the conventional synchronous machine model with constant excitation satisfies a dissipation inequality involving a generalized power supply rate function w_s suitably defined from the active and reactive power and the terminal voltage. The significance of this property is the following. In reference [6] it was shown that this dissipation is also met by classical models of (lossless) transmission lines, classical synchronous machine, constant active power loads and ZIP (constant impedance, current and power) reactive power loads. As a consequence, *detailed models of synchronous generators can be exactly incorporated to the well-known energy function of power systems*. The function w_s have been implicitly employed in the construction of energy functions, see [3], [17].

A Hamiltonian description for the machine model, useful for control purposes, is also derived. Other contribution of this paper is the obtaining, along the lines of previous work [7], of a convex frequency domain condition that is satisfied by the small signal model of the synchronous machine. This frequency domain condition is a special case of Integral Quadratic Constraint (IQC) which allows a very versatile treatment of uncertainties [11].

The excitation control is specially included in the analysis; its effect on the dissipation balance is explicitly treated. However, conventional controllers like AVRs or stabilizers would destroy the dissipativity properties. This is investigated through a numerical example which examines the impact of statorical resistive losses and excitation control.

The structure of the paper is as follows. Section II presents the mathematical model of the synchronous machine. In Section III we analyze its dynamical properties, including the obtaining of a generalized Hamiltonian model. Section IV introduces the corresponding linear model around the equilibrium point and shows how the dissipation inequality can be posed as a convex condition in the frequency domain. Section V presents the frequency domain analysis of the machine model of a benchmark example. We wrap up the paper with some concluding remarks.

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II. Synchronous machine modeling

In this section we recall the well known model for the synchronous machine. It includes three damping circuits at the rotor, salient poles, and the complete stator dynamics. Some notation is introduced to simplify the treatment and some assumptions are established to advance in our development.

The $d-q$ components of the terminal voltage satisfy

$$\begin{cases} E_d &= V\sin(\delta-\theta) \\ E_q &= V\cos(\delta-\theta), \end{cases} \quad (1)$$

where $Ve^{j\theta}$ is the terminal voltage phasor referred to the synchronous reference, and δ is the correspondent axis q rotor position. The entering complex power S_M results:

$$S_M = P_M + jQ_M = (E_d + jE_q)[-(I_d - jI_q)],$$

where the negative signal comes from the convention used to define the stator current as positive when salient. The terminal voltage and the complex power can also be written

$$y := \begin{bmatrix} \theta & V \end{bmatrix}^T; u = \begin{bmatrix} P^M & Q^M \end{bmatrix}^T. \quad (2)$$

We consider here the standard eighth order model for the synchronous machine, see equations 3.120 to 3.134 in [10]:

$$\begin{array}{rcl} E_d &=& \frac{d}{dt}\Phi_d - \Phi_q\omega_r - R_aI_d \\ E_q &=& \frac{d}{dt}\Phi_q + \Phi_d\omega_r - R_aI_q \\ E_{fd} &=& \frac{d}{dt}\Phi_{fd} + R_{fd}I_{fd} \\ 0 &=& \frac{d}{dt}\Phi_{1d} + R_{1d}I_{1d} \\ 0 &=& \frac{d}{dt}\Phi_{1q} + R_{1q}I_{1q} \\ 0 &=& \frac{d}{dt}\Phi_{2q} + R_{2q}I_{2q} \\ \frac{d}{dt}\delta &=& \Omega_0(\omega_r - 1) \\ h\frac{d}{dt}\omega_r &=& T_m - d(\omega_r - 1) - T_e \\ T_e &=& \Phi_dI_q - \Phi_qI_d \end{array} \quad (3)$$

Φ_d, Φ_q represent the $d-q$ components of the stator flux linkage. $\Phi_{1d}, \Phi_{1q}, \Phi_{2q}, \Phi_{fd}$ are the respective rotor flux linkages associated to the damping and field circuits. The currents $I_d.I_q, I_{fd}, I_{1d}, I_{1q}, I_{2q}$ obey to the same notation. We will assume that the damping coefficient and the resistances $d, R_k \geq 0$ and the inertia constant $h > 0$. ω_r denotes the variation of the rotorical angular speed in p.u.

