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Stable Adaptive Neural Network Control of MIMO Nonaffine Nonlinear Discrete-Time Systems

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Abstract— **In this paper, stable adaptive neural network (NN) control, a combination of weighted one-step-ahead control and adaptive NN is developed for a class of multi-input-multi-output (MIMO) nonaffine nonlinear discrete-time systems. The weighted one-step-ahead control is designed to stabilize the nominal linear system, while the adaptive NN compensator is introduced to deal with the nonlinearities. Under the assumption that the inverse control gain matrix has an either positive definite or negative definite symmetric part, the obstacle in NN weights tuning for the MIMO systems is transformed to unknown control direction problem for single-input-single-output (SISO) system. Discrete Nussbaum gain is introduced into the NN weights adaptation law to overcome the unknown control direction problem. It is proved that all signals of the closed-loop system are bounded, while the tracking error converges to a compact set. Simulation result illustrates the effectiveness of the proposed control.**

I. Introduction

In the last decades, adaptive NN control of nonlinear systems has received an increasing attention. Many excellent adaptive NN control approaches have been proposed for discrete-time nonlinear systems [1], [2], [3], [4], [5], [6], and references therein. In [1], a multi-layer NN was used in control of a class of unknown feedback-linearizable discrete-time system, where backpropagation learning algorithm has been adopted to update NN weights and efficient off-line training was required. In [2], NN control was studied for a class of discrete-time nonlinear systems with relative degree of one. The controller singularity problem was excellently solved but not avoided completely. For a class of discrete-time systems in strict feedback form, an effective back-stepping design method was proposed in [3]. For nonaffine systems in nonlinear auto regressive moving average with eXogenous inputs (NARMAX) form, an NN based control method was given in [4], but the stability analysis of the closed-loop system was not given. Based on implicit function theorem, NNs were used to emulate the "inverse controls" in [5] where persistent excitation (PE) condition is needed to guarantee the stability. The implicit function based adaptive NN control has been further developed in [6] where the requirement of PE condition is removed.

The above mentioned results are limited to SISO nonlinear discrete-time systems. For MIMO nonlinear discrete-time systems, the control problem becomes very difficult due to the difficulty in handling the coupling between different inputs, and only a few results are available. In [7], multi-layer NN was used to control a special class of MIMO affine nonlinear discrete-time systems. Direct adaptive neural network control was presented in [8] for a class of MIMO NARMAX systems in affine form. In [9], multivariable neuro-adaptive variable structure control was developed for a very special class of MIMO nonlinear discrete-time systems, in which the output signals were not included in the nonlinear terms. For a class of MIMO nonlinear discrete-time systems in strict feedback form, state and output feedback adaptive NN controls were investigated in [10] and [11], respectively, via backstepping design method. All the above methods are designed for the nonlinear systems that are of affine appearance of control inputs. For MIMO nonaffine nonlinear discrete-time systems, an inverse NN control method was designed in [12]. Since MIMO nonaffine nonlinear systems represent a more general class of nonlinear systems, different control approaches may also be pursued.

In this paper, we investigate MIMO nonaffine nonlinear discrete-time systems described by NARMA model. Inspired by the add-on control strategy of SISO nonlinear systems in [13], add-on adaptive NN control was proposed for MIMO nonaffine nonlinear discrete-time systems, where a linear control was designed for the nominal linear system and an adaptive NN term was employed to deal with nonlinearities. As stated in [13], linear model can often catch dominant dynamics of a nonlinear plant around its operating point and provide good basis for control design. The nominal linear model of systems are usually used in the control design [13], [14]. The main contributions of this paper are listed as follows:

(i) Combining weighted one-step-ahead control and adaptive NN, stable adaptive NN control is proposed for a class of MIMO nonlinear discrete-time systems.
(ii) By introducing discrete Nussbaum gain into the NN weight update law, the assumption on the control gain matrix in previous work is relaxed. At the same time, all signals of the closed-loop system are guaranteed to be bounded, while the tracking error is made to converge to a compact set.

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II. System Description and Preliminaries

A. System Dynamics

Consider the following n-input and n-output nonaffine nonlinear discrete-time system, which can be described by the NARMA (nonlinear auto regressive moving average) model as follows

$$y(k+1)=\Psi(y(k),\cdots,y(k-n_s+1),u(k),\cdots,u(k-m)) \quad (1)$$

where $u(k)=[u_1(k),\cdots,u_n(k)]^T \in R^n$ and $y(k)=[y_1(k),\cdots,y_n(k)]^T \in R^n$ are the system input and output, respectively, $\Psi(\cdot)=[\psi_1(\cdot),\cdots,\psi_n(\cdot)]^T \in R^n$ is a vector-valued smooth nonlinear function, n_s and m are the lengths of system outputs and inputs, respectively.

The control objective is to design a control input $u(k)$, such that the system output $y(k)$ follows a known and bounded trajectory $r(k)=[r_1(k),\cdots,r_n(k)]^T \in R^n$, while all signals in the closed-loop system remain bounded.

Assumption 2.1: The reference trajectory $r(k) \in \Omega_r \subset R^n, \forall k \geq 0$ is bounded and known.

