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Simultaneous IDA-Passivity-based control of a Wound Rotor Synchronous Motor

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Abstract— **This paper presents a new nonlinear passivity-based controller for a wound rotor synchronous machine, acting as a motor drive. From the standard dq-model the control objectives are stated, and the Port-controlled Hamiltonian model is also obtained. A simple power flow study allows to state the control goals in terms of reactive power compensation and ohmic losses reduction. Starting from the Hamiltonian structure, the Simultaneous Interconnection and Damping Assignment (SIDA-PBC) technique is used to develop the control action. The desired robustness of the control action is also taken into account in the design procedure. This results in a globally asymptotically stabilizing controller, which is validated via numerical simulations.**

I. INTRODUCTION

The wound rotor synchronous machine (WRSM) is used for generation and also for drive applications [15]. In the generation case the field voltage is used for regulating the stator voltage, while in motor applications this variable can be used to compensate the power factor of the machine [16]. Several techniques are proposed for controlling the WRSM. Linear techniques are the most used in the industry [9][17], but decoupling methods [8], widely employed for asynchronous machines, have also been extended to the synchronous case, and advanced nonlinear controllers have been applied to this class of machines as well [10].

Passivity-based control (PBC) is a technique that can be used to design controllers for a large kind of systems. Control of a rather general class of electrical machines using PBC methods has been proposed in [11], and the specific cases for synchronous generators and drives can be found also in [5] and [7], respectively. Recently, a new technique based on the PBC properties called Interconnection and Damping Assignment (IDA-PBC) has been proposed in [12]. Using the IDA-PBC approach many electrical machines have been controlled [14], in particular induction machines [1] and permanent magnet synchronous ones [13]. The simultaneous IDA-PBC methodology was proposed in [2], were the induction machine was studied and controlled. The SIDA-PBC technique offers more degrees of freedom than IDA-PBC, and allows to solve more complex interconnected systems and to design output feedback, as opposed to state feedback, controllers (see examples in [2] and [4]).

The main goal of this work is to design a control algorithm for a wound rotor synchronous drive machine, based on the Simultaneous IDA-PBC technique. The paper is organized as follows. In Section II the wound rotor synchronous machine model is introduced and its control goals are described. Section III presents the SIDA-PBC technique and then the control law is obtained. The simulation results are included in Section IV and, finally, conclusions are stated in Section V.

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II. THE WRSM MODEL AND CONTROL GOALS

In this Section we present the dynamical model of the WRSM. From the well-known dynamical equations we also propose a port-Hamiltonian model which allows to describe in a compact form and with a nice physically interpretation the system dynamics. Finally, we compute, in terms of the fixed point values, the active and reactive powers flowing through the stator side of the machine and define the control objectives.

A. dq-model of the WRSM

The state space model in *dq* coordinates of a wound rotor synchronous machine with a field winding (and no damper windings) is given by [3]

$$\dot{\lambda}_d = -R_s i_d + n_p \omega L_s i_q + v_d \quad (1)$$
$$\dot{\lambda}_q = -n_p \omega L_s i_d - R_s i_q - n_p \omega M i_F + v_q \quad (2)$$
$$\dot{\lambda}_F = -R_F i_F + v_F \quad (3)$$

where $\lambda^T = (\lambda_d, \lambda_q, \lambda_F) \in \mathbb{R}^3$ and $i^T = (i_d, i_q, i_F) \in \mathbb{R}^3$ are the fluxes and currents, respectively[1], ω is the mechanical speed, n_p is the number of pole pairs, R_s and R_F are the ohmic resistances of the stator *dq* and rotor field windings, and L_s, L_F and M are the leakage and mutual inductances.

The dynamical model has to be completed with the mechanical equation

$$J_m \frac{\mathrm{d}\omega}{\mathrm{d}t} = n_p M i_F i_q - B_r \omega + \tau_L. \quad (4)$$

Here J_m is the rotor inertia, τ_L is a generic mechanical torque (negative in case of braking), and B_r is a viscous mechanical damping coefficient.

Fluxes and currents are related by

$$\lambda = Li$$

[1] d, q and F subindexes refers to *dq* coordinates and the field variables, respectively.