We now introduce the sub-indices s and r to respectively denote the stator and rotor variables:

$$\Phi_s := \begin{bmatrix} \Phi_d \\ \Phi_q \end{bmatrix}; I_s := \begin{bmatrix} I_d \\ I_q \end{bmatrix}; \Phi_r := \begin{bmatrix} \Phi_{1d} \\ \Phi_{1q} \\ \Phi_{2q} \end{bmatrix}; I_r := \begin{bmatrix} I_{1d} \\ I_{1q} \\ I_{2q} \end{bmatrix}.$$

The relationship between fluxes and currents results

$$\Phi := \begin{bmatrix} \Phi_s \\ \Phi_{fd} \\ \Phi_r \end{bmatrix} = L \begin{bmatrix} -I_s \\ I_{fd} \\ I_r \end{bmatrix} = LI,$$

with

$$L = \begin{bmatrix} L_{ad}+L_l & 0 & L_{ad} & L_{ad} & 0 & 0 \\ 0 & L_{aq}+L_l & 0 & 0 & L_{aq} & L_{aq} \\ L_{ad} & 0 & L_{ffd} & L_{f1d} & 0 & 0 \\ L_{ad} & 0 & L_{f1d} & L_{11d} & 0 & 0 \\ 0 & L_{aq} & 0 & 0 & L_{aq} & L_{aq} \\ 0 & L_{aq} & 0 & 0 & L_{aq} & L_{aq} \end{bmatrix}.$$

Let us introduce an auxiliary matrix

$$J_2 := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

that satisfies $J_2^{-1} = J_2^\top = -J_2$. Thus, the electrical torque T_e and the power S_M can, respectively, be written as:

$$T_e = \Phi_dI_q - \Phi_qI_d = \begin{bmatrix} I_d \\ I_q \end{bmatrix}^T J_2 \begin{bmatrix} \Phi_d \\ \Phi_q \end{bmatrix} = I_s^\top J_2\Phi_s, \quad (4)$$

$$S_M = -(E_d + jE_q)(I_d - jI_q) = -E_s^TI_s + jI_s^TJ_2E_s. \quad (5)$$

The stator equations in (3) can now be written:

$$E_s = \frac{d}{dt}\Phi_s + J_2\Phi_s\omega_r - R_aI_s. \quad (6)$$

We shall state the assumptions we need to proceed with our development.

Assumption 1 The terms $\frac{d}{dt}\Phi$ and R_aI_s in equation (6) are neglected. The term $J_2\Phi_s\omega_r$ is approximated by $J_2\Phi_s$. So, equation (6) will be substituted by

$$E_s = J_2\Phi_s. \quad (7)$$

Notice that the terms $\frac{d}{dt}\Phi$–commonly referred in the literature as $p\Phi$ terms–are typically neglected all along the power system since the electrical network is studied with the help of phasors and a quasi-stationary hypothesis. If we neglect the terms $\frac{d}{dt}\Phi$ in the stator equations, we will treat the machine's stator as the remainder of the network. Taking $\omega_r = 1$ in equation (6) is a typical approximation in power system literature, see [10]. The assumption on R_a is justified by its little significance. Nevertheless, its influence is illustrated in Section V with the help of an example.

The following equation follows from Assumption 1:

$$T_e = I_s^TJ_2\Phi_s = I_s^TE_s = -P_M. \quad (8)$$

With these simplifications, we get the sixth order model for the synchronous machine

$$\Sigma(u,y;v,z): \begin{cases} \frac{d}{dt}\delta &= \Omega_0(\omega_r - 1) \\ h\frac{d}{dt}\omega_r &= T_m - d(\omega_r - 1) + P_M \\ \frac{d}{dt}\Phi_{fd} &= E_{fd} - R_{fd}I_{fd} \\ \frac{d}{dt}\Phi_r &= -R_rI_r \\ E_s &= J_2\Phi_s \end{cases} \quad (9)$$

We denote $x := [\delta \ \omega_r \ \Phi_{fd} \ \Phi_r^\top]^\top \in R^6$ the state vector. At the stator circuit, we take $y := [\theta \ V]^T \in R^2$ and $u := [P_M \ Q_M]^T \in R^2$ as the output and input variables, respectively. At the excitation circuit we denote the output $z = I_{fd}$ and the input $v = E_{fd}$.