Assumption 2.2: [15] The control gain matrix $\mathscr{G}(k) := \frac{\partial \Psi(\cdot)}{\partial u(k)}$, $\forall k \geq 0$, is a full rank matrix, and its inverse $\mathscr{G}^{-1}(k)$ has an either positive definite or negative definite symmetric part, i.e., $\mathscr{G}_{IS}(k)=\frac{\mathscr{G}^{-1}(k)+\mathscr{G}^{-T}(k)}{2}$ is either positive definite or negative definite, with $\mathscr{G}^{-T}(k)$ being the transpose of $\mathscr{G}^{-1}(k)$. In addition, the eigenvalues of $\mathscr{G}_{IS}(k)$ is assumed to be bounded.

Remark 2.1: It should be pointed that matrices $\mathscr{G}(k)$ and $\mathscr{G}^{-1}(k)$ are general real matrices and they are not required to be symmetric. According to the definition of $\mathscr{G}_{IS}(k)$, given a vector $e(k) \in R^n$, we have $e^T(k)\mathscr{G}^{-1}(k)e(k) = e^T(k)\mathscr{G}_{IS}(k)e(k)$. When it is known that $\mathscr{G}_{IS}(k)$ is positive or negative definite, the control design could be great simplified.

Remark 2.2: Assumption 2.2 is quite looser than Assumption 4 in [8], which requires the existence of an orthogonal matrix multiplying the control gain matrix to guarantee the eigenvalues of the product matrix are all positive. It is hard to construct such an orthogonal matrix, especially when the control gain matrix is totally unknown.

A large class of systems (1) can be described by a nominal linear model as follows

$$y(k+1)=-\bar{A}(z^{-1})y(k)+B(z^{-1})u(k) \quad (2)$$

where $\bar{A}(z^{-1})$ and $B(z^{-1})$ are polynomial matrices in terms of the unit back shift operator z^{-1} with $\bar{A}(z^{-1})$ being diagonal. For convenience of analysis, denote $A(z^{-1}) := I+z^{-1}\bar{A}(z^{-1})$, where I is an identity matrix. The matrices $A(z^{-1})$ and $B(z^{-1})$ can be expressed in the following format

$$\begin{aligned} A(z^{-1}) &= I+A_1 z^{-1}+\cdots+A_{n_a}z^{-n_a} \\ B(z^{-1}) &= B_0+B_1 z^{-1}+\cdots+B_{n_b}z^{-n_b} \end{aligned}$$

where $n_a \leq (n_s+1)$ and $n_b \leq m$ are orders of $A(z^{-1})$ and $B(z^{-1})$ are n_a and n_b, respectively. The modeling error is

$$\begin{aligned} &\Delta_f(y(k),\cdots,y(k-n_s+1),u(k),\cdots,u(k-m)) \\ &=\Psi(\cdot)+\bar{A}(z^{-1})y(k)-B(z^{-1})u(k):=\Delta_f(\bar{y}_k,\bar{u}_{k-1},u(k)) \quad (3) \end{aligned}$$

where is a vector-valued unknown nonlinear function with $\Delta_f(\bar{y}_k,\bar{u}_{k-1},u(k)) = [\delta_{f1}(\cdot),\cdots,\delta_{fn}(\cdot)] \in R^n$, $\bar{y}_k = [y^T(k),\cdots,y^T(k-n_s+1)]^T$, and $\bar{u}_{k-1} = [u^T(k-1),\cdots,u^T(k-m)]^T$. Using above notations, the system (1) can be re-written into the following form

$$A(z^{-1})y(k+1)=B(z^{-1})u(k)+\Delta_f(\bar{y}_k,\bar{u}_{k-1},u(k)) \quad (4)$$

Assumption 2.3: The polynomial matrix $B(z^{-1})$ is invertible.

In this paper, our control is designed based on (4) which is equivalent to the nonlinear system (1). It is assumed that matrix $A(z^{-1})$ and $B(z^{-1})$ are known, but the modeling error (3) is unknown. To deal with the effect of the modeling error, an add on control with high order neural network is introduced to deal with nonlinearity in (3), and the discrete Nussbaum gain is employed in the NN adaptation law. To proceed, some preliminaries are given in the next section before control design.

B. Discrete Nussbaum Gain

From Assumption 2.2, we do not know whether $\mathscr{G}_{IS}(k)$ is positive definite or negative definite. It makes the control problem much more difficult since we cannot decide the direction along which the control operates. The following discrete Nussbaum gain is utilized in this paper to deal with this problem.

Definition 2.1: [16] Consider discrete nonlinear function $N(x(k)) \in R$ defined on the sequence $x(k) \in R$ with $x_s(k) := \sup_{\sigma \leq k}\{x(\sigma)\}$. Define $S_N(x(k)) \in R$ as

$$S_N(x(k))=\sum_{\delta=0}^{k} N(x(\delta))\Delta x(\delta) \quad (5)$$

with $\Delta x(k) = x(k+1)-x(k)$. Function $N(x(k))$ is a discrete Nussbaum gain if and only if it satisfies the following two properties:

(i) If $x_s(k)$ increases without bound, then

$$\begin{aligned} \sup_{x_s(k)\geq c_0} \frac{1}{x_s(k)} S_N(x(k)) = +\infty \\ \inf_{x_s(k)\geq c_0} \frac{1}{x_s(k)} S_N(x(k)) = -\infty \end{aligned}$$

(ii) If $x_s(k) \leq c_1$, then $|S_N(x(k))| \leq c_2$ with some positive constants c_1 and c_2.

