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Power Factor Compensation of a Controlled Rectifier with Non–Sinusoidal Generator Voltage using Passive Components

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Abstract— **In a recent paper we introduced a framework for analysis and design of (possibly nonlinear) power factor (PF) compensators for electrical systems operating in non–sinusoidal (but periodic) regimes with nonlinear loads. In particular, under the standard assumption that the generator is a voltage source with no impedance, we characterized all nonlinear loads whose PF is improved with a given nonlinear compensator. In this brief note we use this framework to study the problem of passive PF compensation of a classical half–bridge controlled rectifier (terminated by a resistor) with non–sinusoidal generator voltage. We give necessary and sufficient conditions for improvement of PF with linear inductors or capacitors, which determine ranges for the compensator parameters that depend on the rectifier fire angle and the amplitude of the voltage source. Given the "phase advance" operation of the rectifier it is expected that capacitive compensation improves PF, it is however less obvious that this can also be achieved (under some suitable conditions) with inductors.**

I. Introduction

The optimization of the energy transfer from an AC generator to a load is a classical problem in electrical engineering whose interest has been recently revitalized due to the widespread use of nonlinear loads and switching devices that add distortion to the signals. In a typical scenario it is assumed that the source consists of a voltage generator feeding a load and the problem is to design a compensator, to be placed between the source and the load, to maximize the power transmission efficiency. If the load is scalar linear time–invariant (LTI) and the generator is ideal—that is, with negligible impedance and fixed sinusoidal voltage—it is well known that the optimal compensator minimizes the phase shift between the sources voltage and current waveforms—increasing the so–called sources power factor (PF) [1]. The task of designing compensators that aim at improving PF for nonlinear time–varying loads operating in non–sinusoidal regimes is far from clear.

PF compensators can be implemented using passive elements [2], e.g., capacitor and inductor banks, or active filters [3], [4], [5], [6]. In the latter case, it is assumed that the active filter is a controlated current source, then a desired waveform for the current is defined and compared with the actual signal to generate an error that a compensator (usually an LTI one) tries to drive to zero. The design of passive compensators is typically done via, either a parametric minimization of a figure of merit established through the Fourier expansion of the voltage and current waveforms, or using a "nonlinear non–sinusoidal version" of the well–known concept of reactive power of LTI circuits in sinusoidal regime, e.g., Budeanu's reactive power [7]. Even though the deficiency of Budeanu's definition has been widely documented in the circuits literature, see for example, [11], [12], because of its extreme simplicity it still enjoys a widespread popularity.

In [8] we presented a new framework for analysis and design of (possibly nonlinear) PF compensators for electrical systems operating in non–sinusoidal (but periodic) regimes with nonlinear time–varying loads. To develop the framework we use the standard assumption that the impedance of the voltage generator is negligible and assume the system operates in a steady–state periodic regime. Under these assumptions, the main result of [8] is the characterization of all nonlinear loads whose PF is improved with a given nonlinear compensator.[1] In this brief note we illustrate the application of this result to the practical problem of passive compensation of a classical half–bridge controlled rectifier (terminated by a resistor) with a non–sinusoidal source voltage. We give necessary and sufficient conditions for improvement of PF with linear inductors or capacitors, which determine ranges for the compensator parameters that depend on the rectifier fire angle and the amplitude of the voltage source, and simulations results are presented to show the performance of the proposed compensators.

II. A framework for analysis and design of PF compensators

In this section we briefly review the main results of [8]. We refer the reader to this paper for further details. We consider the classical scenario of energy transfer from an n–phase ac generator to a load as depicted in Fig. 1. The voltage and current of the source are denoted by the column vectors $v_s, i_s \in \mathbb{R}^n$ and the load is described by a (possibly nonlinear and time varying) n–port system described by its admittance operator $Y_\ell : v_s \to i_s$.