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where

$$L = \begin{pmatrix} L_s & 0 & M \\ 0 & L_s & 0 \\ M & 0 & L_F \end{pmatrix}.$$

B. Port-Hamiltonian model of the WRSM

Hamiltonian modeling uses the state dependent energy functions to characterize the dynamics of the different subsystems, and connects them using a Dirac structure, which embodies the power preserving network of relations established by the corresponding physical laws. The result is a mathematical model with an specific structure, called port-controlled Hamiltonian system (PCHS) [18], which lends itself to a natural, physics-based analysis and control design.

Explicit PCHS have the form

$$\left\{ \begin{array}{rcl} \dot{x} & = & (J(x) - R(x))\partial_x H(x) + g(x)u \\ y & = & g^T(x)\partial_x H(x) \end{array} \right. \tag{5}$$

where $x \in \mathbb{R}^n$ is the vector state, $u, y \in \mathbb{R}^m$ are the port variables, and $H(x) : \mathbb{R}^n \to \mathbb{R}$ is the Hamiltonian function, representing the energy function of the system. The ∂_x (or ∂, if no confusion arises) operator defines the gradient of a function of x, and in what follows we will take it as a column vector. $J(x) \in \mathbb{R}^{n\times n}$ is the interconnection matrix, which is skew-symmetric ($J(x) = -J(x)^T$), representing the internal energy flow in the system, and $R(x) \in \mathbb{R}^{n\times n}$ is the dissipation matrix, symmetric and, in physical systems, semi-positive definite ($R(x) = R^T \geq 0$), which accounts for the internal losses of the system. Finally, $g(x) \in \mathbb{R}^{n\times m}$ is an interconnection matrix describing the port connection of the system to the outside the world. It yields the flow of energy to/from the system through the port variables, u and y.

Equations (1), (2), (3) and (4) can be given a port-Hamiltonian form with energy variables, fluxes (λ_d, λ_q, λ_F) and momentum ($p = J_m\omega$). Then, the interconnection and dissipation matrices are, respectively

$$J(x) = \begin{pmatrix} 0 & n_pL_s\omega & 0 & 0 \\ -n_pL_s\omega & 0 & 0 & -n_pMi_F \\ 0 & 0 & 0 & 0 \\ 0 & n_pMi_F & 0 & 0 \end{pmatrix},$$

$$R = \begin{pmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & 0 & R_F & 0 \\ 0 & 0 & 0 & B_r \end{pmatrix},$$

with Hamiltonian (energy) function

$$H(x) = \frac{1}{2}\lambda^T L^{-1}\lambda + \frac{1}{2J_m}p^2,$$

and

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where the external inputs are $u = (v_d, v_q, v_F, \tau_L)$.

C. Fixed points and power flow study

In this subsection we compute the electrical power, in steady-state, flowing through the stator side of the machine in order to compensate the power factor. All this study consider that the three-phase system is sinusoidal and balanced.

Fixed points are solutions to

$$0 = -R_si_d^* + n_p\omega L_si_q^* + v_d \tag{6}$$
$$0 = -n_p\omega L_si_d^* - R_si_q^* - n_p\omega Mi_F^* + v_q \tag{7}$$
$$0 = -R_Fi_F^* + v_F$$
$$0 = n_pMi_F^*i_q^* - B_r\omega^* + \tau_L \tag{8}$$

From the dq-active power definition,

$$P_s = v_di_d + v_qi_q$$

and using the fixed points (6) and (7), we obtain

$$P_s^* = R_s^2(i_d^{*2} + i_q^{*2}) + n_p\omega^* Mi_F^*i_q^*,$$

and tacking into account $\tau_e = n_pMi_Fi_q$, we recover the power balance equation (in steady-state)

$$P_s^* = \underbrace{R_s^2(i_d^{*2} + i_q^{*2})}_{\text{Electrical losses}} + \underbrace{\tau_e\omega^*}_{\text{Mechanical power}} \tag{9}$$

The same study for the reactive power,

$$Q_s = v_di_q - v_qi_d,$$

together with (6) and (7), yields

$$Q_s^* = -n_p\omega^*(L_s(i_d^{*2} + i_q^{*2}) + Mi_d^*i_F^*). \tag{10}$$

Notice that i_F can compensate the power factor (or $Q_s = 0$) with an appropriate value in the third term of (10).