III. Dynamical properties

In this section we will show that the model (9) can be written as a generalized Hamiltonian model, see [18]. Once established this fact, the dissipativity of the model will be defined and proved. In this section and in the sequel, we shall assume that the mechanical torque T_m is constant.

A. Generalized Hamiltonian model

Notice that, in the model (9), the stator flux Φ_s is not a state variable, but a function on δ and y:

$$\Phi_s = -J_2 E_s = -J_2 \begin{bmatrix} V\sin(\delta-\theta) \\ V\cos(\delta-\theta) \end{bmatrix}.$$

Thus, the flux vector Φ is a function of the state vector x and the link variables y. With the help of the auxiliary matrices

$$P_s = \begin{bmatrix} I_{d2} \\ 0_{4\times 2} \end{bmatrix}, P_r = \begin{bmatrix} 0_{2\times 4} \\ I_{d4} \end{bmatrix}$$

we write

$$\Phi = [\Phi_s^T \; \Phi_{fd} \; \Phi_r^T]^T = P_s\Phi_s + P_r[\Phi_{fd} \; \Phi_r^T]^T.$$

We can compute the partial derivatives

$$\frac{\partial\Phi}{\partial y} = \frac{\partial\Phi}{\partial\Phi_s}\frac{\partial\Phi_s}{\partial E_s}\frac{\partial E_s}{\partial y} =$$

$$= P_s[-J_2]\begin{bmatrix} -V\cos(\delta-\theta) & \sin(\delta-\theta) \\ V\sin(\delta-\theta) & \cos(\delta-\theta) \end{bmatrix} =$$

$$= P_s[-J_2][J_2E_s \;\; \frac{1}{V}E_s] = P_s[E_s \;\; -\frac{1}{V}J_2E_s], \quad (10)$$

$$\frac{\partial\Phi}{\partial\delta} = \frac{\partial\Phi}{\partial\Phi_s}\frac{\partial\Phi_s}{\partial E_s}\frac{\partial E_s}{\partial\delta} = P_s[-J_2]\begin{bmatrix} V\cos(\delta-\theta) \\ -V\sin(\delta-\theta) \end{bmatrix} =$$

$$= P_s[-J_2][-J_2E_s] = -P_sE_s. \quad (11)$$

Define the function $S : R^6 \times R^2 \to R$:

$$S(x,y) := \frac{1}{2}\Omega_0 h(\omega_r - 1)^2 + \frac{1}{2}\Phi^\top L^{-1}\Phi - T_m\delta. \quad (12)$$

This function will be shown to meet the role of a storage function, in a sense that will be explained in Section III-B. Notice that function S comprises three terms which possess clear physical interpretation: kinetic energy, magnetic energy and a term of potential energy $T_m\delta$.

The following computations are direct from eqs. (11),(12):

$$\frac{\partial S}{\partial\Phi} = (L^{-1}\Phi)^T = I^T, \quad (13)$$

$$\frac{\partial S}{\partial\Phi}\frac{\partial\Phi}{\partial\delta} = I^T[-P_sE_s] = I_s^TE_s = -P_M. \quad (14)$$

The partial derivative $\frac{\partial S}{\partial y}$ can be computed with the help of equations (13), (10) and (5):

$$\frac{\partial S}{\partial y} = \frac{\partial S}{\partial\Phi}\frac{\partial\Phi}{\partial y} = I^T\frac{\partial\Phi}{\partial y} = I^TP_s[E_s \;\; -\frac{1}{V}J_2E_s] =$$

$$[-I_s]^T[E_s \;\; -\frac{1}{V}J_2E_s] = [-I_s^TE_s \;\; \frac{1}{V}I_s^TJ_2E_s] = [P_M \;\; \frac{Q_M}{V}].$$

Definition (12) and equation (14) imply:

$$\frac{\partial S}{\partial x} = \begin{bmatrix} -T_m + \frac{\partial S}{\partial\Phi}\frac{\partial\Phi}{\partial\delta} \\ h\Omega_0(\omega_r-1) \\ P_r^TL^{-1}\Phi \end{bmatrix}^T = \begin{bmatrix} -T_m - P_M \\ h\Omega_0(\omega_r-1) \\ P_r^TI \end{bmatrix}^T.$$