We make the following assumptions:

Assumption A.1 All the signals in the system are periodic with fundamental period T and belong to the space

$$\begin{aligned}\mathcal{L}_2[0,T) \quad &:= \quad \{x : [0,T) \to \mathbb{R}^n \mid \|x\|^2 \\ &:= \quad \frac{1}{T}\int_0^T |x(\tau)|^2 \, d\tau < \infty \,\}\end{aligned}$$

[1] Or its dual: characterization of all nonlinear compensators that improve the PF for a given nonlinear load.

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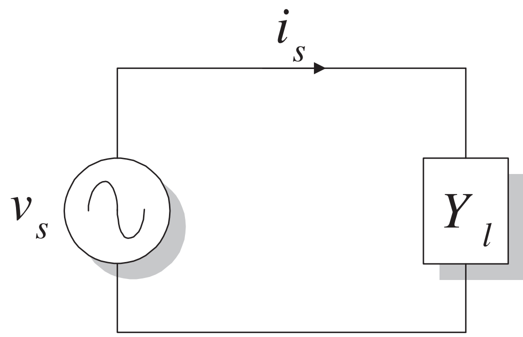

Fig. 1. Circuit schematic of an n–phase ac generator with negligible impedance connected to a (possibly nonlinear and time varying) load.

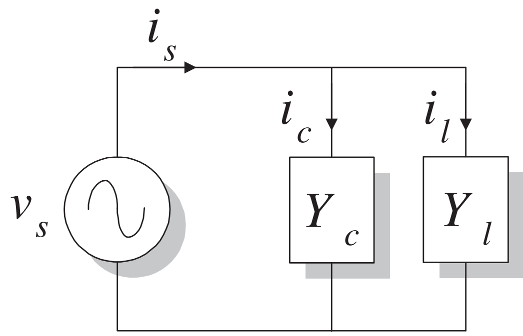

Fig. 2. Circuit schematic of shunt PF compensation configuration.

where $\|\cdot\|$ is called the *rms value* of x and $|\cdot|$ is the Euclidean norm.

Assumption A.2 The source is ideal, in the sense that v_s remains unchanged for all loads Y_ℓ.

Following standard convention [1] we define the PF of the source as

$$PF := \frac{P}{S}, \tag{1}$$

where

$$\begin{aligned} P &:= \langle v_s, i_s\rangle = \frac{1}{T}\int_0^T v_s^\top(t) i_s(t)\, dt \\ S &:= \|v_s\|\,\|i_s\| \end{aligned}$$

are the average and the apparent power, respectively. From Cauchy-Schwartz inequality [13] it is clear that $-1 \leq PF \leq 1$.

Compensation schemes are introduced to maximize PF. A typical compensation configuration is shown in Fig. 2, where $Y_c : v_s \to i_c$ is the compensator admittance operator. To avoid power dissipation, the compensator is restricted to be lossless, that is, it should satisfy

$$\langle v_s, i_c\rangle = \langle v_s, Y_c v_s\rangle = 0. \tag{2}$$

In the absence of a compensator $i_s = i_\ell$ and we consequently define the uncompensated PF as

$$PF_u := \frac{\langle v_s, i_\ell\rangle}{\|v_s\|\,\|i_\ell\|}. \tag{3}$$

Obviously, the compensator Y_c improves PF if and only if $PF > PF_u$.

The main result of [8] is contained in the proposition below.

Proposition 1: Fix the signal v_s and consider the system of Fig. 2. The set of all lossless compensators Y_c that improve the power factor for a given Y_ℓ is given by

$$2\langle Y_\ell v_s, Y_c v_s\rangle + \|Y_c v_s\|^2 < 0. \tag{4}$$

Dually, a given lossless compensator Y_c improves PF for all Y_ℓ that satisfy (4).

