

A 2DOF \mathcal{H}_∞ robust tracking design for a special type of observed state feedback controllers

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Abstract—A control configuration for the design of robust controllers in the general context of LTI systems (SISO, MIMO, stable, unstable) is presented. It consists of a two-degree-of-freedom control architecture constituted by a particular observed state feedback controller arising from a right coprime factorization of the plant together with a prefilter block. Before detailing the general design methodology involving the available two degrees of freedom, the feedback part of the scheme is first analyzed by its own in order to establish connections with respect to a conventional state feedback controller with full state observer. A prefilter block is added afterwards and an optimization based design procedure for the resulting two-degree-of-freedom control scheme is suggested. The feedback part is designed in regulator mode to guarantee robust stability and some performance in terms of disturbance rejection whereas the prefilter controller deals with the servo specifications given in terms of a reference model.

I. INTRODUCTION

The objective of this communication is twofold. On the one hand, it characterizes the somewhat uncommon Observer-Controller configuration appearing in [16]. In what follows this control configuration will be referred to as the O2C for reasons that will become clear later on. Secondly, once the O2C configuration has been presented an \mathcal{H}_∞ two-degree-of-freedom (2DOF) design procedure making use of it is outlined. We limit the treatment in this note to the SISO case, but the design applies to the general LTI framework, including the MIMO case.

The O2C is introduced making a parallel derivation of observed state feedback controllers (OSFC), directly in the transfer function domain, appearing in [9]. It will be seen that the fact of thinking directly in terms of transfer functions instead of in terms of polynomials yields an OSFC which makes use of a full order observer for a non-minimal plant realization of double order with respect to that of a minimal plant realization. This fact gives its name to the presented control configuration. Different approaches for designing the three blocks appearing in the O2C are outlined in this note. First it is shown how the O2C structure can be useful for robustifying a fully observed state feedback controller (FOSFC) following a two-step design in the line of the well-known IMC [11]. On the basis of the pole placement provided by a conventional FOSFC, an input-output relation can be fixed in accordance to a desired closed-loop response. Having done that, the extra degrees of freedom available in the O2C configuration can be used to increase the resulting

robustness margins without changing neither the control law nor affecting the asymptotic observer properties. Adopting the O2C configuration and extending it through the inclusion of a prefilter block, the rest of the note is devoted to suggest an \mathcal{H}_∞ optimization based robust design methodology for the resulting 2DOF scheme. We refer the reader to [6] for an introduction to the \mathcal{H}_∞ control theory. The use of a 2DOF compensator presents the advantage of complete separation between feedback and reference tracking properties [19]. While the approaches found in the literature are mainly based on optimization problems representing different ways of setting simultaneously the two Youla parameters [19] characterizing the controller [16], [19], [7], [10], we present instead a two-step design methodology:

- 1) The O2C configuration is designed first to guarantee robust stability and some levels of performance in terms of disturbance rejection.
- 2) The prefilter controller is designed then on the basis of a reference model to improve the open-loop processing of the controlled system.

The design of controllers on the basis of a reference model has received great attention in the linear control literature [4], being the adaptive case [5] possibly the most widely analyzed. As the system response to a command is an open-loop property [13], [17], no stability margins are necessarily guaranteed when achieving the desired closed-loop response behaviour. Previous approaches that include robustness considerations when designing on the basis of a reference model can be found in the literature [15], [18]. Another more recent approach aimed at overcoming the shortcomings of the well-known IMC [11] can be consulted in [3], where also an \mathcal{H}_∞ optimization problem is posed. Our approach is somehow similar to this. Nevertheless, in [3] the control architecture corresponds to the standard 1DOF feedback configuration for which competing design specifications like the tracking and the disturbance rejection properties have to be traded off. Besides, no specific structure for the controller is assumed. As opposed to this, we employ a 2DOF controller and deal with the tracking specifications in a separate manner. Regarding the controller used in the feedback part (the first DOF) of our scheme, the fixed structure provided by the O2C configuration is adopted and the solution to the corresponding optimization problem is obtained through the use of direct search techniques in the line of [8] and [1]. These are the most remarkable points of the approach presented here. In [2] an extension of the work in [3] using a 2DOF scheme can be found, but the feedback

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part design relies on loop shaping ideas and this makes the design less automatic.