D. Control objectives

Following the results presented above, we can summarize the control goals as follows: to regulate the mechanical speed, ω, and to compensate the power factor, *i.e.* $Q_s^* = 0$. To achieve these objectives we have three control inputs, v_d, v_q and v_F. There is still one degree of freedom which allows to regulate $i_d = i_d^*$ to minimize the loses (see equation (9)).

Unfortunately, both control goals (reactive power and minimization losses) cannot be achieved simultaneously. Equation (10) shows that the term which compensates the reactive power requires a nonzero i_d value. For this reason, the control goal $i_d^* = 0$ is replaced by the objective of getting an small value of the d-current component.

III. CONTROL DESIGN

Figure 1 shows the proposed control scheme. As explained above the control objectives are to regulate ω, i_d and to compensate the reactive power. The SIDA-PBC controller directly regulates ω and i_d, while Q_s is controlled through the rotor current i_F by means of setting $Q_s^* = 0$ in equation (10),[2] implying that

$$i_F^* = \frac{L_s}{Mi_d^*}(i_d^{*2} + i_q^2). \tag{11}$$

[2]The i_q is used instead of i_q^* to improve the robustness (the i_q^* value is strongly dependent on the parameters).

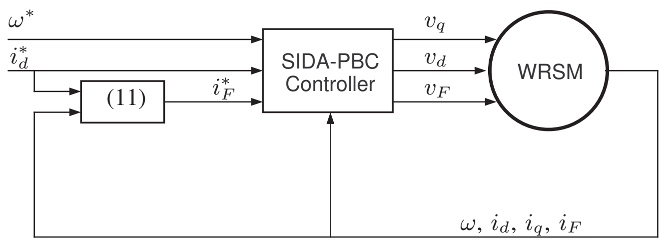

Fig. 1. Control scheme.

A. *SIDA-PBC technique*

The Simultaneous Interconnection and Damping Assignment (SIDA-PBC) was proposed in [2]. It considers the problem of stabilization of an equilibrium point for nonlinear systems in an affine form or, alternatively, in a PCHS form (5).

The key idea behind the SIDA-PBC technique (as it occurs also in the classic IDA-PBC) is to solve the so-called matching equation

$$(J(x,t) - R(x,t))\partial H + gu = F_d(x,t)\partial H_d, \tag{12}$$

where, to enforce dissipativity, the following constraint on the F_d matrix is required

$$F_d(x,t)^T + F_d(x,t) \leq 0. \tag{13}$$

The equilibrium assignment of the desired energy function translates to

$$x^* = \arg\min H_d(x).$$

With appropriate F_d and H_d choices, the control law yields

$$u = (g(x)^T g(x))^{-1} g^T(x)(F_d \partial H_d - (J-R)\partial H).$$

As pointed out in [2], there exists several techniques to solve the matching equation (12), namely the PDE approach of [12] and the so-called algebraic approach of [6]. In this case the problem is solved using the last procedure, *i.e.*, fixing the energy function and then solving the algebraic equations.

B. *SIDA-PBC for the WRSM*

As proposed above, we will use the SIDA-PBC approach to design a controller for the WRSM. First we fix the energy function as

$$H_d = \frac{1}{2}(x - x^*)^T P (x - x^*)$$

where $P = P^T > 0$. To simplify the solution we restrict P to have the form

$$P = \begin{pmatrix} k_d & 0 & 0 & 0 \\ 0 & \gamma_q & 0 & 0 \\ 0 & 0 & \gamma_F & 0 \\ 0 & 0 & 0 & k_\omega \end{pmatrix}.$$

Notice that the positiveness requirement on P implies thet k_d, γ_q, γ_F, $k_\omega > 0$.

The $F(x)$ matrix is chosen in order to facilitate the solution of the resulting algebraic equations. Furthermore, in order to simplify the control structure, we want to assign one output to each control action and also to obtain a controller as robust as possible. The proposed structure is as follows

$$F(x) = \begin{pmatrix} F_{11}(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{24}(x) \\ 0 & 0 & F_{33}(x) & 0 \\ 0 & F_{42}(x) & F_{43}(x) & F_{44}(x) \end{pmatrix},$$

where inequality (13) must be accomplished.