Thus, we get the gradients[1] of function S:

$$\nabla_xS(x,y) = \begin{bmatrix} -T_m - P_M \\ h\Omega_0(\omega_r-1) \\ P_r^TI \end{bmatrix}; \nabla_yS(x,y) = \begin{bmatrix} P_M \\ \frac{Q_M}{V} \end{bmatrix}. \quad (15)$$

We are now in position to state our PCH model

$$\begin{cases} \dot{x} &= (J-R)\nabla_xS(x,y) + B_vE_{fd} \\ 0 &= -\nabla_yS(x,y) + B_u(y)u \end{cases} \quad (16)$$

with

$$J = -J^T = \frac{1}{h}\left[\begin{array}{cc|c} 0 & 1 & \\ -1 & 0 & 0_{2\times 4} \\ \hline \multicolumn{2}{c|}{0_{4\times 2}} & 0_{4\times 4} \end{array}\right], \quad (17)$$

$$R = \left[\begin{array}{cc|c} 0 & 0 & \\ 0 & \frac{d}{h^2\Omega_0} & 0_{2\times 4} \\ \hline \multicolumn{2}{c|}{0_{4\times 2}} & R_4 \end{array}\right] \geq 0, \quad (18)$$

$$R_4 = \begin{bmatrix} R_{fd} & 0 & 0 & 0 \\ 0 & R_{1d} & 0 & 0 \\ 0 & 0 & R_{1q} & 0 \\ 0 & 0 & 0 & R_{2q} \end{bmatrix},$$

$$B_v = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T, B_u(y) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{V} \end{bmatrix}.$$

B. Dissipativity properties

As indicated in the Introduction we adopt the dissipativity framework proposed in [20], see also [8], [18]. To establish our results a slight variation of the classical formulation is needed since the supply rate functions that we consider depend, not only on the port variables (u,y), but also on $\dot{y}$.

Definition 1: Consider a dynamical system Σ given by

$$\begin{cases} \dot{x} &= F(x,u) \\ y &= r(x,u) \end{cases} \quad (19)$$

where $x \in \mathbb{R}^n$ is the state and $(u,y) \in \mathbb{R}^p \times \mathbb{R}^p$ are the port variables. Let $w : \mathbb{R}^p \times \mathbb{R}^p \times \mathbb{R}^p \to \mathbb{R}$ be locally integrable along trajectories of Σ, i.e.

$$\int_{t_1}^{t_2} w(u(t),y(t),\dot{y}(t))dt < \infty, \quad \forall\, t_1,t_2 \in \mathbb{R}.$$

We say that Σ is *cyclo–dissipative* with respect to the supply rate $w(u,y,\dot{y})$ if and only if there exists a differentiable function $S : \mathbb{R}^n \to \mathbb{R}$, called storage function, such that

$$S(x(t_2)) - S(x(t_1)) \leq \int_{t_1}^{t_2} w(u(t),y(t),\dot{y}(t))dt \quad \forall t_2 \geq t_1.$$

If the storage function is *non–negative* we say that Σ is dissipative with respect to the supply rate $w(u,y,\dot{y})$.

As seen from the definition above the distinction between cyclo–dissipative and dissipative systems is the non–negativity of the storage function.[2] It can be shown [8] that a system is cyclo–dissipative when it cannot create (abstract) energy *over closed paths* in the state–space. It

[1]The gradient of a scalar function $S(x,y)$ with respect to variable x will be denoted ∇_xS and represented as a *column* vector.

[2]Actually, as one can always add a constant to the storage function, the question is whether it is bounded from below or not.

might, however, produce energy along some initial portion of such a trajectory; if so, it would not be dissipative. On the other hand, every dissipative system is cyclo–dissipative.