The organization of the paper is as follows. Section 2 presents a short overview and rederivation of the O2C configuration. Section 3 presents possible methodologies for the design of the presented configuration dealing with robust stability. The paper finishes drawing some conclusions and suggesting future research directions.

Throughout the paper, the set of proper and stable transfer functions is denoted with \mathcal{S} , which is a ring with the usual addition and multiplication operations. The elements in \mathcal{S} whose multiplicative inverses are also in the set are its units and $\mathcal{U}_{\mathcal{S}} \in \mathcal{S}$ denotes the set of all of them, i.e., the set of biproper, stable and minimum-phase transfer functions. For convenience, with the name of a polynomial we refer to both the polynomial itself and the vector of its roots. Arguments in transfer functions have been dropped.

II. THE O2C CONFIGURATION

The idea of changing the dynamics of a given plant is at the core of many control approaches like pole placement. Before presenting the O2C configuration, we first review how a pole placement can be achieved directly in the transfer function domain and then point out an interpretation in terms of FOSFC. This analysis follows [9] and serves as the basis for understanding the O2C configuration afterwards. Let

$$P_o = \frac{b}{a} \tag{1}$$

be the transfer function description of a given continuous SISO plant P_o , where it is assumed that a and b are coprime polynomials in the Laplace's variable satisfying $deg(a) = n \geq deg(b)$. We now consider a fictitious signal, ξ , such that

$$\xi = \frac{1}{a}u \quad , \quad y = b\xi \tag{2}$$

where u and y are the input and the output of P_o , respectively. Disregarding physical realizability, consider that we have direct access to ξ and that we feed it back through a $n - 1^{th}$ degree polynomial m as in Figure 1. The input to output relation is now given by

$$T_{yr} = \frac{b}{m+a} \tag{3}$$

It is evident from (3) that by choosing m conveniently we can fix the poles of the resulting input-output relation. It is a well-known fact that ξ and its $n - 1$ derivatives constitute the state of P_o described in controllable canonical form (CCF), so the feedback through m is nothing but a state feedback control law. Obviously, ξ , sometimes referred to as the partial state of the system, is not directly available and has to be estimated. This can be done very easily by finding a solution to the following diophantine equation

$$za + wb = 1 \tag{4}$$

in terms of the polynomials z and w . Once this has been done, the scheme depicted in Figure 1 can be modified as shown in Figure 2.

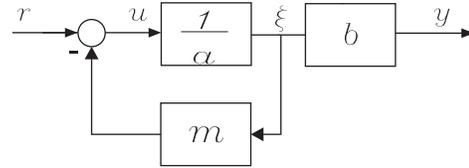


Fig. 1. Unfeasible CCF state feedback thought at the level of polynomials

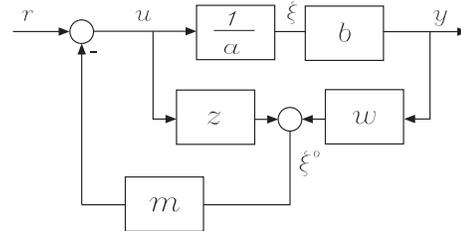


Fig. 2. Unfeasible CCF observed state feedback thought at the level of polynomials

The O2C configuration arises from following exactly the same steps but thinking at the level of transfer functions. The procedure now yields automatically physical realizability. Let

$$P_o = NrMr^{-1} \tag{5}$$

be a right coprime factorization of the plant in (1) over \mathcal{S} , that is $Nr, Mr \in \mathcal{S}$, and consider the fictitious signal ζ such that

$$\zeta = M_r^{-1}u \quad , \quad y = N_r\zeta \tag{6}$$

We can now solve the analogous to (4) Bezout equation

$$X_rM_r + Y_rN_r = 1 \tag{7}$$

and build the analogous to Figure 2 scheme shown in Figure 3

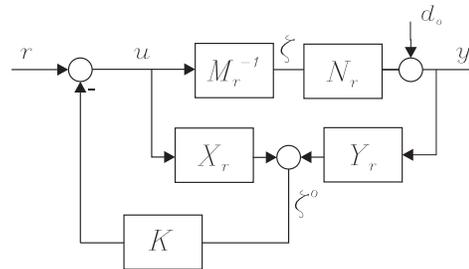


Fig. 3. O2C configuration

where $K \in \mathcal{S}$. In Figure 3 a disturbance in the output has also been drawn for convenient further reference. By defining

