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Reset-free iterative identification based on the finite-dimensional signal subspace

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***Abstract*— This paper proposes a new approach for the iterative identification of continuous-time systems, which is based on the projection type ILC (iterative learning control) concepts. Unlike any other ILC methods, this paper gives a framework to perform ILC without resetting the initial condition at each iteration, which can be achieved by introducing the dynamics into the system representation in the finite-dimensional signal subspace. Therefore, it is not necessary to wait for the equilibrium state patiently or reset the system forcibly. Furthermore, a class of gain decreasing filters are introduced, which plays a crucial role in effective parameter convergence in the presence of heavy noise. Combination of these results gives us the estimates which converge to the true system parameters against measurement noise. A numerical example is given to demonstrate its effectiveness.**

I. INTRODUCTION

This paper is concerned with iterative learning control (ILC) and identification in continuous-time systems.

As for identification, though most of the existing methods are described in discrete-time, it would often be convenient to have continuous-time models directly from the sampled I/O data. In fact, it is often much easier for us to capture the plant dynamics intuitively in continuous-time rather than in discrete-time. A basic difficulty of continuous-time identification is, however, that standard approaches (so called *direct* methods) require to compute the time-derivatives of I/O data. A comprehensive survey about direct methods has been given by [1] and [2]. Furthermore, the Continuous-Time System Identification (CONTSID) tool-box has been developed on the basis of these direct methods [3], [4].

While, iterative learning control (ILC) has attracted much attention over the last two decades as a powerful model-free control methodology, [5], [6], [7], [8]. ILC returns the input which achieves output tracking by iteration of trials for uncertain systems. One of the major drawbacks of ILC in continuous-time is that most approaches need time-derivatives of I/O data [9], therefore it is quite sensitive to measurement noise. In 2001, Hamamoto and Sugie [10], proposed an ILC where the learning law works in a certain finite-dimensional subspace and showed that time-derivatives of the tracking error is not required to achieve perfect tracking in the proposed scheme. Based on this work, Sugie and Sakai [11], [12] proposed an ILC which works in the presence of heavy measurement noise and, moreover, the method was shown to be applicable to identification of continuous-time systems as well. Furthermore, Campi et al., [13] extends the identification method to the general case where the plant has zeros as well as poles. This identification method proved several advantages such as: (i) no time-derivatives of I/O data are required and (ii) it guarantees zero convergence of the parameter estimation error.

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Because of the above merits and its strong tolerance against measurement noise, this paper focuses on the ILC based identification. The method, however, requires us to reset the initial condition at each iteration. Though this is a quite common feature in ILC, it is sometimes very time consuming because we have to wait the equilibrium state patiently or is difficult to reset the plant forcibly in some cases including many process control systems. Furthermore, it is not easy to analyse robust stability quantitatively in [13] because they employ vanishing gain in the *closed* loop. In addition, the method is not suitable for tracking purpose because the reference signal should change at every iteration. The purpose of this paper is to propose a new framework for ILC based identification which does not require the reset of initial state at each iteration and overcomes the above mentioned shortcomings in [13], while enjoying all the merits described above. More concretely, this paper shows a way to perform ILC without resetting the initial condition, which can be achieved by introducing the dynamics into the system representation in the finite-dimensional signal subspace. Then, a class of gain decreasing filters are introduced outside the closed loop, which plays a crucial role in effective parameter convergence in the presence of noise. Combination of these results gives us the estimates which converge to the true system parameters against measurement noise. A numerical example is given to demonstrate its effectiveness.

II. PROBLEM SETTING

Consider the continuous-time SISO plant described by

$$y(t) = \frac{N_p(p)}{D_p(p)}u(t) = \frac{1+\beta_1 p+\cdots+\beta_m p^m}{\alpha_0+\alpha_1 p+\cdots+\alpha_n p^n}u(t), \quad (1)$$

where $u(t)$ and $y(t)$ are the input and the output, respectively, $\alpha_i \in \mathbb{R}(i=0,1,\ldots,n)$ and $\beta_i \in \mathbb{R}(i=1,\ldots,m)$ are coefficient parameters, while p is the differential operator, *i.e.*, $pu(t) = \mathrm{d}u(t)/\mathrm{d}t$. We assume the following:

- We can measure $y_{\mathrm{ob}}(t)$, the output contaminated with noise,

$$y_{\mathrm{ob}}(t) = y(t) + \eta(t) \quad (2)$$

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where $\eta(t)$ is zero-mean measurement noise.