Decoupling control actions and feedback outputs is possible if the F matrix contains only one nonzero term in each row. The three first rows of $F(x)$ have nonzero F_{11}, F_{24} and F_{33} elements, which relate v_d, v_q and v_F to the errors in i_d, ω and i_F, respectively. This is a critical point to improve the robustness of the resulting controller, because in this way only the fixed points of the outputs appear in the control action. Effectively, as it is shown later, the fixed point i_q^*, whose computation requires solving the fixed points equations, (6)–(8), with strong dependence on the parameters, do not appear in the control law.

From the three first rows of (12) we obtain the control actions

$$\begin{aligned} v_d &= R_s i_d - n_p \omega L_s i_q + F_{11} k_d (i_d - i_d^*) \\ v_q &= n_p \omega L_s i_d + R_s i_q + n_p \omega M i_F + F_{24} k_\omega (\omega - \omega^*) \\ v_F &= R_F i_F + F_{33} \gamma_F (i_F - i_F^*), \end{aligned}$$

while from the fourth row,

$$\begin{aligned} & n_p M i_F i_q - B_r \omega + \tau_L - F_{42}\gamma_q (i_q - i_q^*) \\ & - F_{43}\gamma_F (i_F - i_F^*) - F_{44} k_\omega (\omega - \omega^*) = 0. \end{aligned} \tag{14}$$

To solve equation (14) we propose

$$\begin{aligned} F_{42} &= \frac{1}{\gamma_q} n_p M i_F^* \\ F_{43} &= \frac{1}{\gamma_F} n_p M i_q \\ F_{44} &= -\frac{1}{k_\omega} B_r \end{aligned}$$

where the steady-state solution of (4)

$$\tau_L = -n_p M i_F^* i_q^* + B_r \omega^*$$

has been also used. In order to simplify the solution, now we assign

$$\begin{aligned} F_{11} &= -1 \\ F_{24} &= -F_{42}. \end{aligned}$$

With the last choice, it is clear that (13) holds if $F_{33} < 0$ and

$$-2(F_{33} + F_{44}) > F_{43}^2,$$

or substituting,

$$-2\left(F_{33} - \frac{1}{k_\omega} B_r\right) > \left(\frac{1}{\gamma_F} n_p M i_q\right)^2.$$

Finally, with

$$F_{33} = -\frac{1}{4} n_p^2 M^2 i_q^2$$

the previous equation reduces to

$$\frac{1}{k_\omega} B_r > \frac{1}{\gamma_F^2}.$$

In order to simplify the controller the following parameters are assigned

$$\begin{aligned}\gamma_q &= n_p M i_F^*, \\ \gamma_F &= k_F \frac{4}{n_p^2 M^2},\end{aligned}$$

and the control actions are finally

$$\begin{aligned}v_d &= R_s i_d - n_p \omega L_s i_q - k_d (i_d - i_d^*), \\ v_q &= n_p \omega L_s i_d + R_s i_q + n_p \omega M i_F - k_\omega (\omega - \omega^*), \\ v_F &= R_F i_F - k_F i_q^2 (i_F - i_F^*),\end{aligned}$$

while the restriction can be expressed as

$$k_\omega < B_r \frac{(n_p M)^4}{16} k_F^2.$$

From the point of view of energy efficiency, it is desirable for an electrical machine to have a mechanical damping as close to zero as possible, and many control design strategies assume that $B_r = 0$. In our case, mechanical losses play a fundamental role, because the k_ω value is bounded from above by B_r, and this seems to limit the range of available k_ω. However, even if B_r is very small, it cannot be strictly zero and, taking into account that k_F is a free tuning gain, the open set for which the closed loop system is stable can be enlarged.

IV. SIMULATIONS

In this section we present some simulations using the designed controller. The WRSM parameters are: $L_s = 1$mH, $R_s = 0.0303\Omega$, $M = 1.5$mH, $L_F = 8.3$mH, $R_F = 0.0539\Omega$, $n_p = 2$, $J_m = 0.01525$kg·m^2, $B_r = 0.05$N·m·s and $\tau_L = 0$N·m. The control parameters are selected as: $k_d = 1$, $k_F = 10$ and $k_\omega = 0.001$.

The first numerical experiment is performed increasing the desired speed from $\omega^* = 100$rad·s^{-1} to $\omega^* = 150$rad·s^{-1} at $t = 2$s. As pointed out in Section II, the desired d-current must have an small value in order to reduce the electrical losses; in our case, it is fixed at $i_d^* = -0.1$A. Figure 2 shows that the system is perfectly regulated under changes of the desired outputs.