The discrete Nussbaum gain was first proposed in [17]. It is defined as follows. Let $\{x(k)\}$ be a discrete sequence with $x(0)=0, x(k)\geq 0, \forall k>0$ and

$$|\Delta x(k)| := |x(k+1)-x(k)| \leq c_0 \quad (6)$$

where c_0 is a positive constant. Then, the discrete Nussbaum gain is defined on the sequence $x(k)$ as

$$N(x(k)) := x_s(k)s_N(x(k)) \quad (7)$$

where $s_N(x(k))$ is defined in the following manner. Let $s_N(x(0))=+1$. At $k=k_1$, there are two cases:

(i) If $s_N(x(k_1))=+1$, then

$$s_N(x(k_1+1)) = \begin{cases} -1, & S_N(x(k_1)) > x_s^{\frac{3}{2}}(k_1) \\ +1, & \text{otherwise} \end{cases}$$

(ii) If $s_N(x(k_1)) = -1$, then

$$s_N(x(k_1+1)) = \begin{cases} +1, & S_N(x(k_1)) < -x_s^{\frac{3}{2}}(k_1) \\ -1, & \text{otherwise} \end{cases}$$

where $\pm x_s^{\frac{3}{2}}(k)$ defines a pair of switching curves.

Lemma 2.1: [18] Let $V(k)$, $\forall k \geq 0$, be a positive definite function, $N(x(k))$ be a discrete Nussbaum gain, and θ be a nonzero constant. If the following inequality holds:

$$V(k) \leq \sum\nolimits_{\sigma=k_1}^{k} (c_1 + \theta N(x(\sigma)))\Delta x(\sigma) + c_2 x(k) + c_3;\ \forall k \quad (8)$$

where c_1, c_2 and c_3 are some constants, k_1 is a positive integer, then $V(k)$ and $x(k)$ must be bounded, $\forall k$.

Lemma 2.2: [16] Consider discrete Nussbaum gain $N(x(k))$ defined in (7), and the discrete sequence $\{x(k)\}$, $x(0) = 0, x(k) \geq 0, \forall k > 0$, which satisfies (6). Given an arbitrary bounded function $g(k) : R \rightarrow R$, which takes values in the unknown closed intervals $[\underline{g},\ \bar{g}]$, then $N'(x(k)) = g(k)N(x(k))$ is still a discrete Nussbaum gain.

III. Adaptive NN Control of MIMO Systems

A. Stable Adaptive NN Control

Considering the nominal linear system (2), weighted one-step-ahead control with decoupling design can be developed for (2). To design the control, the following cost function is introduced:

$$J = \|P(z^{-1})y(k+1) - R(z^{-1})r(k+1) + Q(z^{-1})\bar{u}(k)\|^2 \quad (9)$$

where $P(z^{-1}), R(z^{-1})$ and $Q(z^{-1})$ are all $n \times n$ diagonal weighting polynomial matrices, and $\bar{u}(k) \in R^n$ is an auxiliary input, which will be defined later. Considering that $P(z^{-1})$ and $A(z^{-1})$ are diagonal and the nominal linear system in (4) is of relative degree one, we have the following Diophantine equation

$$P(z^{-1}) = FA(z^{-1}) + z^{-1}G(z^{-1}) \quad (10)$$

where F is diagonal constant matrix and $G(z^{-1})$ is diagonal polynomial matrix which is of order $n_g = \max\{n_a - 1, n_p - 1\}$.

Assumption 3.1: The choice of $P(z^{-1})$ satisfies that the constant matrix F is a full rank matrix.

The optimal control that minimizes (9) is

$$G(z^{-1})y(k) + FB(z^{-1})u(k) = R(z^{-1})r(k+1) - Q(z^{-1})\bar{u}(k) \quad (11)$$

Denote $H(z^{-1}) = FB(z^{-1})$, and let

$$u(k) = \text{adj}\{H(z^{-1})\}\bar{u}(k) \quad (12)$$

where adj$\{H(z^{-1})\}$ denotes the adjoint matrix of $H(z^{-1})$. Substituting (12) into (11), and using the fact that $H(z^{-1})\text{adj}\{H(z^{-1})\} = \text{adj}\{H(z^{-1})\}H(z^{-1}) = \det\{H(z^{-1})\}I$, we can present the weighted one-step-ahead control with decoupling design as the following form

$$[\det\{H(z^{-1})\}I + Q(z^{-1})]\bar{u}(k) = R(z^{-1})r(k+1) - G(z^{-1})y(k) \quad (13)$$

where $\det\{H(z^{-1})\}$ is the determinant of matrix $H(z^{-1})$.