- Though the true parameters α_i and β_i are unknown, the system is stable, $N_p(p)$ and $D_p(p)$ are co-prime and their order n and m are known.

Let for brevity

$$\gamma^* = [\alpha_0, \ldots, \alpha_n, \beta_1, \ldots, \beta_m]^T . \quad (3)$$

Then, the goal is to find an identification algorithm which produces a series of estimates $\{\hat{\gamma}^k\}$ satisfying

$$\hat{\gamma}^k \to \gamma^* \quad \text{as} \quad k \to \infty \quad (4)$$

III. Reset-free Iterative Learning Control

In this section, we propose a new iterative learning control method which does not require to reset the initial condition.

A. Problem

In this study, it is assumed that the reference signal $r(t)$ is differentiable many times and T-periodic. (i.e. $r(T+t) = r(t), {}^\forall t \geq 0$) Also, $\{r, \dot{r}, \ldots, r^{(n+m)}\}$ is assumed to be linearly independent.

Here, we define

$$u^k(t) = u((k-1)T + t) \quad t \in [0, T] \quad (5)$$
$$y^k(t) = y((k-1)T + t) \quad t \in [0, T] \quad (6)$$

for $k = 1, 2, \cdots$. Then, the objective here is to find an input sequence $\{u^k(t)\}$ such that

$$E\left[r(t) - y^k(t)\right] \to 0 \quad (\text{ as } k \to \infty\text{ }). \quad (7)$$

Also, we let $u^\infty(t)$ be its limit.

B. ILC method without resetting

1) Projection on $\mathcal{F}$: A finite-dimensional subspace $\mathcal{F}$ is set to include $r(t)t \in [0, T]$ and $u^\infty(t)$. Also, n_f is defined as the dimension of $\mathcal{F}$ and $\mathcal{F}$ is set so as to $n_f \geq n+m+1$.

If the plant has no zeros,

$$\text{span}\left\{r(t), \dot{r}(t), \cdots, r^{(n)}(t)\right\} \quad (8)$$

is a candidate of $\mathcal{F}$. If the plant has zeros, $r(t)$ should consist of n_r sinusoidal functions, and

$$\text{span}\left\{r(t), \dot{r}(t), \cdots, r^{(2n_r-1)}\right\} \quad (9)$$

can be a candidate for $\mathcal{F}$.

In this study, we focus on the projection of signals onto $\mathcal{F}$. Now, we introduce $f_1(t), f_2(t), \cdots, f_{n_f}(t) \in L_2[0, T]$ which are the basis of $\mathcal{F}$, and the projection of $u^k(t)$ onto $\mathcal{F}$ is written as

$$u^k(t)\big|_{\mathcal{F}} = \bar{u}_1^k f_1(t) + \cdots + \bar{u}_{n_f}^k f_{n_f}(t). \quad (10)$$

Moreover, $\bar{\boldsymbol{u}}^k \triangleq \left[\bar{u}_1^k, \cdots, \bar{u}_{n_f}^k\right]^T$ is defined as its vector representation. Similarly, $\bar{\boldsymbol{u}}^\infty$, $\bar{\boldsymbol{y}}^k$, $\bar{\boldsymbol{r}}$ are defined as vector representation of the projection of $u^\infty(t)$, $y^k(t)$ and $r(t)t \in [0, T]$, respectively.

2) State space representation in $\mathcal{F}$: We next mention about the system representation which focuses on I/O signals projected onto the finite-dimensional subspace.