Proof: From Kirchhoff's current law, $i_s = i_c + i_\ell$, the relation $i_c = Y_c v_s$, and the losslessness condition (2) we have that $\langle v_s, i_s\rangle = \langle v_s, i_\ell\rangle$. Consequently, for a lossless compensator,

$$PF = \frac{\langle v_s, i_\ell\rangle}{\|v_s\|\|i_s\|}.$$

From the expression above and (3) we have the following chain of equivalences

$$\begin{aligned} PF > PF_u \quad &\Leftrightarrow \quad \|i_\ell\| > \|i_s\| \\ &\Leftrightarrow \quad \|i_\ell\|^2 > \|i_s\|^2 \\ &\Leftrightarrow \quad \|i_\ell\|^2 > \|i_c\|^2 + 2\langle i_c, i_\ell\rangle + \|i_\ell\|^2 \\ &\Leftrightarrow \quad 0 > \|i_c\|^2 + 2\langle i_c, i_\ell\rangle \\ &\Leftrightarrow \quad 0 > \|Y_c v_s\|^2 + 2\langle Y_c v_s, Y_\ell v_s\rangle, \end{aligned} \tag{5}$$

where, to get the fourth equivalence, we have used $i_s = i_c + i_\ell$. ∎

Remark 1: In [8] an alternative formulation of the proposition is given:
The compensator Y_c improves PF if and only if the compensated system with port variables (v_s, i_s) is cyclo–dissipative with respect to the supply rate $w(v_s, i_s) = (Y_\ell v_s + i_s)^\top (Y_\ell v_s - i_s)$. This result is immediately obtained from the equivalences

$$\begin{aligned} \|i_\ell\|^2 > \|i_s\|^2 \quad &\Leftrightarrow \quad \langle i_\ell + i_s, i_\ell - i_s\rangle > 0 \\ &\Leftrightarrow \quad \int_0^T w(v_s(t), i_s(t)) dt > 0, \end{aligned}$$

where the latter inequality is precisely the definition of cyclo–dissipativity [9]. This formulation reveals the key role played by the physical property of cyclo–dissipativity. It also shows that, in the spirit of the classical passivation procedure for stabilization, the PF compensation problem can be recast as one of cyclo–dissipasivation (with respect to the supply rate given above).

Remark 2: From (1) and Cauchy-Schwartz we know that $PF = 1$ if and only if v_s and i_s are collinear—that is, when the load is purely resistive. For scalar LTI RLC circuit loads this fact reveals that the underlying mechanism for PF improvement is one of energy equalization. Indeed, it is well

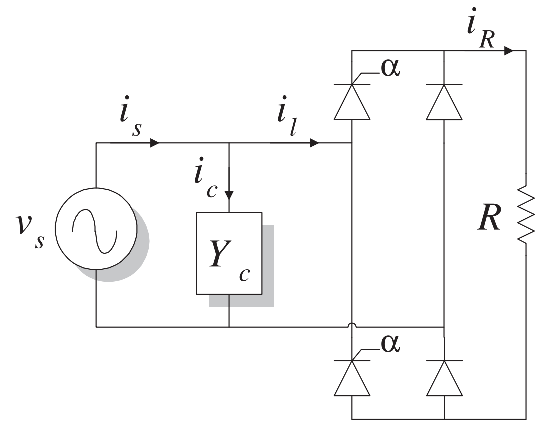

Fig. 3. A semiconverter rectifier load.

known that the frequency response of the impedance of an LTI RLC circuit (with n_R resistors, n_C capacitors and n_L inductors) may be written as

$$\hat{Z}_s(j\omega) = \frac{1}{|\hat{I}_s(j\omega)|^2} \{2P_{av}(\omega) + 4j\omega\,[H_{L_{av}}(\omega) - H_{C_{av}}(\omega)]\}$$

where $\hat{Z}_s(s) = \frac{\hat{V}_s(s)}{\hat{I}_s(s)}$ is the impedance of the circuit with port variables (v_s, i_s),

$$\begin{aligned} P_{av}(\omega) &= \frac{1}{2}\sum_{q=1}^{n_R} R_q|\hat{I}_q(j\omega)|^2 \\ H_{C_{av}}(\omega) &= \frac{1}{4}\sum_{q=1}^{n_C} C_q|\hat{V}_{C_q}(j\omega)|^2 \\ H_{L_{av}}(\omega) &= \frac{1}{4}\sum_{q=1}^{n_L} L_q|\hat{I}_{L_q}(j\omega)|^2, \end{aligned}$$

are the power dissipated in the resistors, and the average electric and magnetic energy stored in the load, respectively. See, e.g., eq. (5.6) of [10]. Roughly speaking, we can then say that the "distance" of the circuit with respect to a "purely resistive" behavior is proportional to the difference between the stored energies. The PF compensator may then be viewed as an energy equalizer, which "adds" the required electric or magnetic energy to bridge the energy differences. Further discussion on this interpretation, and its limitation for switching circuits, may be found in [8].