To deal with the modeling error, an additional control $v(k)$ can be introduced into (13)such that the overall control is given as (4) can be given as

$$\begin{aligned} &[\det\{H(z^{-1})\}I + Q(z^{-1})]\bar{u}(k) = R(z^{-1})r(k+1) - G(z^{-1})y(k) \\ &\qquad + [\det\{H(z^{-1})\}I + Q(z^{-1})](\text{adj}\{H(z^{-1})\})^{-1}v(k) \quad (14) \end{aligned}$$

where $v(k)$ will be designed later in (21). Left-multiplying (4) by $[\det\{H(z^{-1})\}I + Q(z^{-1})]F$, and considering (12) and (14), we can obtain the closed-loop dynamics of the nonlinear system as follows

$$\begin{aligned} &[\det\{H(z^{-1})\}P(z^{-1}) + Q(z^{-1})FA(z^{-1})]y(k+1) \\ &= \det\{H(z^{-1})\}R(z^{-1})r(k+1) \\ &\quad + [\det\{H(z^{-1})\}I + Q(z^{-1})]F\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v(k)) \quad (15) \end{aligned}$$

where

$$\begin{aligned} &\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v(k)) \\ &= B(z^{-1})v(k) + F\Delta_f(\bar{y}_k, \bar{u}_{k-1}, u(k)) \\ &= B(z^{-1})v(k) + F\Delta_f(\bar{y}_k, \bar{u}_{k-1}, v(k) + \text{adj}\{H(z^{-1})\} \times \\ &\quad [\det\{H(z^{-1})\}I + Q(z^{-1})]^{-1}[R(z^{-1})r(k+1) - G(z^{-1})y(k)]) \quad (16) \end{aligned}$$

with $\bar{r}_{k+1} = [r^T(k+1), \cdots, r^T(k+1-n_r)]^T$, and n_r being the order of the polynomial matrix $R(z^{-1})$. If the weighting matrices $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ are chosen as

$$\det\{\det\{H(z^{-1})\}P(z^{-1}) + Q(z^{-1})FA(z^{-1})\} \neq 0, |z| \geq 1 \quad (17)$$

$$\det\{H(1)\}P(1) + Q(1)FA(1) = \det\{H(1)\}R(1) \quad (18)$$

and if the additional control term $v(k)$ is designed such that the last term of the right hand side of (15) is zero, then the nonlinear effects can be canceled, while the closed-loop system is stable and the system output tracks the desired trajectory.

In the following, we will first investigate the existence of $v^*(k)$, which assures that $\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v^*(k)) = 0$. From (16), it can be seen that $\frac{\partial \Delta_F(\cdot)}{\partial v(k)} = B_0 + \frac{\partial \Delta_f(\cdot)}{\partial u(k)} \frac{\partial u(k)}{\partial v(k)}$. Left-multiplying (14) by $\det\{H(z^{-1})\}$, and considering (12), we obtain

$$\begin{aligned} &[\det\{H(z^{-1})\}I + Q(z^{-1})]H(z^{-1})u(k) = R(z^{-1})r(k+1) \\ &\quad - G(z^{-1})y(k) + [\det\{H(z^{-1})\}I + Q(z^{-1})]H(z^{-1})v(k) \quad (19) \end{aligned}$$

which implies that $\frac{\partial u(k)}{\partial v(k)} = I$. Therefore, it can be obtained that $\frac{\partial \Delta_F(\cdot)}{\partial v(k)} = B_0 + \frac{\partial \Delta_f(\cdot)}{\partial u(k)} = \frac{\partial \Psi(\cdot)}{\partial u(k)}$. From Assumption 2.2, we know that $\frac{\partial \Delta_F(\cdot)}{\partial v(k)}$ is a full rank matrix, which implies $\frac{\partial \Delta_F(\cdot)}{\partial v(k)} \neq 0$. According to implicit function theorem, there exists an ideal smooth and unique $v^*(k)$ such that $\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v^*(k)) = 0$. Consider using HONN to approximate $v^*(k)$ as follows

$$\begin{aligned} &v^*(k) = W^{*T}S(\bar{z}(k)) + \epsilon_z,\ W^* \in R^{l \times n} \\ &S(\bar{z}(k)) = [s_1(\bar{z}(k)), s_2(\bar{z}(k)), ..., s_l(\bar{z}(k))]^T \in R^l \\ &s_i(\bar{z}(k)) = \prod\nolimits_{j \in I_i} [s(z_j(k))]^{\mu_j(i)},\ i = 1, 2, ..., l \\ &\bar{z}(k) = [z_1, z_2, \cdots, z_q]^T \\ &\quad = [r^T(k+1), \cdots, r^T(k+1-n_r), y^T(k), \cdots, \\ &\quad y^T(k-n_s+1), u^T(k-1), \cdots, u^T(k-m)]^T \in \Omega_{\bar{z}} \subset R^q \end{aligned} \quad (20)$$

where positive integer l denotes the neural network node number, $\epsilon_z = [\epsilon_{z1}, \cdots, \epsilon_{zn}]^T \in R^n$ is the bounded NN approximation error vector satisfying $\|\epsilon_z\| \leq \epsilon_0$ on the compact set, $\{I_1, I_2, ..., I_l\}$ is a collection of l not-ordered subsets of $\{1, 2, ..., q\}$ and $\mu_j(i)$ are non-negative integers, $s(z_j)$ is chosen as hyperbolic tangent function $s(z_j) = (e^{z_j} - e^{-z_j})/(e^{z_j} + e^{-z_j})$, and $q = n(n_r + n_s + m + 1)$ with n_r being the order of polynomial matrix $R(z^{-1})$. Then the adaptive NN control term $v(k)$ can be constructed as