Let

$$P: \begin{cases} \dot{\boldsymbol{x}}(t) & = A\boldsymbol{x}(t) + Bu(t) \\ y(t) & = C\boldsymbol{x}(t) \end{cases} \quad (11)$$

be a state space representation of the plant model, where $\boldsymbol{x} \in \mathbb{R}^n$ is the state of the system. And, define

$$\boldsymbol{x}^k \triangleq \boldsymbol{x}((k-1)T), \quad (12)$$

which is the initial state of each time span $[(k-1)T, kT]$. Then, we have

$$\bar{P}: \begin{cases} \boldsymbol{x}^{k+1} & = \bar{A}\boldsymbol{x}^k + \bar{B}\bar{\boldsymbol{u}}^k \\ \bar{\boldsymbol{y}}^k & = \bar{C}\boldsymbol{x}^k + \bar{D}\bar{\boldsymbol{u}}^k \end{cases} \quad (13)$$

where $\bar{A} \in \mathbb{R}^{n \times n}, \bar{B} \in \mathbb{R}^{n \times n_f}, \bar{C} \in \mathbb{R}^{n_f \times n}, \bar{D} \in \mathbb{R}^{n_f \times n_f}$ are constant matrices and can be obtained numerically. In case $\{f_1(t), f_2(t), \ldots, f_{n_f}(t)\}$ are an orthonormal basis, these matrices can be written as

$$\bar{A} = e^{AT} \quad (14)$$
$$\bar{B} = \int_0^T e^{A(T-\tau)} B \left[f_1(\tau), \cdots, f_{n_f}(\tau)\right] \mathrm{d}\tau \quad (15)$$
$$\bar{C} = \int_0^T \left[f_1(\tau), \cdots, f_{n_f}(\tau)\right]^T C e^{A\tau} \mathrm{d}\tau \quad (16)$$
$$\bar{D} = \int_0^T \left[f_1(t), \cdots, f_{n_f}(t)\right]^T \times \times \left\{\int_0^t C e^{A(t-\tau)} B \left[f_1(\tau), \cdots, f_{n_f}(\tau)\right] \mathrm{d}\tau\right\} \mathrm{d}t \quad (17)$$

In contrast to the existing iterative learning control method [12] , in which the plant is handled as a static system, the plant can be handled as a discrete time dynamical system. This introduction of dynamical system is crucial to deal with the effect of eliminating reset operation.

3) Controller design in $\mathcal{F}$: In order to achieve (7), it is enough for us to design a robust servo controller $\bar{K}$ as a system represented in $\mathcal{F}$ (see Fig.1) . All of such controllers are characterized in [14]. Note that robust stability is required for uncertainty in the initial plant P_M, and it is shown in [13] how to obtain an initial plant model, and its effectiveness is demonstrated in [15].

Also note that controllers should be strictly proper as the system of projected signal for realizability.

Remark 1: Here, a simple way to design such robust servo controller is shown. Now, we define the transfer function matrix form of the plant in $\mathcal{F}$

$$\bar{P}(z) \triangleq \bar{C}(zI - \bar{A})^{-1}\bar{B} + \bar{D}. \quad (18)$$

Because $\bar{P}(z)$ is stable, if we choose stable and strictly proper system $\bar{Q}(z)$, then

$$\bar{K}: \bar{\boldsymbol{u}}^k = \bar{K}(z)\left(\bar{\boldsymbol{r}} - \bar{\boldsymbol{y}}^k\right) \quad (19)$$

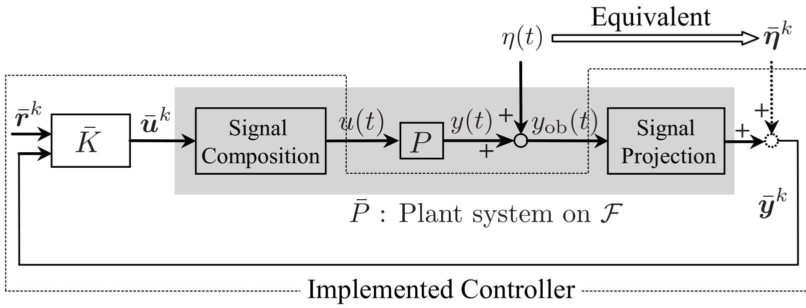


Fig. 1. Block diagram of the control system based on the finite-dimensional signal subspace

where

$$\begin{aligned}\bar{K}(z) =& \bar{P}(1)^{-1}\bar{Q}(1)^{-1}\bar{Q}(z) \\ & \times \left[I - \bar{P}(z)\bar{P}(1)^{-1}\bar{Q}(1)^{-1}\bar{Q}(z)\right]^{-1}\end{aligned} \tag{20}$$

is a stabilizing and strictly proper controller which achieves robust tracking.