$$R \doteq (M_r + K)^{-1} \tag{8}$$

it can be shown (see [16]) that the scheme in Figure 3 is internally stable if and only if $R \in \mathcal{U}_{\mathcal{S}}$. From now on, a

coprime factorization of the plant in (1) of the form

$$M_r = \frac{p_K}{a} \quad N_r = \frac{b}{p_K} \quad (9)$$

where p_K is a Hurwitz polynomial of degree n , and the following structure for the block K

$$K = \frac{m}{p_K} \quad (10)$$

(where m is the same $n - 1^{th}$ degree polynomial defining the state feedback gain in Figures 1,2) are assumed when making reference to the O2C configuration in Figure 3. It is straightforward to see, consider that we have direct access to ζ in Figure 3 (i.e., the blocks X_r and Y_r provide perfect observation of ζ), that with the assignments in (9) and (10) we reach the same input-output relation as in the previous approach, indicated in (3). The link between the two approaches is immediate by observing that

$$\zeta = p_K \xi \quad (11)$$

In terms of the observed state feedback (OSF) interpretation we have that we first observe ζ , which is a known linear combination of the partial state ξ , and then feed it back through (10) in order to get the desired state feedback control law: $m\xi$. The O2C configuration in Figure 3 indeed assumes a non-minimal stabilizable realization of the plant of order $2n$: $P_o = N_r M_r^{-1} = \frac{b}{p_K} \frac{p_K}{a}$, which is full-state observable and has the uncontrollable stable modes given by p_K . As a matter of fact, the realization of the series connection of M_r^{-1} and N_r in (9) assuming that (A_1, B_1, C_1, D_1) and (A_2, B_2, C_2, D_2) are, respectively, their CCF realizations, is given by:

$$P_o = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) = \left(\begin{array}{cc|c} A_1 & 0 & B_1 \\ B_2 C_1 & A_2 & B_2 D_1 \\ \hline D_2 C_1 & C_2 & D_2 D_1 \end{array} \right) \quad (12)$$

In more detail, it can be seen that by applying the following similarity state transformation

$$T = \left(\begin{array}{cc} I_{n \times n} & 0_{n \times n} \\ -I_{n \times n} & I_{n \times n} \end{array} \right) \quad (13)$$

we get the equivalent realization of P_o (A', B', C', D'), where

$$A' = \left(\begin{array}{cc} A_1 & 0_{n \times n} \\ 0_{n \times n} & A_2 \end{array} \right) \quad B' = \left(\begin{array}{c} 0_{(n-1) \times 1} \\ 1 \\ 0_{n \times 1} \end{array} \right) \quad (14)$$

$$C' = (c_{1i})_{i=1..2n} \quad D' = D$$

with $c_{1i} \neq 0 \forall i = 1..2n$. From 14 it is evident that the new realization is build up of two decoupled subsystems, being the associated with the n first state variables observable and controllable and the one associated with the n remaining states observable but uncontrollable. The first subsystem states remain invariant under the similarity transformation (13) and are those of the CCF realization of $\frac{p_K}{a}$, which are obviously equal to those of $\frac{b}{a}$ realized in CCF as well.

Thus, the O2C configuration amounts to observe the $2n$ states of the presented non-minimal realization of P_o but use for state feedback just the first n of them corresponding to the controllable, possibly unstable, modes of the real system. Retaking the unfeasible scheme depicted in Figure (2) it is possible to make it physically realizable by observing that if (4) is satisfied then we have that

$$(zp_L m)a + (wp_L m)b = p_L m \quad (15)$$

where p_L is a Hurwitz polynomial of order n . If we now divide (15) by p_L we arrive at

$$\left(\frac{(zp_L m)^*}{p_L} \right) a + \left(\frac{(wp_L m)^*}{p_L} \right) b = m \quad (16)$$

where $(zp_L m)^*$ and $(wp_L m)^*$ satisfy the following relations

$$\begin{aligned} (wp_L m)^* &= a(wp_L m)^* + r \\ (zp_L m)^* &= (zp_L m)^* + lb \end{aligned} \quad (17)$$

for some r and l . The notation $(zp_L m)^*$ and $(wp_L m)^*$ is used to denote n^{th} order polynomials satisfying (16). This yields the feasible implementation shown in Figure 4, which depicts a conventional FOSFC implementation. The relation between the parameters involved in this conventional implementation and that of the O2C is illustrated in Figure 5.