Figure 3 shows the results of a second test. In this case the external torque is suddenly changed at $t = 2$ from zero to $\tau_L = -0.8$N·m. Even under this change, the system achieves the control goals due to the fact that some robustness has been built-in in the design of the controller.

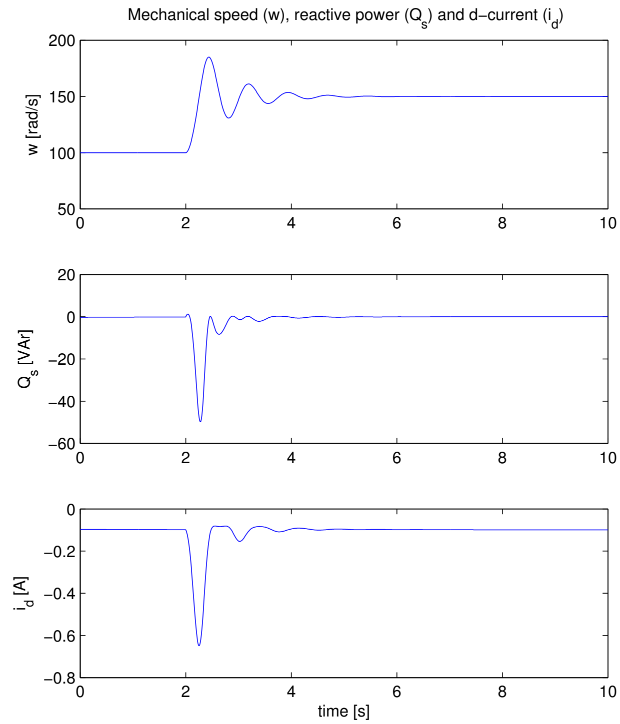

Fig. 2. Simulation results: Mechanical speed, reactive power and i_d current, for a change of the speed reference.

V. CONCLUSIONS

The SIDA-PBC technique has been applied to control a wound rotor synchronous machine for a motor drive application. The SIDA-PBC matching equation has been solved using the algebraic approach and the desired robustness of the resulting controller has been taken into account. The obtained controller is globally asymptotically stable and assures stability for a large range of control gain values. The presented method also allows to decouple the outputs, improving the robustness and facilitating the gain tuning.

Future research includes a dynamical extension keeping the Hamiltonian structure to improve the robustness (basically on the electrical parameters R_s, L_s, M, and R_F) and the behavior. This can be easily done for the i_d and i_F currents (due to the fact that they are passive outputs), but a more complicated task is to design a dynamical extension for the speed loop. Experimental validation with a real plant using the presented control law will be also considered in the future.

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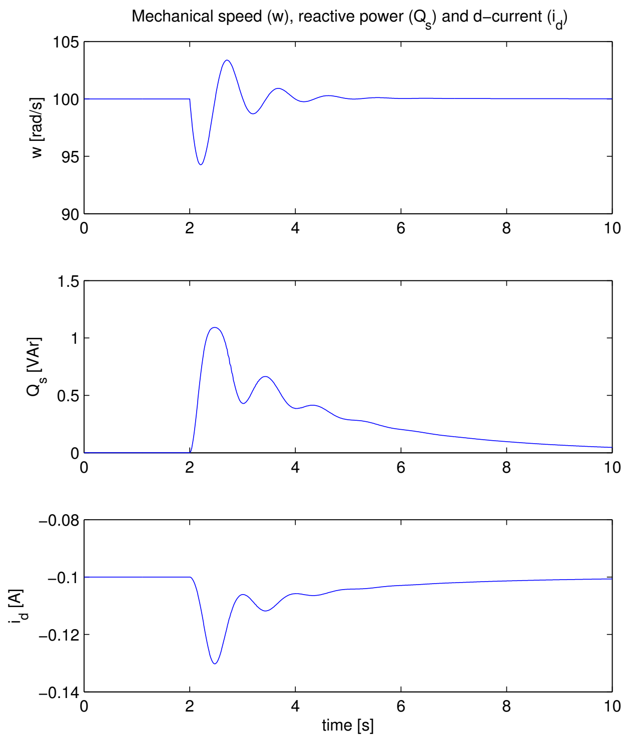

Fig. 3. Simulation results: Mechanical speed, reactive power and i_d current, for a change of the external torque.

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