C. Digital implementation

In this section, we discuss how to approximately implement the iterative learning controller when the I/O are measured at sampling time only.

The structure of controller is shown in Fig. 1. Since $\bar{K}$ can be implemented as ordinal discrete time system, it is enough to describe how to implement the signal composition block and the signal projection block.

1) Preparation: We suppose that the I/O data are $\{u(iT_s), y_{\rm ob}(iT_s)\}\,(i = 0, 1, \dots)$ where T_s is sampling time satisfying $(q+1)T_s = T (q \in \mathbb{N})$.

Given functions $f_1(t), f_2(t), \dots, f_{n_f}(t)$, define $V_{df} \in \mathbb{R}^{(q+1)\times n_f}$ by

$$V_{df} \triangleq \begin{bmatrix} f_1(0) & f_2(0) & \dots & f_{n_f}(0) \\ f_1(T_s) & f_2(T_s) & \dots & f_{n_f}(T_s) \\ \vdots & \vdots & \dots & \vdots \\ f_1(qT_s) & f_2(qT_s) & \dots & f_{n_f}(qT_s) \end{bmatrix}. \tag{21}$$

Let the QR decomposition of V_{df} be

$$V_{df} = QR, \quad Q^T Q = I_{n_f} \tag{22}$$

where $Q \triangleq \left[\boldsymbol{f}_1, \boldsymbol{f}_2, \dots, \boldsymbol{f}_{n_f}\right] \in \mathbb{R}^{(q+1)\times n_f}$ and $R \in \mathbb{R}^{n_f \times n_f}$ is a nonsingular upper triangular matrix. These $\boldsymbol{f}_i$'s constitute an orthogonal basis for projection in the digital implementation.

2) Signal composition block: Signal composition block generates $u((k-1)T + t)\,(t \in [0, T])$ for given $\bar{\boldsymbol{u}}^k$. $u(t)$ is determined at sampling time point $t = (k-1)T, (k-1)T + T_s, \dots, kT - T_s$ by

$$\begin{bmatrix} u\left((k-1)T\right) \\ u\left((k-1)T + T_s\right) \\ u\left((k-1)T + 2T_s\right) \\ \vdots \\ u\left(kT - T_s\right) \end{bmatrix} = V_{df}\bar{\boldsymbol{u}}^k. \tag{23}$$

Between the sampling time points, $u(t)$ can be generated approximately by linear interpolation.

3) Signal projection block: Signal projection block generates $\bar{\boldsymbol{y}}^k$ for measured

$$\boldsymbol{y_d}^k \triangleq \left[y_{\rm ob}\left((k-1)T\right), \dots, y_{\rm ob}\left(kT - T_s\right)\right]^T \in \mathbb{R}^{q+1}. \tag{24}$$

Since $\boldsymbol{f}_i (i = 1, 2, \dots, n_f)$ is an orthogonal set of vectors with unitary norm, the projection of $\boldsymbol{y_d}^k$ onto span$\left\{\boldsymbol{f}_1, \boldsymbol{f}_2, \dots, \boldsymbol{f}_{n_f}\right\}$ is $\left[\boldsymbol{f}_1, \boldsymbol{f}_2, \dots, \boldsymbol{f}_{n_f}\right] Q^T \boldsymbol{y_d}^k = V_{df} R^{-1} Q^T \boldsymbol{y_d}^k$, where $R^{-1} Q^T \boldsymbol{y_d}^k$ is an approximate expression for $\bar{\boldsymbol{y}}^k$. This suggests using

$$R^{-1} Q^T \boldsymbol{y_d}^k \tag{25}$$

as the output $\bar{\boldsymbol{y}}^k$ of digitally implemented signal projection block.