III. PASSIVE PF COMPENSATION OF A SEMICONVERTER CONTROLLED RECTIFIER

In this section we consider the classical single–phase semiconverter controlled rectifier load (terminated by a resistor) shown in Fig. 3. Applying Proposition 1 we give necessary and sufficient conditions for improvement of PF of this load with linear inductors or capacitors.

We make the following reasonable assumptions on v_s:

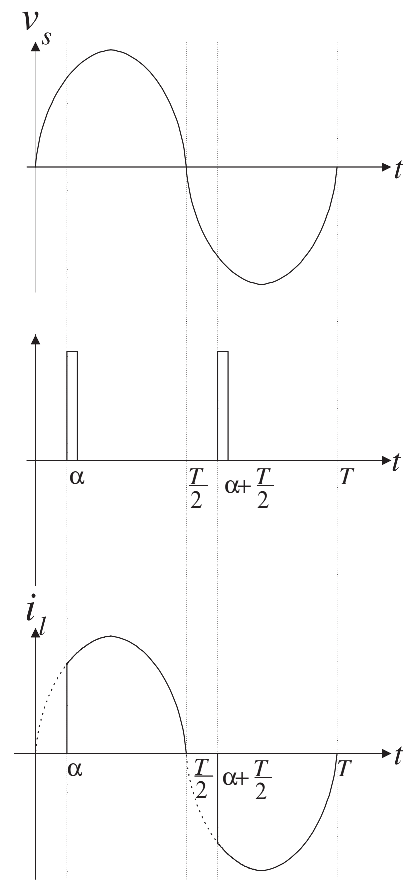

Fig. 4. Waveforms for the semiconverter rectifier.

(A1) Is periodic of period $T \in \mathbb{R}_+$, that is, $v_s(t) = v_s(t+T)$;
(A2) changes sign every half period, i.e., $v_s(kT) = v_s(\frac{kT}{2}) = 0$ for all $k \in \mathbb{Z}_+$;
(A3) is non–negative in the first half–period and non–positive in the second one, that is,

$$v_s(t) \begin{cases} \geq 0, & t \in [0, \frac{T}{2}] \\ \leq 0, & t \in [\frac{T}{2}, T]. \end{cases}$$

We also assume the firing angle α is constant and $\alpha < \frac{T}{2}$. Under these conditions, the load can be modeled as a linear–time varying resistor with admittance operator

$$i_\ell(t) = (Y_\ell v_s)(t) = \begin{cases} 0, & t \in \mathcal{I}_k \\ \frac{1}{R}v_s(t), & \text{otherwise} \end{cases} \quad (6)$$

where the time intervals $\mathcal{I}_k \subset \mathbb{R}_+$ are defined as

$$\mathcal{I}_k = \left[\frac{kT}{2}, \frac{kT}{2} + \alpha\right), \; k \in \mathbb{Z}_+.$$

See Figs. 4 and 5.

Proposition 2: Fix $T \in \mathbb{R}_+$ and $0 < \alpha < \frac{T}{2}$ and consider the half–bridge rectifier of Fig. 3 with capacitive and inductive compensators and v_s verifying assumptions A1, A2 and A3.

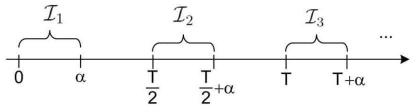

Fig. 5. Time intervals $\mathcal{I}_k$ for the half–bridge rectifier.