$$v(k) = \hat{W}^T(k)S(\bar{z}(k)) \tag{21}$$

where $\hat{W}(k) \in R^{l\times n}$. For convenience of analysis, denote $e(k+1) := \Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v(k))$, where $e(k)$ is used to update the neural network weights. From Assumption 2.2, it is unknown whether the control gain matrix is positive definite or negative definite , thus the discrete Nussbaum gain is introduced into the NN weights adaptation law as follows:

$$\hat{W}(k)=\hat{W}(k-1)-\gamma N(x(k))S(\bar{z}(k-1))a(k)e^T(k)/D(k) \tag{22}$$

$$\Delta x(k)=x(k+1)-x(k)=a(k)e^T(k)e(k)/D(k),\ \ x(0)=0 \tag{23}$$

$$D(k)=(1+|N(x(k))|^2)(1+\|S(\bar{z}(k-1))\|^2+e^T(k)e(k)) \tag{24}$$

$$a(k)=\begin{cases} 1, & \text{if } \frac{\gamma\|e(k)\|}{(1+|N(x(k))|)} > \lambda \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

where $\gamma > 0$ and $\lambda > 0$ can be any positive constants. They can be regarded as the tuning rate and the threshold value of dead zone.

It should be noted that at the k-th step, $e(k)$ can be calculated from the following procedure. Define an extended tracking error $\varepsilon(k)$ as

$$\begin{aligned}\varepsilon(k)=&[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]y(k)\\ &-\det\{H(z^{-1})\}R(z^{-1})r(k)\end{aligned} \tag{26}$$

Considering (15) and (26), we have

$$e(k) = F^{-1}[\det\{H(z^{-1})\}I + Q(z^{-1})]^{-1}\varepsilon(k) \tag{27}$$

B. Stability Analysis

First, let us consider the boundedness of $e(k)$, $\hat{W}(k)$, $N(x(k))$, $x(k)$ and $v(k)$ with NN weights adaptation law (22)-(25).

Theorem 3.1: Consider the adaptive neural network control (21) and the corresponding neural network weights adaptation law (22)-(25). Under Assumption 2.2, the signal $e(k)$, the neural network weights $\hat{W}(k)$, and the discrete Nussbaum gain $N(x(k))$ and $x(k)$ are all uniformly bounded, and $e(k)$ satisfies $\lim_{k\to\infty}\|e(k)\| < C\lambda/\gamma$, with $C = \lim_{k\to\infty}(1 + |N(x(k))|)$.

Proof: Using mean value theorem and considering (20) and (21), we have

$$\begin{aligned}e(k+1)=&\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v(k))\\ =&\Delta_F(\bar{y}_k, \bar{u}_{k-1}, \bar{r}_{k+1}, v^*(k))+\frac{\partial\Delta_F}{\partial v}\Big|_{v_\xi(k)}[v(k)-v^*(k)]\\ =&\mathscr{G}_\xi(k)[\tilde{W}^T(k)S(\bar{z}(k)) - \epsilon_z]\end{aligned} \tag{28}$$

where $\tilde{W}(k) = \hat{W}(k) - W^*$, $\mathscr{G}_\xi(k) = \frac{\partial\Delta_F}{\partial v}\Big|_{v_\xi(k)}$, $v_\xi(k)$ is a point of $L(v(k), v^*(k))$, and $L(v(k), v^*(k))$ denotes the line segment joining two points $v(k)$ and $v^*(k)$, i.e.,

$$L(v(k), v^*(k)) = \{\xi \mid \xi = \theta v(k)+(1-\theta)v^*(k),\ 0 \leq \theta \leq 1\}$$

From (28), it can be obtained that

$$\tilde{W}^T(k-1)S(\bar{z}(k-1)) = \mathscr{G}_\xi^{-1}(k-1)e(k) + \epsilon_z \tag{29}$$

Choose the Lyapunov candidate as follows:

$$V(k) = \text{tr}\{\tilde{W}^T(k-1)\tilde{W}(k-1)\} \tag{30}$$

The difference of $V(k)$ along (29) is

$$\begin{aligned}\Delta V(k) =& V(k+1) - V(k)\\ =& \text{tr}\{\tilde{W}^T(k)\tilde{W}(k) - \tilde{W}^T(k-1)\tilde{W}(k-1)\}\\ =& \text{tr}\{[\tilde{W}(k) - \tilde{W}(k-1)]^T[\tilde{W}(k) - \tilde{W}(k-1)]\\ &+ 2\tilde{W}^T(k-1)[\tilde{W}(k) - \tilde{W}(k-1)]\}\\ =& a(k)\gamma^2N^2(x(k))\frac{S^T(\bar{z}(k-1))S(\bar{z}(k-1))e^T(k)e(k)}{D^2(k)}\\ &- \text{tr}\{2a(k)\gamma N(x(k))\frac{\tilde{W}^T(k-1)S(\bar{z}(k-1))e^T(k)}{D(k)}\}\\ =& a(k)\gamma^2N^2(x(k))\frac{S^T(\bar{z}(k-1))S(\bar{z}(k-1))e^T(k)e(k)}{D^2(k)}\\ &- 2a(k)\gamma N(x(k))\frac{e^T(k)\tilde{W}^T(k-1)S(\bar{z}(k-1)}{D(k)}\\ =& a(k)\gamma^2N^2(x(k))\frac{S^T(\bar{z}(k-1))S(\bar{z}(k-1))e^T(k)e(k)}{D^2(k)}\\ &-2a(k)\gamma N(x(k))\frac{e^T(k)\mathscr{G}_\xi^{-1}(k-1)e(k)}{D(k)}-2a(k)\gamma N(x(k))\frac{e^T(k)\epsilon_z}{D(k)}\end{aligned} \tag{31}$$