IV. Calculation of parameter estimates

In this section, an identification method based on the obtained iterative learning controller is described.

A. Conversion from $\bar{\boldsymbol{u}}^\infty$ to γ^*

First, we clarify the relation between $\bar{\boldsymbol{u}}^\infty$ and γ^*. Here,

$$\left\{r(t), \dot{r}(t), \cdots, r^{(n_f - 1)}(t)\right\} \tag{26}$$

is chosen as the basis to represent projected signals in $\mathcal{F}$ for simplicity.

Case 1: No plant zeros $N_p(p) = 1$: After $y(t)$ converged to periodic reference signal $r(t)$, $u(t) \in \mathcal{F}$ can be written in following linear combination

$$u(t) = \bar{u}_1^\infty r(t) + \bar{u}_2^\infty p r(t) + \dots + \bar{u}_{n+1}^\infty p^n r(t), \tag{27}$$

If the plant has no zeros, $N_p(p) = 1$ and the linear independency of $\left\{r(t), pr(t), \cdots, p^{n_f - 1} r(t)\right\}$ indicates

$$[\alpha_0, \alpha_1, \dots, \alpha_n] = \left[\bar{u}_1^\infty, \bar{u}_2^\infty, \dots, \bar{u}_{n+1}^\infty\right]. \tag{28}$$

Case 2: General case $N_p(p) \neq 1$: In case the plant has zeros and $r(t)$ is composed by n_r sinusoids, system parameters satisfy following relationship.

$$D_p(p) r(t) = N_p(p) u(t) \tag{29}$$

$$\begin{bmatrix} r(t) \\ pr(t) \\ \vdots \\ p^n r(t) \end{bmatrix}^T \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} r(t) \\ pr(t) \\ \vdots \\ p^{2n_r+m-1} r(t) \end{bmatrix}^T U \begin{bmatrix} 1 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}, \quad (30)$$

where

$$U \triangleq \left[U_1 \mid U_2 \right] \triangleq \left[\begin{array}{c|cccc} \bar{u}_1^\infty & 0 & \cdots & 0 \\ \bar{u}_2^\infty & \bar{u}_1^\infty & \ddots & \vdots \\ \bar{u}_3^\infty & \bar{u}_2^\infty & \ddots & 0 \\ \vdots & \vdots & & \bar{u}_1^\infty \\ \bar{u}_{2n_r}^\infty & \bar{u}_{2n_r-1}^\infty & \cdots & \vdots \\ 0 & \bar{u}_{2n_r}^\infty & \cdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \bar{u}_{2n_r}^\infty \end{array}\right]. \quad (31)$$

Since $r(t)$ consists of n_r sinusoidal functions, $p^{2n_r} r(t), \ldots, p^{2n_r+m-1} r(t)$ belong to $\mathcal{F}$ and there exists $P_r \in \mathrm{R}^{2n_r \times m}$ which satisfies

$$\begin{bmatrix} r(t) \\ pr(t) \\ \vdots \\ p^{2n_r+m-1} r(t) \end{bmatrix}^T = \begin{bmatrix} r(t) \\ pr(t) \\ \vdots \\ p^{2n_r-1} r(t) \end{bmatrix}^T \begin{bmatrix} I_{2n_r} & P_r \end{bmatrix}. \quad (32)$$

From above facts and the linear independency of $\left\{ r(t), pr(t), \cdots, p^{2n_r-1} r(t) \right\}$, relationship (30) is equivalent to the following equations

$$\begin{bmatrix} \begin{bmatrix} I_{n+1} \\ \mathbf{0}_{(2n_r-n-1)\times(n+1)} \end{bmatrix}^T \\ -\left([I_{2n_r}, P_r] \cdot U_2 \right)^T \end{bmatrix}^T \cdot \begin{bmatrix} \alpha_0 \\ \vdots \\ \alpha_n \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix} = \begin{bmatrix} I_{2n_r} \\ (P_r)^T \end{bmatrix}^T \cdot U_1, \quad (33)$$

Here, the plant parameters γ^* can be obtained from $\bar{\boldsymbol{u}}^\infty$ by solving these linear equations.