Consider the supply rate functions $W_s : \mathbb{R}^2\times\mathbb{R}^2\times\mathbb{R}^2 \to \mathbb{R}$ and $W_{fd} : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$

$$W_s(u, y, \dot{y}) := u^\top B_u^\top(y)\dot{y} = P\dot{\theta} + \frac{Q}{V}\dot{V}. \quad (20)$$

$$W_{fd}(v, z) := vz = E_{fd}I_{fd}. \quad (21)$$

The cyclo-dissipativity of the operator Σ is a straight-forward consequence of model (16):

Proposition 1: The operator $\Sigma(u, y; v, z)$ defined by the model (9) is cyclo–dissipative with respect to the supply rate $W = W_s(u, y, \dot{y}) + W_{fd}(v, z)$. More precisely,

$$\frac{dS(x, y)}{dt} \leq W_s(u, y, \dot{y}) + W_{fd}(v, z).$$

Proof: Compute the derivative of the storage function S:

$$\frac{dS(x, y)}{dt} = \frac{\partial S}{\partial x}\dot{x} + \frac{\partial S}{\partial y}\dot{y} =$$

$$= [\nabla_x S(x, y)]^\top [(J-R)\nabla_x S(x, y)+B_v E_{fd}]+u^T B_u(y)^T\dot{y} =$$

$$- [\nabla_x S(x, y)]^\top R\nabla_x S(x, y) + E_{fd}I_{fd} + w(u, y, \dot{y}) =$$

$$-d\Omega_0(\omega_r-1)^2-R_{fd}I_{fd}^2-I_r^T R_r I_r+W_{fd}(v, z)+W_s(u, y, \dot{y}) \leq$$

$$\leq W_{fd}(v, z) + W_s(u, y, \dot{y}).$$

The inequality results from equations (17) and (18).

Remark 1: In reference [6] it was shown that Proposition 1 is also met by classical models of (lossless) transmission lines, classical and third order generators models, constant active power loads and ZIP reactive power loads. As a consequence of Proposition 1 and results in [6], detailed models of synchronous generators can be accurately incorporated to the well-known energy function of power systems. The function w_s have been implicitly employed in the construction of energy functions, see [17], [3].

Remark 2: The dissipation is composed by the physically foreseeable terms: mechanical losses, electrical losses. The supply rate function has two components. W_s is the already known supply rate function which rules the energy interchange all along the network [5]. The term $W_{fd} = E_{fd}I_{fd}$ is, naturally, the electrical power supplied to the machine by the excitation system.

Remark 3: Of course, we always can consider the physically natural supply rate function defined as the sum of the supply rate $\hat{w}_s = -E_s^T I_s = P_M$ for the stator and $\hat{w}_{fd} = E_{fd}I_{fd}$ for the field. By doing so, we recover the familiar energy balance at the machine, see [13] and references therein. This study is very interesting, since it establishes links with very well-known physical concepts. However, that election of power supply rate functions faces serious drawbacks when analyzing the stability of a non zero equilibrium, since the dissipativity is not suitably satisfied by the incremental model around a non-zero equilibrium point. Significantly, the power supply rates w_s and w_{fd} defined above have not these drawbacks since they have not first order terms and they vanish at the equilibrium.

C. Incremental properties

When the property of interest is the stability of an equilibrium point, it is necessary to examine the behavior of the supply rate function around the equilibrium [20]. The supply rate function W_s is zero at the equilibrium point which is very important for the equilibrium stability analysis. Although W_s has first order terms ($P_M^\star\dot{\theta}$, etc), they can be easily incorporated to the storage function as additive terms which is also true for the constant and first order terms of W_{fd} around the equilibrium, as it is shown next.

Variables at the equilibrium will be denoted with a supra-index $*$: x^*, I_{fd}^*, etc. Tildes or simply lowercase will denote the incremental variables: $i_{fd} = I_{fd} - I_{fd}^*, \tilde{x} = x - x^*$, etc.

Define the modified storage function $s : R^6 \times R^2 \to R$:

$$s(\tilde{x}, \tilde{y}) = S(x, y) - P_M^*\theta - Q_M^* lnV - I_{fd}^*\Phi_{fd}, \quad (22)$$

and the modified supply rate functions $w_{fd} : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $w_s : \mathbb{R}^2\times\mathbb{R}^2\times\mathbb{R}^2 \to \mathbb{R}$:

$$w_{fd}(\tilde{v}, \tilde{z}) = (E_{fd} - E_{fd}^*)(I_{fd} - I_{fd}^*) = e_{fd}i_{fd},$$

$$w_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}) := (P_M - P_M^*)\dot{\theta} + \frac{Q_M - Q_M^*}{V}\dot{V} = p_M\dot{\theta} + \frac{q_M}{V}\dot{V}. \quad (23)$$