(i) **(Capacitive compensation)** Let $Y_c = Cp$, where $C > 0$ is the value of the capacitance and $p := \frac{d}{dt}$. Then, for all v_s, $\exists C_{max} > 0$ such that

$$PF > PF_u, \quad \text{for all} \quad C < C_{max}.$$

(ii) **(Inductive compensation)** Let $Y_\ell = \frac{1}{Lp}$, where $L > 0$ is the value of the inductance. Assume v_s is such that

$$\phi^2(\alpha) + \phi^2\left(\frac{T}{2} + \alpha\right) > \phi^2\left(\frac{T}{2}\right) + \phi^2(T), \qquad (7)$$

where ϕ is the inductors flux, that is,

$$\phi(t) = \phi(0) + \int_0^t v_s(\zeta)\, d\zeta.$$

Under these conditions, $\exists L_{min} > 0$ such that[2]

$$PF > PF_u, \quad \text{for all} \quad L > L_{min}.$$

Furthermore, if (7) does not hold PF cannot be improved with an inductor.

Proof: In both cases we will verify the third equivalence in (5), that is,

$$PF > PF_u \quad \Leftrightarrow \quad 0 > \|i_c\|^2 + 2\langle i_c, i_\ell\rangle. \qquad (8)$$

(i) For capacitive compensation we have $i_c = C\dot{v}_s$. Hence, $PF > PF_u$ if and only if

$$0 > \|C\dot{v}_s\|^2 + 2\langle C\dot{v}_s, i_\ell\rangle. \qquad (9)$$

Now, since v_s is periodic it admits a Fourier series expansion

$$v_s(t) = \sum_{n=-\infty}^{\infty} \hat{V}_s(n)\exp(jn\omega_0 t), \quad \omega_0 := \frac{2\pi}{T},$$

where $\hat{V}_s(n)$ are the complex Fourier coefficients. Using the properties of periodic signals [15], for the first right hand term in (9) we have

$$C\|\dot{v}_s\|^2 = C\omega_0^2 \sum_{n=-\infty}^{\infty} n^2|\hat{V}_s(n)|^2. \qquad (10)$$

[2]The expressions of C_{max} and L_{min} are given in (11) and (12), respectively.

On the other hand, we can evaluate the second right hand term in (9) as

$$\begin{aligned}
\langle C\dot{v}_s, i_\ell\rangle &= \frac{C}{T}\int_0^T \dot{v}_s(t) i_\ell(t)\, dt \\
&= \frac{C}{RT}\left[\int_\alpha^{\frac{T}{2}} v_s(t)\dot{v}_s(t)\, dt \right.\\
&\quad \left. + \int_{\frac{T}{2}+\alpha}^{T} v_s(t)\dot{v}_s(t)\, dt\right] \\
&= \frac{C}{2RT}\left[v_s^2\left(\frac{T}{2}\right) - v_s^2(\alpha)\right.\\
&\quad \left. + v_s^2(T) - v_s^2\left(\frac{T}{2}+\alpha\right)\right] \\
&= -\frac{C}{2RT}\left[v_s^2(\alpha) + v_s^2\left(\frac{T}{2}+\alpha\right)\right]
\end{aligned}$$

where we have used (6) to get the second equation and the fact that $v_s(T) = v_s\left(\frac{T}{2}\right) = 0$ for the last one. Replacing that last identity above, and (10) in (9), we see that $PF > PF_u$ if and only if

$$v_s^2(\alpha) + v_s^2\left(\frac{T}{2}+\alpha\right) > RTC\omega_0^2 \sum_{n=-\infty}^{\infty} n^2|\hat{V}_s(n)|^2.$$

The proof is completed noting that the inequality holds for all $C < C_{max}$ with

$$C_{max} := \frac{T}{4\pi^2 R}\frac{v_s^2(\alpha) + v_s^2\left(\frac{T}{2}+\alpha\right)}{\sum_{n=-\infty}^{\infty} n^2|\hat{V}_s(n)|^2}. \qquad (11)$$