B. Convergence via decreasing gain filters

By choosing an appropriate basis (26) and using a robust servo controller, $\bar{\boldsymbol{u}}^k$ converges to $\bar{\boldsymbol{u}}^\infty$ even under the existence of model uncertainty. In the presence of measurement noise, $\bar{\boldsymbol{u}}^k$ does not completely converge to $\bar{\boldsymbol{u}}^\infty$ by the controller, but its expected value $E\left[\bar{\boldsymbol{u}}^k\right]$ converges to $\bar{\boldsymbol{u}}^\infty$ by controlling the plant with the robust servo controller.

One possible option is to use arithmetic average of obtained $\left\{\bar{\boldsymbol{u}}^k\right\}$. However, generally speaking, arithmetic average is sensitive to outliers, and early output of iterative learning controller contains much error due to transient dynamics. So, adoption of simple arithmetic average often results in extremely slow convergence.

Here, filter with decreasing gain $\{g_k\}$ is introduced to overcome this problem.

Lemma 1 (decreasing gain filter): Consider the filter with decreasing gain sequence $\{g_k\}$

$$\hat{\boldsymbol{u}}^{k+1} = (1 - g_k)\hat{\boldsymbol{u}}^k + g_k \bar{\boldsymbol{u}}^k \quad (34)$$

Then, its output $\left\{\hat{\boldsymbol{u}}^k\right\}$ converges to $\lim_{k\to\infty} E\left[\bar{\boldsymbol{u}}^k\right]$ if $\{g_k\}$ satisfies,

$$\begin{aligned} & 0 \le g_k < 1 \ (k = 1, 2, \cdots) \\ & \lim_{k\to\infty} g_k = 0, \quad \lim_{n\to\infty} \sum_{k=1}^{n} g_k = \infty. \end{aligned} \quad (35)$$

Here, it is assumed that $\left\{\bar{\boldsymbol{u}}^k\right\}$ satisfies

$$\lim_{k\to\infty} E\left[\bar{\boldsymbol{u}}^k\right] = \bar{\boldsymbol{u}}^\infty \quad (36)$$

$$\left\| E\left[\left(\bar{\boldsymbol{u}}^m - \bar{\boldsymbol{u}}^\infty\right) \left(\bar{\boldsymbol{u}}^n - \bar{\boldsymbol{u}}^\infty\right)^T \right] \right\| < (\text{const}) < +\infty \quad (37)$$

$$\lim_{n\to\infty} \sup_k \left\| E\left[\left(\bar{\boldsymbol{u}}^k - \bar{\boldsymbol{u}}^\infty\right) \left(\bar{\boldsymbol{u}}^{k+n} - \bar{\boldsymbol{u}}^\infty\right)^T \right] \right\| = 0 \quad (38)$$

Proof: For brevity's sake, the detail of the proof is omitted.

Here, we define

$$\hat{P}_k \triangleq \left\| E\left[\left(\hat{\boldsymbol{u}}^k - \bar{\boldsymbol{u}}^\infty\right) \left(\hat{\boldsymbol{u}}^k - \bar{\boldsymbol{u}}^\infty\right)^T \right] \right\| \quad (39)$$

$$R_k \triangleq \left\| E\left[\left(\hat{\boldsymbol{u}}^k - \bar{\boldsymbol{u}}^\infty\right) \left(\bar{\boldsymbol{u}}^k - \bar{\boldsymbol{u}}^\infty\right)^T \right] \right\|. \quad (40)$$

At first, $\lim_{k\to\infty} R_k = 0$ is shown from the definition of the filter (34) and assumptions on its gain sequence $\{g_k\}$ and $\left\{\bar{\boldsymbol{u}}^k\right\}$. Then, we can show $\hat{P}_k \to 0$ using this result . ■

Therefore k-th estimation of the parameter $\hat{\gamma}^k$, which is obtained by replacing $\bar{\boldsymbol{u}}^\infty$ with $\hat{\boldsymbol{u}}^k$, converges to true value

$$\hat{\gamma}^k \to \gamma^* \quad \text{as } k \to \infty \quad (41)$$

in the presence of noise.