It is simple to verify the dissipativity for this formulation:

$$\frac{d}{dt}s(\tilde{x}, \tilde{y}) = \frac{dS(x, y)}{dt} - P_M^*\dot{\theta} - Q_M^*\frac{\dot{V}}{V} - I_{fd}^*\dot{\Phi}_{fd} =$$

$$-d\Omega_0(\omega_r - 1)^2 - R_{fd}I_{fd}^2 - I_r^T R_r I_r + E_{fd}I_{fd}+$$

$$+W_s(u, y, \dot{y}) - P_M^*\dot{\theta} - Q_M^*\frac{\dot{V}}{V} - I_{fd}^*\dot{\Phi}_{fd} =$$

$$= -d\Omega_0(\omega_r - 1)^2 - I_r^T R_r I_r + w_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}) - I_{fd}^*\dot{\Phi}_{fd}+$$

$$+I_{fd}(-R_{fd}I_{fd} + E_{fd}) =$$

$$= -d\Omega_0(\omega_r-1)^2-I_r^T R_r I_r+w_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}})+i_{fd}e_{fd}-R_{fd}i_{fd}^2 \leq$$

$$\leq w_{fd}(\tilde{v}, \tilde{z}) + w_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}).$$

Notice that $s(\tilde{x}, \tilde{y})$ has no first order terms:

$$\frac{\partial s}{\partial \tilde{x}}|_* = \frac{\partial S}{\partial x}|_* - [0\ 0\ I_{fd}^*\ 0\ 0\ 0] = 0$$

$$\frac{\partial s}{\partial \tilde{y}}|_* = \frac{\partial S}{\partial y}|_* - [P_M^*\ \frac{Q_M^*}{V^*}] = 0.$$

IV. Dissipative properties of small signal models

The section III established the cyclo-dissipativity of model (9) . Thus, a fundamental internal property– the balance between the storage of a generalized energy and the interaction with the environment–has been established. We will study how these properties particularize for small signal models. Later, we will exploit the versatility of linear models to study the dissipativity in the frequency domain.

With the help of the Hessian of function s at the equilibrium point:

$$\mathcal{H} := \frac{\partial^2 s(\tilde{x}, \tilde{y})}{\partial(\tilde{x}, \tilde{y})^2}|_\star,$$

we can define $H : \mathbb{R}^6 \times \mathbb{R}^2 \to \mathbb{R}$:

$$H(\tilde{x}, \tilde{y}) := \frac{1}{2} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}^\top \mathcal{H} \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix}, \quad (24)$$

and obtain the linear model

$$\begin{cases} \dot{\tilde{x}} &= (J - R)\nabla_{\tilde{x}} H(\tilde{x}, \tilde{y}) + B_v \tilde{v} \\ 0 &= -\nabla_{\tilde{y}} H(\tilde{x}, \tilde{y}) + [p_M \frac{q_M}{V^*}]^T. \end{cases} \quad (25)$$

We define the supply rate function $\tilde{w}_s : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$

$$\tilde{w}_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}) := [p_M \frac{q_M}{V^*}]\dot{\tilde{y}} = p_M \dot{\theta} + \frac{q_M}{V^*}\dot{V}, \quad (26)$$

the second order term of W_s around the equilibrium point. If we define

$$\mathcal{W} := \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{V^*} \end{bmatrix},$$

we can write

$$\tilde{w}_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}) = [p_M q_M]\mathcal{W}\dot{\tilde{y}} = \tilde{u}^T \mathcal{W}\dot{\tilde{y}}.$$

If the excitation is held constant, i.e. $\tilde{v} \equiv 0$, the dissipation inequality is easily recovered with the help of definitions (22), (24) and (26):

$$\frac{d}{dt}H = \nabla_x^\top H \dot{\tilde{x}} + \nabla_y^\top H \dot{\tilde{y}} \le [p_M \frac{q_M}{V^*}]\dot{\tilde{y}} = \tilde{w}_s(\tilde{u}, \tilde{y}, \dot{\tilde{y}}). \quad (27)$$

Equation (25) is the state space representation of the small signal response of the power system. It also determines an input-output relationship between input $\tilde{u}$ and output $\tilde{y}$ for $\tilde{v} \equiv 0$. Denote $\Sigma(s)$ the transfer matrix: $\tilde{y}(s) = \Sigma(s)\tilde{u}(s)$; being s the Laplace variable[3].