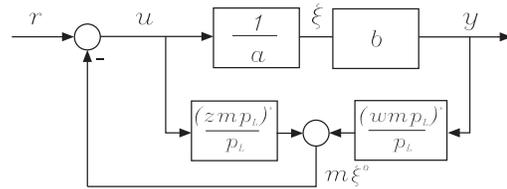


Fig. 4. Feasible implementation of the scheme in Figure 2

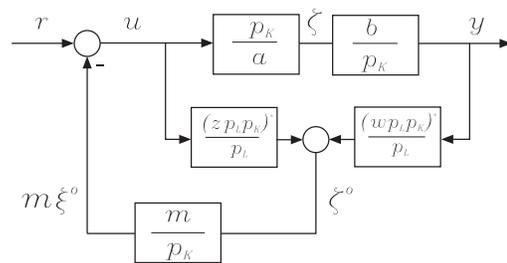


Fig. 5. O2C configuration in terms of the parameters of the scheme in Figure 2

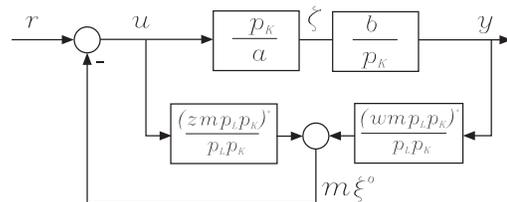


Fig. 6. O2C configuration in the form of a conventional FOSFC, see Figure 4

In figure 6 a rearrangement is performed in order to put the O2C in the form of a conventional FOSFC, see Figure 4. It is readily noticed that the O2C configuration is equivalent to use an observer polynomial of order $2n$ for the original n^{th} order plant, henceforth the name given to the presented configuration. As it is indicated in the next section, some benefits can be obtained from the implementation of a conventional FOSFC via the uncommon O2C configuration.

III. DESIGN METHODOLOGY

In this section we present two controller design methodologies associated with the O2C scheme. Both constitute two-step designs, the first one is conceived to improve the robustness margins of a FOSFC. The second one is more general and adds a prefilter to the basic O2C configuration. In this latter design, the O2C configuration is employed to robustify the closed-loop trying to get the best possible disturbance rejection. The tracking specifications are dealt with at a second stage of the design via the added prefilter, designed making use of the generalized control framework [20].

A. Design based on a standard FOSF controller

From Figure 4 the following loop transfer

$$L_{OC} = P_o \frac{(wmp_L)^*}{p_L} \left(1 + \frac{(zmp_L)^*}{p_L} \right)^{-1} \quad (18)$$

can be obtained. The loop transfer for the O2C configuration in Figure 3 is

$$L_{O2C} = P_o Y_r K (1 + K X_r)^{-1}; \quad (19)$$

We see from (18) and (19) that the loop gains are obviously different. However, when using the O2C configuration we are in fact using the same feedback control law ($u = r - m\xi$) with same asymptotic observer properties than with the conventional FOSFC. As was pointed out in section 2 all the design parameters of the conventional FOSFC implementation (p_L, m) can be taken for an O2C based implementation. Having done this, the p_K polynomial can be tuned for improving the robustness margins without altering the effective state feedback control law. This leads to a more robust implementation of a FOSFC. Thus, the following design steps are suggested:

- 1) Design via conventional techniques a FOSFC in CCF. That is, find convenient state feedback and observer gains satisfying the design requirements. From the observer and the state feedback gains we get values for m and p_L in Figure 4.
- 2) With m and p_L fixed in step 1), find a convenient p_K in order to improve the robustness margins.

B. A 2DOF O2C Design

Having seen that the O2C implementation can lead to more robust implementations of FOSFC we adopt it as the feedback part of a 2DOF control scheme depicted in Figure 7. The methodology to be presented is similar to that in [12], but in this previous work the feedback configuration was

different and the design less automatic, involving a tedious step that has been suppressed in this work. Given a nominal

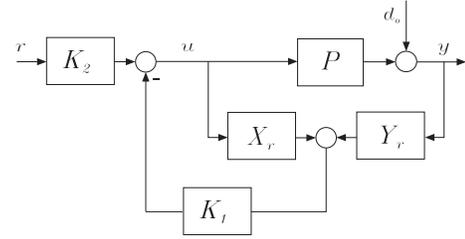


Fig. 7. The general 2DOF proposed scheme

plant model P_o we will consider that the real plant belongs to the set

$$\mathcal{P}_A = \{P : P = P_o + W_1 \Delta\} \quad (20)$$

where W_1 is a scalar frequency weight and $\|\Delta\|_\infty \leq 1$. The design procedure consists of applying the following two steps:

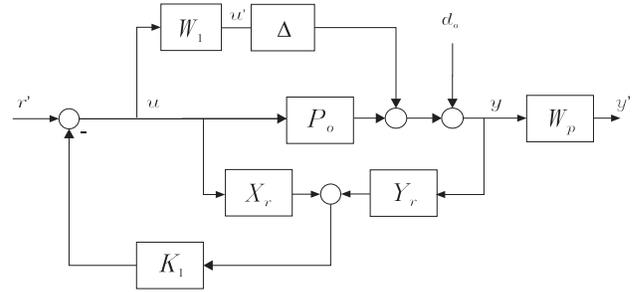


Fig. 8. Scheme for designing the O2C configuration in regulator mode

1) *Design of the O2C part in regulator mode:* In this proposed design the O2C configuration is employed to satisfy the regulator specifications, which amount to provide robust stability according to the uncertainty description in (20) and acceptable output disturbance rejection. The design is based upon the block diagram of Figure 8, where the assumed uncertainty description is displayed. Apart from this, the block W_p represents a frequency weight which is used to indicate in which frequencies the output disturbance suppression is more demanding. From inspection of Figure 8, the relations between the different inputs and outputs are given by:

$$\begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} M_r(1 - RM_r)Y_r & M_r R \\ W_p(1 - N_r(1 - RM_r)Y_r) & N_r R \end{pmatrix} \begin{pmatrix} d_o \\ r' \end{pmatrix} \quad (21)$$

It can be noted from (21) that the relations from r' to u and y are not affected by how we choose p_K . By direct application of the small-gain theorem [20] to Figure (8) the robust stability condition is

$$\|T_{u'd_o}\|_\infty = \|W_1 (M_r(1 - RM_r)Y_r)\|_\infty \leq 1 \quad (22)$$

According to (22), the following optimization problem is posed

$$\begin{aligned} \min_{p_K, p_L, m} \quad & \|W_p(1 - N_r(1 - RM_r)Y_r)\|_\infty \\ \text{subject to} \quad & \|W_1(M_r(1 - RM_r)Y_r)\|_\infty \leq 1 \end{aligned} \quad (23)$$

The optimization problem (23) performs a constrained by robust stability search in the p_K, p_L, m space for optimizing the output disturbance rejection. For solving such a problem, we suggest using direct search techniques. Basically they consist of a method for solving optimization problems that does not require any information about the gradient of the objective function. Unlike more traditional optimization methods that use information about the derivatives to search for an optimal point, a direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. At each step, the algorithm searches a set of points, called a mesh, around the current point. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If the pattern search algorithm finds a point in the mesh that improves the objective function at the current point, the new point becomes the current point at the next step of the algorithm. In [8] a recent application of direct search procedures for solving a specific control problem can be consulted. It is to be noted that (23) could be extended so as to also take into account the T_{ud_i} relation via optimizing a stacked sensitivity function, see [14].

2) *Design of the prefilter* : The tracking specifications are specified via a reference model (T_{ref}) capturing the desired input-output response. A model matching problem is posed as in Figure 9

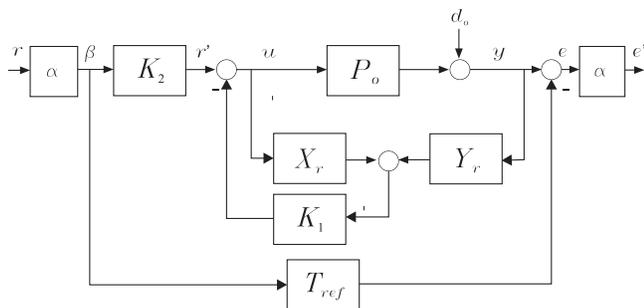


Fig. 9. Model matching problem arrangement for the design of K_2

The idea is to make use of the general control framework [20], see Figure 10, so as to minimize the relation from r to e in an \mathcal{H}_∞ sense. We also want to have certain control over the amount of effort demanded to the controller. Regarding this, the parameter α is introduced for providing a convenient trade-off between good tracking and demanded control action. This is particularly important in the case of the plant $P_o = N_r M_r^{-1}$ having stable lightly-damped zeros within the closed-loop passband since these zeros would be zeros of N_r and would force K_2 to cancel them by placing poles at exact locations to make $T_{yr} = N_r K_2$ close to the reference model T_{ref} , but this would be at the expense of