Remark 2: For any $\alpha > 0$, decreasing gain sequence

$$g_k = \frac{1}{1 + \alpha k} \quad (42)$$

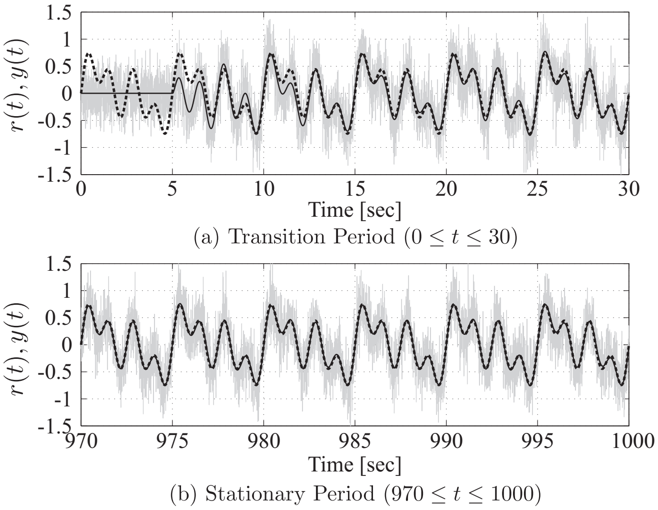
satisfies the condition (35). Especially, $\alpha = 1$ results in arithmetic average

$$\hat{\boldsymbol{u}}^k = \frac{\hat{\boldsymbol{u}}^1 + \bar{\boldsymbol{u}}^1 + \bar{\boldsymbol{u}}^2 + \cdots + \bar{\boldsymbol{u}}^{k-1}}{k}. \quad (43)$$

Moreover, the gain sequence with $0 < \alpha < 1$ tends to weight more recent results ,while the sequence with $\alpha > 1$ weights more early results.

This fact is useful for designing the decreasing gain filter for problems which have some tendency in measurement accuracy.

Remark 3: Decreasing gain sequence is also useful as learning gain of some iterative learning algorithm. For example, Campi et al. [16] proved the learning gain which has decreasing factor $\frac{1}{k+1}$ is suitable, and more general decreasing gain sequence can be proven to be suitable by applying the proof of Lemma 1



(a) Transition Period ($0 \le t \le 30$)

(b) Stationary Period ($970 \le t \le 1000$)