Under mild conditions, the dissipation inequality (27) implies a frequency-dependent inequality.

Proposition 2: If $\Sigma(j\omega) \in R\mathcal{L}_\infty$, then it satisfies

$$\begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix}^* \Pi_d(j\omega) \begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix} \ge 0 \;\; \forall w \in \mathbb{R}, \quad (28)$$

$$\Pi_d(j\omega) := |h(j\omega)|^2 \begin{bmatrix} 0 & -j\omega\mathcal{W}^\top \\ j\omega\mathcal{W} & 0 \end{bmatrix}, \quad (29)$$

for all function $h(s)$ real rational stable and strictly proper.

Proof: The proof rests on a classical argument[4] that consists in considering a perfect sinusoidal input $\tilde{u}(t)$ with angular frequency ω and arbitrary spatial direction:

$$\tilde{u}(t) = Re(u_0 e^{j\omega t}), u_0 \in \mathcal{C}^2.$$

Obtain the sinusoidal functions $\tilde{x}(t), \tilde{y}(t)$ such that the triple $(\tilde{u}, \tilde{x}, \tilde{y})$ satisfies (25). Naturally, $\tilde{y}(t) = Re(\Sigma(j\omega)u_0 e^{j\omega t})$ and the supply rate function $\tilde{w}_s(t)$ is given by

$$\tilde{w}_s(t) = Re(u_0 e^{j\omega t})^T \mathcal{W}\dot{\tilde{y}}.$$

If the dissipation inequality (27) is integrated in one period $T = \frac{2\Pi}{\omega}, \omega \neq 0$, we get:

$$\int_{t_0}^{t_o+T} \tilde{w}_s(t)dt = \frac{T}{4}u_0^* j\omega[\Sigma(j\omega)^\star \mathcal{W} - \mathcal{W}\Sigma(j\omega)]u_0 \ge 0.$$

Thus, since u_0 is arbitrary, it is necessary that

$$\begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix}^\star \begin{bmatrix} 0 & -j\omega\mathcal{W} \\ j\omega\mathcal{W}^T & 0 \end{bmatrix} \begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix} \ge 0 \, \forall \omega.$$

The inclusion of the case $\omega = 0$ is immediate since Π_d vanishes. The factor $|h(j\omega)|^2$ is incorporated in order to ensure the boundedness of Π_d for all ω. □□□

V. Numerical Example

In this section we consider a classical benchmark of power system stability studies: the four machines example of [10], page 813. The objective of this analysis is to verify the fulfillment of the frequency domain condition in Proposition 2.

The system was modeled with the package DSAT [9], including the computation of the linear model around the equilibrium.

Let us compute the eigenvalues of the matricial function

$$\sigma(j\omega) := \begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix}^* \Pi_d(j\omega) \begin{bmatrix} I \\ \Sigma(j\omega) \end{bmatrix} \;\; \forall w \in \mathbb{R}.$$

According to Proposition 2, both eigenvalues of function $\sigma(j\omega)$ must be positive for all $\omega \in \mathbb{R}$. The eigenvalues of $\sigma(j\omega)$ were computed for each generator in the example. The model of generator $G1$ does not include the effect of magnetic saturation nor statorical resistance R_a. The excitation is kept constant. Figure 1 shows the eigenvalues of $\sigma(j\omega)$ for generator $G1$: both are positive, which provide us a computational validation of Proposition 2.

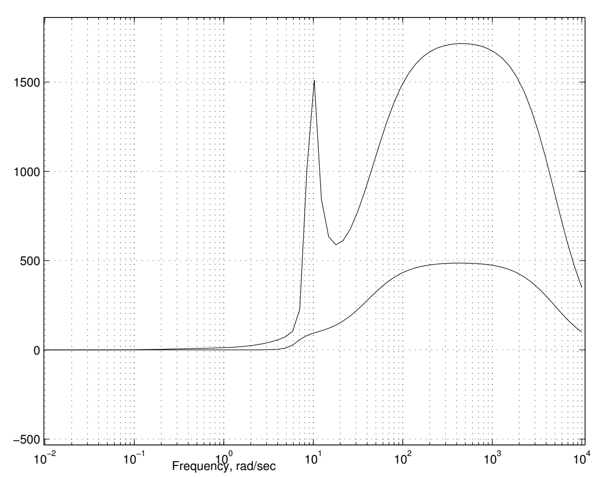

Fig. 1. Eigenvalues of function $\sigma(j\omega)$ for $G1$. No magnetic saturation, no R_a, constant excitation.