According to Assumption 2.2, there always exist two positive constants $\underline{g}$ and $\bar{g}$ such that

$$\underline{g}I\leq\mathscr{G}_\xi^{-1}(k)+\mathscr{G}_\xi^{-T}(k)\leq\bar{g}I \text{ or } -\bar{g}I\leq\mathscr{G}_\xi^{-1}(k)+\mathscr{G}_\xi^{-T}(k)\leq-\underline{g}I$$

where I is the identity matrix. This implies that there always exists a scalar sequence $g(k)$ satisfying $\underline{g}/2 \leq |g(k)| \leq \bar{g}/2$ such that

$$\begin{aligned}e^T(k)\mathscr{G}_\xi^{-1}(k-1)e(k)=&e^T(k)\frac{\mathscr{G}_\xi^{-1}(k-1)+\mathscr{G}_\xi^{-T}(k-1)}{2}e(k)\\ =& g(k)e^T(k)e(k)\end{aligned} \tag{32}$$

Substituting (32) into (31), we obtain

$$\begin{aligned}\Delta V(k)=&a(k)\gamma^2N^2(x(k))\frac{S^T(\bar{z}(k-1))S(\bar{z}(k-1))e^T(k)e(k)}{D^2(k)}\\ &-2g(k)N(x(k))\frac{\gamma a(k)e^T(k)e(k)}{D(k)}-2\frac{a(k)\gamma N(x(k))e^T(k)\epsilon_z}{D(k)}\end{aligned} \tag{33}$$

From (25), we know that $a(k)\|\epsilon_z\| \leq a(k)\frac{\gamma\|e(k)\|}{(1+|N(x(k))|)\lambda}\epsilon_0$, which implies that

$$|a(k)N(x(k))e^T(k)\epsilon_z| \leq a(k)\frac{\gamma\epsilon_0}{\lambda}e^T(k)e(k) \tag{34}$$

Considering (34) and using the fact that $N^2(x(k))S^T(\bar{z}(k-1))S(\bar{z}(k-1)) \leq D(k)$, we have

$$\Delta V(k)\leq\varrho_0\frac{a(k)e^T(k)e(k)}{D(k)} - 2N'(x(k))\frac{a(k)e^T(k)e(k)}{D(k)} \tag{35}$$

where $\varrho_0 = \gamma^2(1 + \frac{2\epsilon_0}{\lambda})$ and $N'(x(k)) = \gamma g(k)N(x(k))$. Since $\gamma > 0$ and $0 < \underline{g}/2 \leq |g(k)| \leq \bar{g}/2$, $N'(x(k))$ is still a discrete Nussbaum gain according to Lemma 2.2. Considering (23) and (35), we have

$$\Delta V(k) \leq \varrho_0\Delta x(k) - 2N'(x(k))\Delta x(k) \tag{36}$$

Applying Lemma 2.1 and 2.2 to (36), the boundedness of $V(k)$, $N(x(k))$ and $x(k)$ can be concluded. It further implies the boundedness of $\hat{W}(k)$. Define a time interval $Z_1 = \{k|a(k) = 1\}$ and suppose Z_1 is an infinite set. Then, from the boundedness of nondecreasing sequence $x(k)$, we have

$$\lim_{k\to\infty, k\in Z_1} \frac{e^T(k)e(k)}{(1+|N(x(k))|^2)(1+\|S(\bar{z}(k-1))\|^2+e^T(k)e(k))}=0$$

which leads to $\lim_{k\to\infty}\|e(k)\| = 0$ according to the Key Technical Lemma [19]. It conflicts with $a(k) = 1$ on Z_1 and therefore we have $\lim_{k\to} a(k) = 0$ because Z_1 must be a finite set. Then, we see that $N(x(k))$ must converge to a constant and let us denote $C = \lim_{k\to\infty}(1 + |N(x(k))|)$. Consequently, we have $\lim_{k\to\infty}\|e(k)\| < C\lambda/\gamma$. This completes the proof. ■

Next, let us analyze the stability of the closed-loop system.

Theorem 3.2: Consider the process (4), the add-on control (14) with (12), and the adaptive NN control (21) with weights adaptation law satisfies (22)-(25). Under Assumptions 2.1, 2.2, 2.3, and 3.1, and the parameters in (14) chosen according to (17) and (18), then all the closed-loop signals are bounded, and the tracking error satisfies

$$\lim_{k\to\infty}\|y(k) - r(k)\| \leq \varrho$$

where ϱ is a positive constant which is adjustable.