$T_{ur} = M_r K_2$ being undesirably large near the frequencies of those zeros. From Figures 8 and 9 we have

$$\begin{pmatrix} u' \\ e' \end{pmatrix} = \begin{pmatrix} T_{u'd_o} & T_{u'r} \\ T_{e'd_o} & T_{e'r} \end{pmatrix} \begin{pmatrix} d_o \\ r \end{pmatrix} \quad (24)$$

with

$$T_{u'd_o} = -W_1(M_r(1 - RM_r)Y_r) \quad (25)$$

$$T_{u'r} = \alpha W_1 M_r K_2 \quad (26)$$

$$T_{e'd_o} = \alpha(1 - N_r(1 - RM_r)Y_r) \quad (27)$$

$$T_{e'r} = \alpha^2(N_r R K_2 - T_{ref}) \quad (28)$$

The 2DOF design problem can be easily cast into the general control configuration seen in Figure 10 by making the following pairings: $w_1 = d_o$, $w_2 = r$, $z_1 = u'$, $z_2 = e'$, $v = \beta$, $u = r'$ and $K = K_2$. The augmented plant G and the controller K_2 are related by the following lower linear fractional transformation (LFT):

$$\mathcal{F}_l(G, K_2) \doteq G_{11} + G_{12}K_2(1 - G_{22}K_2)^{-1}G_{21} \quad (29)$$

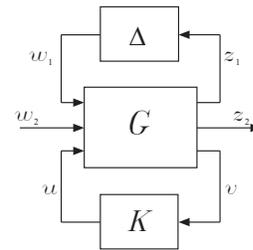


Fig. 10. Generalized control framework

The corresponding partitioned generalized plant G is:

$$\begin{pmatrix} u' \\ e' \\ \beta \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} d_o \\ r \\ r' \end{pmatrix} \quad (30)$$

where

$$G_{11} = \begin{pmatrix} -W_1(1 - M_r(X_r + RN_r Y_r)) & 0 \\ \alpha(1 - N_r(1 - RM_r)Y_r) & -\alpha^2 T_{ref} \end{pmatrix} \quad (31)$$

$$G_{12} = \begin{pmatrix} W_1 M_r R \\ \alpha N_r R \end{pmatrix} \quad (32)$$

$$G_{21} = \begin{pmatrix} 0 & \alpha \end{pmatrix} \quad (33)$$

$$G_{22} = 0 \quad (34)$$

Once the problem has been put in the form of the generalized control configuration shown in Figure 10 K_2 can be readily obtained by using standard software packages.

Remark. The reference signal r must be scaled by a constant W_r to make the closed-loop transfer function from r to y match the desired reference model T_{ref} exactly at steady state. This is not guaranteed by the optimization which is aimed at minimizing the \mathcal{H}_∞ -norm of the error. The required scaling is given by

$$W_r \doteq (K_2(0)N_r(0)R(0))^{-1}T_{ref}(0) \quad (35)$$

Therefore, the resulting reference controller is $K_2 W_r$.

IV. CONCLUSIONS AND FUTURE WORK

A new 2DOF control methodology based on the here called O2C configuration has been presented. The approach applies to general framework of LTI systems and has been introduced as an alternative to the commonly encountered strategy of setting the two controllers arbitrarily, with internal stability being the unique restriction, and parameterizing the controller in terms of the two Youla parameters. First, the O2C configuration has been presented together with its interpretation as a FOSFC. Within the 2DOF design the O2C configuration has been used first to guarantee robust stability and some level of disturbance rejection by solving via direct search techniques a constrained \mathcal{H}_∞ optimization problem for the poles of X_r, Y_r, N_r, M_r and the polynomial m . After that, a prefilter controller to adapt the reference command has been designed using the generalized control framework. Future research is focused on the application of the method presented here to several study cases and on the use of the O2C configuration towards the obtention of two-step design methodologies for a high performance robust tracking control configuration arising from parameterizing the X_r, Y_r blocks which provide a solution to (7).

V. ACKNOWLEDGMENTS

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