The remaining machines were modeled with magnetic saturation. The model of generator G3 includes also the statorical resistance Ra. Eigenvalues of σ for $G3$ are illustrated in Figure 2. As it can be seen, the influence of Ra is negligible for all frequency up to 1000 rad/sec.

Generator $G4$ includes the following AVR:

$$E_{fd} = 200\frac{1+s}{1+10s}(E_t - V_{ref}),$$

[3]The context avoids any potential confusion with the incremental storage function s defined in (22) and the abstract notation for the system $\Sigma(u, y)$.

[4]Proposition 2 can be seen as a special case of the classical KYP lemma.

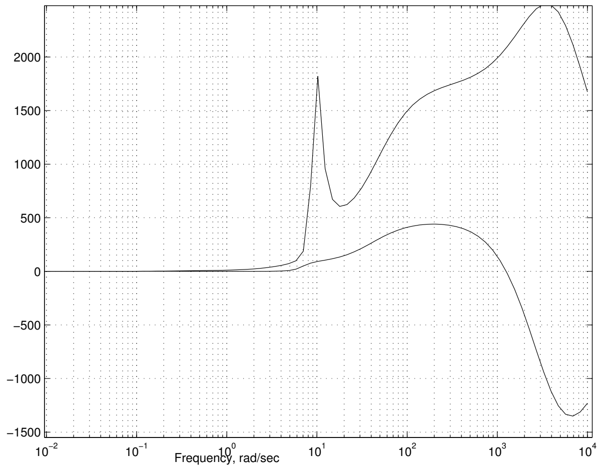

Fig. 2. Eigenvalues of function $\sigma(j\omega)$ for $G2$. Includes the effect of R_a.

which has a relatively high gain. The eigenvalues of $\sigma(j\omega)$ for this case are represented in Figure 3. The AVR naturally affects the frequency response at low frequencies, just below the natural frequencies of the system, around 1 rad/sec.

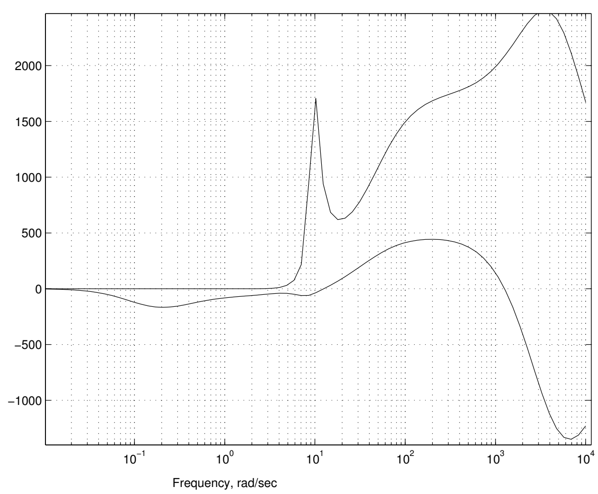

Fig. 3. Eigenvalues of function $\sigma(j\omega)$ for $G4$. Includes the effect of R_a and excitation control.

Figures 1-3 allow us graphically appreciate the influence of R_a and the AVR on the cyclo-dissipativity of the generator model. Notice that this property remains valid in the presence of active excitation control for medium frequencies comprising the basic natural frequencies of electromechanical oscillations. This fact encourages the employment of multiplier Π_d for robustness analysis of power systems.

VI. Concluding remarks

The dissipativity of detailed model of synchronous machines has been precisely stated and demonstrated. Thus, the complete model of the synchronous machine can be incorporated to well-known energy function for stability analysis of power systems. The incremental properties of the supply rate functions around the equilibrium were studied in detail. These properties result in a convex frequency domain condition, which can be used for the stability analysis of interconnected power systems. In this article the dissipativity of the linear model was exploited to analyze the effect of resistive losses and excitation control. Since the computation of the multiplier Π_d derived here is a convex problem, it can be exploited in the synthesis of excitation control for non idealized models, which is currently under research.

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