(ii) In the case of inductor compensation $\dot{\phi} = v_s$ and $\frac{1}{L}\phi = i_c$, where we recall that ϕ is the inductors flux. Hence,

$$\begin{aligned}
\langle i_c, i_\ell\rangle &= \frac{1}{L}\langle \phi, i_\ell\rangle \\
&= \frac{1}{RLT}\left[\int_\alpha^{\frac{T}{2}} \phi(t)\dot{\phi}(t)dt + \int_{\frac{T}{2}+\alpha}^{T} \phi(t)\dot{\phi}(t)dt\right] \\
&= \frac{1}{2RLT}\left[\phi^2\left(\frac{T}{2}\right) - \phi^2(\alpha)\right.\\
&\quad \left. + \phi^2(T) - \phi^2\left(\frac{T}{2}+\alpha\right)\right],
\end{aligned}$$

where we have used (6) to get the second equation. Now, in order to fulfill (8) it is obvious that $\langle i_c, i_\ell\rangle$ must be negative, which is true if and only if (7) is satisfied, this proves the last statement of the claim.

On the other hand,

$$\|i_c\|^2 = \frac{1}{L^2}\|\phi\|^2.$$

Therefore, (8) holds for all $L > L_{min}$ with

$$L_{min} := \frac{RT\|\phi\|^2}{\phi^2\left(\frac{T}{2}\right) - \phi^2(\alpha) + \phi^2\left(\frac{T}{2}+\alpha\right) - \phi^2(T)} \qquad (12)$$

completing the proof. ■

Inductor	PF	Improving PF?
$4.25mH$	0.141	No
$8.5mH$	0.271	No
$17mH$	0.352	Yes
$34mH$	0.343	Yes
$68mH$	0.324	Yes

TABLE I
POWER FACTOR USING DIFFERENT VALUES OF L

Capacitor value	PF	Improving PF?
$32\mu F$	0.345	Yes
$64\mu F$	0.350	Yes
$128\mu F$	0.257	No
$256\mu F$	0.130	No
$512\mu F$	0.061	No

TABLE II
POWER FACTOR USING DIFFERENT VALUES C

Remark 3: Proposition 2 shows that, for any (admissible) waveform v_s, PF will be improved with (sufficiently small) capacitive compensation—where we underscore the qualifier "for any". This property is not very surprising given the obvious "delay advance" action of the rectifier, illustrated in Fig. 4. On the other hand, it is rather unexpected that PF can also be improved with (sufficiently large) inductive compensation, for source voltages v_s such that the magnetic energy stored in the inductor verifies (7). (We recall that the energy stored in a linear inductor is equal to $\frac{1}{2L}\phi^2$.) Observe that, due to the sign constraints imposed on v_s, $\phi(\alpha) \leq \phi(\frac{T}{2})$, while $\phi(T) \leq \phi(\frac{T}{2}+\alpha)$. This means that (7) will hold only if the largest part of the magnetic energy is stored during the second half of the period.

Remark 4: Since the framework allows us to consider nonlinear compensators is interesting to investigate if the PF can be further improved using nonlinear capacitors or inductors. For instance, for a nonlinear capacitor described by $\dot{q} = i_c$ and $v_s = \frac{dH(q)}{dq}$, where q is the capacitor charge and $H(q)$ the stored electrical energy, we have

$$\begin{aligned}\langle i_c, i_\ell \rangle &= \frac{1}{RT}\left[H\left(q\left(\frac{T}{2}\right)\right) + H\left(q(T)\right)\right. \\ &\qquad \left. -H(q(\alpha)) - H\left(q\left(\alpha + \frac{T}{2}\right)\right)\right]\end{aligned}$$

and the question is how to select the energy function to minimize this quantity. Another question of interest is the physical realization of such devices that could, in principle, be approximated with active filters. Current research is under way along these directions.