Proof: Using the definition of $e(k + 1)$, the closed-loop dynamics (15) can be written to read

$$\begin{aligned}&[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]y(k)\\ &=\det\{H(z^{-1})\}R(z^{-1})r(k)+[\det\{H(z^{-1})\}I+Q(z^{-1})]Fe(k)\end{aligned}$$

Considering (17) and the boundedness of $r(k)$, and using Lemma 3.2 in [19], there exist positive constants C_1 and C_2 such that

$$|y_i(k)| \leq C_1+C_2 \max_{0\leq\eta\leq k, 1\leq j\leq n} |e_j(\eta)|,\ i = 1,\cdots,n \quad (37)$$

Since the boundedness of $e(k), \forall k \geq 0$, is guaranteed by Theorem 3.1, it can be concluded from (37) that $y(k)$ is bounded, $\forall k \geq 0$.

Left-multiplying (14) by $FA(z^{-1})$, left-multiplying (4) by $G(z^{-1})F$, combining them and using (12), we obtain

$$\begin{aligned}&[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]\bar{u}(k)\\ &\quad=FA(z^{-1})R(z^{-1})r(k+1)-z^{-1}G(z^{-1})F\Delta_f(\bar{y}_k,\bar{u}_{k-1},u(k))\\ &\quad+FA(z^{-1})[\det\{H(z^{-1})\}I+Q(z^{-1})](\mathrm{adj}\{H(z^{-1})\})^{-1}v(k) \quad (38)\end{aligned}$$

From the definition of $e(k)$ and (16), we have

$$\Delta_f(\bar{y}_k, \bar{u}_{k-1}, u(k))=e(k+1)-B(z^{-1})v(k) \quad (39)$$

Left-multiplying $\det\{H(z^{-1})\}$ on both sides of (38), substituting (39) into (38), considering the equation (12) and using the fact that $\det\{H(z^{-1})\}I=H(z^{-1})\mathrm{adj}\{H(z^{-1})\}$, we obtain

$$\begin{aligned}&[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]H(z^{-1})u(k)\\ &=\det\{H(z^{-1})\}FA(z^{-1})R(z^{-1})r(k+1)-\det\{H(z^{-1})\}G(z^{-1})e(k)\\ &+[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]H(z^{-1})v(k) \quad (40)\end{aligned}$$

From the properties of matrix determinant, we know that

$$\begin{aligned}&\det\{[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]H(z^{-1})\}\\ &=\det\{\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})\}\det\{H(z^{-1})\}\\ &=\det\{\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})\}\det\{F\}\det\{B(z^{-1})\}\end{aligned}$$

From Assumption 2.3, we know that $\det\{B(z^{-1})\} = c \neq 0$, where c is an arbitrary constant. Therefor, from Assumption 3.1, (17) and above equation, we have

$$\det\{[\det\{H(z^{-1})\}P(z^{-1})+Q(z^{-1})FA(z^{-1})]H(z^{-1})\}\neq 0, |z|\geq 1 \quad (41)$$

Considering (41) and the boundedness of $r(k)$, there exist positive constants C_3, C_4 and C_5 such that

$$\begin{aligned}|u_i(k)|\leq C_3+\{&C_4 \max_{0\leq\eta\leq k, 1\leq j\leq n}|e_j(\eta)|\\ &+C_5 \max_{0\leq\eta\leq k, 1\leq j\leq n}|v_j(\eta)|\},\ i = 1,\cdots,n \quad (42)\end{aligned}$$

From Theorem 3.1, we know $e(k)$ and $\hat{W}(k)$ are bounded, thus $v(k)$ is bounded from the boundedness of $\hat{W}(k)$. As a consequence, the boundedness of $u(k)$ can be concluded from (42). Therefore, it can be concluded that all signals of the closed-loop system are bounded.

From (37), the extended tracking error is

$$\limsup_{k\to\infty}\|\varepsilon(k)\| = \|[\det\{H(1)\}+Q(1)]F\|\cdot \limsup_{k\to\infty}\|e(k)\|\leq\varrho_1 \quad (43)$$

where $\varrho_1 = \|[\det\{H(1)\} + Q(1)]F\| \cdot C\lambda/\gamma$, λ and γ are adjustable design parameters. According to (18) and the definition of $\varepsilon(k)$ in (26), (43) implies that $\lim_{k\to\infty}\|y(k) - r(k)\| \leq \varrho$ with $\varrho = \varrho_1/\|\det\{H(1)\}R(1)\|$. ■

IV. SIMULATION

To illustrate the effectiveness of the proposed control, we apply it to the following MIMO nonlinear system.

$$\begin{aligned}y_1(k+1)=&0.9y_1(k)-0.3y_1(k-1)/[1+y_2^2(k-1)]+0.7u_1(k)\\ &+0.1y_1^2(k-1)y_2^2(k)+0.3\sin(u_1(k-1))-0.7u_2(k)+0.6u_2(k-1)\\ y_2(k+1)=&-0.1y_2(k-1)+0.3y_1(k-1)y_2(k)+0.8\sin(u_1(k))\\ &+0.1u_1(k-1)+0.9u_2(k)+0.2u_2(k-1)+0.1u_2^2(k-1)\end{aligned}$$

The control objective is to make the outputs $y_1(k)$ and $y_2(k)$ track the trajectories $r_1(k) = 0.6\mathrm{sign}(\cos(\pi k/200)) + 0.4$ and $r_2(k) = 0.5\mathrm{sign}(\sin(\pi k/200))$, respectively.