Fig. 2. Output $y(t)$ with measurement noise in tracking control

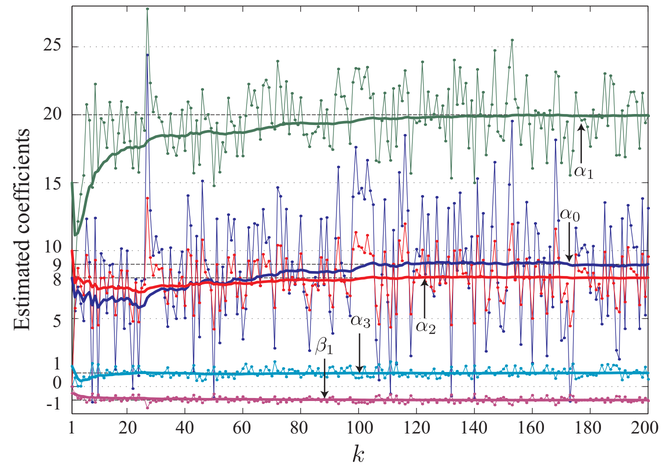


Fig. 3. Identified coefficients in each cycle

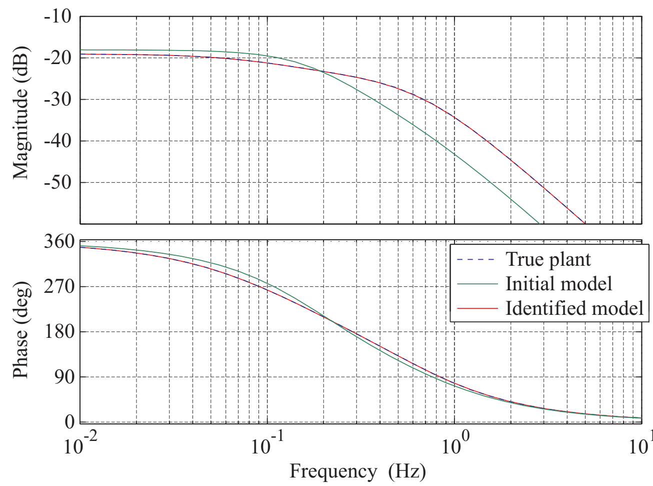


Fig. 4. Bode plots of the true plant, initial model and identified model

V. Numerical example

In this section, a numerical example is shown to confirm the proposed method. The target plant in this example is described as

$$P(s) = \frac{1-s}{9+20s+8s^2+s^3}. \tag{44}$$

But now, we assume the case in which designer only have an inaccurate model of the plant

$$P_M(s) = \frac{1-0.5s}{8+15s+10s^2+1.5s^3}, \tag{45}$$

and obtain an accurate model of the plant by the proposed method.

At first, cycle of the reference signal is set as $T = 5$[sec] and the periodic reference signal $r(t)$ is set as

$$r(t) = \frac{1}{3}\sin\left(\frac{2\pi}{T}t\right) + \frac{1}{3}\sin\left(\frac{2\pi}{T}2t\right) + \frac{1}{3}\sin\left(\frac{2\pi}{T}4t\right), \tag{46}$$

so $n_r = n = 3$, $m = 1$. For the convenience in identification, the basis is chosen as

$$\{f_1(t), f_2(t), \cdots, f_6(t)\} = \left\{r(t), \dot{r}(t), \cdots, r^{(5)}\right\}. \tag{47}$$

Secondly, a robust servo controller is designed according to Remark 1 with the parameter $\bar{Q}(z) = \frac{1}{1-2z} \cdot I_{2n_r}$. This controller can stabilize the plant $P(s)$, in spite it is designed with inaccurate model $P_M(s)$. In this example, sampling time T_s is set to 5×10^{-3}[sec] and $y(t)$ is measured with the white noise with zero mean and standard deviation 0.3.

Fig. 2 shows the response $y(t)$ with the designed controller. In the figure, black solid line shows the response of $y(t)$ without the measurement noise, thin gray line shows $y(t)$ with the measurement noise, and dashed line shows the periodic reference signal $r(t)$.

Finally, we introduce a gain decreasing filter to estimate accurate plant model. Here, decreasing gain

$$g_k = \frac{1}{0.8k+1} \tag{48}$$

is adopted to suppress the error during transition period (See Remark 2). And, the initial estimation $\hat{\boldsymbol{u}}^1$ is obtained from initial plat model $P_M(s)$.

Fig. 3 shows the history of estimated plant parameter. In the figure, thin line with dots shows the estimation which directly obtained from $\bar{\boldsymbol{u}}^k$ and thick line shows the estimation from output of the decreasing gain filter $\hat{\boldsymbol{u}}^k$. After 200 cycles, obtained estimation of the plant is

$$\frac{1-0.997s}{9.003+19.98s+8.02s^2+0.997s^3}. \tag{49}$$

and comparison with the true plant in frequency domain is shown in Fig. 4.

These results show that the proposed method can achieve accurate identification even under heavy measurement noise.

VI. Conclusions

In this study, a novel iterative identification method is presented. The proposed method is based on the novel system representation which focuses on I/O signals projected onto the finite-dimensional subspace, and achieves reset-free iterative identification. This reset-free iterative learning control scheme should be distinguished from repetitive control by the fact that it does not perform any feedback action during each iteration. In the proposed method, a class of gain decreasing filters are introduced outside the closed loop. The filter plays a crucial role in effective parameter convergence in this method, and has high flexibility to deal with various estimation problems with some tendency in measurement accuracy. Since the proposed method inherits advantages of the projection type ILC method, it requires no time derivatives of measured data and no data pre-processing, and has high tolerance to measurement noise. A numerical example was shown to confirm these property of the proposed method.

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