IV. SIMULATION RESULTS

In this section we present some preliminary simulation results to evaluate the performance of the proposed compensators (inductor and capacitor). The converter parameters are chosen as $R = 10\Omega$, $T = 0.02s$ and $\alpha = 0.0075s$ and the voltage source is $V_s = 280V cos(50\pi t)$. This corresponds to $PF_u = 0.3014$. Furthermore, $L_{min} = 17mH$ and $C_{max} = 128\mu F$.

Tables I and II shows the values of the compensated PF for different choices of L and C, above and below their critical values. As expected the results are consistent with the terms computed via (11) and (12), respectively.

V. CONCLUDING REMARKS AND FUTURE WORK

Using the framework developed in [8] we have analyzed in this paper a practical example of a PF compensation problem that includes nonlinear and switching phenomena, namely, the semiconverter controlled rectifier. The analysis is carried out for general periodic, but not necessarily sinusoidal, source voltages. Although we restrict ourselves to the simplest case of single–phase rectifier and linear capacitive or inductive compensation, the analysis can be easily extended to more general cases. We give physically interpretable necessary and sufficient conditions for PF improvement that may be used for the design of a passive compensator—showing, in this way, the usefulness of the proposed framework.

Several open questions are being currently investigated:

- Characterization of nonlinear switching loads whose PF can be compensated with LTI inductors or capacitors—for a suitable class of v_s. As discussed in [8], and also indicated here, this analysis corresponds to the determination of loads that are cyclo–dissipative with respect to supply rates $v_s^\top \int i_\ell$ and $\dot{v}_s^\top i_\ell$ respectively.
- Definition of an "optimal" LC circuit for a given nonlinear load.
- Investigate the advantages of using nonlinear passive compensation instead of linear, that is, to consider nonlinear capacitors and inductors as discussed in Remark 4. Also, the realization with active filters of a given admittance Y_c, for instance a nonlinear inductor with a specified energy function, should be studied in the future.
- At this point we have only given a test on whether PF is improved or not. However, for practical purposes a definition of a figure of merit to quantify the PF improvement is needed. Also, we have concentrated our attention on PF, with no explicit reference to the harmonic pollution reduction. Even though it is obvious that increasing PF will reduce the harmonic distortion, it would be desirable to reveal some explicit relations between the two objectives.
- It would be interesting to investigate the PF compensation abilities of active filters, which currently dominate high–performance applications. Towards this end, we can consider the diagram of Fig. 6 where the PF compensator is a parallel active filter approximated by an ideal voltage source e in series with an inductor with

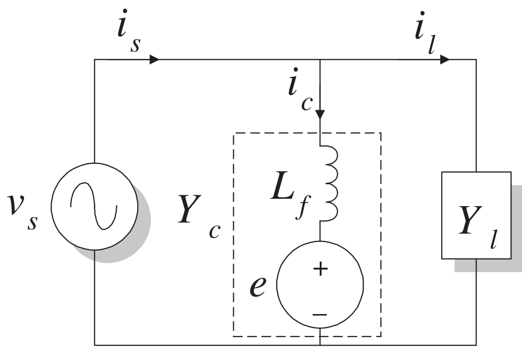

Fig. 6. Circuit schematic of a shunt active filter operating as a PF compensator.

inductance L_f.
The voltage e represents the control law that, in practice, will determine the position of the switches in the active filter. Typically, the control objective in active filters is to render i_s collinear with v_s. Hence, an error signal $i_s - gv_s$ is defined, where g is a desired equivalent admittance for the compensated circuit, which is an additional design parameter that maybe time varying but we take here to be a positive constant. To try to drive the error to zero an LTI filter is used, leading to $e = F(p)(i_s - gv_s)$, where $F(p) \in \mathbb{R}(p)$—which are commonly taken to be PI or resonant filters. Unfortunately, although it is possible in this case to define the corresponding admittance operator Y_c, this will not be (in general) a lossless operator and the existing framework does not apply.

- A big open question concerns our key assumption of ideal source. It is not clear at this point how to even formulate the problem when the impedance of the generator cannot be neglected. Notice that in this case, even if v_s remains unchanged, due to the voltage drop in the sources surge impedance the actual voltage applied to the load changes is a function of i_c.

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