The control gain matrix of the system can be obtained as

$$\mathscr{G}(k) = \begin{bmatrix} 0.7 & -0.7 \\ 0.8\cos(u_1(k)) & 0.9 \end{bmatrix}$$

Obviously, $\mathscr{G}(k)$ is always positive definite. In addition, it can be seen that the origin is an equilibrium point of the system. Around the origin, the linearized model for controller design can be obtained by Taylor's expansion, which is described by

$$A(z^{-1}) = \begin{bmatrix} 1 - 0.9z^{-1} + 0.3z^{-2} & 0 \\ 0 & 1 + 0.1z^{-2} \end{bmatrix}$$

$$B(z^{-1}) = \begin{bmatrix} 0.7 + 0.3z^{-1} & -0.7 + 0.6z^{-1} \\ 0.8 + 0.1z^{-1} & 0.9 + 0.2z^{-1} \end{bmatrix}$$

Selecting $P(z^{-1})=I$ and solving the Diophantine equation (10), we obtain $G(z^{-1})=\mathrm{diag}\{0.9-0.3z^{-1}, -0.1z^{-1}\}$ and $F=$

I. While matrices $Q(z^{-1}) = (1 - z^{-1}) \cdot \mathrm{diag}\{0.6242, 0.2202\}$ and $R(z^{-1}) = I$, which satisfy the conditions (17) and (18). The initial system states are $y_1(i) = 0, y_2(i) = 0, u_1(i) = 0, u_2(i) = 0, e_1(i) = 0, e_2(i) = 0$, $i = -2, -1, 0$, and the initial NN weights are $\hat{W}(0) = 0$ and $S(0) = 0$. The number of neurons used is $l = 118$. The tuning factor and the threshold value are chosen as $\gamma = 0.8$ and $\lambda = 0.0001$. The simulation results are shown in Figs. 1-3. Fig. 1 shows the outputs track their desired trajectories. The control inputs $u_1(k)$ and $u_2(k)$ are shown as solid lines in Fig. 2, while the norm of the NN weights, $\|\hat{W}(k)\|$, the discrete Nussbaum gain $N(x(k))$ and its argument $x(k)$ are demonstrated in Fig. 3. From Figs. 1-3, it can be seen that the system outputs can track their desired trajectories well under the proposed control, while $\hat{W}(k)$, $N(x(k))$ and $x(k)$ are all bounded.

For comparison, the results obtained by only using the weighted one-step-ahead control are displayed as dash-dotted lines in Figs. 1 and 2. Obviously, the system outputs can not track their desired trajectories under the weighted one-step-ahead control without NNs. Therefore, it can be concluded that the tracking performance is greatly improved by using add-on adaptive NN.

V. Conclusion

In this paper, stable adaptive NN control has been developed for a class of MIMO nonaffine nonlinear discrete-time systems. Based on the nominal linear model of the system, weighted one-step-ahead control has been designed. Then, adaptive NN control term has been introduced to deal with the nonlinearities. All the closed-loop signals are guaranteed to be bounded, and the tracking errors are made to converge to a compact set. Simulation result has shown the effectiveness of the proposed control.

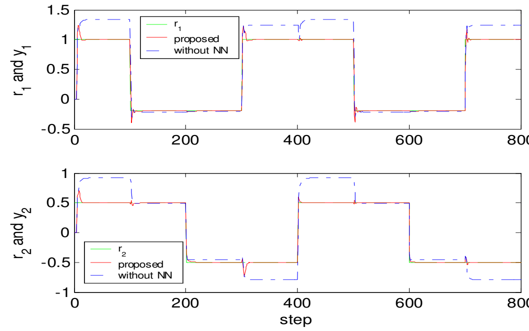

Fig. 1. System outputs

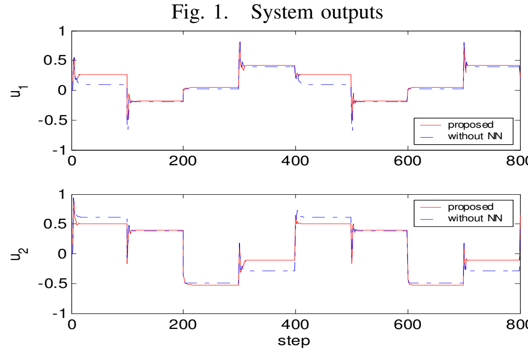

Fig. 2. Control inputs

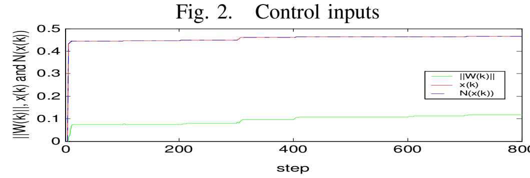

Fig. 3. $x(k)$, $N(x(k))$ and Norm of NN weights ($\|\hat{W}(k)\|$)

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